Path-integral quantization of a Dirac string Lagrangian in the $A^0 = 0$ gauge

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It is shown that the path-integral quantization of a magnetic-monopole string Lagrangian can be carried out in the $A^0 = 0$ gauge without imposing constraints or fixing the gauge completely. Longitudinal modes and the string variables associated with the gauge freedom are not eliminated, but the quantum fluctuations of these variables are integrated out in the Feynman path integral, and the electric Coulomb interaction potential and the charge-monopole interaction potential are obtained as effective potentials. The constraints are imposed on the state vectors. It is shown how to construct such state vectors obeying constraints at all times by using the path integral.

I. INTRODUCTION

Path-integral quantization for theories with first-class constraints is usually done following the method of Faddeev and Popov.[1] In this method one imposes the first-class constraints on the phase-space path integral and also fixes a corresponding gauge. However, it was shown by Gribov[2] that in non-Abelian theories there are ambiguities in gauge fixing. Several authors have tried to overcome this problem.[3] One such method was successfully used by Chang[4] for the case of ordinary electrodynamics with point electric charges. In this method the $A_0 = 0$ gauge was used but the Coulomb gauge $\vec{\nabla}\cdot\vec{A} = 0$ was not imposed on the path integral. Longitudinal modes associated with the latter gauge conditions were integrated out and the Coulomb interaction between electric charges was obtained as an effective potential.

In the present paper we will use the above method for another Lagrangian. The Lagrangian contains massive strings in addition to point electric charges and vector fields. In the limit when the strings become massless, we have effectively the Dirac Lagrangian[5] which describes a system of electric charges and magnetic monopoles. As in Ref. 4 we use the $A_0 = 0$ gauge but we do not impose the gauge $\vec{\nabla}\cdot\vec{A} = 0$. We obtain the Coulomb interaction between the electric charges as before. We then investigate the limit in which the strings become massless. In this limit the string variables become gauge variables. However, in the path integral, we do not eliminate these gauge variables. The interaction between monopoles and electric charges is obtained as an effective potential after integrating out the string variables.

The content of this article is organized as follows: In Sec. II, we discuss the classical dynamics and find the Schrödinger state vectors satisfying the constraint equation. In Sec. III, we discuss the path-integral quantization. We carry out the momenta integrations and the integrations with respect to the longitudinal mode variables associated with the gauge freedom. We also discuss the time evolution of the Schrödinger state vectors that satisfy the constraint equations initially. In Sec. IV, we consider the limit in which the mass of the string becomes zero. We integrate the string variables associated with the gauge freedom in this limit and find the time evolution of the state vectors satisfying all the constraints including the ones that appear in this limit. This gives us the effective action. Section V is devoted to some concluding remarks. In Appendix A we introduce new variables and write the action in a form convenient for evaluating the path integral. Schrödinger state vectors satisfying all the constraints are derived in Appendix B.

II. LAGRANGIAN AND STATE VECTORS SATISFYING IMPLIED CONSTRAINTS

We consider the following Lagrangian:

$$L = -\tfrac{1}{4}\int F_{\mu\nu}F^{\mu\nu}d^3x + \tfrac{1}{2}\sum_a m_a \dot{\vec{r}}_a^{\,2} + \tfrac{1}{2}\sum_b \int \rho_b(\sigma)\dot{\vec{y}}_b(\sigma,\tau)^2 d\sigma - \sum_a e_a A_\mu \dot{r}_a^\mu , \tag{2.1a}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \tfrac{1}{2}\epsilon_{\mu\nu\lambda\sigma}\sum_b g_b \int (\dot{y}_b^\lambda y_b'^\sigma - \dot{y}_b^\sigma y_b'^\lambda)[\epsilon(\sigma)/2]\delta^4(x-y)\,d\sigma\, d\tau , \tag{2.1b}$$

and $y_b(\sigma,\tau)$ are the coordinates of the sheet spanned by the string, $\dot{y}_b$ and y_b' are the derivatives with respect to the timelike parameter τ and spacelike parameter σ, respectively. $r_a(t)$ are the coordinates of the electric charges. m_a and e_a denote the masses and the magnitudes of the electric charges. g_b are the monopole charges and $\rho_b(\sigma)$ is the mass density of the string $y_b(\sigma)$ such that

$$\rho_b(\sigma) > 0 \quad \text{for all } \sigma \text{ and } b\,. \tag{2.2}$$

Notice that apart from the fact that in the Lagrangian (2.1a), electric charges and strings are nonrelativistic, L differs from the usual Dirac Lagrangian[5,6] in that we have introduced kinetic energy to string variables. In Sec. IV we will consider the interesting limit in which $\rho_b(\sigma)=m_b\delta(\sigma)$. In this limit L goes over to the usual nonrelativistic Dirac string Lagrangian. However, in this and the next sections we will assume (2.2). The reason for this is twofold: First, the path integral over the string variables becomes well-defined, and secondly, we will avoid the constraints corresponding to the string variables and thereby we will avoid the problem of imposing these constraints on the path integral, as we shall see in Sec. III.

For the development of the Hamiltonian formalism, it is convenient to identify τ with real time t and we will also set $y^0(\sigma,t)=r^0(t)=t$. To obtain the Hamiltonian we calculate the canonical momenta $\pi_\mu(x)$, $\eta_{bi}(\sigma,t)$, and $p_{ai}(t)$ corresponding to the variables $A^\mu(x)$, $y_b^i(\sigma,t)$, and $r_a^i(t)$. From L we obtain

$$\pi_\mu(x)=-F_{0\mu}(x)\,, \tag{2.3a}$$

$$\vec{\eta}_b(\sigma)=-g_b\frac{\epsilon(\sigma)}{2}\vec{y}_b'\times\vec{\pi}(\vec{y}_b)+\rho_b(\sigma)\dot{\vec{y}}_b(\sigma)\,, \tag{2.3b}$$

$$\vec{p}_a=m_a\dot{\vec{r}}_a+e_a\vec{A}(\vec{r}_a)\,. \tag{2.3c}$$

The only primary constraint is

$$\pi_0(x)=0\,. \tag{2.4a}$$

Because of (2.2) we do not get any constraints from (2.3b). However, as was pointed out earlier, in the limit when $\rho_b(\sigma)=m_b\delta(\sigma)$,

$$\vec{X}_b(\sigma)\equiv\vec{\eta}_b(\sigma)+g_b\frac{\epsilon(\sigma)}{2}\vec{y}_b'\times\vec{\pi}(\vec{y}_b)=0 \quad \text{for } \sigma\neq 0 \tag{2.4b}$$

also become primary constraints.[6] We will take care of constraints (2.4b) in Sec. IV when we take this limit.

The Hamiltonian then becomes

$$\begin{aligned}H&=\sum_a\vec{p}_a\cdot\dot{\vec{r}}_a+\sum_b\int\vec{\eta}_b(\sigma)\cdot\dot{\vec{y}}_b(\sigma)d\sigma+\int\vec{\pi}\cdot\dot{\vec{A}}d^3x-L\\&=\int d^3x\left[\tfrac{1}{4}F_{ij}(x)^2+\tfrac{1}{2}\vec{\pi}(x)^2+\left(\vec{\nabla}\cdot\vec{\pi}+\sum_a e_a\delta^3(\vec{x}-\vec{r}_a)\right)A_0(x)\right]\\&\quad+\sum_a\frac{1}{2m_a}[\vec{p}_a-e_a\vec{A}(\vec{r}_a)]^2+\sum_b\int\frac{1}{2\rho_b(\sigma)}\left(\vec{\eta}_b(\sigma)+g_b\frac{\epsilon(\sigma)}{2}\vec{y}_b'\times\vec{\pi}(\vec{y}_b)\right)^2d\sigma+\int v(x)\pi^0(x)d^3x\,,\end{aligned} \tag{2.5}$$

where $v(x)$ is the Lagrange multiplier corresponding to the constraint (2.4a). When we require that $\pi_0=0$ be preserved in time, i.e., its Poisson brackets with H vanishes weakly, we get the following secondary constraint:

$$\vec{\nabla}\cdot\vec{\pi}+\sum_a e_a\delta^3(\vec{x}-\vec{r}_a)=0\,, \tag{2.6}$$

Both the constraints (2.4a) and (2.6) are first class. Moreover, one can eliminate A^0 and π^0 from the theory without changing the dynamics in any way. This can be seen by the fact that $\dot{A}^0(x)=[H,A^0(x)]_{\text{PB}}=v(x)$ is completely arbitrary. Therefore, in the following we will use the $A_0=0$ gauge.

