PHYSICAL REVIEW D VOLUME 20, NUMBER 1 1 JULY 1979

Consequences of the u-d quark mass difference for $\pi^0 \to \gamma\gamma$ and $\pi^+ \to e^+\nu\gamma$

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(Received 19 December 1978)

Radiative pion decay is studied in the framework of several relativistic quark models with or without a σ scalar meson, within the one-loop approximation. The new feature that we consider is that the quark masses are nondegenerate. It is found that the form factors are very sensitive to the quark mass ratio, m_d/m_u, and that the Vaks-Ioffe relation is severely broken in a model-dependent way. Therefore, the experimental values of $\gamma = F_A/F_V$ must be corrected if $m_u \neq m_d$. All the models considered appear to indicate that the smallest value of γ is preferred if $m_d > m_u$. However, none is in good agreement with the experiment within 1 standard deviation. The best values for $\pi \to e\nu\gamma$ are in the SU(2) $\times$ SU(2) σ model.

INTRODUCTION

The idea of having a relatively large mass ratio between u and d quarks has been considered recently as a promising alternative for the strong symmetry-breaking mechanism.[1] The u, d mass ratio has been calculated in the quantum-chromodynamics (QCD) framework to be of the order of $m_d/m_u \sim 1.8$ or 1.5 depending on whether one considers the pseudoscalar mass spectra[1] or the neutron-proton mass difference.[2]

The SU(2) $\times$ SU(2) and isospin symmetries would then be a result of having relatively small quark masses. Many measurable quantities would be insensitive to the large quark mass ratio. Nonetheless, there are physical quantities that strongly depend on the mass ratio, for example, the $\pi^0 n \to \pi^0 n$ and $\pi^0 p \to \pi^0 p$ scattering lengths at threshold could have a variation of as much as 30% from those predicted for $m_u = m_d$.

In this work we study the consequences of the nondegeneracy of the quark masses for the radiative pion decays $\pi^0 \to \gamma\gamma$ and $\pi^+ \to e\nu\gamma$. The presence of the pion as an asymptotic particle in these processes implies that one must introduce the pion explicitly in the Lagrangian of the problem, as long as the bound-state problem of the quarks is not already solved. In this work, we consider several models. Firstly, we take the most naive extension of the quark model that includes the pion field. This model is explicitly discussed in Sec. I. Results for other models are simply stated. The common features of the models are (i) they give reasonable values for $\pi^0 \to \gamma\gamma$ at least in the limit $m_u = m_d$ through the Adler theorem,[3,4] and (ii) the quark mass difference is always introduced by hand as in the model considered in Sec. I. We will consider (1) a fractionally-charged-quark color-triplet model with and without a scalar σ meson, (2) the SU(2) $\times$ SU(2) σ model of Gell-Mann and Lévy[5] modified by the quark mass splitting term, and (3) a Han-Nambu[6]-triplet-type quark model with and without the σ meson.

There is a well known theorem by Adler[3] and Bell and Jackiw[4] that states that the $\pi^0 \to \gamma\gamma$ decay amplitude, $\mathfrak{F}_{\pi^0 \to \gamma\gamma}$, is given, in the quark and σ models, by the triangle graph solely. It has been shown[7] that in the framework of extended partial conservation of the axial-vector current (EPCAC) this process can be used to give constraints on the quark mass ratio. In Sec. II we compute again for completeness the amplitude using different masses for the u and d quarks without reference to the EPCAC hypothesis.

The process $\pi^+ \to e\nu\gamma$ has two form factors F_A and F_V.[8] The vector form factor, F_V, can be related[9] to the $\pi^0 \to \gamma\gamma$ amplitude, $\mathfrak{F}_{\pi^0 \to \gamma\gamma}$, assuming among other things that isospin is a good symmetry of nature. In Sec. III we calculate how much this relation is violated by the hypothesis of nondegeneracy of the quark masses. In Sec. IV we calculate and compare the result of our calculation to the experimental data.[10]

This model can be straightforwardly extended to the $\eta \to \gamma\gamma$ and K radiative decays. In this work we restrict ourselves to the π case where we met all the technical difficulties and where the results are not sensitive if one includes the Cabibbo angle and the quark mass difference at the same time because $\cos\theta_C \sim 1$. We finally remark that it is not obvious at this time whether the masses we are using are exactly the same as those of QCD, because, according to the current ideas, QCD should be able to produce the pion as a bound state of quarks. Anyhow, using the hypothesis that there is a large quark mass ratio, we have been able to prove that the relative sign of F_V and F_A can be negative, a result that has been obtained in the past[11] only by using an unelegant cutoff procedure that somehow reproduces the quark-confinement mechanism.

