Neutrino oscillations and stellar collapse

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It is shown that even if vacuum neutrino oscillations exist, they are effectively inhibited from occurring in collapsing stars because of the high matter density.

Neutrinos play an important role in the calculations of stellar collapse.[1] While such calculations are based on our present experimental and theoretical knowledge of neutrino interactions, one needs to take into account present uncertainties concerning neutrino physics. One intriguing possibility is that neutrinos of one type, for example ν_e, are converted into other types, such as ν_μ and ν_τ, as they travel through the vacuum. Such "neutrino oscillations" are the subject of a number of experimental searches and have been suggested as a cause of the deficiency of solar neutrinos.[2] Mazurek[3] has considered the consequences that would follow if ν_e trapped during stellar collapse were to oscillate into ν_μ and ν_τ during a time scale of the order of milliseconds with the consequence that the lepton Fermi energy is reduced and there is some conversion of lepton energy to baryon energy. A more extreme possibility would be the oscillation of ν_e into a new type of neutrino ν_x which does not have the normal weak interactions[4]; as a result, some of the trapped neutrino energy could escape rapidly. Here we wish to point out that even if vacuum oscillations do occur, the oscillations are so modified by the dense matter of the stellar core that they become insignificant.

To illustrate our point, we consider two neutrino types ν_e and ν_μ. Vacuum oscillations occur if the eigenvectors of the mass matrix are mixtures:

$$\begin{aligned} |\nu_1\rangle &= \cos\theta_v |\nu_e\rangle + \sin\theta_v |\nu_\mu\rangle , \\ |\nu_2\rangle &= -\sin\theta_v |\nu_e\rangle + \cos\theta_v |\nu_\mu\rangle . \end{aligned} \tag{1}$$

As a result, a state originally $|\nu_e\rangle$ becomes after propagating a distance x in vacuum

$$|\nu_e t\rangle \sim \cos\theta_v |\nu_1\rangle - \sin\theta_v |\nu_2\rangle \exp(i2\pi x/l_v) , \tag{2}$$

where l_v is the vacuum oscillation length

$$l_v = 4\pi k/(m_1{}^2 - m_2{}^2) = 250 \frac{k\ (\text{MeV})}{\Delta m^2\ (\text{eV})^2}\ \text{cm} , \tag{3}$$

and k is the neutrino momentum. The probability that ν_e has oscillated into ν_μ is then

$$|\langle \nu_\mu | \nu_e t\rangle|^2 = \tfrac{1}{2}\sin^2 2\theta_v\left(1 - \cos\frac{2\pi x}{l_v}\right). \tag{4}$$

For propagation in matter[5] it is necessary to take into account the index of refraction n which is related to the forward scattering by the optical theorem. If we consider only the charged-current scattering of ν_e from electrons, then

$$k(n-1) = \frac{2\pi N_e}{k} f(0) = GN_e , \tag{5}$$

where N_e is the number of electrons per unit volume and G is the Fermi constant. The characteristic length for a phase change of 2π due to this index of refraction is

$$l_0 = \frac{2\pi}{k(n-1)} = \frac{2.7\times 10^9\ \text{cm}}{\rho_e} , \tag{6}$$

where $\rho_e = Y_e \rho$ is N_e divided by $6\times 10^{23}\ \text{cm}^{-3}$. In considering ν_e and ν_μ, the neutral-current couplings are believed to be the same, but there is no charged-current contribution to the forward scattering of ν_μ as there is for ν_e. Thus, Eq. (5) gives the difference between the index of refraction of ν_e and that of ν_μ, and l_0 represents the distance over which ν_e and ν_μ get out of phase. The vacuum-oscillation mechanism becomes ineffective if l_0 is much less than l_v. A simple calculation[5] yields the result that in matter we must use Eqs. (1), (2), and (4) with θ_v, l_v replaced by θ_m, l_m given by

$$\sin^2 2\theta_m = \sin^2 2\theta_v \left(\frac{l_m}{l_v}\right)^2 , \tag{7a}$$

$$l_m{}^2 = l_v{}^2\left[1 + \left(\frac{l_v}{l_0}\right)^2 - 2\frac{l_v}{l_0}\cos 2\theta_v\right]^{-1} . \tag{7b}$$

For collapsing stellar cores we are interested in $\rho_e > 10^{10}$ so that $l_0 < 1$ cm. Present experiments[2] indicate $\Delta m^2 < 4\ \text{eV}^2$ so that for neutrinos with $k \sim 10$ to 100 MeV we have $l_v > 600$ cm. Thus $l_0 \ll l_v$, and we can approximate Eqs. (7) as

$$l_m \approx l_0 < 1\ \text{cm} ,$$

$$2\theta_m < \frac{l_0}{l_v} < \frac{1}{600} .$$

It follows that the oscillations have a negligible

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magnitude, less than 10^{-5} in probability.

The same arguments clearly hold for the case of oscillations of ν_e into ν_x. Indeed, oscillations from any weakly interacting ν to one which has no weak interactions are effectively inhibited by the index-of-refraction effect. Thus, trapped neutrinos can never oscillate into freely escaping neutrinos.

A more extreme oscillation mechanism we have suggested is one in which there are no vacuum oscillations but there are oscillations induced by matter.[5] This occurs if the neutral-current interaction changes neutrinos from one type to another; this is contrary to the basic idea of the $SU(2) \times U(1)$ gauge theories that agree so beautifully with experiment; therefore, it is unlikely, but it is not ruled out experimentally. In this case it is possible to get a large mixing angle in matter independent of the density[6] with an oscillation length given approximately by Eq. (6). Thus the oscillations are effectively instantaneous and their only effect would be to share the neutrino energy among several neutrino types, all of which are trapped.

It would be important[3] if it were possible for ν_e to transform into $\bar{\nu}_e$. However, normal $\bar{\nu}_e$ are right-handed and oscillations do not change left-handed particles into right-handed particles. However, if ν_e has a mass it has both left-handed (ν_{eL}) and right-handed (ν_{eR}) states and can possibly have a small magnetic moment. Thus, it is conceivable that large magnetic fields could transform ν_{eL} into ν_{eR}. However, ν_{eR} must have very different weak interactions than ν_{eL}, since the usual charged current only couples electrons to ν_{eL}. Therefore, because of the index-of-refraction effect, the transformation of ν_{eL} into ν_{eR} would have to take place over a distance less than l_0, which would require absurdly large magnetic fields.

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[1]For a review, see K. A. Van Riper in *Neutrinos—78*, edited by E. Fowler (Purdue University Press, W. Lafayette, Indiana 1978).

[2]For a review see S. Bilenky and B. Pontecorvo, Usp. Fiz. Nauk. 123, 181 (1977) [Sov. Phys. Usp. 20, 776 (1977)].

[3]T. Mazurek, in Proceedings of the Neutrino-79 Conference, Bergen, Norway (unpublished).

[4]Within the framework of $SU(2) \times U(1)$ theories, ν_x could be a left-handed singlet.

[5]L. Wolfenstein, Phys. Rev. D 17, 2369 (1978).

[6]The mixing angle θ is given by Eq. (15) of Ref. 5. For the neutral-current couplings of the Weinberg-Salam mode, this gives $\tan 2\theta_m = (1 - Y_e)/Y_e$ provided the neutral current is completely off-diagonal.