Do non-asymptotically-free known interactions invalidate the unification hypothesis?

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The running coupling constants and possible unification of the known interaction group product are considered. For fractional and integral quark charges, non-asymptotically-free U(1) gauge and quartic scalar coupling constants are shown not to exceed order unity prior to appropriate embedding mass scales for unification with gravity and for lepton-hadron unification, respectively. Empirical bounds on the mass scale for lepton-hadron unification are shown to be somewhat lower than previously estimated. Essential quartic scalar-field couplings within proposed $[SU(4)]^4$ embedding symmetry are shown to be non-asymptotically-free.

I. THE UNIFICATION HYPOTHESIS

The idea that strong, electromagnetic, weak, and gravitational interactions may be contained in a single embedding group structure is an extremely attractive one, for it suggests fundamental unity in nature.[1,2] The symmetry of such a structure is, of course, broken in order to account for the different particle interactions, and that is where difficulties begin. We note, however, that the unification hypothesis is somewhat more than wishful thinking. The hypothesis has already been shown to provide insight into why the strong interactions are stronger than the weak-electromagnetic interactions,[3] the origins of lepton-hadron universality in the weak charged-current interaction,[1,4] and the relation between empirical weak-angle values and hierarchical symmetry breaking.[3,5,6]

It is no accident that these successes pertain to the magnitudes of gauge coupling constants, rather than the scalar-field self-couplings or the Yukawa couplings of the embedding theory. There is consensus among physicists that the strong interactions are described by local chromodynamic SU(3) invariance and the weak-electromagnetic interactions by some product of SU(2) and U(1) local gauge groups. The decoupling theorem[7] allows us to see how corresponding subgroup gauge coupling constants can evolve from a single gauge coupling constant of a larger embedding theory.[3] Thus, the fundamental couplings of nature become equal at sufficiently large Euclidean momenta. Since the embedding theory is non-Abelian, all U(1) factors in the weak-electromagnetic sector eventually become asymptotically free.[8]

Therefore, the unification of known interaction subgroups allows gauge coupling constants to be eventually anchored to a single UV-stable fixed point (zero); is it possible that nongauge couplings show similar behavior? Unfortunately, there is presently no consensus as to the nature or structure of symmetry breaking, except that realistic spontaneously broken gauge theories are seldom asymptotically free.[9-11] The problem is further complicated by the fact that the decoupling theorem is no longer useful when considering scalar-field multiplets of the embedding symmetry, because light scalars do not always decouple within a given multiplet. Since there is some agreement about the Higgs structure of the known interactions, it may be more useful to concentrate on the embedded rather than the embedding symmetry.

In this paper we assume that the known interactions are based on the local gauge symmetry $SU(2)\times U(1)\times SU(3)$. For the theory with unbroken color, the single quartic scalar coupling constant present is shown [in addition to the U(1) gauge coupling constant] to always be non-asymptotically-free. If color is broken, the corresponding quartic coupling is shown to be non-asymptotically-free as well. Coupling-constant values at low momentum scales, however, are constrained to be small ($\sim\alpha^{1/2}$) by the relative strength of the weak interactions, i.e., by the empirical Fermi constant and weak angle. Thus, a picture emerges in which running non-asymptotically-free known interaction coupling constants increase above their physical values with an increasing scale of momenta. For *all* couplings to be eventually UV-finite, some kind of embedding must occur before those non-asymptotically-free couplings diverge. Specifically, we want scalar [and U(1)] known interaction particle multiplets to be embedded in the multiplets of a larger unifying group whose coupling constants are asymptotically free. The larger group structure manifests itself at appropriately large momenta, henceforth denoted as the "embedding mass scale." Presumably, this mass scale [corresponding to the mass of non-$SU(2)\times U(1)\times SU(3)$ gauge bosons in the embedding group] corresponds to the scale of momenta at which diverging known interaction coupling constants "turn around" and begin to diminish in magnitude with subsequent increases in momentum scale.[12] The statement that all nature (not just the gauge couplings) is eventually UV-finite corresponds to what we will mean henceforth by the "unification hypothesis."[13]

In Sec. II of this paper we review the minimal known interaction model $[SU(2)\times U(1)\times SU(3)]$ for fractional and integral quark charge assignments. The non-asymptotically-free behavior of the U(1) gauge coupling constant is examined for both choices of quark charge and for an arbitrary number of quark flavors. By tying the running U(1) gauge coupling constant g' to α and the physical weak angle at $t=0$ ($\mu \lesssim m_W$), we are able to show that g' will not exceed order unity until $\mu \gg G_{\rm grav}^{-1/2}$, the momentum scale at which unification with gravity is expected.[3] We claim therefore, that the U(1) coupling constant remains UV-finite over a sufficiently large range of momenta that no intermediate embedding symmetry is necessary prior to unification with gravity.

In Sec. III we examine the quartic scalar-field couplings of the $SU(2)\times U(1)\times SU(3)$ theory for fractional (unbroken color) and integer (broken color) quark charges. At least one minimal-model quartic scalar-field coupling constant is proven to always be non-asymptotically-free, regardless of whether color is broken or unbroken. For either case, however, the domain of "temporary asymptotic freedom" is shown to extend past appropriate embedding mass scales. For fractional quark charges, such a mass scale would be the Planck mass ($G_{\rm grav}^{-1/2}$); for integer quark charges the embedding mass scale is the mass of the leptoquark boson (which mediates the decay of quarks).[14]

Empirical bounds on the leptoquark-boson mass are shown to be lower than has been estimated previously in Sec. IV. Moreover, the coupling of leptoquark bosons to fermions is demonstrated to be an archetypal example of a coupling constant that appear to be non-asymptotically-free at low momenta, but which becomes identified with an asymptotically-free embedding coupling constant at high momenta.

We conclude Sec. IV by showing how the proposed $[SU(4)]^4$ embedding symmetry[6] contains at least one non-asymptotically-free quartic scalar coupling constant, associated with $(4,\bar{4})$-type scalar-field multiplets. Such multiplets are essential for the realization of a low unification mass scale in the $[SU(4)]^4$ theory.[6] We further show how an increase in the number of flavors will salvage asymptotic freedom for such coupling constants, but argue that additional scalar fields necessary to break resultant higher symmetry will contribute other non-asymptotically-free couplings. Thus, we end this paper pessimistically, suggesting that either the unification hypothesis is untenable (not all couplings are UV-finite) or that some kind of eigenvalue[15] or induced-scalar-coupling prescription[16] is necessary for symmetry breaking to remain consistent with the unification hypothesis.

II. THE MINIMAL MODEL

We assume that the known interactions of nature are described by a gauge theory based on spontaneously broken local $SU(2)\times U(1)\times SU(3)$ symmetry. Such a choice is justified both by the success of the Salam-Weinberg model[17] in explaining neutral-current phenomenology[18] as well by the considerable qualitative insight into leptoproduction scaling phenomena obtained by gauging the color group.[19] We shall denote the $SU(2)\times U(1)\times SU(3)$ model to be the "minimal model" for the known interactions. The minimal model may contain fractionally or integrally charged quarks. For the former case, color is unbroken and quarks are presumably confined. For the latter case, additional spontaneous symmetry breaking is necessary to break the color group. In either case, however, there exist non-asymptotically-free couplings that will diverge unless the minimal model is embedded in a larger asymptotically-free theory. We seek to obtain some insight into the momentum scale at which this embedding should be manifest.

We begin by listing essential features of the $SU(2)\times U(1)\times SU(3)$ minimal model. We borrow heavily from the 1973 paper of Pati and Salam that describes an $SU(2)\times U(1)\times SU(3)$ model with integer charge quarks,[1] for the notation of that paper is appropriate for fractionally charged quarks as well. Moreover, the $SU(4)\times SU(4)$ classification symmetry of fermions presented in that paper is suggestive of higher embedding symmetry, implications of which are considered in Sec. IV.