I. THE MODEL

In order to sketch our computation framework and notation let us take, for example, the following strong-interaction Lagrangian[12]:

$$\mathcal{L}_{\text{strong}}=\bar{q}_j(i\not\partial-m_q)q_j+\tfrac{1}{2}(\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}-m_\pi^2\vec{\pi}\cdot\vec{\pi})+g\bar{q}_j i\vec{\pi}\cdot\vec{\tau}\gamma_5 q_j\,, \quad (1)$$

where $\vec{\pi}$, m_π, q, m_q are an isotriplet of pion fields, the pion mass, an isodoublet of quark fields, and the quark mass matrix respectively; g is the strong-interaction coupling constant, and $\vec{\tau}$ are the Pauli matrices. j is a color index that runs from 1 to 3 or is equal to 1 if no color is considered.

The weak interactions will be considered of the current-current type because there is no use in introducing W-boson effects in view of the low momentum transfer of the pion decay processes. The hadronic currents are generated by the gauge transformations

$$q_j\to(1+\tfrac{1}{2}i\vec{\tau}\cdot\vec{u})q_j\,,\qquad \vec{\pi}\to\vec{\pi}-\vec{u}\times\vec{\pi}\,,$$

for the vector current and

$$q_j\to(1+\tfrac{1}{2}i\vec{\tau}\cdot\vec{v}\gamma_5)q_j\,,\qquad \vec{\pi}\to\vec{\pi}\,,$$

for the axial-vector current. These currents are then given by

$$\vec{V}_\mu=\tfrac{1}{2}\bar{q}_j\vec{\tau}\gamma_\mu q_j+\vec{\pi}\times\partial_\mu\vec{\pi}\,, \quad (2a)$$

$$\vec{A}_\mu=\tfrac{1}{2}\bar{q}_j\vec{\tau}\gamma_\mu\gamma_5 q_j\,. \quad (2b)$$

The divergences of the currents are

$$\partial_\mu\vec{V}^\mu=\tfrac{1}{2}i\bar{q}_j[m_q,\vec{\tau}]q_j\,, \quad (3a)$$

$$\partial_\mu\vec{A}^\mu=\tfrac{1}{2}i\bar{q}_j\{m_q,\vec{\tau}\}\gamma_5 q_j-g\bar{q}_j\vec{\pi}q_j\,, \quad (3b)$$

where square (curly) brackets denote (anti-) commutation. We can see that CVC is explicitly violated by the quark mass difference and that PCAC is not satisfied as an operator relation.

The hadronic weak current is given by

$$j_\mu=[(V_1-iV_2)_\mu-(A_1-iA_2)_\mu]\cos\theta_C\,. \quad (4)$$

The leptonic weak current is the usual

$$l_\mu=\bar{u}_\nu\gamma_\mu(1-\gamma_5)v_l\,, \quad (5)$$

where l is a lepton and ν its neutrino. Electromagnetic interactions are included in the strong and weak Lagrangians via the minimal substitution $\partial_\mu\to\partial_\mu-ie\Delta_\mu$. Of course, the Lagrangian of Eq. (1) is not the ultimate one, but for the processes we are to consider it has the essential ingredients, and in the one-loop approximation it makes no difference if we take this or a more complete Lagrangian[6,7,13] (except for F_A if one considers a σ-type model).

II. $\pi^0\to\gamma\gamma$

The amplitude for $\pi^0\to\gamma\gamma$ is defined by

$$\mathfrak{M}(\pi^0(p)\to\gamma(k)\gamma(k'))=e^2\mathcal{E}'^\mu(k')\mathcal{E}^\nu(k)\epsilon_{\mu\nu\rho\sigma}k^\rho p^\sigma\mathfrak{F}_{\pi^0\gamma\gamma}(p^2)\,, \quad (6)$$

where $\mathcal{E}$ and $\mathcal{E}'$ are the polarization vectors of the photons which correspond to momenta k and k', and p is the pion momentum. Note that we include the normalization factors in the phase space.

