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Constraints due to partial conservation of axial-vector current and charmed-baryon couplings

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The PCAC (partial conservation of axial-vector current) consistency conditions are derived for inelastic processes involving charmed baryons. Numerical estimates of the conditions for the processes $\pi C_0 \to \pi C_0$, $\pi C_1 \to \pi C_1$, $\pi C_0 \to \pi C_1$, and $\pi C_0 \to \pi C_1^*$, determined with the help of observed charmed-baryon contributions, are used in determining the pion–charmed-baryon coupling strengths. We find that the value of $g_{\pi C_1 C_1}{}^2/4\pi$ agrees well with the previously determined values obtained with the potential-model calculations. The decay widths of C_1 and C_1^* into $C_0\pi$ channel also compare well with the results of Lee, Quigg, and Rosner. We also note that the remaining couplings $g_{\pi C_1 C_1^*}$ and $g_{\pi C_1^* C_1^*}$ have very small values and thus C_1^* completely decouples from $C_1\pi$ and $C_1^*\pi$ channels.

I. INTRODUCTION

Recent experimental discoveries of charmed mesons and baryons have sparked an unusual interest in the study of particle physics and now almost no one doubts about the existence of charm. As in the case of charmed mesons, the discovery of charmed baryons permits us to use the properties of the observed states to get at the dynamics of strong as well as electromagnetic and weak interactions. At Fermilab, the lowest-mass charmed baryon C_0^+ (or Λ_c, i.e., the charmed analog of Λ by replacing s quark by c quark) was experimentally identified[1] with the state at 2.26 GeV observed in the nonleptonic decay mode $\Lambda\pi^+\pi^-\pi^-$ via photoproduction. This state was already reported in the experiment involving neutrino interactions and now also found in the electron-positron annihilation experiment.[1] C_0^+ combined with an additional $\pi^\pm$ gives enhancements at 2.43 GeV for C_1 (or Σ_c) and at 2.48 GeV for C_1^* (or Σ_c^*), which are the charmed analogs of Σ and Y_1^*, respectively. However, the experimental data for these states are still incomplete. It, therefore, appears very significant to search experimentally for the production of these states in hadronic interactions and also warrants theoretical investigations to determine the hadronic properties of these states. In this paper, we want to calculate the coupling strengths $g_{\pi C_0 C_1}$, $g_{\pi C_0 C_1^*}$, $g_{\pi C_1 C_1}$, and $g_{\pi C_1^* C_1^*}$ in order to find out the hadronic nature of these states.

Recently, the methods of current algebra and the partially conserved axial-vector current (PCAC) hypothesis have proved to be highly useful in the study of various weak, electromagnetic-, and strong-interaction processes. These methods are based on the fact that the Born approximation with a certain derivative coupling of the pion field gives results equivalent to PCAC. One of the main consequences of the PCAC hypothesis is the Adler consistency condition[2] on the covariant amplitudes, which relates the amplitudes involving the emission of a "soft pion" to the amplitude in the absence of "soft-pion" emission. The divergence of the axial-vector current, when used as an interpolating field for the pseudoscalar π meson, yields the smoothest possible off-shell continuation for matrix elements involving such mesons. The purpose of this paper is to exploit the Adler consistency conditions on the covariant amplitudes associated with the matrix elements of the elastic as well as inelastic processes,

$$\pi + C_0 \to \pi + C_0, \tag{1}$$

$$\pi + C_1 \to \pi + C_1, \tag{2}$$

$$\pi + C_0 \to \pi + C_1, \tag{3}$$

$$\pi + C_0 \to \pi + C_1^*, \tag{4}$$

for obtaining valuable information about the coupling strengths of the charmed baryons with pions.

The consistency conditions have been used extensively in the past for the determination of strong-interaction parameters. The consistency condition for πN scattering[2] has been tested by Adler and is found to be satisfied experimentally. Similarly, Martin has derived[3] and tested the condition for $\pi\Lambda$ scattering and later on Chan and Smalley[4] used the recent experimental data to determine the couplings involved by testing the consistency condition for $\pi\Lambda$ and $\pi\Sigma$ scattering. Similar calculations[5,6] have also been made for strangeness-changing currents. Singh *et al.*[7] have extended the utility of the PCAC hypothesis in deriving and testing the consistency conditions for the inelastic process $\pi N \to \pi N^*$. Here we try to determine the charmed-baryon–pion couplings by deriving similar conditions for the processes (1)–(4) and testing them with the help of known charmed-baryon states.