We assume there are four quark flavors, corresponding to two weak doublets in any given color. In addition, let there be two weak lepton doublets. We list the fermions of the model in such a way as to exhibit $SU(4)\times SU(4)$ classification symmetry[1] [extensions to $SU(F)\times SU(4)$ for $F>4$ are straightforward to obtain[20]]:

$$\begin{aligned}\Psi_{i\alpha}&=\frac{1+\gamma_5}{2}\Psi_{i\alpha}+\frac{1-\gamma_5}{2}\Psi_{i\alpha}\\&=\Psi^L_{i\alpha}+\Psi^R_{i\alpha}\\&=\begin{bmatrix}u_r & u_y & u_b & \nu_e\\ d_r & d_y & d_b & e\\ s_r & s_y & s_b & \mu\\ c_r & c_y & c_b & \nu_\mu\end{bmatrix}.\end{aligned} \tag{2.1}$$

Gauge bosons of SU(3)×SU(2)×U(1) are denoted by V^A, W^B, and U, respectively ($A=1,\ldots,8$; $B=1,2,3$); the interaction of fermions with gauge bosons is given by $\mathcal{L}_F=\bar{\Psi}\gamma_\mu D^\mu\Psi$ (Ref. 1):

$$\begin{aligned}D_\mu\Psi_{i\alpha}=\partial_\mu\Psi_{i\alpha}&+ig_3V^A_\mu\begin{pmatrix}\frac{1}{2}\lambda^A_{\alpha\beta}\Psi_{i\beta}; & \alpha\le 3\\ 0 & \alpha=4\end{pmatrix}\\&+ig_2W^A_\mu\begin{pmatrix}\frac{1}{2}\tau^A_{ij}\Psi^L_{j\alpha} & i\le 2\\ \frac{1}{2}\tau^A_{5-i,j}\Psi^L_{5-j,\alpha}; & i\ge 3\end{pmatrix}\\&+ig'U_\mu\left[\begin{pmatrix}\frac{1}{2}\tau^3_{ij}\Psi^R_{j\alpha}; & i\le 2\\ \frac{1}{2}\tau^3_{5-i,j}\Psi^R_{5-j,\alpha}; & i\ge 3\end{pmatrix}+\tfrac{1}{6}(1-4\delta_{\alpha 4})\Psi_{i\alpha}\right].\end{aligned} \tag{2.2}$$

[g_3, g_2, and g' are the gauge coupling constants of SU(3)×SU(2)×U(1).] The Salam-Weinberg model[17] is realized by introducing one or more scalar-field doublets ϕ_i transforming in the (2,1,1) representation of SU(2)×U(1)×SU(3) (Ref. 1):

$$D_\mu\phi_i=\partial_\mu\phi_i+ig_2(\tfrac{1}{2}\tau^A_{ij})W^A_\mu\phi_j+ig'(\tfrac{1}{2}U_\mu)\phi_i \tag{2.3}$$

($\mathcal{L}_\phi=|D_\mu\phi_i|^2$). Spontaneous symmetry breaking through the ϕ multiplet gives appropriate masses to $W^\pm$ and Z, where

$$\begin{aligned}Z&=(g_2W^3-g'U)/(g_2{}^2+g'^2)^{1/2}\\&=\cos\theta_W\,W^3-\sin\theta_W\,U.\end{aligned} \tag{2.4}$$

If quarks are fractionally charged, the photon is orthonormal to Z and color is unbroken:

$$A=\sin\theta_W\,W^3+\cos\theta_W\,U \tag{2.5}$$

$$e^{-2}=g_2{}^{-2}+g'^{-2}. \tag{2.6}$$

If quarks are integrally charged, an additional scalar-field representation is necessary. Pati and Salam[1] introduced a scalar-field multiplet σ that transforms in the $(2+2,1,\bar{3})$ representation of SU(2)×U(1)×SU(3):

$$\begin{aligned}D_\mu\sigma_{i\alpha}=\partial_\mu\sigma_{i\alpha}&+ig_2W^A_\mu\begin{pmatrix}\frac{1}{2}\tau^A_{ij}\sigma_{j\alpha}; & i\le 2\\ \frac{1}{2}\tau^A_{5-i,j}\sigma_{5-j,\alpha}; & i\ge 3\end{pmatrix}\\&+ig'U_\mu\sigma_{i\alpha}/6-ig_3\tfrac{1}{2}V^A_\mu\lambda^A_{\beta\alpha}\sigma_{i\beta}.\end{aligned} \tag{2.7}$$

The structure of this representation can accomodate vacuum expectation values (VEV's) from which color gluons are able to obtain equal masses, consistent with global color symmetry.

When σ acquires a nonzero VEV, the color gluons V^A acquire mass. Flavor couplings in (2.7) attach flavor components to the color gluons coupling to diagonal currents. The photon remains orthonormal to these (now massive) states by acquiring a color component:

$$\begin{aligned}&A/e=W^3/g_2+U/g'-(V^3+V^8/\sqrt{3})/g_3,\\&e^{-2}=g_2{}^{-2}+g'^{-2}+4g_3{}^{-2}/3.\end{aligned} \tag{2.8}$$

We shall now consider the running value for the non-asymptotically-free U(1) coupling constant g' (running values for quartic couplings in the scalar-field potential will be considered in Sec. III). Let μ_0 be the subtraction mass at which g_3, g_2, and g' are at their renormalized (physical) values. If the momenta of external lines are rescaled from p_i to $p_i(\mu/\mu_0)$, the rescaling of g' with $t\equiv\ln\mu/\mu_0$ can be obtained from the fermionic couplings of Eq. (2.2)[9]:

$$16\pi^3\dot{g}'=40g'^3/9. \tag{2.9}$$

We have neglected contributions to running g' arising from scalar-field multiplets. [The role of scalars is inconsequential compared to that of fermions. For fractional- and integral-quark-charge models, the right-hand side of (2.9) acquires additive factors of $g'^3/12$ and $5g'^3/36$, respectively, from scalar-field multiplets.] Equation (2.9) is easily generalized to a minimal model containing $2n$ flavors. If the number of lepton and quark flavors is the same,

$$16\pi^2\dot{g}'=20ng'^3/9. \tag{2.10}$$

Since g' varies quite slowly with μ, we may choose $\mu_0=m_W$ as the value at which g' and g_2 have their physical values. From Eqs. (2.4), (2.5), and (2.6) we see that for $t=0$

$$g'(0)=\sqrt{4\pi\alpha}/\cos\theta_W, \tag{2.11}$$

where $\alpha=\frac{1}{137}$ and $\cos\theta_W$ is empirically determined. For the case of integer quark charges, α in (2.11) is replaced by $\alpha/(1-4\alpha/3\alpha_s)$ ($\alpha_s\equiv g_3{}^2/4\pi$). If $\alpha_s(\mu=3\text{ GeV})\gtrsim 0.2$, $\alpha_s(\mu=m_W)$ is sufficiently greater than α that this correction (~5%) may be ignored.

We now apply the boundary condition (2.11) to Eq (2.10) and obtain

$$g'^2(\mu)=\frac{4\pi\alpha}{\cos^2\theta_W-[10\alpha n/9\pi)\ln(\mu/m_W)}. \tag{2.12}$$

For example, if there are four flavors and we let $\sin^2\theta_W=\frac{1}{4}$, we find that

$$g'^2(\mu) = 0.12/[1 - 0.007\ln(\mu/m_W)]\,. \tag{2.13}$$

In Eq. (2.13) g' does not approach order unity until μ/m_W is of order 10^{60}. Even if there are a total of eight flavors (corresponding to the existence of an additional four-flavor mirror multiplet in the $[SU(4)]^4$ embedding theory)[6] and we choose $\sin^2\theta_W = \frac{3}{8}$, $g'^2(\mu)$ does not grow large until $\mu/m_W \sim 10^{25}$.