We will impose the first-class constraints on the state vectors but not on the path integral. In other words, in evaluating the path integral we will not use the first-class constraint equation but once the path integral is evaluated, we will use it to find the time evolution of an initial state vector which satisfies the constraints. For consistency, the final state vector must also satisfy these constraints. In view of this, we will find the subspace $\mathfrak{M}$ of the Hilbert space $\mathcal{H}$ of state vectors such that any state vector $\psi\in\mathfrak{M}$ satisfies the constraint (2.6), i.e.,

$$\left[\vec{\nabla}\cdot\vec{\pi}+\sum_a e_a\delta^3(\vec{x}-\vec{r}_a)\right]\psi=0\,. \tag{2.7}$$

To find this subspace $\mathfrak{M}$ we find a differential operator corresponding to $[\vec{\nabla}\cdot\vec{\pi}+\sum_a e_a\delta^3(\vec{x}-\vec{r}_a)]$ as in the usual canonical procedure. For this it is convenient to go over to the normal-mode coordinates defined by expanding the fields $\vec{A}(x)$ in a large cubic box of volume $\Omega=(2L)^3$

$$\vec{A}(\vec{x},t)\equiv\sum_{\vec{k}\lambda}\vec{q}_{\vec{k}\lambda}(t)\phi_{\vec{k}\lambda}(\vec{x})\,, \tag{2.8}$$

where

$$\phi_{\vec{k}\lambda}(\vec{x})=\begin{cases}\frac{1}{\sqrt{2\Omega}}[(1+\lambda)\cos\vec{k}\cdot x+(1-\lambda)\sin\vec{k}\cdot x] \\ \qquad \text{for } \vec{k}\neq 0,\ \lambda=\pm 1 \\ \frac{1}{\sqrt{\Omega}} \text{ for } \vec{k}=0,\ \lambda=0,\end{cases}$$

$$k\in\left\{\pm\frac{n\pi}{L},\pm\frac{m\pi}{L},\frac{l\pi}{L}: l,m,n,=0,1,2,3,\dots\right\}.$$

For $\vec{k}\neq 0$ one can decompose $\vec{q}_{k\lambda}$ into the longitudinal- and transverse-mode coordinates $q^L_{\vec{k}\lambda}$ and $\vec{q}^{\,T}_{\vec{k}\lambda}$, respectively, as follows:

$$\vec{q}_{\vec{k}\lambda}=\hat{k}q^L_{\vec{k}\lambda}+\vec{q}^{\,T}_{\vec{k}\lambda},\quad \hat{k}\cdot\vec{q}^{\,T}_{\vec{k}\lambda}=0\,.$$

In the Schrödinger picture the canonical momentum $\vec{\pi}(x)$ in terms of normal-mode coordinates then becomes

$$\vec{\pi}(x)=\sum_{\vec{k}\neq 0}\phi_{\vec{k}\lambda}(\vec{x})\left(\hat{k}\frac{\partial}{\partial q^L_{\vec{k}\lambda}}+\frac{\partial}{\partial\vec{q}^{\,T}_{\vec{k}\lambda}}\right)+\phi_{00}(\vec{x})\frac{\partial}{\partial\vec{q}_{00}}\,. \quad (2.9)$$

Using (2.9) in (2.7), one obtains the solution[4]

$$\psi(\{q^L_{\vec{k}\lambda}\},\xi)=f(\{q^L_{\vec{k}\lambda}\},\{\vec{r}_a\})\tilde{\psi}(\xi)\,, \quad (2.10a)$$

where ξ symbolically denotes all the variables $\vec{q}^{\,T}_{\vec{k}\lambda}$, $\vec{q}_{00}$, $\vec{y}_b(\sigma)$, and $\vec{r}_a$, and

$$f(\{q^L_{\vec{k}\lambda}\},\{\vec{r}_a\})=\exp\left[\frac{i}{\hbar}\sum_{\substack{\vec{k}\neq 0\\ \lambda,a}}\frac{\lambda}{k}e_a q^L_{\vec{k}\lambda}\phi_{\vec{k}-\lambda}(\vec{r}_a)\right], \quad (2.10b)$$

and $\tilde{\psi}$ is an arbitrary function of the indicated variables. Equations (2.10a), (2.10b) define the subspace $\mathfrak{M}$. In the next section we will evaluate the path integral partially in order to investigate the time evolution of a state vector $\psi\in\mathfrak{M}$. For this it will be sufficient to carry out the integration with respect to all the canonical momenta and the $q^L_{\vec{k}\lambda}$.

III. PATH-INTEGRAL QUANTIZATION

We write down the usual phase-space path integral.[7] The functional integration variables are $\vec{A}(x)$, $\vec{\pi}(x)$, $\vec{y}_b(\sigma,t)$, $\vec{r}_a(t)$, $\vec{\eta}_b(\sigma,t)$, and $\vec{p}_a(t)$. The path integral is

$$K(t_2,t_1)\equiv\prod_{\vec{x}}\int_{\vec{A}(\vec{x},t_1)}^{\vec{A}(\vec{x},t_2)}d\vec{A}(\vec{x})\int_{-\infty}^{\infty}d\vec{\pi}(\vec{x})\prod_a\int_{\vec{r}_a(t_1)}^{\vec{r}_a(t_2)}d\vec{r}_a\int_{-\infty}^{\infty}d\vec{p}_a\prod_{b,\sigma}\int_{\vec{y}_b(\sigma,t_1)}^{\vec{y}_b(\sigma,t_2)}d\vec{y}_b(\sigma)\int_{-\infty}^{\infty}d\vec{\eta}_b(\sigma)$$
$$\times\exp\left[\frac{i}{\hbar}\int_{t_1}^{t_2}dt\left(\sum_a\vec{p}_a\cdot\dot{\vec{r}}_a+\sum_b\int\vec{\eta}_b\cdot\dot{\vec{y}}_b d\sigma+\int\vec{\pi}\cdot\dot{\vec{A}}d^3x-H\right)\right], \quad (3.1)$$

where H is given by (2.5) with $A_0=\pi_0=0$. Notice that we have not included the Lagrange multiplier term corresponding to the constraint $[\vec{\nabla}\cdot\vec{\pi}+\sum_a e_a\delta^3(\vec{x}-\vec{r}_a)]$ in the Hamiltonian H. In this respect our approach is different from that of Faddeev and Popov.[1] In their approach one would introduce the Lagrange multiplier with the above constraint in the path integral and would fix the corresponding gauge ($\vec{\nabla}\cdot\vec{A}=0$). For the Abelian problem under discussion, the two procedures will yield the same results. However, in view of the well-known Gribov ambiguities in the non-Abelian theories, we would like to pursue our procedure of not fixing the gauge completely.

A. Integration over the momentum variables

All the momenta integrations are just Gaussian integrals and the result after carrying out these integrations is

$$K(t_2,t_1)=\prod_{\vec{x}}\int_{\vec{A}(\vec{x},t_1)}^{\vec{A}(\vec{x},t_2)}\mathfrak{D}\vec{A}(\vec{x})\prod_a\int_{\vec{r}_a(t_1)}^{\vec{r}_a(t_2)}\mathfrak{D}\vec{r}_a\prod_{b,\sigma}\int_{\vec{y}_b(\sigma,t_1)}^{\vec{y}_b(\sigma,t_2)}\mathfrak{D}\vec{y}_b(\sigma,t)\exp\left(\frac{i}{\hbar}S_{21}\right), \quad (3.2a)$$

where S_{21} is the action $\int_{t_1}^{t_2}Ldt$,

$$L=\int d^3x\left[-\tfrac{1}{4}F_{ij}{}^2+\tfrac{1}{2}\left(\dot{\vec{A}}-\sum_b g_b\int\dot{\vec{y}}_b\times\vec{y}_b'\delta^3(\vec{x}-\vec{y}_b)\frac{\epsilon(\sigma)}{2}d\sigma\right)^2\right]+\sum_a e_a\vec{A}(\vec{r}_a)\cdot\dot{\vec{r}}_a+\tfrac{1}{2}\sum_a m_a\dot{\vec{r}}_a{}^2+\sum_b\tfrac{1}{2}\int\rho_b\dot{\vec{y}}_b{}^2d\sigma\,. \quad (3.2b)$$

B. Longitudinal-mode path integration

To carry out the integration with respect to the longitudinal part of $\vec{A}(x)$, it is convenient to go over to the normal-mode coordinates defined in Eq. (2.8). To write the integral in a convenient form, we define the following variables:

$$Q^L_{\vec{k}\lambda}\equiv q^L_{\vec{k}\lambda}+B^L_{\vec{k}\lambda}\,, \quad (3.3a)$$

$$B^L_{\vec{k}\lambda}\equiv-\int_{t_1}^{t_2}d\tau\sum_b g_b\int\dot{\vec{y}}_b(\sigma,\tau)\times\vec{y}_b'(\sigma,\tau)\cdot\hat{k}\times\phi_{\vec{k}\lambda}(\vec{y}_b(\sigma,\tau))\frac{\epsilon(\sigma)}{2}d\sigma\,, \quad (3.3b)$$

$$\vec{Q}^T_{\vec{k}\lambda}\equiv\vec{q}^{\,T}_{\vec{k}\lambda}+\sum_b g_b\frac{\lambda}{k}\int\vec{y}_b'\times\hat{k}\phi_{\vec{k}-\lambda}(\vec{y}_b)\frac{\epsilon(\sigma)}{2}d\sigma\,, \quad (3.3c)$$

$$\vec{Q}_{00}\equiv\vec{q}_{00}+\tfrac{1}{2}\sum_b g_b\phi_{00}\int\vec{y}_b'\times\vec{y}_b\frac{\epsilon(\sigma)}{2}d\sigma\,, \quad (3.3d)$$

$$\vec{C}^T_{\vec{k}\lambda} \equiv \frac{\lambda}{k}\sum_b g_b \dot{\vec{z}}_b \times \hat{k}\phi_{\vec{k}-\lambda}(\vec{z}_b), \quad \vec{z}_b = \vec{y}_b(0,t), \tag{3.3e}$$

$$J^L_{\vec{k}\lambda} \equiv \sum_a e_a \dot{\vec{r}}_a \cdot \hat{k}\phi_{\vec{k}\lambda}(\vec{r}_a), \tag{3.3f}$$

$$\vec{J}_{\vec{k}\lambda} \equiv \sum_a e_a \dot{\vec{r}}_a \phi_{\vec{k}\lambda}(\vec{r}_a). \tag{3.3g}$$