$\mathfrak{F}_{\pi^0\to\gamma\gamma}$ is then given by a sum of triangle loops, see Fig. 1. We obtain

$$\mathfrak{F}_{\pi^0\to\gamma\gamma}(p^2)=\frac{g}{2\pi^2e^2}\frac{1}{p\cdot k}\sum_i\left\{e_{u_i}^2m_{u_i}\left[\sin^{-1}\left(\frac{p^2}{4m_{u_i}^2}\right)^{1/2}\right]^2-e_{d_i}^2m_{d_i}\left[\sin^{-1}\left(\frac{p^2}{4m_{d_i}^2}\right)^{1/2}\right]^2\right\}, \quad (7)$$

where i is a color index, and the integrals have been performed subject to the restriction $p^2<4m_u^2$ and $p^2<4m_d^2$. In the soft-pion limit, $p^2\to0$, we get

$$\mathfrak{F}_{\pi^0\to\gamma\gamma}(0)=\frac{g}{4\pi^2e^2}\sum_i\left[\frac{e_{u_i}^2}{m_{u_i}}-\frac{e_{d_i}^2}{m_{d_i}}\right]. \quad (8)$$

We note that in the soft-pion limit $\mathfrak{F}_{\pi^0\to\gamma\gamma}$ depends hyperbolically with respect to $\kappa=m_d/m_u$ if $e_d\neq0$. In this paper we shall normalize all the form factors with respect to $g/4\pi^2m_u\equiv K$. If $m_d=m_u$, this can be related to the pion decay constant f_π. See Adler, Ref. 3, or a σ model.[4] With this notation Eq. (8) is rewritten as

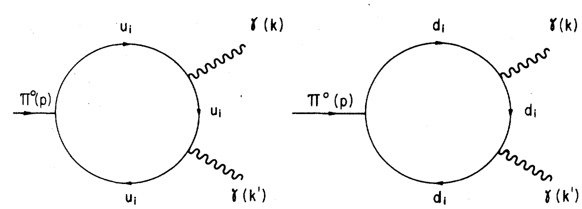

FIG. 1. The decay amplitude of $\pi^0\to\gamma\gamma$ is given by the sum of these two loops and those with k and k' interchanged.

$$\mathfrak{F}_{\pi^0\to\gamma\gamma}(0)=\frac{K}{e^2}\sum_i\left(e_{u_i}{}^2-\frac{e_{d_i}{}^2}{\kappa}\right). \tag{9}$$

This is the usual result provided that $\kappa=1$. We remark the generality of Eq. (9). This result is modified neither by higher-order diagrams nor by a renormalization of the quark masses, provided that this renormalization is multiplicative.

III. $\pi\to e\nu\gamma$ AND THE BREAKDOWN OF THE VAKS–IOFFE RELATION

The most general form of the amplitude of $\pi\to e\nu\gamma$ consistent with Lorentz covariance, gauge invariance and a four-fermion $V-A$ current-current interaction is

$$\mathfrak{M}(\pi(p)\to l(s)\nu_l(r)\gamma(k))=\frac{G\cos\theta_C}{\sqrt{2}}e\mathcal{E}^\nu(k)\Big\{l^\mu\Big[\epsilon_{\mu\nu\rho\sigma}k^\rho p^\sigma F_V(t)-i(k_\mu p_\nu-g_{\mu\nu}p\cdot k)F_A(t)-i\Big((k-p)_\mu p_\nu\frac{2\sqrt{2}f_\pi}{m_\pi{}^2-t}-\sqrt{2}f_\pi g_{\mu\nu}\Big)\Big]-l_{\mu\nu}i\sqrt{2}f_\pi p^\mu\Big\}, \tag{10}$$

where G_F is the Fermi constant, $\mathcal{E}(k)$ is the photon polarization vector, f_π is the pion decay constant defined by

$$\langle 0|A_\mu|\pi^+(p)\rangle=-i\sqrt{2}f_\pi p_\mu \tag{11}$$

and is equal to $f_\pi\simeq 0.96m_\pi$, θ_C is the Cabibbo angle. This should perhaps not be included if we are to consider nondegenerate quark masses,[14] but we include it here anyhow because it does not make much difference, since $\cos\theta_C\simeq 1$; and the following definitions hold:

$$t=(p-k)^2,$$

$$l^\mu=\bar{u}_\nu(s)\gamma^\mu(1-\gamma_5)v_l(r),$$

and

$$p_\mu l^{\mu\nu}=\bar{u}_\nu(s)\not{p}(1-\gamma_5)\frac{\not{r}+\not{k}-m_l}{(r+k)^2-m_l{}^2}\gamma^\nu v_l(r).$$