Thus, the running value for the U(1) coupling constant remains small over enormous changes in momentum scale, provided it is anchored to its small physical value at empirical energies. We note that the scale at which gravitation is expected to play a role is $\mu_P = G_{\text{grav}}^{-1/2} \sim 10^{17} m_W$.[3] The value of g' at μ_P remains quite small, so that perturbation theory remains valid over the entire subgravitational range of μ. We conclude, therefore, that the non-asymptotic freedom of the U(1) gauge coupling is not really a problem for the minimal model; limits obtained from g' on the size of an embedding mass scale are truly cosmic.

III. FURTHER LIMITS ON THE EMBEDDING MASS

The scalar-field potential in the SU(2)×U(1) ×SU(3) minimal model is given by

$$\begin{aligned} V(\sigma,\phi) = &-\mu_1^2\Phi/2 - \mu_2^2 C/2 + \lambda_1 C^2/2 \\ &+ \lambda_2 C_{\alpha\beta}C_{\beta\alpha}/2 + \lambda_3\Phi^2/2 + \lambda_4\Phi C/2 \\ &+ \lambda_5[(|\phi_i^*\sigma_{i\alpha}|^2 + |\phi_i^*\sigma_{5-i,\alpha}|^2)]\,, \end{aligned} \tag{3.1}$$

where $\Phi \equiv \phi_i^*\phi_i$, $C_{\alpha\beta} \equiv \sigma_{i\alpha}\sigma_{i\beta}^*$, and $C \equiv C_{\alpha\alpha}$. The term in square brackets contracts the SU(2) indices of ϕ with the $SU(2)^{I+II}$ indices of σ (all summations in the square brackets are $\sum_{i=1}^2$). If color is unbroken in the minimal model, the σ field is not present and

$$V(\sigma,\phi) = -\mu_1^2\Phi/2 + \lambda_3\Phi^2/2\,. \tag{3.2}$$

For the present we shall consider the minimal model with broken color. In their discussion of this model, Pati and Salam assumed that ϕ_i and $\sigma_{i\alpha}$ acquire vacuum expectation values (VEV's) of the form[1]

$$\langle\phi\rangle_i = \begin{pmatrix}\phi\\0\end{pmatrix}, \quad \langle\sigma\rangle_{i\alpha} = \begin{bmatrix}\sigma & 0 & 0\\ 0 & \sigma & 0\\ 0 & 0 & \sigma\\ 0 & 0 & 0\end{bmatrix}. \tag{3.3}$$

These VEV's are possible only if λ_5 is chosen to be zero. Hence λ_5 will be chosen to be zero henceforth.[21] The VEV $\langle\phi\rangle$ breaks the weak sector into the Weinberg-Salam theory. The VEV $\langle\sigma\rangle$ was chosen because it gives equal masses to all color gluons, consistent with global color symmetry. The gluon mass matrix arising from the $|\partial_\mu\sigma_{i\alpha}|^2$ term in Eq. (2.7) is just

$$M_{AB}{}^2 V^A V^B/2 = g_3{}^2 V^A V^B \lambda^B_{\gamma\alpha}\lambda^A_{\alpha\beta}C_{\gamma\beta}/4\,. \tag{3.4}$$

If $C_{\gamma\beta}$ is proportional to the identity matrix, then $m_{AB}{}^2 = (g_3{}^2 C/3)\delta_{AB}$, and global color symmetry is preserved. The VEV (3.3) satisfies this property.

We seek to obtain physical constraints on the quartic scalar couplings $\lambda_1 - \lambda_4$ before we shall consider the magnitude of their running values. For the theory to have a bounded energy spectrum, we require $V(\sigma,\phi)$ to be positive as classical fields σ and ϕ become infinite, independent of the relative magnitudes of σ and ϕ. Thus, if $|\sigma| \to \infty$, $|\phi| \to \infty$, and $\sigma/\phi \to 0$, $V(\sigma,\phi)$ is greater than zero only if $\lambda_3 > 0$. Similarly, if $\phi/\sigma \to 0$, $\lambda_1 C^2 + \lambda_2 C_{\alpha\beta}C_{\beta\alpha}$ must be greater than zero, in which case $\lambda \equiv \lambda_1 + \lambda_2/3 > 0$.

Similarly, we can obtain a lower bound on the magnitude of λ_4. First note that for $\lambda_2 > 0$, $\sigma \to \pm\infty, \phi \to \pm\infty$

$$V(\sigma,\phi) \geq \lambda C^2/2 + \lambda_3\Phi^2/2 + \lambda_4 C\Phi/2 \geq 0\,. \tag{3.5}$$

Let $\xi \equiv C/\Phi$. Since C and Φ are positive-definite,

$$F(\xi) \equiv \lambda\xi^2 + \lambda_4\xi + \lambda_3 \geq 0\,. \tag{3.6}$$

$F(\xi)$ has a minimum with respect to ξ when $\xi = -\lambda_4/2\lambda$. We require $F(\xi_{\min}) \geq 0$, in which case

$$4\lambda\lambda_3 - \lambda_4{}^2 \geq 0\,. \tag{3.7}$$

[If $\lambda_2 < 0$, (3.7) becomes $4(\lambda_1 + \lambda_2)\lambda_3 - \lambda_4{}^2 \geq 0$. Since $\lambda > (\lambda_1 + \lambda_2)$ if $\lambda_2 < 0$, Eq. (3.7) remains valid.]

Further constraints on the quartic scalar couplings are obtained from spontaneous symmetry breaking. If we choose not to break local color, corresponding to the potential (3.2), it is well known that

$$m_W = g_2\langle\phi\rangle/\sqrt{2}\,, \tag{3.8}$$

where $\langle\phi\rangle = (\mu_1{}^2/2\lambda_3)^{1/2}$.[17] The parameter μ_1 corresponds to the mass of the physical Higgs particle after spontaneous symmetry breaking. λ_3 is related to physically accessible parameters by the relation

$$\lambda_3 = g_2{}^2\mu_1{}^2/4m_W{}^2\,. \tag{3.9}$$

Thus the empirical lower bound (~10 GeV) on the mass μ_1 of the physical Higgs field[22] limits the magnitude of the renormalized λ_3 coupling. Since $g_2{}^2 = 4\pi\alpha/\sin^2\theta_W$, $\lambda_3 \approx 0.1\,\mu_1{}^2/m_W{}^2$ $(\sin^2\theta_W \approx \frac{1}{4})$.

