Dynamics of spontaneous symmetry breaking in the Weinberg-Salam theory

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We argue that the existence of fundamental scalar fields constitutes a serious flaw of the Weinberg-Salam theory. A possible scheme without such fields is described. The symmetry breaking is induced by a new strongly interacting sector whose natural scale is of the order of a few TeV.

I. WHY NOT FUNDAMENTAL SCALARS?

The need for fundamental scalar fields in the theory of weak and electromagnetic forces[1] is a serious flaw. Aside from the subjective esthetic argument, there exists a real difficulty connected with the quadratic mass divergences which always accompany scalar fields.[2] These divergences violate a concept of naturalness which requires the observable properties of a theory to be stable against minute variations of the fundamental parameters.

The basic underlying framework of discussion of naturalness assumes the existence of a fundamental length scale κ^{-1} which serves as a real cutoff. Many authors[3] have speculated that κ should be of order 10^{19} GeV corresponding to the Planck gravitational length. The basic parameters of such a theory are some set of dimensionless bare couplings g_0 and masses. The dimensionless bare masses are defined as the ratio of bare mass to cutoff:

$$\mu_0 = m_0/\kappa . \tag{1}$$

The principle of naturalness requires the physical properties of the output at low energy to be stable against very small variations of g_0 and μ_0. One such striking property is the existence of a "light" mass spectrum of order 1 GeV. From a dimensionless viewpoint the light spectrum has mass 10^{19} times smaller than the fundamental scale. It is in order to ask what kind of special adjustments of parameters must be made in order to ensure such a gigantic ratio of mass scales.

To illustrate a case of an unnatural adjustment, consider a particle which receives a self-energy which is quadratic in κ. To make the discussion simple, suppose the form of the mass correction is

$$m^2 = m_0^2 + \Delta m^2$$
$$= m_0^2 + \kappa^2 g_0^2 . \tag{2}$$

Solving for μ_0^2 gives

$$\mu_0^2 = \frac{m_0^2}{\kappa^2} = \frac{m^2}{\kappa^2} - g_0^2 . \tag{3}$$

Now if m is a physical mass of order 1 GeV and $\kappa \sim 10^{19}$ GeV, then

$$\mu_0^2 = -g_0^2(1 - 10^{-38}) . \tag{4}$$

Equation (4) means that μ_0^2 must be adjusted to the 38th decimal place. What happens if it is not? Then the mass will come out to be of order 10^{19} GeV.

Such adjustments are unnatural and will be assumed absent in the correct theory. Unfortunately, all present theories contain such unnatural adjustments because of the quadratic divergences in the scalar-particle masses.

Not all theories in which the physical and cutoff scales are vastly different are unnatural. Fortunately there exists a class of non-Abelian gauge theories where enormous scale ratios may occur naturally. These are the asymptotically free theories.[4]

In asymptotically free theories the scale-dependent "running" coupling constant satisfies

$$q \frac{\partial g}{\partial q} = -Cg^3 + O(g^5), \tag{5}$$

where q is the momentum scale at which the coupling is measured. The constant C is positive so that the coupling increases toward the infrared. It is believed that such theories spontaneously generate masses corresponding to values of q for which g becomes large. Integrating Eq. (5) gives

$$\frac{1}{g^2(q)} = 2C \ln(q/\kappa) + \frac{1}{g_0^2}, \tag{6}$$

where $g^2(K)$ is identified with the bare coupling g_0. Of course, Eq. (6) is inaccurate when g becomes large, but we may use it as a guide to where the coupling becomes large. It is evident that the value of m which makes $g(m)$ become large satisfies

$$\frac{m}{\kappa} = \exp\left(-\frac{1}{2Cg_0^2}\right). \tag{7}$$

Evidently, to make $m/\kappa \sim 10^{-19}$ requires the bare coupling g_0 to be

$$g_0^2 = \frac{1}{38C \ln 10} = \frac{0.012}{C}. \tag{8}$$

As an example, consider pure non-Abelian SU_3 Yang-Mills theory. The constant C is given by[4]

$$C = \frac{11}{16\pi^2} \tag{9}$$

so that

$$g_0^2 \sim 0.2.$$

This is hardly an unnatural value for g_0^2. Furthermore, the value of m/κ is not violently sensitive to small variations of g_0.