In the model considered f_π is divergent and cannot be computed. However, F_A and F_V turn out to be finite. They can be obtained from the graphs in Fig. 2, subject to the restriction $t<p^2<(m_u+m_d)^2$. The vector form factor is given by

$$F_V=\frac{\sqrt{2}gg_V}{(2\pi)^2}\sum_i\left\{\frac{e_{u_i}}{e}[m_uI_0(m_u,m_d,t,p^2)+\Delta mI_1(m_u,m_d,t,p^2)]+\frac{e_{d_i}}{e}[m_u\to m_d]\right\}, \tag{12}$$

where

$$\Delta m=m_d-m_u,$$

$$I_n(m_u,m_d,t,p^2)=\frac{1}{2p\cdot k}\int_0^1 dy\,y^{n-1}\ln\frac{m_u{}^2+(m_d{}^2-m_u{}^2)y-ty^2}{m_u{}^2+(m_d{}^2-m_u{}^2)y-p^2y^2}, \tag{13}$$

and the term in the second set of square brackets in Eq. (12) means interchange of m_u and m_d. Explicit expressions of $I_n(m_u,m_d,t,p^2)$ are given in the Appendix. The main feature of them is that they are rather soft functions of p^2 and t, therefore, we discuss here the soft-pion limit of them

$$F_V=\frac{gg_V}{2\sqrt{2}\pi^2 e m_u(\kappa^2-1)}\times\left\{e_u\left[\frac{2\kappa^2}{(\kappa^2-1)(\kappa+1)}\ln\kappa+\frac{1}{2}\frac{\kappa^2+1}{\kappa+1}-1\right]-\kappa e_d[m_u\leftrightarrow m_d]\right\}, \tag{14}$$

where

$$\kappa=\frac{m_d}{m_u}. \tag{15}$$

Comparing this expression to the one in Eq. (8) we see that while the Vaks-Ioffe relation $F_V(t=0)=(1/\sqrt{2})\mathfrak{F}_{\pi^0\to\gamma\gamma}(0)$ is true for $m_u=m_d$, this relation is broken down if $m_u\neq m_d$. In Fig. 3 the ratio $1/\sqrt{2}\mathfrak{F}_{\pi^0\to\gamma\gamma}(0)/F_V(t=0)$ is given for several models in the soft-pion limit $p^2\to 0$; the result for nonzero values of p^2 can be directly obtained using the relations for I_n in the Appendix.[15]

We would like to remark that the expressions for $\mathfrak{F}_{\pi^0\to\gamma\gamma}$ and F_V are true not only for the models

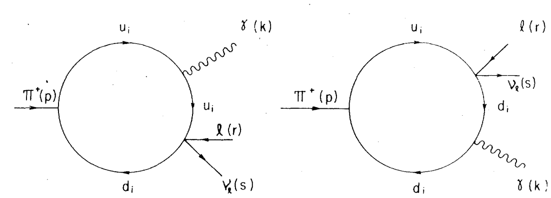

FIG. 2. From these diagrams one can obtain the quark contribution to F_A and F_V in the one-loop approximation.

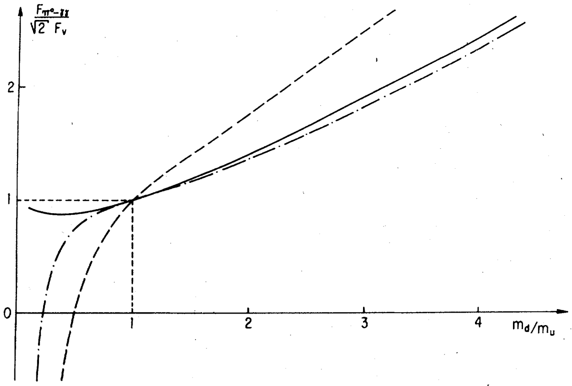

FIG. 3. Curves showing the breakdown of the Vaks-Ioffe relation, $\mathfrak{F}_{\pi^0\gamma\gamma}=\sqrt{2}\,F_V$, in the fractionally-charged-quark model (point-dashed line) and in the Han-Nambu-triplet-type model (dashed line), with and without the σ meson, and in the SU(2) ×SU(2) model of Gell-Mann and Lévy (continuous line).

presented here, but also in the usual σ models because in these models there is no σ-π coupling to the weak vector current. The result also holds for chiral models such as the used by Pervushin and Volkov.[13]

One might ask if the condition $p^2<(m_u+m_d)^2$ or $p^2<4m_q^2$, which have been used in order to prevent the appearance of an absortive part in the amplitudes (6) and (10), imply that one is using constituent quark masses. This is not clear because:

(i) We are using the masses that appear in the Lagrangian (1) without any reference to a specific renormalization.