Using these variables the path integral (3.2a) can be written as (for detailed proof see Appendix A)

$$K(t_2,t_1) = \sum_{\substack{k\neq 0\\ \lambda}} \int_{q^L_{\vec{k}\lambda}(t_1)}^{q^L_{\vec{k}\lambda}(t_2)} \mathfrak{D}q^L_{\vec{k}\lambda} \int_{\xi(t_1)}^{\xi(t_2)} \mathfrak{D}\xi \exp\left(\frac{i}{\hbar}S_{21}\right). \tag{3.4}$$

Here by $\int \mathfrak{D}\xi$ we mean integration over all the variables $\vec{q}^T_{\vec{k}\lambda}$, $\vec{q}_{00}$, $\vec{y}_b(\sigma)$, $\vec{r}_a$. S_{21} in (3.4) can be written in the form of a sum of terms,

$$S_{21} = S^0_{21} + S^T_{21} + S^L_{21} + S^y_{21} + S^z_{21} + S^e_{21}, \tag{3.5}$$

where S^0_{21} contains all $\vec{q}_{00}$ terms and is given by

$$S^0_{21} = \int_{t_1}^{t_2} dt \left[\tfrac{1}{2}\dot{\vec{Q}}_{00}{}^2 + \tfrac{1}{2}\dot{\vec{Q}}_{00}\sum_b g_b \phi_{00}\dot{\vec{z}}_b \times \vec{z}_b + \sum_a e_a \dot{\vec{r}}_a \cdot \dot{\vec{Q}}_{00}\phi_{00}\right]. \tag{3.6a}$$

S^L_{21} contains all $q^L_{\vec{k}\lambda}$ terms and is given by

$$S^L_{21} = \int_{t_1}^{t_2} dt \sum_{\substack{\vec{k}\neq 0\\ \lambda}} \left[\tfrac{1}{2}(\dot{Q}^L_{\vec{k}\lambda})^2 + J^L_{\vec{k}\lambda}Q^L_{\vec{k}\lambda}\right] - \sum_{\substack{\vec{k}\neq 0\\ \lambda}} B^L_{\vec{k}\lambda}(t_2)\sum_a e_a \frac{\lambda}{k}\phi_{\vec{k}-\lambda}(\vec{r}_a(t_2)). \tag{3.6b}$$

S^T_{21} contains all $\vec{q}^T_{\vec{k}\lambda}$ terms and is given by

$$S^T_{21} = \int_{t_1}^{t_2} dt \sum_{\substack{\vec{k}\neq 0\\ \lambda}} \left[\tfrac{1}{2}(\dot{\vec{Q}}^T_{\vec{k}\lambda})^2 + \dot{\vec{Q}}^T_{\vec{k}\lambda}\cdot \vec{C}^T_{\vec{k}\lambda} - \tfrac{1}{2}k^2(\vec{Q}^T_{\vec{k}\lambda})^2 + \vec{J}_{\vec{k}\lambda}\cdot \vec{Q}^T_{\vec{k}\lambda}\right]. \tag{3.6c}$$

S^y_{21} contains all the $\vec{y}_b(\sigma)$, $\sigma \neq 0$ terms that are not included in S^0_{21}, S^L_{21}, and S^T_{21}:

$$S^y_{21} = \int_{t_1}^{t_2} dt \left[\tfrac{1}{2}\sum_b \rho_b \dot{\vec{y}}_b{}^2 d\sigma + \sum_{b,a}\frac{e_a g_b}{4\pi}\int \frac{\dot{\vec{Y}}_{ba}\times \vec{Y}'_{ba}\cdot \vec{Y}_{ba}}{|\vec{Y}_{ba}|^3}\frac{\epsilon(\sigma)}{2}d\sigma\right], \tag{3.6d}$$

with

$$\vec{Y}_{ba} \equiv \vec{y}_b(\sigma) - \vec{r}_a. \tag{3.6e}$$

Finally S^e_{21} and S^z_{21} contain the remaining $\vec{r}_a$ and $\vec{z}_b$ terms, respectively:

$$S^e_{21} = \int_{t_1}^{t_2} dt \tfrac{1}{2}\sum_a m_a \dot{\vec{r}}_a{}^2, \tag{3.6f}$$

$$S^z_{21} = \int_{t_1}^{t_2} dt \left[-\sum_{b>b'}\frac{g_b g_{b'}}{4\pi|\vec{Z}_b - \vec{Z}_{b'}|} + \tfrac{1}{2}\sum_{\substack{\vec{k}\neq 0\\ \lambda}} \vec{C}^{T\,2}_{\vec{k}\lambda}\right]. \tag{3.6g}$$

In the last expression we have dropped the time-independent infinite Coulomb self-energy terms of monopoles.

Now we carry out the integration with respect to the longitudinal-mode variables $q^L_{\vec{k}\lambda}$. Since only S^L_{21} contains $q^L_{\vec{k}\lambda}$, we can take out all the other terms of the S_{21} outside this integration. S^L_{21} is the action corresponding to a zero-frequency-mode particle subject to an external force term $J^L_{\vec{k}\lambda}$ and the path integral for this case has been worked out, for example, in Ref. 4. Making use of the result from Ref. 4 one can write

$$\prod_{\substack{k\neq 0\\ \lambda}} \int_{q^L_{\vec{k}\lambda}(t_1)}^{q^L_{\vec{k}\lambda}(t_2)} \mathfrak{D}q^L_{\vec{k}\lambda}\exp\left(\frac{i}{\hbar}S^L_{21}\right) = \left\{\prod_{\substack{\vec{k}\neq 0\\ \lambda}} \frac{1}{[2\pi i\hbar(t_2-t_1)]^{1/2}}\exp\left[\frac{i}{\hbar}\frac{\left(Q^L_{\vec{k}\lambda}(t_2) - Q^L_{\vec{k}\lambda}(t_1) - \sum_a e_a \frac{\lambda}{k}\int_{t_1}^{t_2} dt\,\phi_{\vec{k}-\lambda}(\vec{r}_a)\right)^2}{2(t_2-t_1)}\right]\right\}$$
$$\times f(\{Q^L_{\vec{k}\lambda}(t_2)\},\{\vec{r}_a(t_2)\}) f^*(\{Q^L_{\vec{k}\lambda}(t_1)\},\{\vec{r}_a(t_1)\}) \exp\left(\frac{i}{\hbar}\int_{t_1}^{t_2} dt \sum_{a,a'}\frac{-e_a e_{a'}}{4\pi|\vec{r}_a - \vec{r}_{a'}|}\right)$$
$$\times \exp\left[-\frac{i}{\hbar}\sum_{\substack{k\neq 0\\ \lambda}} B^L_{\vec{k}\lambda}(t_2)\sum_a e_a \frac{\lambda}{k}\phi_{\vec{k}-\lambda}(\vec{r}_a(t_2))\right]. \tag{3.7}$$

Making use of (3.3a) and (3.7) in (3.4), we get for the path integral

$$K(t_2,t_1) = \int_{\xi(t_1)}^{\xi(t_2)} \mathfrak{D}\xi \exp\left[\frac{i}{\hbar}(S^T_{21} + S^0_{21} + S^y_{21} + S^z_{21} + \tilde{S}^e_{21})\right]$$
$$\times\left\{\prod_{\substack{k\neq 0\\ \lambda}} \frac{1}{[2\pi i\hbar(t_2-t_1)]^{1/2}}\exp\left[\frac{i}{\hbar}\left(q^L_{\vec{k}\lambda}(t_2) + B^L_{\vec{k}\lambda}(t_2) - q^L_{\vec{k}\lambda}(t_1) - \sum_a e_a \frac{\lambda}{k}\int_{t_1}^{t_2} dt\,\phi_{\vec{k}-\lambda}(\vec{r}_a)\right)^2\right]\right\}$$
$$\times f(\{q^L_{\vec{k}\lambda}(t_2)\},\{\vec{r}_a(t_2)\}) f^*(\{q^L_{\vec{k}\lambda}(t_1)\},\{\vec{r}_a(t_1)\}), \tag{3.8a}$$

where

$$\tilde{S}^e_{21}=S^e_{21}-\int_{t_1}^{t_2}dt\sum_{a>a'}\frac{e_a e_{a'}}{4\pi|\vec{r}_a-\vec{r}_{a'}|}. \tag{3.8b}$$

The second term in (3.8b) is the electric Coulomb interaction term where we have dropped the time-independent infinite self-energy terms.

C. Time evoluation of state vectors belonging to $\mathfrak{M}$

Time evolution of a state vector $\psi(\{q^L_{\vec{k}\lambda}(t_1)\},\xi(t_1))$ is given by

$$\psi(\{q^L_{\vec{k}\lambda}(t_2)\},\xi(t_2))=\int d\xi(t_1)\sum_{\substack{\vec{k}\neq 0\\ \lambda}}\int dq^L_{\vec{k}\lambda}(t_1)K(t_2,t_1)\psi(\{q^L_{\vec{k}\lambda}(t_1)\},\xi(t_1)).$$

If $\psi(\{q^L_{\vec{k}\lambda}(t_1)\},\xi(t_1))\in\mathfrak{M}$, then from (2.10a), (2.10b) we see that $\psi(\{q^L_{\vec{k}\lambda}(t_1)\},\xi(t_1))$ contains a factor $f(\{q^L_{\vec{k}\lambda}(t_1)\},\{\vec{r}_a(t_1)\})$, which cancels with the term $f^*(\{q^L_{\vec{k}\lambda}(t_1)\},\{\vec{r}_a(t_1)\})$ in $K(t_2,t_1)$, and the $q^L_{\vec{k}\lambda}(t_1)$ integration becomes a Gaussian integral whose value is one. Thus

$$\psi(\{q^L_{\vec{k}\lambda}(t_1)\},\xi(t_2))=f(\{q^L_{\vec{k}\lambda}(t_2)\},\{\vec{r}_a(t_2)\})\times\tilde{\psi}(\xi(t_2)), \tag{3.9a}$$

where

$$\tilde{\psi}(\xi(t_2))=\int d\xi(t_1)\tilde{K}(t_2,t_1)\tilde{\psi}(\xi(t_1)) \tag{3.9b}$$

and

$$\tilde{K}(t_2,t_1)=\int_{\xi(t_1)}^{\xi(t_2)}\mathfrak{D}\xi\exp\left[\frac{i}{\hbar}(S^T_{21}+S^0_{21}+S^y_{21}+S^z_{21}+\tilde{S}^e_{21})\right]. \tag{3.9c}$$

$\tilde{K}(t_2,t_1)$ is the effective Feynman path integral, which does not contain any longitudinal-mode variables. Equation (3.9a) implies that the Schrödinger wave function, which initially satisfied the constraint Eq. (2.9), will continue to do so at all later times. The Coulomb potential [in (3.8b)] between electric charges is obtained as an effective potential by integrating out all the longitudinal-mode variables.