II. A NATURAL SCENARIO

Let us assume that at the smallest distances (Planck length) nature is described by a very symmetric "grand unified" theory. The grand unifying group is called G. Let us suppose that G is spontaneously broken. This might occur for a variety of reasons, including the existence of scalar fields or gravitational attraction. Since we shall forbid unnatural adjustments of constants we must assume that any masses which are generated in the first round of symmetry breaking are of order 10^{19} GeV.

At a somewhat larger distance scale, say 10^{17} GeV, a phenomenological description should exist. It will contain those survivors of the first symmetry breakdown which gained no mass. Furthermore, it will have a symmetry group

$$G_1 \otimes G_2 \otimes G_3 \otimes \cdots$$

consisting of factors which are not broken by the first breakdown.

The survivor fields will include
(1) the gauge bosons for the group $G_1 \otimes G_2 \otimes \cdots$,
(2) some subset of fermions which were protected by unbroken γ_5 symmetries,
(3) some Goldstone bosons. In what follows we assume no such Goldstone bosons are present.

If not for the fermions, the different G_i would define uncoupled gauge sectors. These sectors are, in general, coupled by fermions having non-trivial transformation properties under more than one G_i. For example, quarks form the bridge which couples quantum-chromodynamic (QCD) gluons with the photon, intermediate-vector-boson sector in the standard theory.

Thus, to specify a theory we must give a set of G_i and a set of fermion fields along with their transformation properties under all G_i. Furthermore, we will also need a set of coupling constants g_i. These may be taken to be the running couplings at a low enough energy so that the effects of the very heavy masses have disappeared. Henceforth we assume this to be 10^{+17} GeV. Henceforth we define $K = 10^{17}$.

Consider next the evolution of the running couplings. Some of them will increase and some will decrease as the energy scale is lowered. From studying examples it is clear that different g_i may blow up and produce masses at rather different scales. To see why this is so consider the case of two uncoupled gauge theories G_1 and G_2. Each will have its bare coupling g_1, g_2 and will evolve to give a mass scale

$$\frac{m_1}{K} \approx \exp\left(-\frac{1}{2C_1 g_1^2}\right),$$
$$\frac{m_2}{K} \approx \exp\left(-\frac{1}{2C_2 g_2^2}\right), \tag{10}$$

and

$$\frac{m_1}{m_2} = \exp\left[-\frac{1}{2}\left(\frac{1}{C_2 g_2^2} - \frac{1}{C_1 g_1^2}\right)\right]. \tag{11}$$

If we now assume $1/C_i g_i^2$ is large enough to make m_i/K very small then a few present differences between C_1 and C_2 or g_1 and g_2 can easily make $m_1/m_2 \sim 10^{-3}$.

Thus our expectation is for a $G_1 \otimes G_2 \otimes G_3 \cdots$ gauge theory with a set of Fermi fields connecting the G_i and a set of dynamically produced mass scales fairly well separated. The question to which this paper is addressed is: Can this type of theory produce the required kinds of spontaneous symmetry breakdown needed to understand weak, electromagnetic, and strong interactions?

III. A WARMUP EXAMPLE

The set of subgroups G_i must include SU_3 (color) and the electromagnetic-weak group $SU_2 \otimes U_1$ which we will call flavor. The fermion content must include quarks and leptons. As our simplest example we consider a theory with the massless flavor doublet (u, d) of color-triplet quarks. The quarks interact with an octet of color gluons and the four flavor gauge fields W^α and B. The coupling constants are chosen as they would be in realistic models so that the QCD coupling becomes ~ 1 at 1 GeV and the electromagnetic charge is $\sim \frac{1}{3}$. We call the SU_3, SU_2, and U_1 coupling constants g_3, g_2, and g_1. This theory is the standard theory of a single quark doublet with the exception that no fundamental scalar Higgs field are included.

