Magnitude of the cosmological baryon asymmetry

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(Received 6 June 1979)

We have examined the magnitude of the cosmological baryon asymmetry arising in several of the standard models of *CP* violation. Agreement with the experimental baryon to photon number ratio $n_B/n_\gamma \approx 10^{-8}$ is obtained in models where superheavy Higgs mesons decay with complex amplitude into other Higgs mesons. By contrast, in the Kobayashi-Maskawa model $n_B/n_\gamma \approx O(10^{-20})$.

I. INTRODUCTION

It was generally believed that the explanation for the matter-antimatter asymmetry of the universe lay in the simple fact that the universe originated with a nonzero net baryon number, which was conserved for all time. Recent examinations of this question[1-7] have led, however, to a possible new understanding of this asymmetry as having evolved from an originally symmetric state The chief new ingredient has been the introduction of grand unified models of strong, weak, and electromagnetic interactions. In these models quarks and leptons are placed on a similar footing and interactions are present which violate baryon (and lepton) number. As we shall discuss briefly there are at least two scenarios one can envision for the early universe ($T > M_{\text{Planck}}$): Either the net baryon number n_B is originally zero or baryon-number-violating interactions lead to an equilibrium state in which n_B becomes zero. In either case the universe must evolve from an $n_B \approx 0$ state into the present universe in which[8] $n_B \approx 10^{80}$. This is to be compared to the photon number, $n_\gamma \approx 10^{88}$, so that

$$n_B/n_\gamma \approx 10^{-8}. \tag{1.1}$$

There are three key ingredients[2] necessary for the evolution from the $n_B \approx 0$ state to the present asymmetric universe:

(a) microscopic violation of baryon number,
(b) CP (or equivalently T) violation,
(c) departure from thermal equilibrium.

The first occurs, as mentioned earlier, in grand unified models and is mediated by the interaction of superheavy, color-triplet bosons.[9] These, which we generically call X, may be either gauge bosons or Higgs bosons; they couple to both the ql and the $\overline{q}\overline{q}$ fermion channels (q is a quark and l a lepton) and have characteristic masses of order $10^{15\pm2}$ GeV.[10] CP violation is model dependent and may be introduced in a variety of ways, as we shall illustrate. The third key ingredient, departure from thermal equilibrium, was shown by Toussaint, Treiman, Wilczek, and Zee[2] to require something other than the scattering of ordinary (i.e., effectively massless) fermions since such processes have no mass threshold. Toussaint et al.[2] and Weinberg[6] have pointed out that the needed departure from equilibrium could be due to the decay of X particles as the temperature T falls below their mass M_X.

A possible scenario is the following[11]: (1) Starting at a temperature T of the order of the Planck mass M_P, X-mediated collisions have a rate Γ_c which is faster than H, the expansion rate of the universe. Since these processes include baryon-number-violating interactions, any baryon asymmetry originally present in the big bang will be effectively erased. During this period, characterized by $M_P > T > M_X$, the X decay rate and the X production rate by inverse decay are small compared to H. (2) As the temperature falls both H and Γ_c decrease while the X decay rate Γ_x increases. When $\Gamma_x \gtrsim H$ we reach a regime in which X decay is important. For M_X sufficiently heavy, e.g., $M_X \sim 10^{15}$ GeV, the inverse decay will always be smaller than the expansion rate and any baryon asymmetry due to X decays will persist throughout the later evolution.[12]

To relate the asymmetry to microscopic parameters we suppose, for example, that the X baryons decay into two channels with branching ratios r, $1-r$ and baryon number B_1, B_2; $\overline{X}$ bosons decay with different branching ratios $\overline{r}, 1-\overline{r}$ into channels with baryon number $-B_1, -B_2$. The average baryon number produced by decays of X and $\overline{X}$ is

$$\Delta B = \tfrac{1}{2}(r-\overline{r})(B_1 - B_2). \tag{1.2}$$

The observed baryon asymmetry is determined by ΔB and the density of X's at the time of decay:

$$\frac{n_B}{n_\gamma} = \left(\frac{N_X}{N}\right)\Delta B, \tag{1.3}$$

where N is the total number of helicity states and N_X is the number of type X.

With the above setting it is the task of particle physics to explain the magnitude of n_B/n_γ. For definiteness we shall consider the standard SU(5) grand unified model of Georgi and Glashow.[13] Typically $N_X/N \approx 10^{-2}$ in this model so that we need $\Delta B \approx 10^{-6}$ in order to obtain a value for n_B/n_γ in agreement with (1.1). Any tree approximation to the decay amplitude will give $r=\bar{r}$ because of CPT. We therefore look for an interference between a tree amplitude g_0 and an amplitude with one (or more) quantum loops. If we denote the one-loop amplitude by the product of a Feynman integral $I(s+i\epsilon)$ and a coupling strength g_1 then

$$\Delta B \propto \int d\omega |g_0 + g_1 I(s+i\epsilon)|^2 - \int d\omega |g_0^* + g_1^* I(s+i\epsilon)|^2$$

$$= 4\int d\omega \, \mathrm{Im}(g_1 g_0^*)\, \mathrm{Im}[I(s+i\epsilon)]\,. \tag{1.4}$$

Thus we require both complex couplings and an s-channel discontinuity to obtain $\Delta B \neq 0$.