IV. PATH INTEGRAL AND THE EFFECTIVE ACTION IN THE LIMIT OF MASSLESS STRINGS

In Sec. II we noted that in the limit

$$\rho_b(\sigma)=m_b\delta(\sigma), \tag{4.1}$$

there are additional primary constraints (2.4b). It can be verified that these primary constraints do not give rise to any secondary constraints, when the condition that these constraints be preserved in time is required. It can also be verified that the set of constraints (2.4a), (2.4b), and (2.6) form a set of first-class constraints, that is, the Poisson brackets among these constraints vanish weakly, provided the electric charges do not lie on the strings. We will assume that the latter condition holds. As before, we will impose these additional constraints on the initial state vectors and then, using the effective path integral $\tilde{K}(t_2,t_1)$, we will find out how these initial state vectors evolve in time.

We also note that a naive substitution of (4.1) in $\tilde{K}(t_2,t_1)$ will give rise to infinities. However, we will follow the consistent prescription of taking the limit (4.1) only in the effective path integral, that will be obtained after finding the time evolution of an initial state vector, which satisfies all the constraints. In this way the infinities will not appear. To do this we will carry out the integration in $\tilde{K}(t_2,t_1)$ one step further.

A. Path integration over the string variables in $\tilde{K}(t_2,t_1)$

To carry out this path integration it will be convenient to consider the set of variables $(\vec{Q}^T_{\vec{k}\lambda},\vec{Q}_{00},\vec{y}_b(\sigma),\vec{r}_a)$ as the independent variables instead of the original independent variables $(\vec{q}^T_{\vec{k}\lambda},\vec{q}_{00},\vec{y}_b(\sigma),\vec{r}_a)$. The path integral (3.9c) can be expressed in terms of these new independent variables as follows:

$$\tilde{K}(t_2,t_1)=\int_{\chi_1}^{\chi_2}\mathfrak{D}\chi\prod_{\substack{\sigma\neq 0\\ b}}\int_{\vec{y}_b(\sigma,t_1)}^{\vec{y}_b(\sigma,t_2)}\mathfrak{D}\vec{y}_b(\sigma)\exp\left[\frac{i}{\hbar}(S^T_{21}+S^0_{21}+S^y_{21}+\tilde{S}^e_{21}+S^z_{21})\right], \tag{4.2}$$

where, for brevity, we denote by χ all the variables $\vec{Q}^T_{\vec{k}\lambda}$, $\vec{Q}_{00}$, $\vec{z}_b$, $\vec{r}_a$ and by $\int_{\chi_1}^{\chi_2}\mathfrak{D}\chi$ we mean

$$\prod_{\substack{k\neq 0\\ \lambda}}\int_{\vec{Q}^T_{\vec{k}\lambda}(t_1)}^{\vec{Q}^T_{\vec{k}\lambda}(t_2)}\mathfrak{D}\vec{Q}^T_{\vec{k}\lambda}\int_{\vec{Q}_{00}(t_1)}^{\vec{Q}_{00}(t_2)}\mathfrak{D}\vec{Q}_{00}\prod_b\int_{\vec{z}_b(t_1)}^{\vec{z}_b(t_2)}\mathfrak{D}\vec{z}_b\prod_a\int_{\vec{r}_a(t_1)}^{\vec{r}_a(t_2)}\mathfrak{D}\vec{r}_a.$$

S_{21}^T, S_{21}^0, S_{21}^z, and $\tilde{S}_{21}^e$ do not depend on $\vec{y}_b(\sigma)$, except for $\sigma=0$. Therefore, these terms can be taken out of the $\Pi_{\sigma\neq 0}\int \mathfrak{D}\vec{y}_b(\sigma)$ integrations. Only S_{21}^y depends on $\vec{y}_b(\sigma)$ $(\sigma\neq 0)$ as can be seen from (3.6d). We first consider the last term in (3.6d). We note that

$$\Omega_{ba}^{\pm}(t_2,t_1)\equiv \pm\int_{t_1}^{t_2}dt\int_0^{\pm\infty}d\sigma\frac{\dot{\vec{Y}}_{ba}\times\vec{Y}'_{ab}}{|\vec{Y}_{ba}|^3}\cdot\vec{Y}_{ba} \qquad (4.3)$$

are the solid angles subtended at the origin, by the sheet spanned by the semistrings $\vec{Y}_{ba}(\sigma)$ $(\sigma=\pm|\sigma|)$, when they go from $\vec{Y}_{ba}(\sigma,t_1)$ to $\vec{Y}_{ba}(\sigma,t_2)$. These solid angles will depend on the boundary of these sheets [that is, on $\vec{Y}_{ba}(\sigma,t_1)$, $\vec{Y}_{ba}(\sigma,t_2)$ and the path of $\vec{Y}_{ba}(0,t)$], as well as on the number of loops the sheet makes around the origin. For example, if for $\sigma>0$, $\vec{Y}_{ba}(\sigma)$ while going from $\vec{Y}_{ba}(\sigma,t_1)$ to $\vec{Y}_{ba}(\sigma,t_2)$ loops around the origin once, then compared to the case when no loops are made, $\Omega_{ba}^{+}(t_2,t_1)$ will get an additional factor of 4π. However, since in the integrand in (4.2), $\Omega_{ba}^{\pm}(t_2,t_1)$ appears in the form

$$\exp\left(\frac{i}{\hbar}\sum_{b,a}\frac{e_a g_b}{8\pi}[\Omega_{ba}^{+}(t_2,t_1)-\Omega_{ba}^{-}(t_2,t_1)]\right),$$

the additional factor in $\Omega_{ba}^{+}(t_2,t_1)$ will modify the integrand in (4.5) by a phase

$$\exp\left(\frac{i}{\hbar}\frac{e_a g_b}{2}\right).$$

This means that if Schwinger quantization condition

$$e_a g_b = 2nh, \quad n=0,\pm 1,\pm 2,\ldots \qquad (4.4)$$

is assumed for all a and b, then this phase factor in the integrand in (4.2) will be just unity. In the following we will assume that the Schwinger quantization condition (4.4) holds for all a and b. Therefore, the term

$$\exp\left(\frac{i}{\hbar}\sum_{a,b}\frac{e_a g_b}{8\pi}[\Omega_{ba}^{+}(t_2,t_1)-\Omega_{ba}^{-}(t_2,t_1)]\right)$$

can be taken out of the $\Pi_{\sigma\neq 0,b}\int\mathfrak{D}\vec{y}_b(\sigma)$ integrations. The only remaining term in the integrand (4.2), which depends on the path of $\vec{y}_b(\sigma)$ $(\sigma\neq 0)$, is the kinetic energy term

$$\exp\left(\frac{i}{\hbar}\sum_b\int_{t_1}^{t_2}dt\int\tfrac{1}{2}\rho_b\dot{\vec{y}}_b^{\,2}d\sigma\right).$$

To carry out the integration we discretize the string variables as follows:

$$\sigma_n = n\Delta\sigma, \quad n=0,\pm 1,\pm 2,\ldots,$$
$$\rho_{bn}=\frac{1}{\Delta\sigma}\int_{\sigma_n-\Delta\sigma/2}^{\sigma_n+\Delta\sigma/2}d\sigma\rho_b, \qquad (4.5)$$

where $\Delta\sigma$ is a small real number. Using (4.5), and noting that the resulting integrals with respect to $\vec{y}_b(\sigma_n)$ $(n\neq 0)$ are just Gaussian integrals, we obtain the effective path integral $\tilde{K}(t_2,t_1)$,

$$\tilde{K}(t_2,t_1)=\left[\prod_{\substack{n\neq 0\\ b}}\left(\frac{\rho_{bn}\Delta\sigma}{2\pi i\hbar(t_2-t_1)}\right)^{1/2}\right]\exp\left(\frac{i}{\hbar}\sum_b\sum_{n\neq 0}\tfrac{1}{2}\rho_{bn}\Delta\sigma\frac{[\vec{y}_b(\sigma_n,t_2)-\vec{y}_b(\sigma_n,t_1)]^2}{t_2-t_1}\right)\bar{K}(t_2,t_1), \qquad (4.6a)$$

where

$$\bar{K}(t_2,t_1)=\int_{\chi(t_1)}^{\chi(t_2)}\mathfrak{D}\chi\exp\left[\frac{i}{\hbar}(S_{21}^T+S_{21}^0+\tilde{S}_{21}^z+S_{21}^{ze}+\tilde{S}_{21}^e)\right], \qquad (4.6b)$$

where

$$\tilde{S}_{21}^z=S_{21}^z+\Delta\sigma\int_{t_1}^{t_2}dt\sum_b\tfrac{1}{2}\rho_{b0}\dot{\vec{z}}_b^{\,2} \qquad (4.6c)$$

and

$$S_{21}^{ze}=\sum_{a,b}\frac{e_a g_b}{8\pi}[\Omega_{ba}^{+}(t_2,t_1)-\Omega_{ba}^{-}(t_2,t_1)]. \qquad (4.6d)$$