(ii) The triangle-anomaly theorem,[3] which is the key of our analysis, is strictly valid only for the soft-pion limit, $p^2\to 0$. Thus, the true restriction in this limit is that $(m_u+m_d)>0$.

(iii) It has been shown that when a quark-confinement mechanism is supplied,[11] the masses that appear in the inequalities are changed from the m_q's to $m_q'^2=m_q^2+s^2$, where s^2 is the size of the "box." In the limit $s=0$, which we are using, m_q is the "bare" mass.

IV. F_A; THE RATIO $\gamma=F_A/F_V$

The evaluation of F_A follows the same scheme presented above for F_V (see Fig. 2). We get in the one-loop approximation

$$F_A=\frac{\sqrt{2}\,gg_A}{(2\pi)^2}\sum_i\left\{\frac{e_{u_i}}{e}\left[m_u(I_0-2I_1)-\Delta m\left(I_1-\frac{1}{p\cdot k}\;m_u^2\left(I_0+\frac{m_d^2-m_u^2-t}{m_u^2}I_1+\frac{t}{m_u^2}I_2-\frac{1}{2m_\mu^2}\right)\right)\right]-\frac{e_{d_i}}{e}\,[m_u\leftrightarrow m_d]\right\},\tag{16}$$

where the factor that multiplies e_{d_i} is obtained from the one that multiplies e_{u_i} interchanging m_u and m_d. In the soft-pion limit this expression reduces to

$$F_A=\frac{gg_A}{2\sqrt{2}\pi^2 e}\,\frac{1}{m_u(\kappa^2-1)}\sum_i\left\{e_{u_i}\left[\frac{2}{\kappa^2-1}\left(2\,\frac{\kappa^2-2}{\kappa^2-1}+\kappa^2\left(\frac{\kappa^2}{2}\,\frac{1-\kappa}{(1-\kappa^2)}+1\right)\right)\ln\kappa\right.\right.$$
$$\left.\left.-\frac{1-\kappa}{2(\kappa^2-1)}\left(\frac{1}{3}(\kappa^2-1)+\frac{1}{\kappa^2-1}+\frac{3}{2}\right)-2\,\frac{\kappa^2}{\kappa^2-1}\right]+\kappa e_{d_i}[m_u\leftrightarrow m_d]\right\}.\tag{17}$$

In Fig. 4, we show the soft-pion limit for several models and compare it with the experimental value.[10] We remark that in order to compare $\gamma_{\rm exp}$ with $\gamma_{\rm model}$, one has to take into account the breakdown of the Vaks-Ioffe relation given in Sec. III.

The corrected values for $\gamma^+_{\rm exp}$ and $\gamma^-_{\rm exp}$, have been obtained using the formula

$$\gamma^\pm\left(\frac{m_d}{m_u}\right)=\frac{\gamma^++\gamma^-}{2}\pm\left[\left(\frac{\gamma^++\gamma^-}{2}\right)^2-1+(1-\gamma^+\gamma^-)\frac{F_V^2(m_d/m_u)}{F_V^2(1)}\right]^{1/2},\tag{18}$$

where γ^+ and γ^- in the right-hand side are those obtained from the experimental data using the Vaks-Ioffe relation. In obtaining Eq. (18) use was made of the fact that the interference term between the inner bremsstrahlung and structure-dependent parts of the amplitude is negligible for large electron and photon energies and large angles between them in $\pi\to e\nu\gamma$.[8]

In numerical estimations we show the results for $\gamma_{\rm exp}=-0.15\pm0.11$ or 2.07 ± 0.11 of the first paper in Ref. 10. If one takes $\gamma_{\rm exp}=-0.44\pm0.12$ or -2.36 ± 0.12, conclusions are not strongly modified.