The Lagrangian for our warmup model is

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F_{\mu\nu} - \tfrac{1}{4}W_{\mu\gamma}W_{\mu\gamma} - \tfrac{1}{4}B_{\mu\nu}B_{\mu\nu}$$
$$+ i\bar{\psi}\gamma_\mu(\partial_\mu + ig_3\hat{f} + ig_2\hat{W}_\mu + ig_1\hat{B}_\mu)\psi. \tag{12}$$

The objects $\hat{F}_\mu$, $\hat{W}_\mu$, and $\hat{B}_\mu$ are constructed from the SU_3, SU_2, U_1 vector potentials, Dirac matrices, 3×3 color matrices, and 2×2 flavor matrices. For example,

$$\hat{W}_\mu = W_\mu^\alpha \frac{\tau^\alpha}{2}\left(\frac{1 - \gamma_5}{2}\right), \tag{13}$$

where τ^α are flavor Pauli matrices. Similarly,

$$\hat{B}_\mu = \left[\frac{1 - \gamma_5}{4} - \frac{1}{3} + \left(\frac{1 + \tau_3}{2}\right)\left(\frac{1 + \gamma_5}{2}\right)\right]B_\mu. \tag{14}$$

In analyzing the above model we will make use of a number of standard assumptions. We now list them:

(1) The weak-electromagnetic sector can be treated as a small perturbation. The remaining assumptions apply to the pure SU_3 (color) sector when g_1 and g_2 are switched off.

(2) The strong interactions are invariant under chiral $SU_2 \otimes SU_2$ in the limit of vanishing bare quark mass.[6]

(3) Chiral $SU_2 \times SU_2$ is spontaneously broken and realized in the Nambu-Goldstone mode. The pion is the Goldstone boson. The "order parameter" signaling the spontaneous breakdown is $\langle 0 | \bar{\psi}\psi | 0 \rangle = \langle 0 | \bar{u}u + \bar{d}d | 0 \rangle$, which is nonzero.

(4) The chiral limit $(m_\pi^2 \to 0)$ is a smooth one in which all strong-interaction quantities (other than m_π) change by only a few percent. In particular, this includes f_π—the pion decay constant.

Our problem is to determine the behavior of the

(a)

W   W

(b)

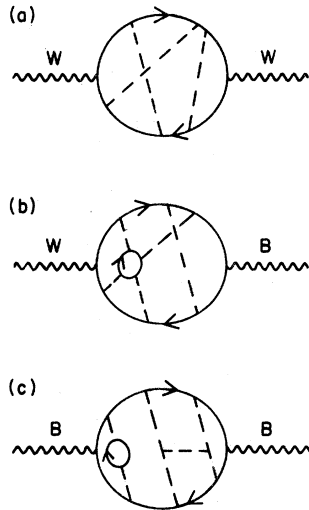W   B

(c)

B   B

FIG. 1. Contributions to $\pi_{\mu\nu}$ from quark-gluon states. Solid lines indicate quarks. Broken lines are gluons.

W, Z, and photon masses in this theory. In particular, we would like to know if the strong interactions can somehow replace the Higgs scalars and provide masses for the intermediate vector bosons. To this end we must examine the effects of quarks and SU_3 gluons on the W and B propagators. The relevant processes are shown in Figs. 1(a)–1(c).

Let us first ignore the B field and concentrate on the class of processes illustrated in Fig. 1(a). Invoking the familiar arguments of gauge invariance, we write the one-particle-irreducible vacuum polarization as

$$\pi_{\mu\nu}^{\alpha\beta} = \delta^{\alpha\beta}\left(\frac{g_2}{4}\right)^2(k^2 g_{\mu\nu} - k_\mu k_\nu)\pi(k^2), \tag{15}$$

where α, β indicate SU_2 indices. Evidently the W propagator is modified from

$$\delta^{\alpha\beta}\left(\frac{k^2 g_{\mu\nu} - k_\mu k_\nu}{k^4}\right) \tag{16}$$

to

$$\frac{\delta^{\alpha\beta}(g_{\mu\nu} - k_\mu k_\nu/k^2)}{k^2[1 + g_2^2\pi(k^2)/4]}. \tag{17}$$

Unless $\pi(k^2)$ is singular at $k^2 = 0$ the W propagator will have a pole at $k^2 = 0$ indicating a massless vector boson. From this point of view, the role of the fundamental scalar Goldstone bosons is to provide a pole in π at $k^2 = 0$.