B. Time evolution of state vectors satisfying all the constraints

Let $\bar{\mathfrak{M}}$ be the subspace of state vectors satisfying both the constraint Eqs. (2.6) and (2.4b). Clearly $\bar{\mathfrak{M}}\subset\mathfrak{M}$. Any $\psi\in\bar{\mathfrak{M}}$ is given by (see Appendix B for proof)

$$\psi(\{q_{\vec{k}\lambda}^L\},\xi)=f(\{q_{\vec{k}\lambda}^L\},\{\vec{r}_a\})\tilde{\psi}(\xi), \qquad (4.7a)$$

where

$$\tilde{\psi}(\xi)=F(\{\vec{Y}_{ba}\})\bar{\psi}(\chi), \qquad (4.7b)$$

where χ denotes the variables $\vec{Q}_{\vec{k}\lambda}^T$, $\vec{Q}_{00}$, $\vec{r}_a$, and $\vec{z}_b$. $\bar{\psi}$ is an arbitrary function of these variables. $\vec{Q}_{\vec{k}\lambda}^T$, $\vec{Q}_{00}$, $\vec{Y}_{ba}$, and f are as defined in Eqs. (3.3c), (3.3d), (3.6e), and (2.10b), respectively. $F(\{\vec{Y}_{ba}\})$ is defined as follows:

$$F(\{\vec{Y}_{ba}\})=\exp\left(\frac{i}{\hbar}\sum_{a,b}\frac{e_a g_b}{8\pi}[\Omega^{+}(\vec{z}_b-\vec{r}_a+\vec{y}_b^{\,0},\vec{Y}_{ba}) - \Omega^{-}(\vec{z}_b-\vec{r}_a+\vec{y}_b^{\,0},\vec{Y}_{ba})]\right), \qquad (4.8)$$

where $\Omega^{\pm}(\vec{z}_b-\vec{r}_a+\vec{y}_b^{\,0},\vec{Y}_{ba})$ are the solid angles subtended at the origin by the sheet spanned by the semistrings $\vec{Y}_{ba}(\sigma)$ $(\sigma=\pm|\sigma|)$ in going from $\vec{z}_b-\vec{r}_a+\vec{y}_b^{\,0}(\sigma)$ to $\vec{Y}_{ba}(\sigma)$ along paths which without any loss of generality can be taken to be straight lines joining $\vec{z}_b-\vec{r}_a+\vec{y}_b^{\,0}(\sigma)$ and $\vec{Y}_{ba}(\sigma)$ for each σ. $\vec{y}_b^{\,0}(\sigma)$ is an arbitrary fixed string such that $\vec{y}_b^{\,0}(0)=0$.

Now $\psi\in\bar{\mathfrak{M}}$ is also contained in $\mathfrak{M}$. Hence as shown in Sec. III C, if at $t=t_1$ $\psi\in\bar{\mathfrak{M}}$, then ψ at $t=t_2$ is given by

$$\psi(\{q^L_{\vec{k}\lambda}(t_2)\},\xi(t_2)) = f(\{q^L_{\vec{k}\lambda}(t_2)\},\{\vec{r}_a(t_2)\})\tilde{\psi}(\xi(t_2)), \quad (4.9a)$$

where

$$\tilde{\psi}(\xi(t_2)) = \int dx(t_1)\,\overline{\psi}(\chi(t_1)) \times \prod_{\substack{n\neq 0\\ b}} \int d\vec{y}_b(\sigma_n,t_1)\tilde{K}(t_2,t_1)F(\{\vec{Y}_{ba}(t_1)\}). \quad (4.9b)$$

In obtaining (4.9b), we have used Eq. (4.7b) in Eq. (3.9b) and have also changed the variables from $(\vec{q}^L_{\vec{k}\lambda},\vec{q}_{00},\vec{y}_b,\vec{r}_a)$ to $(\vec{Q}^T_{\vec{k}\lambda},\vec{Q}_{00},\vec{y}_b,\vec{r}_a)$ in the right-hand side. From (4.6a)–(4.6d) one can see that $\tilde{K}(t_2,t_1)$ contains a factor $\exp[(i/\hbar)S^{ze}_{21}]$, which together with $F(\{\vec{Y}_{ba}(t_1)\})$, gives

$$F(\{\vec{Y}_{ba}(t_2)\})\exp\left(\frac{i}{\hbar}\sum_{a,b}\frac{e_a g_b}{8\pi}[\Omega^{+0}_{ba}(t_2,t_1)-\Omega^{-0}_{ba}(t_2,t_1)]\right).$$

$\Omega^{\pm 0}_{ba}(t_2,t_1)$ are the solid angles subtended at the origin by the sheets defined by $\vec{z}_b(t)-\vec{r}_a(t)+\vec{y}^0_b(\sigma)$, with $t\in[t_1,t_2]$ and $\sigma\in[0,\pm\infty]$. Since $\vec{y}^0_b(\sigma)$ are fixed strings, $\Omega^{\pm 0}_{ba}(t_2,t_1)$ depends only on the path of $\vec{Z}_b-\vec{r}_a$ and consequently terms involving $\Omega^{\pm 0}_{ba}(t_2,t_1)$ can be taken outside the $\Pi_{n\neq 0,\,b}\int d\vec{y}_b(\sigma_n,t_1)$ integrations. From (4.6a) it can be seen that the integrations with respect to $y_b(\sigma_n,t_1)$ $(n\neq 0)$ in (4.9b) are just Gaussian integrals and their values are just one. The final result is

$$\tilde{\psi}(\xi(t_2)) = F(\{\vec{Y}_{ba}(t_2)\})\overline{\psi}(\chi(t_2)), \quad (4.10a)$$

where

$$\overline{\psi}(\chi(t_2)) = \int d\chi(t_1)K_{\text{eff}}(t_2,t_1)\overline{\psi}(\chi(t_1)), \quad (4.10b)$$

$$K_{\text{eff}}(t_2,t_1) = \int_{\chi(t_1)}^{\chi(t_2)}\mathfrak{D}\chi\,\exp\left(\frac{i}{\hbar}S^{\text{eff}}_{21}\right), \quad (4.10c)$$

$$S^{\text{eff}}_{21} = S^0_{21}+S^T_{21}+\tilde{S}^e_{21}+\tilde{S}^z_{21}+\tilde{S}^{Ze}_{21}, \quad (4.10d)$$

$$\tilde{S}^{Ze}_{21} = \sum_{a,b}\frac{e_a g_b}{8\pi}[\Omega^{+0}_{ba}(t_2,t_1)-\Omega^{-0}_{ba}(t_2,t_1)] = \int_{t_1}^{t_2}dt\sum_{a,b}\frac{e_a g_b}{4\pi}(\dot{\vec{Z}}_b-\dot{\vec{r}}_a) \times\int d\sigma\frac{\epsilon(\sigma)}{2}\frac{\vec{y}^{0\prime}_b\times(\vec{Z}_b-\vec{r}_a+\vec{y}^0_b)}{|\vec{Z}_b-\vec{r}_a+\vec{y}^0_b|^3}. \quad (4.10e)$$

When we compare Eqs. (4.9a), (4.10a) with Eqs. (4.7a), (4.7b) we see that at t_2, $\psi\in\mathfrak{M}$. If we now take the limit (4.1), that is $\rho_b(\sigma)=m_b\delta(\sigma)$, then

$$\rho_{b0} = \frac{1}{\Delta\sigma}\int_{-\Delta\sigma/2}^{\Delta\sigma/2}m_b\delta(\sigma)d\sigma = \frac{m_b}{\Delta\sigma}.$$

In this limit, therefore, Eqs. (4.10a)–(4.10e) remain the same, except in Eq. (4.10d) $\tilde{S}^z_{21}$ will have to be replaced by $\overline{S}^Z_{21}$, where

$$\overline{S}^Z_{21} = \int_{t_1}^{t_2}dt\left[\sum_{\substack{\vec{k}\neq 0\\ \lambda}}\tfrac{1}{2}\dot{\vec{C}}^{T\,2}_{\vec{k}\lambda} - \sum_{b>b'}\frac{g_b g_{b'}}{4\pi|\vec{Z}_b-\vec{Z}_{b'}|} + \sum_b\tfrac{1}{2}m_b\dot{\vec{Z}}_b^{\,2}\right], \quad (4.11a)$$

and the effective action is given by

$$S^{\text{eff}}_{21} = S^0_{21}+S^T_{21}+\tilde{S}^e_{21}+\overline{S}^z_{21}+\tilde{S}^{Ze}_{21}. \quad (4.11b)$$