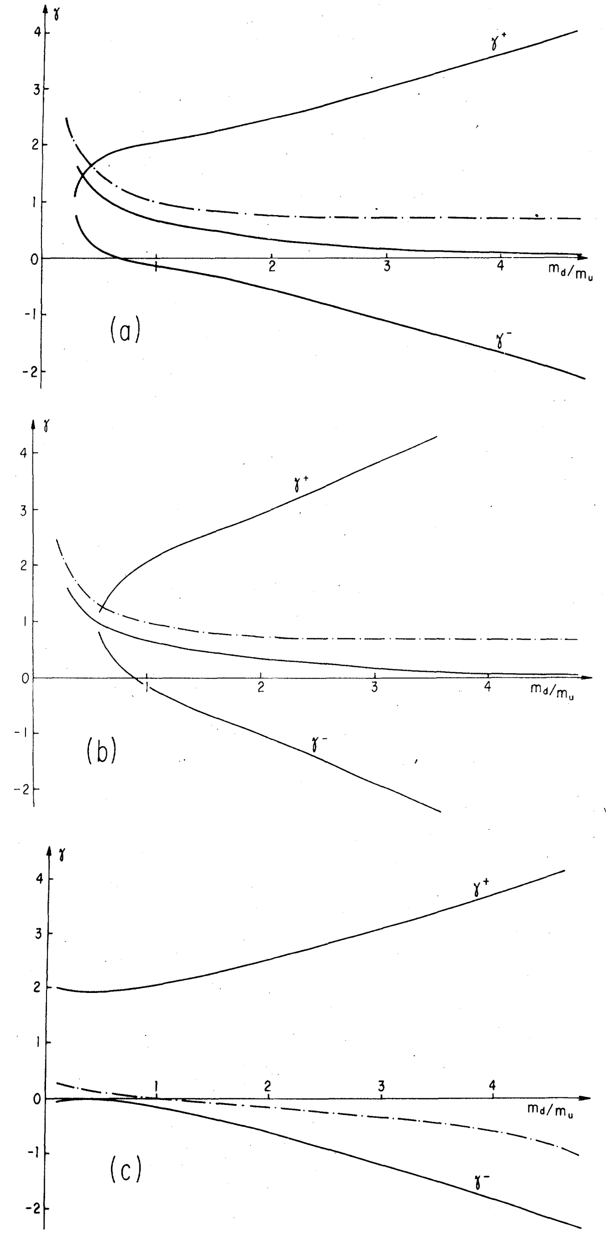

FIG. 4. Curves showing the corrected experimental γ values and the predictions in the soft-pion limit as function of m_d/m_u in the fractionally-charged-quark color-triplet model (a) and in the Han-Nambu-triplet-type model (b), with (continuous line) and without (point-dashed line) the σ meson, and in the SU(2) ×SU(2) σ model (c). Although there are allowed values in some of the models, they are in a region where $m_d < m_u$.

V. CONCLUDING REMARKS

In this work we show that the form factors $\mathfrak{F}_{\pi^0\to\gamma\gamma}$, F_A and F_V, relevant in the decays

$$\pi^0 \to \gamma\gamma ,$$

$$\pi \to e\nu\gamma ,$$

are very sensitive to the quark-mass ratio, m_d/m_u.

Because conserved vector current (CVC) is explicitly violated by the quark mass difference, the Vaks-Ioffe relation is severely broken. We show that this breaking is very important regardless of the absolute value of the quark mass parameter. This has immediate consequences for the experimental values of $\gamma = F_A/F_V$, which make use of the Vaks-Ioffe relation.[10]

Although none of the studied models explains the γ experimental values for $m_d > m_u$, all appear to indicate that the smallest γ is preferred. Moreover, it is possible to obtain negative theoretical values for this parameter without using any confinement mechanism. The better approximation is given by the σ SU(2) × SU(2) model.

It seems that the treatment developed here is applicable to the kaons and η radiative decays. But, it is not clear whether it is consistent or redundant to consider simultaneously the Cabibbo angle and the nondegeneracy of the quarks masses. This angle is possibly already taken into account by the quark-mass-difference term. This is irrelevant for the pion decay, but not for the other cases.