Now consider the contribution of the pion to $\pi(k^2)$. Since no explicit scalars are included, the quarks must be massless. (Recall that the only source of quark mass in the Weinberg-Salam theory is the Yukawa couplings.) It then follows that the pion is massless, at least insofar as it is regarded as an unperturbed state of the pure strong interaction. Thus we can immediately write the pion contribution to π as a massless pole in the vicinity of $k^2 = 0$:

$$\pi(k^2) \approx f_\pi^2/k^2. \tag{18}$$

Accordingly, the pion replaces the usual scalar fields and shifts the mass of the W to

$$M_W^2 = \left(\frac{g_2}{2}\right)^2 f_\pi^2 \approx (30 \text{ MeV})^2. \tag{19}$$

Next consider the contribution of the pion to the processes in Figs. 1(b) and 1(c). For this we need to know the coupling of the pion to the Abelian U_1 current. From Eq. (14) we see that this current is

$$\bar{\psi}\gamma_\mu\left[\frac{1}{2}\left(\frac{1 - \gamma_5}{2}\right) - \frac{1}{3} + \left(\frac{1 + \tau_3}{2}\right)\left(\frac{1 + \tau_5}{2}\right)\right]\psi. \tag{20}$$

The term which couples to the neutral pion is

$$-\tfrac{1}{4}\bar{\psi}\tau_3\gamma_5\gamma_\mu\psi. \tag{21}$$

Thus Figs. 1(b) and 1(c) receive pion-pole contributions

$$\pi_{WB} = \frac{g_1 g_2}{4k^2} f_\pi^2 , \tag{22}$$

$$\pi_{BB} = \left(\frac{g_1}{4k^2}\right)^2 f_\pi^2 . \tag{23}$$

All of this is summarized by a mass matrix

$$M^2 = \begin{bmatrix} g_2^2 & 0 & 0 & 0 \\ 0 & g_2^2 & 0 & 0 \\ 0 & 0 & g_2^2 & g_1 g_2 \\ 0 & 0 & g_1 g_2 & g_1^2 \end{bmatrix} \tfrac{1}{4} f_\pi^2 , \tag{24}$$

where the labeling of the rows and columns is (W^+, W^-, W^0, B).

The mass matrix in Eq. (24) is identical to that in the Weinberg-Salam (WS) theory with the exception that f_π would be replaced by the vacuum expectation value of the scalar field ϕ. Thus the masses of Z and $W^\pm$ are in the same ratio as in the WS theory but are scaled down by the factor

$$\frac{f_\pi}{\langle\phi\rangle} \approx \frac{1}{3000} . \tag{25}$$

Naturally, in the model we are considering the pion is absent from the real spectrum, being replaced by the longitudinal $W^\pm$ and Z.

The correspondence between the pion and the usual scalar doublet ϕ may be made manifest. Define

$$\begin{aligned} \pi^\alpha &= \bar\psi \gamma_5 \tau^\alpha \psi , \\ \sigma &= \bar\psi\psi . \end{aligned} \tag{26}$$

The two component field ϕ of WS may be replaced by

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \longrightarrow \begin{pmatrix} \pi_1 + i\pi_2 \\ \pi_3 + i\sigma \end{pmatrix} . \tag{27}$$

It is easily seen that such a two-component object transforms as a spinor under left-handed SU_2 and has the same Abelian charge as ϕ. Lastly, the spontaneous breaking of the symmetry is accomplished by the usual strong interactions which (we believe) give rise to $\langle\sigma\rangle \neq 0$.

An interesting point we wish to emphasize before attempting a realistic example involves the $SU_2 \times SU_2$ symmetry of the hadron sector (before g_2 and g_1 are switched on). In general, to consistently couple a sector to the weak-electromagnetic interaction that sector need only have $SU_2 \times U_1$ symmetry. The extra symmetry under $SU_2 \times SU_2$ is also present in the WS model. To see it we write

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} \alpha_1 + i\alpha_2 \\ \alpha_3 + i\alpha_4 \end{pmatrix} \tag{28}$$

and then note that the scalar field Lagrangian in WS has symmetry under the four-dimensional rotations ($=SU_2 \times SU_2$) in the ($\alpha_1, \alpha_2, \alpha_3, \alpha_4$,) space.