II. METHOD OF CALCULATION

The derivation of the Adler consistency conditions for the processes (1) and (2) can be done on the pattern of $\pi\Lambda$ and $\pi\Sigma$ scattering, respectively. If we decompose the matrix elements for the meson-baryon scattering $M(K)+B(p_i)\to M(q)+B(p_f)$ into the usual invariant amplitudes,

$$\bar{u}(p_f)Mu(p_i)=\bar{u}(p_f)[-A(s,t,K^2)+\hat{K}B(s,t,K^2)]u(p_i)\,, \tag{5}$$

where

$$\nu=-K\cdot(p_i+p_f)/2m_{C_0}\,,$$
$$\nu_B=K\cdot q/2m_{C_0}\,,$$
$$s=-(K+p_i)^2\,,$$
$$t=-(K-q)^2\,.$$
$$\hat{K}=\gamma\cdot K$$

Here we have shown K^2 explicitly in A and B, since we want to explore the consequences when the incident pion is taken off its mass shell. The Adler consistency condition for the process (1) can then be obtained if we take $s=m_{C_0}^2$, $t=m_\pi^2$, and $K^2=0$:

$$A(\nu=0,\nu_B=0,K^2=0)=0\,. \tag{6}$$

For the reaction (2), consider[4] the following crossing-even combination:

$$M_{\pi C_1}=2M_{\pi C_1}(I=1)-M_{\pi C_1}(I=0)\,. \tag{7}$$

Therefore, the consistency condition for the process (2) can be written in a fashion analogous to that derived for $\pi\Sigma$ scattering,[4]

$$A(\nu=0,\nu_B=0,K^2=0)=K_{C_1C_1\pi}(0)\,\frac{16\pi}{m_{C_1}}\,\frac{g_{\pi C_1C_1}{}^2}{4\pi}\,. \tag{8}$$

Here $K_{C_1C_1\pi}$ is the pion form factor for the πC_1C_1 vertex evaluated at $K^2=0$. This form factor is normalized as $K_{C_1C_1\pi}(K^2=m_\pi^2)$. Expanding $K_{C_1C_1\pi}$ at $K^2=m_\pi^2$ and assuming the deviation at $K^2=0$ to be small, we can take $K_{C_1C_1\pi}(0)\simeq 1$. The result of Adler in the πN case supports this conclusion.

For the numerical evaluation of these conditions we assume unsubtracted dispersion relations for the above amplitudes. In order to find the resonance contributions to the condition (6), we make use of the relation of A with f_1 and f_2:

$$A=4\pi\left\{\frac{f_1(W+m_{C_0})}{[(E_1+m_{C_0})(E_2+m_{C_0})]^{1/2}}-\frac{f_2(W-m_{C_0})}{[(E_1-m_{C_0})(E_2-m_{C_0})]^{1/2}}\right\}, \tag{9}$$

where $W=\sqrt{s}$ is the total center-of-mass energy and

$$E_1=\frac{W^2+m_{C_0}^2}{2W}\,,$$
$$E_2=\frac{W^2+m_{C_0}^2-m_\pi^2}{2W}\,.$$

We can now express f_1 and f_2 in terms of partial-wave amplitudes as follows:

$$f_1=\sum_{l=0}^{\infty}[f_{l+}P'_{l+1}(x)-f_{l-}P'_{l-1}(x)]\,,$$
$$f_2=\sum_{l=1}^{\infty}[(f_{l-}-f_{l+})P'_l(x)]\,, \tag{10}$$

where

$$x=\cos\theta=\left(1+\frac{m_\pi^2}{E_2^2-m_{C_0}^2}\right)^{1/2}. \tag{11}$$

The resonance contributions to partial-wave amplitudes $f_{l\pm}$ are determined in the narrow-width approximation. To take the off-mass-shell nature of pions into consideration, we multiply by a factor $(q_{\rm off}/q_{\rm on})^l$ the corresponding expressions for f_1 and f_2. Here $q_{\rm off}$ and $q_{\rm on}$ are the magnitudes of the c.m. three-momenta of initial and final particles, respectively.