For the minimal model with broken color, the masses m_V (of colored vector gluons) and m_W are given by $(\sin^2\theta_W \approx \frac{1}{4})$:[23]

$$m_V = g_3\sqrt{C/6} \lesssim 2\ \text{GeV}\,,$$
$$m_W = g_2\sqrt{\Phi/2} \approx 75\ \text{GeV}. \tag{3.10}$$

Assuming $\alpha_s \gtrsim 0.2$ at appropriate vertex momenta ($\mu \approx m_V$),[24]

$$\sqrt{C/3} \approx \sqrt{2}\, m_W/g_3 < 1.8\ \text{GeV}$$

and

$$\sqrt{\Phi} \approx \sqrt{2}\, m_W/g_2 \approx 170\ \text{GeV}\,.$$

If $\lambda_5 = 0$, the VEV's minimizing $V(\sigma, \phi)$ that are consistent with constraints $\lambda > 0$, $\lambda_3 > 0$ and Eq. (3.7) are

$$\Phi = (2\lambda\mu_1{}^2 - \lambda_4\mu_2{}^2)/(4\lambda\lambda_3 - \lambda_4{}^2)\,,$$
$$C = (2\lambda_3\mu_2{}^2 - \lambda_4\mu_1{}^2)/(4\lambda\lambda_3 - \lambda_4{}^2)\,, \tag{3.11}$$

in which case

$$\frac{C}{3\Phi} = \frac{2\lambda_3\mu_2{}^2 - \lambda_4\mu_1{}^2}{6\lambda\mu_1{}^2 - 3\lambda_4\mu_2{}^2} \lesssim 10^{-4}\,. \tag{3.12}$$

We now consider the behavior of the running quartic scalar coupling constants $\lambda_i(\mu)$. The renormalization mass μ corresponds to the scale of momenta entering the interaction vertex. By using the physical constraints obtained above for λ_i to establish boundary conditions for $\mu < m_W$, we are able to determine how large μ must be before any non-asymptotically-free couplings (NAFC) diverge (or, more relevantly, until perturbation theory ceases to be valid). The unification hypothesis requires that any larger embedding group must manifest itself before the NAFC's become too large. Thus the value of μ at which NAFC's do become too large is an upper bound on the embedding mass scale. [Recall that growth of NAFC's is curtailed in the unification hypothesis by the appearance of the larger (asymptotically free) group structure at values of μ exceeding the mass of any gauge particles in the embedding theory which are not also contained in the known interaction subgroup.]

Let us first consider the minimal model in which color is unbroken; the σ scalar-field multiplet is absent. The only quartic scalar coupling constant present is λ_3. The differential equation for the associated running coupling constant is straightforward to obtain from the potential (3.2) and the interaction of scalars with gauge particles [Eq. (2.3)][10]:

$$16\pi^2\dot{\lambda}_3 = 12\lambda_3{}^2 - 9g_2{}^2\lambda_3 - 3g'^2\lambda_3 + \tfrac{9}{4}(g_2{}^4) + \tfrac{3}{2}(g'^2g_2{}^2) + \tfrac{3}{4}(g'^4)\,. \tag{3.13}$$

We show below that $\dot{\lambda}_3 > 0$, independent of the relative magnitudes of λ_3, g_2, and g'. Therefore, the minimal theory of the known interactions with unbroken color has two NAFC's, g' and λ_3. To demonstrate that λ_3 is a NAFC, we parametrize Eq. (3.13) as follows:

$$16\pi^2\dot{\lambda}_3 = g_2{}^4(\xi_1 + \xi_2 r + \xi_3 r^2)\,, \tag{3.14}$$

where $\lambda_3(t) = [a(t)g_2{}^2(t)]$, $r(t) = g'^2(t)/g_2{}^2(t)$, and where

$$\xi_1 = 12a^2 - 9a + \tfrac{9}{4}\,,$$
$$\xi_2 = -3a + \tfrac{3}{2}\,, \tag{3.15}$$
$$\xi_3 = \tfrac{3}{4}\,.$$

Since r is real, $\dot{\lambda}_3$ cannot be less than zero unless r is between the real roots of the polynomial in Eq. (3.14). However, the discriminant of that polynomial,

$$\xi_2{}^2 - 4\xi_1\xi_3 = 9(-3a^2 + 2a - \tfrac{1}{2})\,, \tag{3.16}$$

is less than zero for all real a. Thus, roots of $(\xi_1 + \xi_2 r + \xi_3 r^2)$ are complex. Moreover $\xi_3 > 0$, in which case $\xi_1 + \xi_2 r + \xi_3 r^2$ must be greater than zero for all real r, and $\dot{\lambda}_3$ is greater than zero regardless of the magnitude (or sign) of $a(t)$ and $r(t)$.

We stress that the breaking of weak symmetry alone is sufficient to destroy asymptotic freedom, even if g'^2 is chosen to be zero or is otherwise argued away. The only way noneigenvalue asymptotic freedom can be realized for λ_3 is by eventually embedding the weak scalar-field doublet in a representation of some larger embedding group.

To determine a bound on the mass scale at which such embedding must manifest itself, we combine Eq. (3.14) with expressions obtained for the running gauge coupling constants from the scalar and fermion interaction Lagrangians of Sec. II[9]:

$$16\pi^2\dot{g}' = 163g'^3/36\,,$$
$$16\pi^2\dot{g}_2 = -55g_2{}^3/12\,, \tag{3.17}$$
$$16\pi^2\dot{g}_3 = -25g_3{}^3/3\,.$$

(Four flavors are assumed to be present. Color is unbroken.) The renormalized coupling constants are empirically known for $\mu < m_W$; we know that $\sin^2\theta_W = \frac{1}{4}$, $\alpha = \frac{1}{137}$, and for $\mu \cong 3$ GeV, $\alpha_s(\mu) \cong 0.2$ (this last constraint is sufficient to establish a value for g_3 at $\mu = m_W$).[24] By choosing $t = \ln(\mu/m_W)$, the running coupling constants are anchored to their renormalized values at $t = 0$, and we thereby obtain boundary conditions for the running-coupling-constant differential equations. We find from the minimal model relations $e^{-2} = g_2{}^{-2} + g'^{-2}$, $\tan\theta_W = g'/g_2$, that $[g'(0)]^2 = 0.125$ and $[g_2(0)]^2 = 0.375$. The renormalized ($t = 0$)

value for λ_3 depends on the mass of the physical scalar field [Eq. (3.9)]. In Fig. 1(a) we have plotted the running values of λ_3 for two different choices for the scalar-field mass. The behavior of g_2 and g' is also shown. One important feature of this figure is that if the scalar-field mass is m_W (or smaller), perturbation theory remains valid out to values of μ exceeding the Planck mass ($\mu_P > G_{\text{grav}}^{-1/2}$), the natural mass scale for combining gravity with the particle interactions. However, if the scalar-field mass is $4m_W$, perturbation theory falls apart at $\mu \sim 10^{10}$ GeV $\ll G_{\text{grav}}^{-1/2}$. Empirical limits on the leftover physical scalar-field mass are thus shown to be important in the construction of models attempting to unify the known particle interactions with gravity.

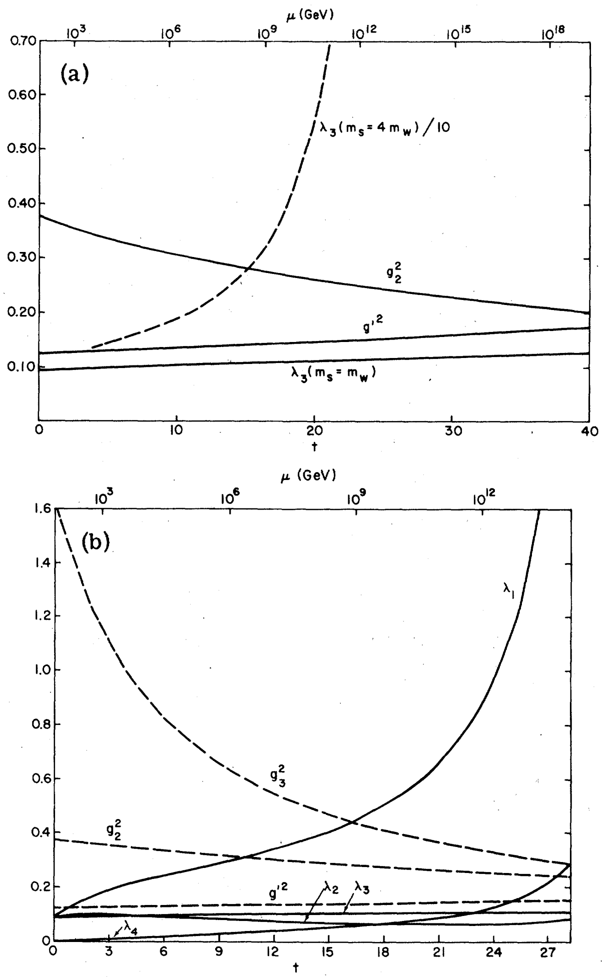

FIG. 1. Behavior of running coupling constants is displayed as a function of the momentum scale μ [$t \equiv \ln(\mu/m_W)$]. In (a) the evolution of coupling constants from their physcial values is displayed for the minimal known interaction model with fractional quark charges. The quartic scalar-field coupling constant λ_3 is shown for the cases where the surviving Higgs-field mass m_S is equal to m_W as well as $4m_W$. (In the latter case, the figure displays $\lambda_3/10$.) In (b), the evolution of coupling constants is displayed for integer quark charges. Boundary conditions for $\lambda_i(0)$ are discussed in the text.