In the WS model the additional symmetry is accidental and may be eliminated if nonrenormalizable interaction or additional scalar multiplets are introduced. In our case it is entirely natural, following from the symmetries of a multiplet of Dirac fermions.

It is interesting to ask what evidence exists for the $SU_2 \times SU_2$ symmetry. In our example the extra symmetry implies ordinary isospin [SU_2 (left) $+ SU_2$ (right)] symmetry and guarantees that f_π is the same for neutral and charged pions. If the symmetry were reduced to $SU_2 \times U_1$ then in general $f_{\pi^\pm} \neq f_{\pi^0}$. The result would be a modification of the structure of the mass matrix in Eq. (24). The success of the WS model in neutral-current phenomenology is rather sensitive to this structure. Therefore, a large deviation from $SU_2 \times SU_2$ symmetry in the scalar field sector will be inconsistent with observed neutral currents.

A final point involves the existence of more than one quark multiplet. If the number of quark doublets is increased from one to N, the hadronic chiral symmetry becomes $SU_{2N} \times SU_{2N}$. Since mass terms are forbidden when the fundamental scalars are absent this will necessarily be a symmetry of the hadronic sector. The number of Goldstone bosons will be $N^2 - 1$. The longitudinal Z and $W^\pm$ will again absorb three of these, leaving $N^2 - 4$ spin-zero objects. These objects will gain mass because the weak interactions explicitly violate the symmetries which correspond to them. In other words, they are what Weinberg calls pseudo-Goldstone bosons. Their mass will in general be of the same order of magnitude as that of Z and W.

IV. A MORE REALISTIC EXAMPLE

Let us now consider the possible existence of a new undiscovered strongly interacting sector, similar to ordinary strong interactions except with a mass scale of order 10^3 GeV. To be specific we introduce a new family of fermions called "heavy-color" quarks and an associated field χ. The heavy-color quarks form a flavor $SU_2 \times U_1$ doublet and an n-tuple in a new SU_n symmetry space called heavycolor. Heavycolor is a gauge symmetry requiring a multiplet of gauge bosons G_μ. The symmetry of the theory is then

$$SU_n(\text{HC}) \otimes SU_3(C) \otimes SU_2 \otimes U_1$$

with couplings g_n, g_3, g_2, g_1. The fermion content includes

(1) *Leptons*. These are flavor doublets and color–heavy-color singlets.

(2) *Quarks*. These are flavor doublets, color triplets, heavy-color singlets.

(3) *Heavy-color quarks*. These are flavor doublets, color singlets, and heavy-color n-tuples.

The coupling g_n is chosen so that a mass scale of order 1 TeV—the heavy-color (HC) interaction—becomes strong. To make this precise we first consider the pure HC theory ignoring quarks, leptons, color, and flavor. The bare g_n is then adjusted so that the lightest nonzero mass of a heavy-color hadron is ~1 TeV.

The Lagrangian of our model is

$$\mathcal{L} = -\tfrac{1}{4}G_{\mu\nu}G_{\mu\nu} - \tfrac{1}{4}F_{\mu\nu}F_{\mu\nu} - \tfrac{1}{4}W_{\mu\nu}W_{\mu\nu} - \tfrac{1}{4}B_{\mu\nu}B_{\mu\nu}$$
$$+ \bar{\chi}\gamma_\mu(\partial_\mu + ig_n\hat{G}_\mu + ig_2\hat{W}_\mu + ig_1\hat{B}_\mu)\chi$$
$$+ \bar{\psi}\gamma_\mu(\partial_\mu + ig_3\hat{F}_\mu + ig_2\hat{W}_\mu + ig_1\hat{B}_\mu)\psi. \qquad (29)$$

In speculating about the solution of this model we will make use of the following observations and assumptions.