The necessary CP violation can be introduced into X decay in a number of ways. We will show that the values obtained for n_B/n_γ are usually too small by several orders of magnitude because the needed complex coupling constants are Yukawa couplings, whose magnitude is typically $O(G_F{}^{1/2}M_q) \lesssim 10^{-2}$. One model, originally proposed by Weinberg,[14] in which the CP violation is introduced via Higgs quartic self-couplings [which can be $O(1)$], has the potential to give a larger value of n_B/n_γ. The relevant processes are the decay of superheavy Higgs bosons into lighter Higgs bosons, which then decay to fermions. We show that one can obtain through this mechanism the experimental value of $n_B/n_\gamma \approx 10^{-8}$. In fact, a value as large as 10^{-4} is possible. The chain of decays we propose, in which the CP violation occurs directly in the Higgs self-couplings, may occur in a wide variety of models.

II. THE KOBAYASHI-MASKAWA (KM) MODEL

If CP is not imposed on the Lagrangian then the Yukawa couplings will be complex. Kobayashi and Maskawa[15] observed that in a model with six or more quarks this will generally lead to CP violation in the $K^0-\bar{K}^0$ mass matrix. In SU(5) we have

$$\mathfrak{L}_{\mathrm{Yuk}} = (f_1^\dagger)_{mn}\bar{\psi}_{m,\alpha\beta}\chi_n^\alpha \phi^\beta + (f_2)_{mn}\epsilon_{\alpha\beta\mu\nu\lambda}\psi_m^{\alpha\beta}C\psi_n^{\mu\nu}\varphi^\lambda + \mathrm{H.c.} \tag{2.1}$$

where m,n are summed over the generations of fermion and C is the charge-conjugation matrix. Each generation consists of a right-handed, five-dimensional representation χ^α and a left-handed, ten-dimensional representation $\psi^{\alpha\beta}$. We divide the SU(5) indices $\alpha,\beta = 1,2,\ldots,5$ into flavor in-

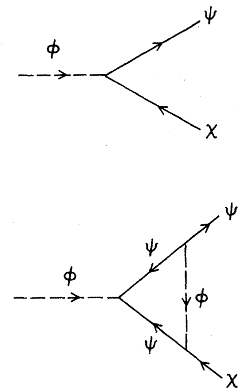

FIG. 1. A tree and one-loop diagram whose interference fails to give a ΔB in the KM model.

dices $i,j,k = 1,2$ and color indices $a,b,c = 3,4,5$.

The color-triplet, Higgs boson φ^a is superheavy ($\sim 10^{15}$ GeV) and decays into both ql and $\overline{qq}$. If r is the branching ratio into ql and $1-r$ the branching ratio into $\overline{qq}$, one might expect that the tree and one-loop diagrams in Fig. 1 would interfere to give $r \neq \bar{r}$. However, the tree diagram is proportional to $f_1^\dagger$ and the one-loop diagram to $f_2^\dagger f_2 f_1^\dagger$ so that summing over fermions gives

$$\Delta B \propto \mathrm{Im}\,\mathrm{Tr}(f_2^\dagger f_2 f_1^\dagger f_1) = 0\,.$$

To obtain a complex interference with the tree diagram one must actually go to a three-loop diagram[5] such as Fig. 2, which gives

$$\Delta B(\varphi\ \mathrm{decay}) \sim \frac{\mathrm{Im}\,\mathrm{Tr}(f_2^\dagger f_1 f_1^\dagger f_2 f_2^\dagger f_2 f_1^\dagger f_1)}{16\pi(8\pi^2)^2[\mathrm{Tr}(f_1^\dagger f_1) + \mathrm{Tr}(f_2^\dagger f_2)]}\,. \tag{2.2}$$

Even if we optimistically put all the $f_i \approx 10^{-2}$ and assume the imaginary part is maximal, this gives a hopelessly small $\Delta B \sim 10^{-18}$.

The color-triplet vector bosons $\tilde{W}_i^a$ are also superheavy and decay into ql and $\overline{qq}$. However, because the gauge couplings are real and diagonal in the original Lagrangian basis, it still requires eight Yukawa couplings to have an asymmetry.

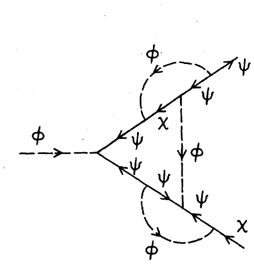

FIG. 2. A typical contribution to ΔB from ϕ decay in the KM model.

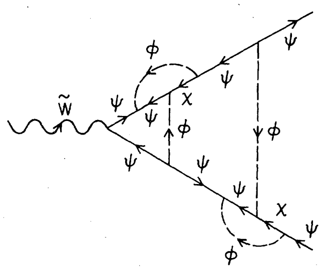

FIG. 3. A typical contribution to ΔB from $\tilde{W}$ decay in the KM model.