The PCAC consistency condition for the strong-interaction inelastic process $\pi C_0\to\pi C_1$ can be derived (see Appendix A) and is found to be

$$A_{\rm inel}(\nu=0,\nu_B=0,K^2=0)=\frac{g_{\pi C_0C_1}{}^2K_{\pi C_0C_1}(0)}{m_{C_0}+m_{C_1}}\,. \tag{12}$$

Here the definitions of ν and ν_B are modified as

$$\nu=-\frac{K\cdot(p_i+p_f)}{m_{C_0}+m_{C_1}}\,,$$
$$\nu_B=\frac{K\cdot q}{m_{C_0}+m_{C_1}}\,.$$

Finally, for the process $\pi C_0\to\pi C_1^*$ we get four invariant amplitudes because C_1^* has spin $\frac{3}{2}$. The matrix element for the process has the following general structure:

$$\bar{\psi}_\alpha(p_f)M_\alpha\gamma_5 u(p_i)=\bar{\psi}_\alpha(p_f)(a_1K_\alpha+a_2q_\alpha+\hat{K}b_1K_\alpha+\hat{K}b_2q_\alpha)\gamma_5u(p_i)\,. \tag{13}$$

We get the consistency condition involving the invariant amplitude a_2 alone, and it can be written as (see Appendix B)

$$a_2(\nu=0, \nu_B=0, K^2=0)=0\,. \tag{14}$$

Here

$$\nu=-\frac{K\cdot(p_i+p_f)}{m_{C_0}+m_{C_1^*}},$$

$$\nu_B=\frac{K\cdot q}{m_{C_0}+m_{C_1^*}}\,.$$

III. RESULTS AND DISCUSSION

Saturating the consistency condition (6) with the help of known[8] isospin-1 charmed-baryon resonances C_1 and C_1^* only in the narrow-width approximation, we get

$$\Gamma_{C_1^*}-4.68\Gamma_{C_1}=0\,, \tag{15}$$

where $\Gamma_{C_1^*}$ and Γ_{C_1} are the widths of C_1^* and C_1 for the decay into $C_0\pi$, respectively. We find that the above result agrees well with the result obtained by Lee, Quigg, and Rosner.[8] Using[8] $\Gamma_{C_1^*}=20$ MeV, we get $\Gamma_{C_1}=4.3$ MeV. Using these values of the decay widths, the corresponding coupling constants can be obtained with the help of the following relations[9,10]:

$$\Gamma_{C_1}=\frac{g_{C_1C_0\pi}{}^2}{4\pi}\,\frac{(E-m_{C_0})|\vec{p}|}{m_{C_1}}\,,$$
$$\Gamma_{C_1^*}=\frac{g_{C_1^*C_0\pi}{}^2}{4\pi}\,\frac{(E+m_{C_0})|\vec{p}|^3}{3m_{C_1^*}}\,, \tag{16}$$

where E (E') and $|\vec{p}|$ ($|\vec{p}\,'|$) are the energy and magnitude of the three-momentum of C_0 in the rest frame of the resonance C_1 (C_1^*). Finally, we get

$$\frac{g_{C_1C_0\pi}{}^2}{4\pi}\simeq 58.3\,,$$
$$\frac{g_{C_1^*C_0\pi}{}^2}{4\pi}\simeq 0.15\,. \tag{17}$$