(1) The evolutions of the heavy-color and color couplings with scale are only slightly different from what they would be if each sector were completely isolated. The justification for this is that heavy color and color are only coupled by their *weak* interactions with B and W. If g_1 and g_2 were zero, g_n and g_3 would evolve completely separately. In fact, the quark-gluon and heavy-color-quark–heavy-color-gluon worlds would be completely noninteracting.

(2) The isolated heavy-color sector is essentially similar to the color sector except scaled up in energy by ~3000. This means that $\langle 0|\bar{\chi}\chi|0\rangle \neq 0$. This implies the existence of a family of massless heavy-color pions with decay constants $F_\pi \sim f_\pi \times 3000$. It also means that there exists a rich spectrum of heavy-color hadrons.

As in our warmup example the Z and $W^\pm$ gain a mass. This time the mass is mainly due to the mixing of the HC-pion with W and B. The ordinary pion becomes very slightly mixed with the HC-pion but remains exactly massless. This is so because the ordinary and HC axial-vector currents are separately conserved. The longitudinal components of Z and W can only absorb one linear combination of the Goldstone bosons associated with these currents.

The model described here is certainly incomplete. As it stands it cannot account for the masses of leptons and quarks. We shall discuss this further in Sec. V.
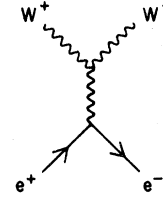

FIG. 2. The process $e^+e^- \to W^+W^-$.

V. IMPLICATIONS OF HEAVY COLOR

The behavior of processes at and above the TeV range is very different in the usual and present theories. By the usual theory I will always mean two things. First, symmetry breaking is caused by fundamental scalar fields. Second, all coupling constants including the scalar self-coupling are small so that perturbation theory is applicable.

Our first problem is to determine the mass scale of the heavy-color hadrons. To this end we observe that Eq. (19) will be replaced by

$$M_W^2 = \frac{g_2^2}{4}F_\pi^2, \qquad (30)$$

where F_π is the HC-pion decay constant. Since we know that $M_W \sim 90$ GeV, we find

$$F_\pi \sim 250 \text{ GeV}. \qquad (31)$$

Since we have assumed that the heavy-color sector is simply a scaled-up version of the usual strong interactions, it follows that heavy-color-hadron masses are F_π/f_π times their hadronic counterparts. Since $F_\pi/f_\pi \sim 3 \times 10^3$, the mass of a low-lying HC-hadron will be ~3 TeV.

The main differences in behavior of this and the usual model involve processes in which longitudinally polarized Z's and W's are produced at energies above a TeV. For example, consider e^+e^- annihilation. As in the usual theory the e^+e^- pair can form a virtual photon or Z boson which can then materialize as a pair of transverse $W^\pm$ bosons. This process is illustrated in Fig. 2. The transverse $W^\pm$ are weakly coupled and therefore contribute a smooth nonresonant contribution to R that can be computed in perturbation theory. (See Fig. 3.)

In the usual weakly coupled scalar version of the Weinberg-Salam theory the virtual γ or Z can also materialize as a pair of charged scalars disguised as longitudinal $W^\pm$. Since the scalars are also weakly coupled the contribution to R is smooth, nonresonant, and similar to Fig. 3.

In the present theory the scalar sector is replaced by the heavy-color sector and the virtual γ-Z may decay into a pair of heavy-color quarks. The resulting behavior of R will exhibit all the
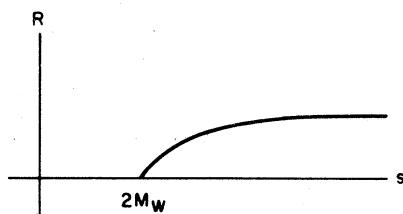
FIG. 3. R as a function of energy for transverse gluon production.

characteristics of resonances and final-state interaction which characterized R at ordinary energies ~0-3 GeV. It should exhibit the bumps of the HC-ρ, HC-Ω, and so on. (See Fig. 4.) The only difference is that the entire scale of masses, widths, and level separations will be of order 1 TeV instead of 1 GeV.