To compare the effective action (4.11b) obtained above, with the usual action for theories with monopole strings, we choose $\vec{y}^0_b(\sigma)=\hat{n}\sigma$, where $\hat{n}$ is a fixed unit vector. We also take the static monopole limit, $m_b\to\infty$, $\dot{\vec{Z}}_b=0$. In this limit $\vec{C}^T_{\vec{k}\lambda}\to 0$, and

$$\frac{g_b g_{b'}}{4\pi|\vec{Z}_b-\vec{Z}_{b'}|}$$

becomes a constant which can be set to zero. For simplicity, let us consider a single monopole situated at $Z_1=0$ and a single electric charge at $\vec{r}$. Further, if we ignore the terms involving $\vec{Q}^T_{\vec{k}\lambda}$ and $\vec{Q}_{00}$, that is, if we neglect the radiation effects, then the effective action (4.11b) becomes

$$S^{\text{eff}}_{21} = \int_{t_1}^{t_2}dt\left[\tfrac{1}{2}m\dot{\vec{r}}^{\,2} + \frac{eg}{4\pi}\frac{\dot{\vec{r}}\cdot(\hat{n}\times\vec{r})(\hat{n}\cdot\vec{r})}{[\vec{r}^{\,2}-(\hat{n}\cdot\vec{r})^2]}\right].$$

This is the usual action for a nonrelativistic electric charge in a static monopole field and has been studied in detail by Balachandran *et al.*[8]

V. CONCLUSION

In view of the Gribov ambiguities associated with gauge fixing in the non-Abelian theories, we pursued a different approach to quantize gauge theories. The main feature of this approach is that the gauge is not fixed completely and the first-class constraints are imposed only on the initial state vectors and not on the path integral. This approach was used by Chang[4] for the case of electromagnetic field with charged particles. In the present paper we have used the same method for a more complicated but Abelian Lagrangian involving monopole strings. The complication consisted in the fact that besides the Gauss's-law constraint we had additional constraints corresponding to the string variables. The former constraint was handled in the same way as was done in Ref. 4. For the latter constraints, we first introduced a kinetic energy term to the string variables and thereby removed these constraints from the path integral. However, these constraints were imposed on the initial state vectors, and it was shown that the path integral preserves these constraints on the state vectors under time evolution. After integrating out all the string variables, we obtained the expected effective action.

This approach seems to work for the Abelian gauge theories. Whether it can be extended to the non-Abelian gauge theories or not, is yet to be seen.

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APPENDIX A

In this appendix we will derive Eqs. (3.5), (3.6a)–(3.6g). The Lagrangian (3.2b) in terms of the normal-mode coordinates defined in Eq. (2.8) is

$$L=\sum_{\vec k,\lambda}\left[\tfrac{1}{2}\dot{\vec q}_{\vec k\lambda}^{\,2}-\tfrac{1}{2}k^2\vec q_{\vec k\lambda}^{\,T\,2}-\sum_b g_b\int\dot{\vec y}_b\times\vec y_b{}'\cdot\dot{\vec q}_{\vec k\lambda}\phi_{\vec k\lambda}(\vec y_b)\frac{\epsilon(\sigma)}{2}d\sigma+\sum_a e_a\dot{\vec r}_a\cdot\vec q_{\vec k\lambda}\phi_{\vec k\lambda}(\vec r_a)-\sum_b g_b\lambda\int\vec y_b{}'\cdot\vec k\times\vec q_{\vec k\lambda}^{\,T}\frac{\epsilon(\sigma)}{2}\phi_{\vec k,-\lambda}(\vec y_b)d\sigma\right]$$
$$+\int d^3x\left[\tfrac{1}{2}\left(\sum_b g_b\int\dot{\vec y}_b\times\vec y_b{}'\delta^3(\vec x-\vec y_b)\frac{\epsilon(\sigma)}{2}d\sigma\right)^2-\tfrac{1}{2}\left(\sum_b g_b\int\vec y_b{}'\delta^3(\vec x-\vec y_b)\frac{\epsilon(\sigma)}{2}d\sigma\right)^2\right]$$
$$+\tfrac{1}{2}\sum_a m_a\dot{\vec r}_a^{\,2}+\tfrac{1}{2}\sum_b\int\rho_b(\sigma)\dot{\vec y}_b^{\,2}d\sigma\,. \tag{A1}$$

Here $\int d\sigma$ is carried out over those values of σ for which $\vec y_b(\sigma)$ is in the volume Ω. We also fix the strings on the boundary of the volume Ω.

We note that in the limit $\Omega\to\infty$, replacing the sum by an integral over $\vec k$, we obtain

$$\sum_{\vec k\lambda}\phi_{\vec k\lambda}(\vec x)\phi_{\vec k\lambda}(\vec y)\xrightarrow[\Omega\to\infty]{}\delta^3(\vec x-\vec y)\,, \tag{A2a}$$

$$\sum_{\substack{k\neq 0\\ \lambda}}\frac{1}{k^2}\phi_{\vec k\lambda}(\vec x)\phi_{\vec k\lambda}(\vec y)\xrightarrow[\Omega\to\infty]{}-\frac{1}{\nabla_x^2}\delta^3(\vec x-\vec y)=\frac{1}{4\pi|\vec x-\vec y|}\,, \tag{A2b}$$

$$\sum_{\substack{\vec k\neq 0\\ \lambda}}\frac{\lambda}{k^2}\phi_{\vec k\lambda}(\vec x)\Phi_{\vec k-\lambda}(\vec y)\xrightarrow[\Omega\to\infty]{}-\nabla_x\frac{1}{\nabla_x^2}\delta^3(\vec x-\vec y)=-\frac{1}{4\pi}\frac{\vec x-\vec y}{|\vec x-\vec y|^3}\,. \tag{A2c}$$

We first consider the first term in $\int d^3x$ in Eq. (A1). After integrating with respect to $\vec x$ and using (A2a), one can write

$$\int d^3x\left[\sum_b g_b\int\dot{\vec y}_b\times\vec y_b{}'\delta^3(\vec x-\vec y_b)\frac{\epsilon(\sigma)}{2}d\sigma\right]^2=\sum_{\vec k\lambda}\left[\sum_b g_b\int d\sigma\frac{\epsilon(\sigma)}{2}\dot{\vec y}_b\times\vec y_b{}'\phi_{\vec k\lambda}(\vec y_b)\right]^2. \tag{A3}$$

Similarly

$$\int d^3x\left[\sum_b g_b\int\vec y_b{}'\delta^3(\vec x-\vec y_b)\frac{\epsilon(\sigma)}{2}d\sigma\right]^2=\sum_{\vec k\lambda}\left[\sum_b g_b\int d\sigma\frac{\epsilon(\sigma)}{2}\vec y_b{}'\phi_{\vec k\lambda}(\vec y_b)\right]^2. \tag{A4}$$

Substituting (A3) and (A4) in (A1) one gets

$$L=\sum_{\vec k\lambda}\tfrac{1}{2}\left[\dot{\vec q}_{\vec k\lambda}-\sum_b g_b\int\dot{\vec y}_b\times\vec y_b{}'\phi_{\vec k\lambda}(\vec y_b)\frac{\epsilon(\sigma)}{2}d\sigma\right]^2-\sum_{\substack{k\neq 0\\ \lambda}}\tfrac{1}{2}\left[\vec k\times\vec q_{\vec k\lambda}^{\,T}+\lambda\sum g_b\int\vec y_b{}'\phi_{\vec k,-\lambda}(\vec y_b)\frac{\epsilon(\sigma)}{2}d\sigma\right]^2$$
$$+\sum_{\vec k\lambda}\sum_a e_a\dot{\vec r}_a\cdot\vec q_{\vec k\lambda}\phi_{\vec k\lambda}(\vec r_a)+\tfrac{1}{2}\sum_a m_a\dot{\vec r}_a^{\,2}+\tfrac{1}{2}\sum_b\int\rho_b\dot{\vec y}_b^{\,2}d\sigma\,. \tag{A5}$$

Here we have dropped a term proportional to $\phi_{00}{}^2$ because $\phi_{00}{}^2$ goes as $1/L^3$, whereas the coefficient of $\phi_{00}{}^2$ namely $\frac{1}{2}\{\sum_b g_b\vec y_b{}'[\epsilon(\sigma)/2]d\sigma\}^2$ goes at most as L^2.

We define new variables for $\vec k\neq 0$ as

$$Q_{\vec k\lambda}^L=q_{\vec k\lambda}^L+B_{\vec k\lambda}^L\,, \tag{A6}$$

$$B_{\vec k\lambda}^L=-\sum_b g_b\int_{t_1}^t d\tau\int d\sigma\frac{\epsilon(\sigma)}{2}\dot{\vec y}_b(\tau)\times\vec y_b{}'(\tau)\cdot\hat k\phi_{\vec k\lambda}(\vec y_b(\tau))\,, \tag{A7}$$

and

$$\vec{Q}^T_{\vec{k}\lambda} \equiv \vec{q}^T_{\vec{k}\lambda} + \vec{D}^T_{\vec{k}\lambda}, \tag{A8}$$

$$\vec{D}^T_{\vec{k}\lambda} = \sum_b g_b \frac{\lambda}{k} \int \vec{y}_b' \times \hat{k} \phi_{\vec{k},-\lambda}(\vec{y}_b) \frac{\epsilon(\sigma)}{2} d\sigma . \tag{A9}$$

Differentiating (A9), with respect to time and carrying out partial integration with respect to σ, we obtain

$$\dot{\vec{D}}^T_{\vec{k}\lambda} = \sum_b g_b \frac{\lambda}{k} \Big[\int \left(\frac{\epsilon(\sigma)}{2} \dot{\vec{y}}_b \times \hat{k} \phi_{\vec{k}-\lambda}(\vec{y}_b) \right)' d\sigma - \dot{\vec{Z}}_b \times \hat{k} \phi_{\vec{k}-\lambda}(\vec{Z}_b)$$
$$- \int d\sigma \frac{\epsilon(\sigma)}{2} \dot{\vec{y}}_b \times k \vec{y}_b' \cdot \vec{k} \lambda \phi_{\vec{k}\lambda}(\vec{y}_b) + \int d\sigma \frac{\epsilon(\sigma)}{2} \vec{y}_b' \times \hat{k} \dot{\vec{y}}_b \cdot \vec{k} \lambda \phi_{\vec{k}\lambda}(\vec{y}_b) \Big]. \tag{A10}$$