Similarly estimating the consistency conditions (8), (12), and (14) with the known charmed baryon states, we get the following relations:

$$\frac{4g_{\pi C_1C_1^*}{}^2}{m_{C_1^*}m_\pi{}^2}\left[\left(m_{C_1}{}^2-\frac{m_\pi{}^2}{2}\right)\left(\frac{2m_{C_1}}{3m_{C_1^*}}+1\right)-\frac{m_{C_1}{}^2}{3}\right]+\frac{2g_{\pi C_1C_0}{}^2}{m_{C_0}+m_{C_1}}=\frac{4g_{\pi C_1C_1}{}^2}{m_{C_1}}, \tag{18}$$

$$\left[(m_{C_0}{}^2+m_{C_1}{}^2-m_\pi{}^2)\left\{\frac{1}{m_{C_1^*}}+\frac{(m_{C_0}+m_{C_1})}{3m_{C_1^*}{}^2}\right\}-\frac{2m_{C_0}m_{C_1}}{3m_{C_1^*}}\right]\frac{g_{\pi C_0C_1^*}g_{\pi C_1C_1^*}}{m_\pi{}^2}-\frac{1}{m_{C_0}+m_{C_1}}g_{\pi C_0C_1}g_{\pi C_1C_1}=\frac{g_{\pi C_0C_1}{}^2}{m_{C_0}+m_{C_1}}\,, \tag{19}$$

$$-\frac{g_{\pi C_1C_1^*}g_{\pi C_0C_1}}{m_\pi(m_{C_1}+m_{C_0})}+\frac{g_{\pi C_1^*C_1^*}g_{\pi C_0C_1^*}}{m_\pi(m_{C_1^*}-m_{C_0})}\left(1-\frac{2}{3}\frac{m_{C_0}{}^2}{m_{C_1^*}{}^2}-\frac{m_{C_0}}{3m_{C_1^*}}\right)=0\,. \tag{20}$$

In finding these relations we have assumed[11] unsubtracted dispersion relations and evaluated them by the pole-term contributions in s as well as u channels at fixed $t=m_\pi{}^2$ (i.e., $\nu_B=0$). Also in (20), we have ignored $g'_{\pi C_1^*C_1^*}$ coupling appearing in the full Lagrangian,[12]

$$L_{\pi C_1^*C_1^*}=g_{C_1^*C_1^*\pi}\overline{C}{}^*_{1\mu}\gamma_5C_1^{*\mu}\pi+g'_{C_1^*C_1^*\pi}\overline{C}{}_1^{*\mu}\gamma_5C_1^{*\nu}\partial_\mu\partial_\nu\pi\,, \tag{21}$$

because it is purely an F-wave coupling and we are working in the low-energy approximation. Here $C_{1\mu}^*$ is the Rarita-Schwinger spin-$\frac{3}{2}$ wave function for C_1^*. Using the values of the couplings given in Eq. (17), we get the following values for the remaining coupling constants:

$$\frac{g_{C_1C_1\pi}{}^2}{4\pi}=28.40\,, \tag{22}$$

$$\frac{g_{C_1^*C_1\pi}{}^2}{4\pi}=0.034\,, \tag{23}$$

$$\frac{g_{C_1^*C_1^*\pi}{}^2}{4\pi}=1.43\,. \tag{24}$$

We find that $g_{C_1C_1\pi}/\sqrt{4\pi}\simeq 5.33$, which is in agreement with the previous values of 3.98 and 3.55, as determined by Dover *et al.*[13] with calculations based on the models of Bryan and Phillips[14] and Nagels, Rijken, and de Swart,[15] respectively. The smallness of C_1^* couplings indicates that C_1^* almost decouples with the $C_1\pi$ and $C_1^*\pi$ channels. We hope that future experimental as well as theoretical investigations on the hadronic productions of charmed-baryon processes will test these predictions, which are based on the simple use of current algebra and PCAC hypothesis.