Let us now consider the broken-color minimal model. The running scalar couplings are obtained using the methods of Ref. 10 on the potential (3.1) and the interactions (2.3) and (2.7) of scalar fields ϕ and σ with gauge particles.

$$\begin{aligned}16\pi^2\dot{\lambda}_1 = {} & 32\lambda_1^2 + 28\lambda_1\lambda_2 + 6\lambda_2^2 + \lambda_4^2 \\ & - (16g_3^2 + 9g_2^2 + g'^2/3)\lambda_1 \\ & + 11g_3^4/6 + 7g_2^2g_3^2 + 9g_2^4/4 \\ & - g'^2g_3^2/9 - g'^2g_2^2/6 + g'^4/108\,,\end{aligned} \tag{3.18}$$

$$\begin{aligned}16\pi^2\dot{\lambda}_2 = {} & 12\lambda_1\lambda_2 + 14\lambda_2^2 \\ & - (16g_3^2 + 9g_2^2 + g'^2/3)\lambda_2 \\ & + 15g_3^4/6 - 5g_2^2g_3^3 + g'^2g_3^2/3 + g'^2g_2^2/3\,,\end{aligned} \tag{3.19}$$

$$\begin{aligned}16\pi^2\dot{\lambda}_3 = {} & 12\lambda_3^2 + 6\lambda_4^2 - (9g_2^2 + 3g'^2)\lambda_3 \\ & + 9g_2^4/4 + 3g'^2g_2^2/2 + 3g'^4/4\,,\end{aligned} \tag{3.20}$$

$$\begin{aligned}16\pi^2\dot{\lambda}_4 = {} & 26\lambda_1\lambda_4 + 14\lambda_2\lambda_4 + 6\lambda_3\lambda_4 + 2\lambda_4^2 \\ & - (8g_3^2 + 9g_2^2 + 5g'^2/3)\lambda_4 \\ & + 9g_2^4/2 - g'^2g_2^2 + g'^4/6\,.\end{aligned} \tag{3.21}$$

We first note that Eq. (3.20) differs from Eq. (3.14) only by the presence of a positive $6\lambda_4^2$ term on the right-hand side. Thus, $\dot{\lambda}_3$ remains positive; λ_3 is unequivocally an NAFC, dashing any hopes that the greater complexity of the broken-color minimal model might somehow permit asymptotic freedom in its quartic scalar couplings. We reiterate that at least one quartic scalar NAFC has been shown to occur whether or not local color is preserved.

The broken-color running gauge coupling constants are easily obtained from Eqs. (2.2), (2.3), and (2.7):

$$\begin{aligned}16\pi^2\dot{g}' &= 165g'^3/36\,, \\ 16\pi^2\dot{g}_2 &= -\,49g_2^3/12\,, \\ 16\pi^2\dot{g}_3 &= -\,8g_3^3\,.\end{aligned} \tag{3.22}$$

These differ somewhat from the unbroken-color case because of additional one-loop contributions from the σ scalar-field multiplet. We obtain $t=0$ boundary conditions from renormalized gauge-coupling-constant values in precisely the same way as before, except that now $e^{-2} = g_2^{-2} + g'^{-2} + 4g_3^{-2}/3$; the photon has a colored component.

Boundary conditions $\lambda_i(t=0)$ are more difficult to ascertain because the number of parameters available exceeds the number of VEV and bound-energy-spectrum constraints. The most favorable circumstances consistent with the VEV constraint (3.12) and the continued validity of perturbation theory are those in which $\mu_2{}^2 \ll \mu_1{}^2$, $\lambda_4 = 2\lambda_3\mu_2{}^2/\mu_1{}^2$. For this choice of parameters, $\lambda_3(t=0) = g_2{}^2(0)\mu_1{}^2/4m_W{}^2$, as was the case for unbroken color [see Eq. (3.9)]. Moreover, $\lambda_4(0) = 2\lambda_3(0)\mu_2{}^2/\mu_1{}^2 \ll \lambda_3(0)$, thereby permitting $\lambda_1(0)$ and $\lambda_2(0)$ to also be small compared to $\lambda_3(0)$ without contradicting the constraint of Eq. (3.7). For this choice of boundary conditions, the results are approximately the same as those obtained for unbroken color: The embedding mass scale may be as large as the Planck mass provided the physical weak scalar boson has a mass $\mu_1 < m_W$. Since $\mu_2 \ll \mu_1$ this parametrization requires the existence of many light ($\mu_2 \ll m_W$) colored scalar particles, corresponding to sixteen unabsorbed degrees of freedom in $\sigma_{i\alpha}$.

In fact, it is quite difficult to avoid the presence of light colored scalars in the broken-color theory. Both the numerator and denominator of Eq. (3.12) must be separately positive since the numerators and denominators of the (positive-definite) VEV parameters Φ and C are separately positive [compare Eqs. (3.7) and (3.11)]. Therefore, $\lambda_4 < 2\lambda_3\mu_2{}^2/\mu_1{}^2$ and $\lambda_4 < 2\lambda\mu_1{}^2/\mu_2{}^2$. Suppose $\mu_1{}^2$ and $\mu_2{}^2$ are comparable in magnitude and no miraculous cancellations (such as in the case considered in the previous paragraph) occur in the numerator of Eq. (3.12). If $\mu_1{}^2 = \mu_2{}^2$,

$$\lambda_4 < 2\lambda_3 < 6\times10^{-4}\lambda \ . \tag{3.23}$$

We see from (3.11) that if λ dominates λ_3 and λ_4, then

$$\Phi \approx \mu_1{}^2/2\lambda_3 \tag{3.24}$$

and $\lambda_3(0) = g_2{}^2(0)\mu_1{}^2/4m_W{}^2 \sim e^2/\sin^2\theta_W$. From (3.23), $\lambda(0)$ is at least 3000 times as large as $\lambda_3(0)$, and "asymptotic freedom" is impossible, even at low energies. If $\mu_2{}^2 > \mu_1{}^2$ the results are even worse. The only way to avoid premature scaling violation is to have λ less than or comparable to λ_3, in which case consistency with (3.12) and (3.7) requires either $\mu_1{}^2 \gg \mu_2{}^2$ or a miraculous cancellation in the numerator of (3.12).