But $\dot{\vec{y}}_b$ at the end points of the string is zero. Therefore, if we define

$$\vec{C}^T_{\vec{k}\lambda} \equiv \sum_b g_b \frac{\lambda}{k} \dot{\vec{Z}}_b \times \hat{k} \phi_{\vec{k}-\lambda}(\vec{Z}_b), \tag{A11}$$

then from (A10) and (A11) we obtain

$$\dot{\vec{D}}^T_{\vec{k}\lambda} + \vec{C}^T_{\vec{k}\lambda} = -\hat{k} \times \left[\sum_b g_b \int d\sigma \frac{\epsilon(\sigma)}{2} (\dot{\vec{y}}_b \times \vec{y}_b') \times \hat{k} \phi_{\vec{k}\lambda}(\vec{y}_b) \right]. \tag{A12}$$

Using the new variables we can rewrite the first term in the right-hand side of (A5), for $\vec{k} \neq 0$, as

$$\left[\dot{\vec{q}}_{\vec{k}\lambda} - \sum_b g_b \int \dot{\vec{y}}_b \times \vec{y}_b' \phi_{\vec{k}\lambda}(\vec{y}_b) \frac{\epsilon(\sigma)}{2} d\sigma \right]^2 = \dot{Q}^{L^2}_{\vec{k}\lambda} + [\dot{\vec{Q}}^T_{\vec{k}\lambda} + \vec{C}^T_{\vec{k}\lambda}]^2 . \tag{A13}$$

Using the definition (A8), (A9) and partially integrating with respect to σ, we can rewrite the second term in the right-hand side of (A5) as

$$\sum_{\substack{\vec{k}\neq 0 \\ \lambda}} \left[\vec{k} \times \vec{q}^T_{\vec{k}\lambda} + \lambda \sum_b g_b \int \vec{y}_b' \phi_{\vec{k}-\lambda}(\vec{y}_b) \frac{\epsilon(\sigma)}{2} d\sigma \right]^2 = \sum_{\substack{k\neq 0 \\ \lambda}} k^2 \vec{Q}^{T^2}_{\vec{k}\lambda} + \sum_{\substack{k\neq 0 \\ \lambda}} \frac{1}{k^2} \left[\sum_b g_b \int \left(\phi_{\vec{k}\lambda}(\vec{y}_b) \frac{\epsilon(\sigma)}{2} \right)' d\sigma - \sum_b g_b \phi_{\vec{k}\lambda}(\vec{Z}_b) \right]^2$$
$$= \sum_{\substack{k\neq 0 \\ \lambda}} k^2 \vec{Q}^{T^2}_{\vec{k}\lambda} + \sum_{b_1 > b_2} \frac{2 g_{b_1} g_{b_2}}{4\pi} \frac{1}{|\vec{Z}_{b_1} - \vec{Z}_{b_2}|}, \tag{A14}$$

where in the last step we have used (A2b) and the fact that as $\Omega \to \infty$, $1/|\vec{Z}_b - \vec{y}_b| \to 0$, if $\vec{y}_b$ is the point of the string on the boundary of Ω. We have also ignored the time-independent self-energy terms of monopoles.

Next we consider the third term in the right-hand side of (A5), for $\vec{k} \neq 0$:

$$\sum_{\substack{\vec{k}\neq 0 \\ \lambda}} \sum_a e_a \dot{\vec{r}}_a \cdot \vec{q}_{\vec{k}\lambda} \phi_{\vec{k}\lambda}(\vec{r}_a) = \sum_{\substack{k\neq 0 \\ \lambda}} [Q^L_{\vec{k}\lambda} J^L_{\vec{k}\lambda} + \vec{Q}^T_{\vec{k}\lambda} \cdot \vec{J}_{\vec{k}\lambda} - B^L_{\vec{k}\lambda} J^L_{\vec{k}\lambda} - \vec{D}^T_{\vec{k}\lambda} \cdot \vec{J}_{\vec{k}\lambda}], \tag{A15}$$

where

$$\vec{J}_{\vec{k}\lambda} = \sum_a e_a \dot{\vec{r}}_a \phi_{\vec{k}\lambda}(\vec{r}_a), \tag{A16}$$

$$\vec{J}^L_{\vec{k}\lambda} = \sum_a e_a \dot{\vec{r}}_a \cdot \hat{k} \phi_{\vec{k}\lambda}(\vec{r}_a) = \frac{\lambda}{k} \sum_a e_a \dot{\phi}_{\vec{k}-\lambda}(\vec{r}_a). \tag{A17}$$

Now the third term in (A15), using the definition (A7) and (A17) and integrating partially with respect to time, can be written as

$$\sum_{\substack{\vec{k}\neq 0 \\ \lambda}} B^L_{\vec{k}\lambda} J^L_{\vec{k}\lambda} = \sum_{\substack{k\neq 0 \\ \lambda}} \left[\frac{d}{dt} \sum_a e_a \frac{\lambda}{k} B^L_{\vec{k}\lambda} \phi_{\vec{k}-\lambda}(\vec{r}_a) + \sum_{a,b} e_a g_b \int \dot{\vec{y}}_b \times \vec{y}_b' \cdot \vec{k} \frac{\lambda}{k^2} \phi_{\vec{k}\lambda}(\vec{y}_b) \phi_{\vec{k}-\lambda}(\vec{r}_a) \frac{\epsilon(\sigma)}{2} d\sigma \right]$$
$$= \frac{d}{dt} \sum_{\substack{\vec{k}\neq 0 \\ \lambda}} \sum_a e_a \frac{\lambda}{k} B^L_{\vec{k}\lambda} \phi_{\vec{k}-\lambda}(\vec{r}_a) - \sum_{a,b} \frac{e_a g_b}{4\pi} \int \frac{\dot{\vec{y}}_b \times \vec{y}_b' \cdot \vec{Y}_{ba}}{|\vec{Y}_{ba}|^3} \frac{\epsilon(\sigma)}{2} d\sigma , \tag{A18}$$

where $\vec{Y}_{ba}(\sigma) = \vec{y}_b(\sigma) - \vec{r}_a$ and in the last step we have used Eq. (A2c). Similarly using the definitions (A9) and (A16), and the relation (A2c), one can rewrite the last term of (A15)

$$\sum_{\substack{\vec{k}\neq 0\\ \lambda}} \vec{D}^T_{\vec{k}\lambda}\cdot\vec{J}_{\vec{k}\lambda} = \sum_{a,b}\frac{e_a g_b}{4\pi}\int\frac{\dot{\vec{r}}_a\times\vec{y}_b'\cdot\vec{Y}_{ba}}{|\vec{Y}_{ba}|^3}\frac{\epsilon(\sigma)}{2}d\sigma\,. \tag{A19}$$

Combining (A15), (A18), and (A19) we get

$$\sum_{\substack{\vec{k}\neq 0\\ \lambda}} e_a\dot{\vec{r}}_a\cdot\vec{q}_{\vec{k}\lambda}\phi_{\vec{k}\lambda}(\vec{r}_a) = \sum_{\substack{\vec{k}\neq 0\\ \lambda}}\left[Q^L_{\vec{k}\lambda}J^L_{\vec{k}\lambda} + \dot{\vec{Q}}^T_{\vec{k}\lambda}\cdot\vec{J}_{\vec{k}\lambda} - \frac{d}{dt}\sum_a e_a\frac{\lambda}{k}B^L_{\vec{k}\lambda}\phi_{\vec{k},-\lambda}(\vec{r}_a)\right] + \sum_{a,b}\frac{e_a g_b}{4\pi}\int d\sigma\,\frac{\epsilon(\sigma)}{2}\frac{\dot{\vec{Y}}_{ba}\times\vec{Y}_{ba}'\cdot\vec{Y}_{ba}}{|\vec{Y}_{ba}|^3}\,. \tag{A20}$$

Finally, we consider the $\vec{q}_{00}$ and ϕ_{00} terms. Let L_{00} be the part of L containing $\vec{q}_{00}$ and ϕ_{00} terms, then from (A5) one obtains

$$L_{00} = \tfrac{1}{2}\left[\dot{\vec{q}}_{00} - \sum_b g_b\int\dot{\vec{y}}_b\times\vec{y}_b'\phi_{00}\frac{\epsilon(\sigma)}{2}d\sigma\right]^2 + \sum_a e_a\dot{\vec{r}}_a\cdot\vec{q}_{00}\phi_{00}\,.$$

If we define

$$\vec{Q}_{00} = \vec{q}_{00} + \tfrac{1}{2}\phi_{00}\sum_b g_b\int\vec{y}_b'\times\vec{y}_b\frac{\epsilon(\sigma)}{2}d\sigma\,, \tag{A21}$$

then differentiating (A21) with respect to time and integrating partially, and noting that $\vec{y}_b = 0$ on the boundary of volume Ω, we obtain

$$\dot{\vec{Q}}_{00} = \dot{\vec{q}}_{00} - \phi_{00}\sum_b g_b\int\dot{\vec{y}}_b\times\vec{y}_b'\frac{\epsilon(\sigma)}{2}d\sigma - \tfrac{1}{2}\phi_{00}\sum_b g_b\dot{\vec{Z}}_b\times\vec{Z}_b\,.$$

Therefore, one can rewrite L_{00} as

$$L_{00} = \tfrac{1}{2}\dot{\vec{Q}}_{00}{}^2 + \tfrac{1}{2}\dot{\vec{Q}}_{00}\sum_b g_b\phi_{00}\dot{\vec{Z}}_b\times\vec{Z}_b + \sum_a e_a\dot{\vec{r}}_a\cdot\vec{Q}_{00}\phi_{00}\,, \tag{A22}$$

where we have dropped out the $\phi_{00}{}^2$ terms.