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APPENDIX A: DERIVATION OF THE CONSISTENCY CONDITION FOR THE PROCESS $\pi C_0 \to \pi C_1$

The matrix element of the axial-vector current J^A_μ can be written as (see Fig. 1)

$$\left(\frac{p_{i0}p_0'}{m_{C_0}m_{C_1}}\right)^{1/2}\langle C_1(p')|J^A_\mu|C_0(p_i)\rangle = \bar{u}(p')[G(K^2)\gamma_\mu\gamma_5 - F(K^2)\sigma_{\eta\mu}K_\eta\gamma_5 - iH(K^2)K_\mu\gamma_5]u(p_i)\,. \quad \text{(A1)}$$

Assuming the PCAC relation we get

$$\partial_\mu J^A_\mu = C_\pi \phi_\pi\,, \quad \text{(A2)}$$

where ϕ_π is the renormalized pion field operator which is related to the meson-source operator j_π as follows:

$$j_\pi = (\Box^2 - m_\pi{}^2)\phi_\pi\,. \quad \text{(A3)}$$

The matrix element of the meson-source operator is expressed as

$$\left(\frac{p_{i0}p_0'}{m_{C_0}m_{C_1}}\right)^{1/2}\langle C_1(p')|j_\pi|C_0(p_i)\rangle = g_{C_1C_0\pi}K_{C_1C_0\pi}(K^2)\bar{u}(p')K_\alpha u(p_i)\,. \quad \text{(A4)}$$

Therefore, taking the matrix element of each side of (A2) in the $K^2=0$ limit gives

$$G(0) = \frac{C_\pi}{m_\pi{}^2}\,\frac{g_{\pi C_0C_1}K_{\pi C_0C_1}(0)}{(m_{C_0}+m_{C_1})}\,, \quad \text{(A5)}$$

where $g_{\pi C_0C_1}$ is the renormalized coupling constant and $K_{\pi C_0C_1}$ is the pionic form factor of the $C_1C_0\pi$ vertex.

Let us now adopt the usual decomposition[2] of the matrix element $\langle \pi C_1|J^A_\mu|C_0\rangle$ in eight covariant amplitudes A_j as given by

$$\left(\frac{p_{i0}p_{f0}{}^2q_0}{m_{C_0}m_{C_1}}\right)^{1/2}\langle \pi C_1|J^A_\mu|C_0\rangle = \bar{u}(p_f)\sum_{j=1}^{8} O^j_\mu A_j(\nu,\nu_B,K^2)u(p_i)\,, \quad \text{(A6)}$$

where

$$O^1_\mu = \tfrac{1}{2}(\gamma_\mu\hat{q} - \hat{q}\gamma_\mu)\,,$$
$$O^2_\mu = (p_i + p_f)_\mu\,,$$
$$O^3_\mu = q_\mu\,,$$
$$O^4_\mu = -\tfrac{1}{2}(m_{C_0}+m_{C_1})\gamma_\mu\,,$$
$$O^5_\mu = -\hat{K}(p_i+p_f)_\mu\,,$$
$$O^6_\mu = -\hat{K}q_\mu\,,$$
$$O^7_\mu = K_\mu\,,$$
$$O^8_\mu = -\hat{K}K_\mu\,.$$

Since there is only one isospin state involved, we shall not specify the isospin indices explicitly. We decompose the amplitude A_j as a sum of pole-term contributions plus the residual amplitudes

$$A_j = A^P_j + A^R_j\,. \quad \text{(A7)}$$

$$\bar{u}(p_f)\sum_{j=1}^{8} O^j_\mu A^P_j u(p_i) = \bar{u}(p_f)\; g_{\pi C_1C_1}\gamma_5\frac{\hat{p}'+m_{C_1}}{p'^2-m_{C_1}{}^2}J^A_\mu + J^A_\mu\frac{p''+m_{C_1}}{p''^2-m_{C_1}{}^2}g_{\pi C_0C_1}\gamma_5\; u(p_i)\,. \quad \text{(A8)}$$

Thus we find contributions to the following amplitudes only:

The pole-term contributions may be evaluated from Fig. 1:

$$A^P_1 = \frac{g_{\pi C_0C_1}G(K^2)}{(m_{C_0}+m_{C_1})(\nu+\nu_B)} - \frac{g_{\pi C_1C_1}G(K^2)}{(m_{C_0}+m_{C_1})(\nu_B-\nu+\Delta)}\,,$$

$$A^P_3 = \frac{g_{\pi C_0C_1}G(K^2)}{(m_{C_0}+m_{C_1})(\nu+\nu_B)} - \frac{g_{\pi C_1C_1}G(K^2)}{(m_{C_0}+m_{C_1})(\nu_B-\nu+\Delta)}\,, \quad \text{(A9)}$$