Since $\mu_1{}^2$ cannot be too much larger than $4m_W{}^2$ without $\lambda_3(0)$ acquiring too large a value (3.9), μ_2 is expected to be considerably smaller than m_W. Hence, we expect the broken-color theory to contain light colored scalars in addition to light unconfined gluons and quarks. The detection of such particles is beyond the scope of this paper.[25]

In Fig. 1(b) we have assumed that $\lambda(0)$ and $\lambda_3(0)$ are comparable, and have plotted the evolution of running coupling constants. Specifically, we have chosen $\lambda_1(0) = \lambda_2(0) = \lambda_3(0) = g_2{}^2(0)/4$, consistent with a choice $\mu_2{}^2 \ll \mu_1{}^2 = m_W{}^2$ for the scalar masses, a choice that allows (3.12) to be satisfied. Moreover, $\lambda_4(0) < 2\lambda_3(0)\mu_2{}^2/\mu_1{}^2 \ll \lambda_3(0)$. Figure 1(b) illustrates how perturbation theory remains valid out to $\mu \gtrsim 10^{13}$ GeV. This mass scale is below the Planck mass, but well above the embedding mass necessary to explain the instability (and consequent nondetection) of integer charge quarks in broken-color theories.[26]

The salient point of this analysis is to demonstrate the presence of NAFC's in the Higgs sector of the minimal model, as well as to show how such couplings are still compatible with perturbation theory for a large range of $t = \ln(Q^2/m_W{}^2)$. The connection between unabsorbed Higgs field masses and the embedding mass scale is demonstrated. Improved limits on the mass of the surviving Salam-Weinberg Higgs field[27] (or, for that matter, detection of such a particle) would provide insight into how the known interactions can be eventually combined with gravity. Finally, we stress how light colored Higgs fields in the broken-color model seem to be necessary for $\lambda \sim \lambda_3 \sim e^2/\sin^2\theta_W$; otherwise $\lambda \gg \lambda_3$, and partons cannot appear to be relatively free (i.e., scaling should not occur) in the present energy range.

IV. LOW EMBEDDING MASSES

We now change our focus from the embedded theory to the embedding theory. The results of the previous three sections indicate that we may choose boundary conditions for embedded-theory coupling constants that allow all NAFC's to remain small out to extremely large values of t. Hence, the phrase "temporary asymptotic freedom" is shown to be more than a slogan. We wish to consider here whether the embedding symmetry can (or should) appear at lower unifying mass scales.

Low unifying mass scales are, in fact, required for integral-quark-charge models.[26] In such models lepton number is the fourth color, and the chromodynamic SU(3) is contained within an SU(4) group.[4] Gauge bosons $V^1 - V^8$ of this SU(4) correspond to SU(3) gluons; $V^9 - V^{14}$ are leptoquark bosons (LQB) that couple quarks directly to leptons [V^{15} is a diagonal generator which eventually donates a component to the U(1) survivor of symmetry breaking].[4,28] The bosons $V^9 - V^{14}$ also mediate the decays of integer charge quarks which must be sufficiently unstable to explain their present nonobservation ($\tau_q \sim 10^{-11}-10^{-12}$ sec).[29] Sufficient instability will occur provided the LQB's are not too heavy; the value $m_{\rm LQB} \leq 10^5$ GeV has been quoted in the literature as compatible with quark-lifetime estimates.[29] Moreover, the leptoquark boson cannot be too light or else the unobserved decay $K^0 \to e^\pm \mu^\mp$ should have been seen [Fig. 2(a)]. Let f denote the coupling of an LQB to a quark and a lepton. The nonobservation of $K^0 \to e^\mp \mu^\pm$ implies that

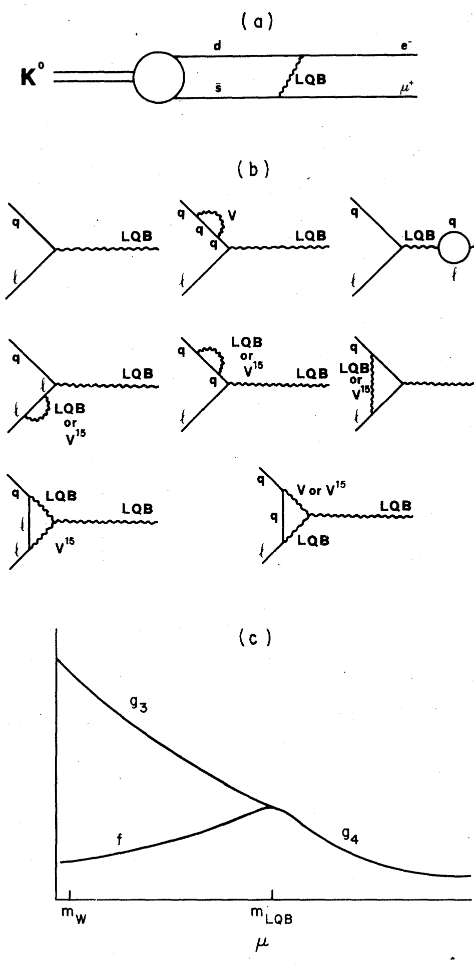

FIG. 2. The leptoquark-boson (LQB) mass and coupling strength (f) to fermions is subject to empirical and analytical constraints. (a) demonstrates how the unobserved $K^0 \to e\mu$ decay is mediated by LQB exchange, thereby providing a lower bound on the LQB mass. (b) lists one-loop diagrams contributing to the renormalization of f. If SU(4) is broken to SU(3) and $\mu \ll m_{\rm LQB}$, only the top three diagrams contribute nonvanishing contributions to $\beta(f)$; all other diagrams contain massive particles in their internal loops. (c) shows how f and g_3 are to be identified with the embedding SU(4) gauge coupling constant g_4 if $\mu \gg m_{\rm LQB}$. The behavior of the chromodynamic coupling constant g_3 is contrasted with that of f.

$f^2/m_{\rm LQB}{}^2 \lesssim G_F\alpha^2$.[4] If $f^2/4\pi$ is order unity (as proposed in Ref. 4), $m_{\rm LQB}$ should be greater than or of order 10^5 GeV, thereby providing a lower bound on the embedding mass scale barely compatible with quark instability.

We show here that this lower bound on the leptoquark-boson mass should be reduced. Moreover, we also show that the coupling constant f is an archetypal example of an NAFC that becomes asymptotically free at momenta where embedding symmetry is manifest. In Fig. 2(b) we show the one-loop terms contributing to the running value of f. Only the first three diagrams do not contain massive LQB's in the UV-divergent loops. We consider renormalization subtractions from the point $(p_i, -p_i, 0)$ where the fermion momenta p_i are at the Euclidean point $p_i{}^2 = -\mu^2$. If $\mu \gg m_{\rm LQB}$, the μ dependence of divergent renormalization constants Z_i may be expressed as $\ln(\Lambda/\mu) + O(m_{\rm LQB}{}^2/\mu^2)$. Thus, the LQB mass may be ignored and the coefficients of the divergent parts of each graph may be used to show that the β function for f is the same as that of a massless SU(4) theory (to lowest order). However, if $\mu \ll m_{\rm LQB}$, no contributions to the running value of f is obtained from any loops containing LQB's. For these loops, the μ dependence of renormalization constants Z_i is of the form $\ln(\Lambda/m_{\rm LQB}) + O(\mu^2/m_{\rm LQB}{}^2)$, and $\mu\partial Z_i/\partial\mu \to \mu^2/m_{\rm LQB}{}^2 \to 0$.[30] Only the first three diagrams of Fig. 2(b) give nonvanishing contributions to $\beta(f)$. Since fermion self-energy becomes zero in the Landau gauge, only the quark-lepton-loop contribution to the LQB vacuum-polarization diagram contributes, and the contribution of this term to $\beta(f)$ is positive: $\dot{f} = \beta(f) = \frac{4}{3} S_3(F) f^3/16\pi^2 + O(\mu^2/m_{\rm LQB}{}^2)$.[31] Thus, the running value of f grows with μ until μ approaches $m_{\rm LQB}$, at which point f behaves like an asymptotically-free SU(4) coupling [Fig. 2(c)].

f should therefore appear to be a NAFC in the present empirical range of momentum transfers. In Fig. 2(c) we show schematically how f and g_3 devolve from an embedding SU(4) coupling constant—f is expected to be much less than g_3 at present energies. In particular, the bound on the LQB mass obtained from Fig. 2(a) must be revised. The momentum transfer is only ~1 GeV, in which case f^2 is much less than unity. The lower bound on $m_{\rm LQB}$ must correspondingly decrease below 10^5 GeV.[32]

Earlier calculations have stressed the need for a mass scale much larger than $m_{\rm LQB}$, claiming that embedding must occur near the Planck mass.[3,5] Otherwise, the quantity α_s/α is too small at known energies to account for its divergence at 0.2–0.5 GeV.[24] We emphasize that while α_s must be small enough at SLAC momentum transfers to explain scaling, it must still be sufficiently large to bind quarks into hadrons and preclude the premature onset of scaling at even lower energies. Data in the scaling region suggest that $\alpha_s(\mu \approx 3\ \text{GeV}) \approx 0.2 \gg \alpha$.