Combining Eqs. (A5), (A13), (A14), (A20), and (A22), we get Eqs. (3.5), (3.6a)–(3.6g) for the action.

APPENDIX B: STATE VECTORS SATISFYING ALL THE CONSTRAINTS

In this appendix we will find the subspace $\mathfrak{M}$ of state vectors satisfying both the constraints (2.6) and (2.4b). The differential operator corresponding to the constraint (2.4b) can be obtained by the standard canonical procedure, and the corresponding equation is

$$\left[\frac{\partial}{\partial\vec{y}_b(\sigma)} + g_b\sum_{\vec{k}\lambda}\phi_{\vec{k}\lambda}(\vec{y}_b)\frac{\epsilon(\sigma)}{2}\vec{y}_b'\times\frac{\partial}{\partial\vec{q}_{\vec{k}\lambda}}\right]\psi = 0,\ \text{for } \sigma\neq 0\,. \tag{B1}$$

ψ satisfies the constraint (2.6) also. Therefore, from (2.10a),

$$\psi(\{q^L_{\vec{k}\lambda}\},\xi) = f(\{q^L_{\vec{k}\lambda}\},\{\vec{r}_a\})\tilde{\psi}(\xi)\,. \tag{B2}$$

Substituting this in (B1), dividing by f and noting that

$$\sum_{\vec{k},\lambda}\phi_{\vec{k}\lambda}(\vec{y}_b)\frac{\partial}{\partial\vec{q}_{\vec{k}\lambda}} = \phi_{00}\frac{\partial}{\partial\vec{q}_{00}} + \sum_{\substack{\vec{k}\neq 0\\ \lambda}}\phi_{\vec{k}\lambda}(\vec{y}_b)\left(\hat{k}\frac{\partial}{\partial q^L_{\vec{k}\lambda}} + \frac{\partial}{\partial\vec{q}^T_{\vec{k}\lambda}}\right),$$

we obtain

$$\left[\frac{\partial}{\partial\vec{y}_b(\sigma)} + \frac{\epsilon(\sigma)}{2}g_b\vec{y}_b'\times\left(\phi_{00}\frac{\partial}{\partial\vec{q}_{00}} + \sum_{\substack{k\neq 0\\ \lambda}}\phi_{\vec{k}\lambda}(\vec{y}_b)\frac{\partial}{\partial\vec{q}^T_{\vec{k}\lambda}}\right)\right]\tilde{\psi} = \sum_a g_b e_a\frac{i}{\hbar}\tilde{\psi}\frac{\epsilon(\sigma)}{2}\frac{1}{4\pi}\frac{\vec{y}_b'\times\vec{Y}_{ba}}{|\vec{Y}_{ba}|^3}\,. \tag{B3}$$

In obtaining (B3) we have used Eq. (A2c). Equation (B3) suggests that $\tilde{\psi}$ can be written in the form

$$\tilde{\psi}(\xi) = F(\{\vec{Y}_{ba}\})\bar{\psi}(\xi)\,, \tag{B4}$$

where

$$F(\{Y_{ba}\}) = \exp\left[\frac{i}{\hbar}\left(\sum_{a,b}e_a g_b\int d\sigma\frac{\epsilon(\sigma)}{2}\frac{1}{4\pi}\int_{\vec{Z}_b-\vec{r}_a+\vec{y}^0_b(\sigma)}^{\vec{Y}_{ba}(\sigma)}\frac{\delta\vec{y}_{ba}(\sigma)\times\vec{y}_{ba}'(\sigma)}{|\vec{y}_{ba}(\sigma)|^3}\cdot\vec{y}_{ba}(\sigma)\right)\right].$$

In this expression $\vec{y}^0_b(\sigma)$ is an arbitrary fixed string such that $\vec{y}^0_b(0)=0$ and the path of $\vec{y}_{ba}(\sigma)$ may be conveniently taken to be the straight line joining $\vec{Z}_b - \vec{r}_a + \vec{y}^0_b(\sigma)$ and $\vec{Y}_{ba}(\sigma)$. The integral is carried out over only those values such that $\vec{y}_{ba}(\sigma) + \vec{r}_a$ lies inside the volume Ω. In fact, one can see that

$$\pm\int_0^{\pm\infty}d\sigma\int_{\vec{Z}_b-\vec{r}_a+\vec{y}^0_b(\sigma)}^{\vec{Y}_{ba}(\sigma)}\frac{\delta\vec{y}_{ba}(\sigma)\times\vec{y}_{ba}'(\sigma)}{|\vec{y}_{ba}(\sigma)|^3}\cdot\vec{y}_{ba}(\sigma) = \Omega^{\pm}(\vec{Z}_b - \vec{r}_a + \vec{y}^0_b, \vec{Y}_{ba})\,,$$

where $\Omega^{\pm}(\vec{Z}_b - \vec{r}_a + \vec{y}^0_b, \vec{Y}_{ba})$ are the solid angles subtended at the origin by the sheet spanned by the semistrings $\vec{y}_{ba}(\sigma)$ for $\sigma = \pm|\sigma|$ in going from $\vec{Z}_b - \vec{r}_a + \vec{y}^0_b(\sigma)$ to $\vec{Y}_{ba}(\sigma)$. Therefore, F can be written as

$$F(\{Y_{ba}\}) = \exp\left[\frac{i}{2\hbar}\sum_{a,b}\frac{e_a g_b}{4\pi}[\Omega^+(\vec{Z}_b - \vec{r}_a + \vec{y}^0_b, \vec{Y}_{ba}) - \Omega^-(\vec{Z}_b - \vec{r}_a + \vec{y}^0_b, \vec{Y}_{ba})]\right]. \tag{B5}$$

F is well defined provided the Schwinger quantization condition is assumed to satisfy, i.e., $e_a g_b = 2nh$ when n is some integer. We will assume that this condition is satisfied.

We notice that

$$\frac{\partial}{\partial \vec{y}_b(\sigma)} F = F \frac{i}{\hbar} \sum_a e_a g_b \int d\sigma \frac{\epsilon(\sigma)}{2} \frac{1}{4\pi} \frac{\vec{Y}'_{ba} \times \vec{Y}_{ba}}{|\vec{Y}_{ba}|^3}. \quad \text{(B6)}$$

Substituting (B4) in (B3) and making use of (B6), one gets

$$\left[\frac{\partial}{\partial \vec{y}_b(\sigma)} + \frac{\epsilon(\sigma)}{2} g_b \vec{y}'_b \times \left(\phi_{00} \frac{\partial}{\partial \vec{q}_{00}} + \sum_{\substack{k \neq 0 \\ \lambda}} \phi_{\vec{k}\lambda}(\vec{y}_b) \frac{\partial}{\partial \vec{q}^T_{\vec{k}\lambda}}\right)\right] \psi = 0 . \quad \text{(B7)}$$

We change the variables from $(\vec{q}^T_{\vec{k}\lambda}, \vec{q}_{00}, \vec{y}_b(\sigma), \vec{r}_a)$ to $(\vec{Q}^T_{\vec{k}\lambda}, \vec{Q}_{00}, \vec{y}_b(\sigma), \vec{r}_a)$, where

$$\begin{aligned} \vec{Q}^T_{\vec{k}\lambda} &= \vec{q}^T_{\vec{k}\lambda} + \sum_b g_b \frac{\lambda}{k} \int \vec{y}'_b \times \hat{k} \phi_{\vec{k}-\lambda}(y_b) \frac{\epsilon(\sigma)}{2} d\sigma , \\ \vec{Q}_{00} &= \vec{q}_{00} + \tfrac{1}{2} \sum_b g_b \phi_{00} \int \vec{y}'_b \times \vec{y}_b \frac{\epsilon(\sigma)}{2} d\sigma . \end{aligned} \quad \text{(B8)}$$

Let

$$\bar{\psi}(\xi) = \bar{\Psi}(\{\vec{Q}^T_{\vec{k}\lambda}\}, \vec{Q}_{00}, \{\vec{y}_b(\sigma)\}, \{\vec{r}_a\}). \quad \text{(B9)}$$

Then in terms of the new variables defined in (B8) one can show that Eq. (B7) becomes

$$\frac{\partial}{\partial \vec{y}_b(\sigma)} \bar{\Psi} = 0 \quad \text{for } \sigma \neq 0 . \quad \text{(B10)}$$

(B10) implies that $\bar{\Psi}$ is independent of $\vec{y}_b(\sigma)$ ($\sigma \neq 0$). Therefore, $\bar{\Psi}$ is an arbitrary function of $\vec{Q}^T_{\vec{k}\lambda}$, $\vec{Q}_{00}$, $\vec{Z}_b$, and $\vec{r}_a$. For brevity, we will denote all the latter variables by χ,

$$\bar{\Psi} = \bar{\Psi}(\chi) . \quad \text{(B11)}$$

Equations (B2), (B4), (B5), (B9), and (B11) define the subspace $\mathfrak{M}$ of state vectors satisfying both the constraints (2.6) and (2.4b).

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