$$A^P_4 = \frac{m_{C_0}-m_{C_1}}{m_{C_0}+m_{C_1}}\,\frac{G(K^2)}{(\nu_B+\nu)}\,g_{\pi C_0C_1}\,,$$

where

$$\Delta = \frac{m_{C_1}{}^2 - m_{C_0}{}^2}{m_{C_1}+m_{C_0}}\,.$$

We can also write the matrix element

$$\langle \pi C_1|\partial_\mu J^A_\mu|C_0\rangle = K_\mu\langle \pi C_1|J^A_\mu|C_0\rangle|_{K^2=0}\,, \quad \text{(A10)}$$

where the covariant amplitudes for meson-baryon scattering are related as follows:

$$\left(\frac{p_{f0}p_{i0}2q_0}{m_{C_1}m_{C_0}}\right)^{1/2}\langle \pi C_1|\partial_\mu J^A_\mu|C_0\rangle|_{K^2=0} = \bar{u}(p_f)Mu(p_i)\,, \quad \text{(A11)}$$

with

$$M = -A + \hat{K}B\,. \quad \text{(A12)}$$

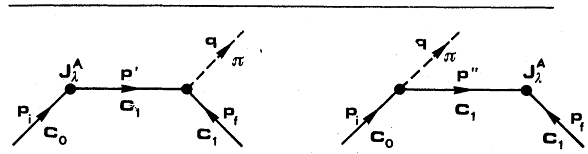

FIG. 1. Born diagram for the process $\pi + C_0 \to \pi + C_1$.

The nonpole terms are found to be

$$A^R=(m_{C_0}+m_{C_1})\nu(A_1^R+A_2^R)+(m_{C_0}+m_{C_1})\nu_B A_3^R\,,$$

$$B^R=(m_{C_0}+m_{C_\tau})(A_1^R-\tfrac{1}{2}A_4^R+\nu A_5^R-\nu_B A_6^R)\,. \quad \text{(A13)}$$

Thus we get the following consistency condition for A, which takes the pole term contributions from $A_1^{P'}$ and A_3^P:

$$A(\nu\to 0,\nu_B\to 0,K^2\to 0)=\frac{g_{\pi C_0C_1}{}^2K_{\pi C_0C_1}(0)}{m_{C_0}+m_{C_1}}\,. \quad \text{(A14)}$$

In deriving this relation, we have assumed that nonpole amplitudes A_j^R have no kinematical singularities.

APPENDIX B: DERIVATION OF CONSISTENCY CONDITION FOR $\pi C_0\to\pi C_1^*$

In this case the PCAC hypothesis with the relation of the renormalized pion field operator ϕ_π to the source operator j_π can be used to get

$$\langle C_1(p')|\partial_\mu J_\mu^A|C_0(p_i)\rangle\!\rangle=C_\pi\langle C_1(p')|\phi_\pi|C_0(p_i)\rangle\!\rangle\,, \quad \text{(B1)}$$

and thus we find

$$C_\pi=\frac{(m_{C_0}+m_{C_1})m_\pi{}^2G_A(0)}{g_{\pi C_0C_1}F_{\pi C_0C_1}(0)}\,, \quad \text{(B2)}$$

where $g_{\pi C_0C_1}$ is the renormalized coupling constant and $F_{\pi C_0C_1}(0)$ is the pionic form factor at πC_0C_1 vertex. The matrix element of the axial-vector current J^A can be written as

$$\left(\frac{p_{i0}p_0'}{m_{C_0}m_{C_1}}\right)^{1/2}\langle C_1(p')|J_\lambda^A|C_0(p_i)\rangle\!\rangle=\bar u(p')[G(K^2)\gamma_\lambda\gamma_5-F(K^2)\sigma_{\eta\lambda}K_\eta\gamma_5-iH(K^2)K_\lambda\gamma_5]u(p_i)\,. \quad \text{(B3)}$$