The final states of such processes will involve increasing multiplicities of HC-hadrons. If our experience in hadron physics is a good guide then most of the final HC-hadrons will be HC-pions. Of course real HC-pions do not exist, having been replaced by the longitudinal $W^{\pm}$, Z states. Indeed, the following theorem is easy to prove: To lowest order in α the amplitude for producing a given state including some set of Z_{long} and W_{long} is equal to the amplitude for a state in which the Z_L, W_L are replaced by HC-pions.

The longitudinal bosons decay in a conventional way to leptons and hadrons. However, the distribution of the longitudinal bosons will not resemble the usual theory. In the usual theory each boson, longitudinal or transverse, costs a factor of α since all couplings are small. In the present theory longitudinal bosons proliferate like pions once the energy exceeds a few TeV.

Perhaps the most interesting consequence of the new theory is the existence of a new conservation law—heavy-color baryon number $= \int \chi^* \chi \, d^3x$. The lightest HC-hadron carrying HC-baryon number will be stable and have a mass ~1-2 TeV. If the heavy-color group SU_n has odd (even) n this particle will be a fermion (boson). Its only interaction with ordinary matter will be weak-electro-
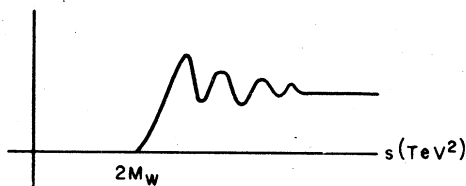

FIG. 4. Conbributions to R from the heavy-color sector.

magnetic. It may be charged like the proton or neutral like the neutron. If, like the proton, it is charged and found in any abundance in the universe, it may be detectable as a component of cosmic rays.

VI. CONCLUSIONS

One aspect of the scalar-boson problem has not been mentioned in this paper. The usual scalar-boson mechanism provides masses not only for the vectors but also the leptons and quarks. In the present example the only way to mimic the fermion mass mechanism would involve four-Fermi nonrenormalizable interactions. Indeed, if the scalar field ϕ is replaced by HC-quark bilinears in the Yukawa couplings, a quartic coupling of the form $\chi^\dagger \chi \, \psi^\dagger \psi$ is generated. This coupling produces the conventional fermion mass matrix when $\langle \bar{\chi} \chi \rangle$ gets a nonvanishing value.

The inability to generate mass without four-Fermi couplings is due to the chiral γ_5 symmetry of vector couplings. In the present theory this symmetry is a continuous symmetry if we ignore weak instantons. Therefore, any dynamical fermion mass generation would require massless Goldstone bosons.

In general, by adding more sectors, including a gauge group which mixes e and μ as well as strange and nonstrange quarks, we can reduce the γ_5 symmetry to a discrete symmetry. In order to do this we must make use of the instantons of this new sector which means the coupling must be significantly greater than α. If this theory can be made to work then no Goldstone bosons would be required by dynamical mass generation.

Notes added.

(1) After submitting this paper for publication I became aware of the work of S. Weinberg in which many of the motivations for heavy-color are described.[5]

(2) Very recently Weinberg[6] and Georgi, Lane, and Eichten have been led to consider a very similar proposal for replacing Higgs bosons by dynamically bound pionlike objects originating in a strong interaction whose scale is ~1 TeV. I would like to thank S. Weinberg for a copy of his report and H. Georgi for communicating his interest in the problem to me.

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[1] For a review of the conventional theory see S. Weinberg, in *Proceedings of the International Symposium on Lepton and Photon Interactions at High Energies, Hamburg, 1977*, edited by F. Gutbrod (DESY, Hamburg, 1977).

[2] The particular concept of naturalness and the objections to scalar fields described in this paper are due to K. Wilson, private communication.

[3] H. Georgi, H. Quinn, and S. Weinberg, Phys. Rev. Lett. **33**, 451 (1974).

[4] H. D. Politzer, Phys. Rev. Lett. **30**, 1346 (1973); D. Gross and F. Wilczek, *ibid.* **30**, 1343 (1973).

[5] S. Weinberg, Phys. Rev. D **13**, 974 (1975). This paper contains many of the conceptual motivations described here.

[6] S. Weinberg, Phys. Rev. D **19**, 1277 (1979).