Recent work has shown that α_s/α can be sufficiently large without employing an astronomical unifying mass scale.[6] The calculation assumes that an $[\text{SU}(4)]^4 \equiv \text{SU}(4)_L \times \text{SU}(4)_R \times \text{SU}(4)_L \times \text{SU}(4)_R$ color-flavor, left-right-symmetric theory is broken to an intermediate stage at which the weak $\text{SU}(2) \times \text{U}(1)$ group and a chiral color group $\text{SU}(3)_L \times \text{SU}(3)_R$ are preserved. This chiral color group must subsequently break into conventional vector chromodynamics, but there is no reason for this breaking not to occur at or below the weak interaction mass scale m_W. Hence, the known interaction subgroup is enlarged to include chiral color, doubling the number of gluon exchanges that contribute to the decline of g_3 with increasing μ. Consequently, α_s/α requires a much smaller range of μ to become appropriately large; if $\alpha_s(\mu = 3\ \text{GeV}) \approx 0.1-0.2$, the unifying mass scale is found to be as low as 10^5 GeV, corresponding to energies accessible to cosmic-ray experiments.[6]

A critical ingredient in this split-color prescription is the existence of a unified-theory scalar-field multiplet that is capable of breaking intermediate chiral $\text{SU}(3) \times \text{SU}(3)$ symmetry into conventional SU(3) chromodynamics. Such a multiplet transforms as $(3, \overline{3})$ under $\text{SU}(3) \times \text{SU}(3)$. Thus it was proposed that the $\text{SU}(4)_L \times \text{SU}(4)_R$ color-containing portion of the $[\text{SU}(4)]^4$ model include a $(4, \overline{4})$ scalar-field multiplet capable of giving mass to axial-vector combinations of canonical SU(4) gluons.[6] Had we chosen the unifying group to be $[\text{SU}(N)]^4$, corresponding to an N-flavor theory, the same reasoning would require an $(N, \overline{N})$ scalar multiplet of the $\text{SU}(N) \times \text{SU}(N)$ subgroup containing chiral color.

We show below that an $(N, \overline{N})$ scalar-field multiplet of $\text{SU}(N) \times \text{SU}(N)$ cannot have asymptotically free couplings unless $N \geq 5$.[33] This result not only implies that a $(3, \overline{3})$ scalar-field multiplet of $\text{SU}(3) \times \text{SU}(3)$ breaks asymptotic freedom; it also implies that the proposed embedding of such a multiplet in a $(1, 1, 4, \overline{4})$ multiplet of $[\text{SU}(4)]^4$ remains inconsistent with asymptotic freedom, thereby contradicting the unification hypothesis.

To demonstrate these results consider a scalar-field multiplet $\sigma_{i\alpha}$ that transforms as $(N, \overline{N})$ under $\text{SU}(N) \times \text{SU}(N)$ (the indices i and α both run from 1 to N). The terms in the potential involving quartic $\sigma_{i\alpha}$ self-couplings are

$$V(\sigma) = \lambda_1 (\sigma_{i\alpha}\sigma_{i\alpha}^*)^2/2 + \lambda_2 \sigma_{i\alpha}\sigma_{i\beta}^*\sigma_{j\alpha}^*\sigma_{j\beta}/2 . \quad (4.1)$$

We neglect for now couplings of σ with any other scalar fields, since these can only destabilize asymptotically free behavior.[10] The differential equations for the running coupling constants can be derived[10] using the potential (4.1) and the scalar-gauge-field interaction

$$\begin{aligned} \mathcal{L}_I &= (g^2/4) M_{i\alpha} M_{i\alpha}^* , \\ M_{i\alpha} &= [(\lambda^A V_L^A)_{ij}\sigma_{j\alpha} - (\lambda^A V_R^A)_{\beta\alpha}\sigma_{i\beta}] . \end{aligned} \quad (4.2)$$

The resulting equations for the running quartic scalar coupling constants are

$$16\pi^2 \dot{\lambda}_1 = (2N^2+8)\lambda_1^2 + 8N\lambda_1\lambda_2 + 6\lambda_2^2 - 12[(N^2-1)/N]\lambda_1 g^2 + 3[(3N^2+4)/N^2] g^4 , \quad (4.3a)$$

$$16\pi^2 \dot{\lambda}_2 = 12\lambda_1\lambda_2 + 4N\lambda_2^2 - 12[(N^2-1)/N]\lambda_2 g^2 + 3[(N^2-8)/N] g^4 , \quad (4.3b)$$

where g is the SU(N) gauge coupling constant. Let $\lambda_1 \equiv k_1 g^2$ and $\lambda_2 \equiv k_2 g^2$. Since g is assumed to be asymptotically free, $(\dot{g}^2) = -bg^4$, where $22N/48\pi^2 > b > 0$. Fixed points occur when $\dot{k}_i = 0$, in which case

$$0 = (2N^2+8)k_1^2 + 8Nk_1k_2 + 6k_2^2 + [16\pi^2 b - 12(N^2-1)/N]k_1 + 3(3N^2+4)/N^2 , \quad (4.4a)$$

$$0 = 12k_1k_2 + 4Nk_2^2 + [16\pi^2 b - 12(N^2-1)/N]k_2 + 3(N^2-8)/N . \quad (4.4b)$$

We optimize the range of real solutions to Eqs. (4.4) by letting the terms linear in k_i be as negative as possible. Thus, we let b approach zero (corresponding to the saturation of asymptotic-freedom behavior by other fermion or scalar-field multiplets). In the $b=0$ limit, we solve (4.4b) for k_1 and substitute that solution into (4.4a):

$$k_1 = -Nk_2/3 + (N^2-1)/N - (N^2-8)/4Nk_2 , \quad (4.5a)$$

$$P_6 N^6 + P_4 N^4 + P_2 N^2 + P_0 = 0 , \quad (4.5b)$$

where

$$P_6 = 2k_2^4/9 - 4k_2^3/3 + 7k_2^2/3 - k_2 + \tfrac{1}{8} , \quad (4.6a)$$

$$P_4 = -16k_2^4/9 + 8k_2^3 - 34k_2^2/3 + 8k_2 - \tfrac{3}{2} , \quad (4.6b)$$

$$P_2 = (6k_2^3 - 20k_2^2/3 + 73k_2/3 + 1)k_2 , \quad (4.6c)$$

$$P_0 = 8k_2^2 - 8k_2 + 32 . \quad (4.6d)$$

We have grouped the quartic equation in k_2 by powers of N in order to consider the large-N behavior of the solutions to (4.6). The coefficient of N^6 has double zeros at $k_2=\frac{3}{2}\pm(\frac{3}{2})^{1/2}$:

$$P_6=2[k_2-\tfrac{3}{2}+(\tfrac{3}{2})^{1/2}]^2[k_2-\tfrac{3}{2}-(\tfrac{3}{2})^{1/2}]^2/9\,. \quad (4.7)$$

P_6 is greater than zero everywhere except its roots. Thus we must look at the coefficient of N^4 to see whether a locus of real zeros can be maintained. Curiously, the coefficients of N^4 share the same zeros of P_6:

$$P_4=[k_2-\tfrac{3}{2}+(\tfrac{3}{2})^{1/2}][k_2-\tfrac{3}{2}-(\tfrac{3}{2})^{1/2}] \times(-16k_2{}^2/9+8k_2/3-2)\,. \quad (4.8)$$

In the directed neighborhood of the real zeros defined by $k_2=\frac{3}{2}\pm(\frac{3}{2})^{1/2}\pm|\epsilon|$, N^6P is $\sim+|\epsilon|^2N^6$, while $N^4P_4\sim-|\epsilon|N^4$. Thus, there always exists a sufficiently small value of $|\epsilon|$ such that

$$P_6N^6+P_4N^4<0\,. \quad (4.9)$$

Since $N^6P_6\geq 0$, we see that (4.5b) always contains a locus of four real zeros for sufficiently large N. The second and fourth largest zeros are UV-stable in k_2.