Here the matrix element $\langle\pi C_1^*|J_\lambda^A|C_0\rangle$ can be decomposed into 18 covariant amplitudes A_j as follows:

$$\left(\frac{p_{i0}p_{f0}{}^2q_0}{m_{C_0}m_{C_1^*}}\right)^{1/2}\langle\pi C_1^*|J_\lambda^A|C_0\rangle=\bar\psi_\alpha(p_f)\sum_{j=1}^{18}O_{\lambda\alpha}^jA_j(\nu,\nu_B,K^2)\gamma_5u(p_i)\,, \quad \text{(B4)}$$

where $O_{\lambda\alpha}^j$ are given in Ref. 7. Splitting the amplitude as

$$A_j=A_j^B+A_j^R\,, \quad \text{(B5)}$$

and extracting the Born term contributions A_j^B from Fig. 2, we get

$$\bar\psi_\alpha(p_f)\sum_{j=1}^{18}O_{\lambda\alpha}^jA_ju(p_i)=\bar\psi_\alpha(p_f)\frac{1}{(m_{C_0}+m_{C_1^*})}\left\{g_{\pi C_1C_1^*}2q_\alpha\left(\frac{m_{C_1}+m_{C_1^*}+\hat q}{\nu_B+\nu-\Delta-\Delta'}\right)G(K^2)\gamma_\lambda\right.$$
$$+\left[f_1(K^2)\delta_{\lambda\alpha}+f_2(K^2)\gamma_\lambda K_\alpha+f_4(K^2)\left(\frac{p_f+p_i-q}{2}\right)_\lambda K_\alpha\right]$$
$$\left.\times\left(\frac{-m_{C_0}+m_{C_1}-\hat q}{\nu_B+\nu-\Delta-\Delta'}\right)g_{\pi C_0C_1}\right\}\gamma_5u(p_i)\,, \quad \text{(B6)}$$

where

$$\Delta=\frac{m_{C_1^*}{}^2-m_{C_0}{}^{*2}}{m_{C_1^*}+m_{C_0}}\,,$$

$$\Delta'=\frac{m_{C_1}{}^2-m_{C_0}{}^2}{m_{C_1^*}+m_{C_0}}\,.$$

Comparing the coefficients of operators O_λ^j on both sides, we get

$$A_2^B=\frac{g_{\pi C_0C_1}}{2(m_{C_0}+m_{C_1^*})}f_4\left(\frac{m_{C_1}-m_{C_0}}{\nu+\nu_B-\Delta-\Delta'}\right)\,,$$

$$A_3^B=\frac{2g_{\pi C_1C_1^*}}{(m_{C_0}+m_{C_1^*})}G(K^2)\frac{1}{(\nu_B-\nu-\Delta')}\,,$$

$$A_4^B=-\frac{g_{\pi C_0C_1}}{m_{C_0}+m_{C_1^*}}\left[f_2+\left(\frac{m_{C_0}-m_{C_1}}{2}\right)f_4\right]\times\frac{1}{(\nu+\nu_B-\Delta-\Delta')}\,,$$

$$A_6^B=-\frac{g_{\pi C_0C_1}}{m_{C_0}+m_{C_1^*}}f_4\frac{1}{(\nu_B+\nu-\Delta-\Delta')}\,,$$

$$A_7^B=\frac{2g_{\pi C_1C_1^*}}{m_{C_0}+m_{C_1^*}}G(K^2)\frac{m_{C_1}+m_{C_1^*}}{(\nu_B-\nu-\Delta')}\,,$$

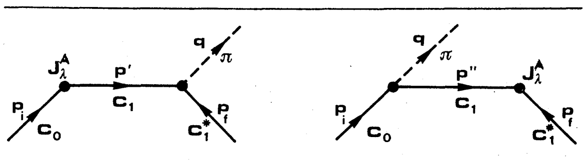

FIG. 2. Born diagram for the process $\pi+C_0\to\pi+C_1^*$.