Finally, we would like to comment that if the quark masses that we are using in this problem are indeed the same as those in QCD, then Eq. (8) implies that the only possibility to get a zero u-quark mass[15] is to have at the same time $g = 0$. This will, of course, indicate that one must "decouple," because $g = 0$, the physical pion from the quarks, and that the only nonvanishing amplitudes would be those proportional to g/m_u, such as the ones in $\pi^0 \to \gamma\gamma$ or $\pi \to e\nu\gamma$.

Note added. While this manuscript was being typed we became aware of a paper by J. Bernabéu, R. Tarrach, and F. J. Ynduraîn [Phys. Lett. 79B, 464 (1978)]. They obtain a large isospin-breaking effect of 50% when $\xi = m_d/m_u$ is changed from 0 to infinity. Our Fig. 3 shows a much greater effect for this variation of ξ.

ACKNOWLEDGMENTS

We are pleased to acknowledge many illuminating discussions with G. Cocho on the subject of this paper. We also thank H. Moreno and A. Zepeda for clarifying discussions on the interpretation of the quark masses. We also thank the computational assistance we received from A. Mendoza.

APPENDIX

In this appendix we give the explicit expressions for the integrals that appear in Eq. (13). Defining

$$\delta=\frac{m_d^{\,2}-m_u^{\,2}}{m_u^{\,2}}\,,\quad \pi=\frac{p^2}{m_u^{\,2}}\,,\quad \tau=\frac{(k-p)^2}{m_u^{\,2}}\,,$$

we have

$$I_n=\frac{(-1)^n}{m_u^{\,2}}\,\frac{1}{\pi-\tau}\,(j_n^{(\pi)}-j_n^{(\tau)})\,, \tag{19}$$

where we have two cases.[15]

(i) For $(\alpha-\delta)^2\leq 4\alpha$, where $\alpha=\pi,\tau$, we get

$$j_0^{(\alpha)}=2\mathrm{Li}_2\left[\sqrt{\alpha}\,,\,\mathrm{arc\,cos}\frac{\alpha-\delta}{2\sqrt{\alpha}}\right]\,, \tag{20}$$

$$j_1^{(\alpha)}=\frac{1}{\alpha}\left[[4\alpha-(\alpha-\delta)^2]^{1/2}\,\mathrm{arc\,tan}\,\frac{[4\alpha-(\alpha-\delta)^2]^{1/2}}{2-(\alpha-\delta)}+\frac{\delta}{2}\ln(\delta+1)\right]\,, \tag{21}$$

$$j_2^{(\alpha)}=\frac{1}{2\alpha}\left[(\alpha+\delta)j_1^{(\alpha)}-\tfrac{1}{2}(\delta+2)\ln(\delta+1)-\delta\right]\,. \tag{22}$$

(ii) For $(\alpha-\delta)^2>4\alpha$ we obtain

$$j_0^{(\alpha)}=\mathrm{Li}_2\left[\frac{\alpha-\delta}{2}+\frac{1}{2}[(\alpha-\delta)^2-4\alpha]^{1/2}\right]+\mathrm{Li}_2\left[\frac{\alpha-\delta}{2}-[(\alpha-\delta)^2-4\alpha]^{1/2}\right]\,, \tag{23}$$

$$j_1^{(\alpha)}=\frac{1}{\alpha}\left[[(\alpha-\delta)^2-4\alpha]^{1/2}\times\ln\frac{\alpha-\delta+2-[(\alpha-\delta)^2-4\alpha]^{1/2}}{2[\delta+1]^{1/2}}+\frac{\delta}{2}\ln(\delta+1)\right]\,, \tag{24}$$

$$j_2^{(\alpha)}=\frac{1}{2\alpha}\left[(\alpha+\delta)j_1^{(\alpha)}-\tfrac{1}{2}(\delta+2)\ln(\delta+1)-\delta\right]\,, \tag{25}$$

where[16]

$$\mathrm{Li}_2(x)=-\int_0^x\frac{\ln(1-z)}{z}\,dz\,. \tag{26}$$

In the limit $p^2\to 0$, for which $0\leq\tau<\pi\to 0$, we get

$$I_0=\frac{1}{\delta^2 m_u^{\,2}}\,[(\delta+1)\ln(\delta+1)-\delta]\,, \tag{27}$$

$$I_1=\frac{1}{\delta^2 m_u^{\,2}}\,[(1+1/\delta)\ln(\delta+1)-(\tfrac{1}{2}\delta+1)]\,. \tag{28}$$

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