This result is quite useful in that it shows an economical way to break half the generators of $SU(N)\times SU(N)$ (for sufficiently large N) without destroying asymptotic freedom. Unfortunately, $N=4$ is not sufficiently large. It is easily verified by minimizing the left-hand side (4.5b) that the left-hand-side polynomial is greater than zero everywhere if $N=4$.[34] If $N=4$, the coefficients of N^2 and N^0 are sufficiently positive to remove the locus of real zeros; N must be greater than or equal to 5 for the locus to exist. Thus, the $[SU(4)]^4$ unified theory is manifestly inconsistent with conventional asymptotic freedom[35]—the quartic scalar couplings that must occur to break chiral chromodynamics cannot be asymptotically free.

The above may be construed to be an argument for more flavors in a unified model; for example, an $[SU(6)]^4$ theory can contain asymptotically free couplings of $(6,\bar{6})$ scalar-field multiplets. However, no embedding theory that spontaneously breaks into the known interactions has been constructed that does not destroy asymptotic freedom.[35] For example, the flavor $SU(N)\times SU(N)$ sector of an $[SU(N)]^4$ model must eventually break down to the U(1) group of electromagnetism. It is well known that no "noneigenvalue" asymptotically free solutions have been found for the spontaneous breakdown of even SU(3) to U(1).[10] It is doubtful that $SU(N>3)$ can be spontaneously broken to $U(1)_{em}$ without disrupting asymptotic freedom; the larger N is, the more doubtful the possibility.

In light of these remarks, is the unification hypothesis tenable? The idea of unification has more profound motivations than having all particles act free at short distances. It may be that hadronic constituents are subject to strong forces at extremely short distances from the exchange of scalar fields. We also note that asymptotically free theories have been found in which all coupling constants keep a fixed numerical proportionality to the gauge coupling constant, provided there exists a sufficiently rich structure of Yukawa couplings.[15] Models which employ these so-called "eigenvalue" solutions have had to employ exotic fermion multiplets to obtain the needed Yukawa structure, in addition to all the scalar fields necessary to break a given symmetry. The requirement that couplings be related by strict numerical proportionalities to obtain desired solutions seems fundamentally unphysical, unless more profound reasons exist for nature to variationally select appropriate eigenvalues.

It is also possible that symmetries are not broken by scalar fields at all, but by some other dynamical means. Present and continuing successes of the Salam-Weinberg model in explaining neutrino-interaction phenomenology,[18] as well as the occurrence of the same phenomenology in the spontaneous breakdown of larger groups,[36] are powerful arguments for the existence of the Higgs mechanism; we cannot lightly dismiss the idea of spontaneous symmetry breaking through scalar-field vacuum expectation values.[37] Perhaps, the quartic scalar-field couplings are themselves induced and need not occur in the basic Lagrangian. Such an approach has been proposed by Salam and Strathdee[16] and may well resolve present inconsistencies between asymptotic freedom and the eventual unification of the interactions of nature.

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[27]Such limits are revised in J. S. Kang, University of Maryland Technical Report No. 77-083 (unpublished); M. Veltman, Phys. Lett. 70B, 253 (1977); and B. W. Lee, C. Quigg, and H. Thacker, Phys. Rev. Lett. 38, 883 (1977).

[28]V. Elias and A. R. Swift, Phys. Rev. D 15, 1937 (1977).

[29]J. C. Pati, A. Salam, and S. Sakakibara, Phys. Rev. Lett. 36, 1229 (1976).

[30]The reasoning here is the same as that used in applying the decoupling theorem (Ref. 7). LQB's do not, however, belong to the gauge multiplet of a preserved subgroup [SU(3)], as is the case for external particles in previous applications of the decoupling theorem. Thus a "running mass" must also be considered (Politzer, Ref. 19). The physical mass, considered here, is the boundary condition ($t = 0$) for the running mass.

[31]H. D. Politzer, Phys. Rev. Lett. 30, 1346 (1973).

[32]Coupling-constant magnitudes obtained in Ref. 12 suggest only an order-of-magnitude decrease in the bound on m_{LQB}. Should the strong interactions acquire their strength from a chiral color intermediate level of symmetry (Ref. 6), the bound on m_{LQB} may decrease further.

[33]The same conclusion appears in Ref. 9. However, fixed-point equations obtained in that paper disagree with those obtained here by nontrivial scale factors—the equations cannot be reconciled. Results concerning the asymptotic freedom of $(4, \overline{4})$ scalar-field quartic couplings are judged to be of sufficient importance to the $[\mathrm{SU}(4)]^4$ model (Ref. 6) to justify the calculations presented here.

[34]This result is unaffected by allowing the parameter b appearing in Eq. (4.4) to be in the range $11/6\pi^2 > b > 0$.

[35]Conventional asymptotic freedom is to be distinguished from eigenvalue prescriptions for asymptotic freedom (see Ref. 15). Note also that the symmetry-breaking prescription proposed by A. Buras *et al.*, Nucl. Phys. B135, 66 (1978) in which one adjoint and one vector representation of SU(5) are employed to obtain the known interaction group product has already been shown to contain non-asymptotically-free scalar-field coupling (Ref. 10). Such a pair of scalar-field multiplets of SU(N) can have asymptotically-free quartic scalar couplings only if $N \geq 7$.

[36]J. C. Pati, S. Rajpoot, and A. Salam, Phys. Rev. D 17,

131 (1978); H. Georgi and S. Weinberg, *ibid.* 17, 275 (1978).

[37]Models without Higgs mechanisms have been proposed that are consistent with Weinberg-Salam neutrino phenomenology but which suffer from other problems. In the unrenormalizable model proposed by J. D. Bjorken [in *The Unification of Elementary Forces and Gauge Theories* (Ref. 13); Phys. Rev. D 19, 335 (1978)] the weak angle is inversely proportional to the neutrino charge radius. The calculated neutrino charge radius [J. Bernstein and T. D. Lee, Phys. Rev. Lett. 11, 512 (1963)] is presently too large, corresponding to $\sin^2\theta_W \cong 0$. Moreover, the present consistency of the Weinberg-Salam model with SLAC atomic parity experiments (in which the electron's coupling to Z is measured) implies that the electron and the neutrino have charge radii of equal magnitude. (I am grateful to A. Salam for pointing this out.) The model of P. Q. Hung and J. J. Sakurai [Nucl. Phys. B143, 81 (1978)] also dispenses with Higgs particles and suffers from unitarity problems in $e^+e^- \to W^+W^-$ and in WW scattering that are discussed by C. H. Llewellyn Smith, LEP Summer Study 1–2 (CERN/ISR LEP Summer Study, 1978) (unpublished).