$$A_8^B=\frac{g_{\pi C_0 C_1}}{m_{C_0}+m_{C_1^*}}f_2\frac{m_{C_1}-m_{C_0}}{(\nu_B+\nu-\Delta-\Delta')},$$

$$A_{10}^B=\frac{g_{\pi C_1 C_1^*}}{2(m_{C_0}+m_{C_1^*})}f_4\frac{1}{(\nu_B+\nu-\Delta-\Delta')},$$

$$A_{12}^B=\frac{g_{\pi C_0 C_1}}{2(m_{C_0}+m_{C_1^*})}f_4\frac{1}{(\nu_B+\nu-\Delta-\Delta')},$$

$$A_{14}^B=-\frac{2g_{\pi C_1 C_1^*}}{m_{C_0}+m_{C_1^*}}\frac{G(K^2)}{(\nu_B-\nu-\Delta')},$$

$$A_{16}^B=-\frac{g_{\pi C_0 C_1}}{m_{C_0}+m_{C_1^*}}f_2\frac{1}{(\nu_B+\nu-\Delta-\Delta')},$$

$$A_{17}^B=\frac{g_{\pi C_0 C_1}}{m_{C_0}+m_{C_1^*}}f_1\frac{m_{C_1}-m_{C_0}}{\nu_B+\nu-\Delta-\Delta'}\ .$$

Now let us evaluate the matrix element

$$\langle\pi C_1^*|\partial_\lambda J_\lambda^A|C_0\rangle=K_\lambda\langle\pi C_1^*|J_\lambda^A|C_0\rangle|_{K^2=0}\,. \tag{B7}$$

We can write

$$\left(\frac{p_{i0}p_{f0}{}^2q_0}{m_{C_0}m_{C_1^*}}\right)^{1/2}\langle\pi C_1^*|\partial_\lambda J_\lambda^A|C_0\rangle|_{K^2=0}=\bar\psi_\alpha(p_f)M_\alpha\gamma_5 u(p_i)\,, \tag{B8}$$

where

$$M_\alpha=a_1K_\alpha+a_2q_\alpha+\hat K(b_1K_\alpha+b_2q_\alpha). \tag{B9}$$

From (B8) we get nonpole terms as follows:

$$\begin{aligned}a_1^R&=(m_{C_0}+m_{C_1^*})(\nu A_2^R+A_4^R+\nu A_{16}^R-A_{18})\\&\quad-(m_{C_0}+m_{C_1^*})^2(\nu A_{10}^R+\nu_B A_{12}^R)+A_{17}^R\,,\\a_2^R&=(m_{C_0}+m_{C_1^*})(\nu A_1^R+\nu_B A_3^R+\nu A_{14}^R)\\&\quad-(m_{C_0}+m_{C_1^*})(\nu A_9^R+\nu_B A_{11}^R)\,,\\b_1^R&=A_8^R+\Delta A_{16}^R+A_{18}^R\\&\quad+(m_{C_0}+m_{C_1^*})(\nu A_{10}^R+\nu_B A_{12}^R)\,,\\b_2^R&=A_7^R+\Delta A_{14}^R\\&\quad+(m_{C_0}+m_{C_1^*})(\nu A_9^R+\nu_B A_{11}^R)\,.\end{aligned} \tag{B10}$$

Similarly we can write the pole-term contributions in terms of A_j:

$$\begin{aligned}a_1^B&=(m_{C_0}+m_{C_1^*})(\nu_B A_4^B+\nu A_{16}^B-A_{18}^B)\\&\quad-(m_{C_0}+m_{C_1^*})^2(\nu A_{10}^B+\nu_B A_{12}^B)\,,\\a_2^B&=(m_{C_0}+m_{C_1^*})(\nu_B A_3^B+\nu A_{14}^B)\,,\\b_1^B&=(m_{C_0}+m_{C_1^*})(\nu A_{10}^B+\nu_B A_{12}^B)+\Delta A_{16}^B+A_{18}^B\,,\\b_2^B&=A_7^B+\Delta A_{14}^B.\end{aligned} \tag{B11}$$

Evidently, with the assumption that the a_j^R do not possess any kinematical singularity, the following consistency condition can be obtained for the process $\pi C_0\to\pi C_1^*$ in the "soft-pion" limit":

$$a_2(\nu\to 0,\nu_B\to 0,K^2\to 0)=0\,. \tag{B12}$$

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