One such diagram is shown in Fig. 3. It involves precisely the same trace as (2.2) but has one more loop so that

$$\Delta B(\tilde{W}\text{ decay}) \sim \frac{g^2\,\mathrm{Im}\mathrm{Tr}(f_2^\dagger f_1 f_1^\dagger f_2 f_2^\dagger f_2 f_1^\dagger f_1)}{16\pi(8\pi^2)^3 g^2}\,.$$

This seems to be a general result, valid in all models of CP violation, that the vector-boson decay gives a smaller asymmetry ($f^2/8\pi^2$) than the scalar decay. In the later models we will therefore not discuss the $\tilde{W}$ interactions at all.

III. THE WEINBERG (THREE-HIGGS) MODEL

Weinberg pointed out in a four-quark model that the quartic self-couplings of Higgs mesons could violate CP invariance if there were three or more Higgs multiplets.[14] To ensure the natural conservation of quark flavors in neutral currents, only two of the Higgs can couple to fermions:

$$\mathcal{L}_{\mathrm{Yuk}} = (f_1^\dagger)_{mn}\bar{\psi}_{m,\alpha\beta}\chi_n^\alpha\varphi_1^\beta + (f_2)_{mn}\epsilon_{\alpha\beta\mu\nu\lambda}\psi_m^{\alpha\beta}C\psi_n^{\mu\nu}\varphi_2^\lambda + \text{H.c.} \tag{3.1}$$

The superheavy color triplets φ_1^a and φ_2^a have different fermionic decays. In particular, φ_1^a has four decay modes ($u^a e^-$, $d^a\nu$, $\bar{u}_b\bar{d}_c$, $\bar{u}_c\bar{d}_b$ with a,b,c cyclic) but φ_2^a has only three modes ($u_a e^-$, $\bar{u}_b\bar{d}_c$, $\bar{u}_c\bar{d}_b$) because ν appears only in the χ multiplet. The average baryon numbers produced by φ_1^a and φ_2^a, respectively, are

$$\begin{aligned} B_1 &= \tfrac{1}{4}(\tfrac{1}{3}+\tfrac{1}{3}-\tfrac{2}{3}-\tfrac{2}{3}) = -\tfrac{1}{6}\,,\\ B_2 &= \tfrac{1}{3}(\tfrac{1}{3}-\tfrac{2}{3}-\tfrac{2}{3}) = -\tfrac{1}{3}\,. \end{aligned} \tag{3.2}$$

As yet we have not specified the source of CP violation. If this were just an extended Kobayashi-Maskawa scheme then the baryon asymmetry would be much smaller than (2.2) because $\varphi_1 \neq \varphi_2$ prevents the existence of even three-loop diagrams such as Fig. 2. However, in the Weinberg scheme there is a third Higgs field φ_3, that does not couple to fermions, with interactions

$$\begin{aligned} V(\varphi) = {}& M_r^{\,2}(\varphi_r^\dagger\varphi_r) + a_{rs}(\varphi_r^\dagger\varphi_r)(\varphi_s^\dagger\varphi_s)\\ &+ b_{rs}(\varphi_r^\dagger\varphi_s)(\varphi_s^\dagger\varphi_r) + c_{rs}(\varphi_r^\dagger\varphi_s)(\varphi_r^\dagger\varphi_s)\,, \end{aligned} \tag{3.3}$$

where r,s are summed from 1 to 3. Hermiticity requires that a_{rs} and b_{rs} be real and symmetric but only that c_{rs} be Hermitian. One can always define away the phase in c_{12}, for example, but the product $c_{12}c_{23}c_{31}$ is generally complex after all redefinitions. The mass terms for the color triplets φ_1^a, φ_2^a, φ_3^a come from the superheavy vacuum expectation value (~10^{15} GeV) of the $\underline{24}$ and are automatically diagonal because of the discrete symmetries necessary to ensure that φ_1, φ_2, φ_3 have distinct Yukawa couplings. The vacuum expectation values of the φ are so small (~100 GeV) that we may safely ignore their contribution to masses of the color triplets.

A particularly simple scenario is obtained if we choose the superheavy masses to satisfy $M_3 > M_2 > M_1$.[16] Then φ_3^a has two decay channels: φ_1^a plus two massless Higgs mesons or φ_2^a plus two massless Higgs mesons; and these channels go to different baryon numbers B_1 and B_2. The decays of φ_3^a will violate CP invariance because of the complex c_{rs} in (3.3). The Born amplitude for $\varphi_3^a \to \varphi_1^a + \varphi_1^j + \bar{\varphi}_3^j$ is c_{13} and will interfere with the one-loop diagram in Fig. 4(a) to give

$$r_{3\to1} - \bar{r}_{3\to1} \propto 4\int d\omega\,\mathrm{Im}(c_{12}c_{23}c_{31})\,\mathrm{Im}[I_2(s+i\epsilon)]\,, \tag{3.4}$$

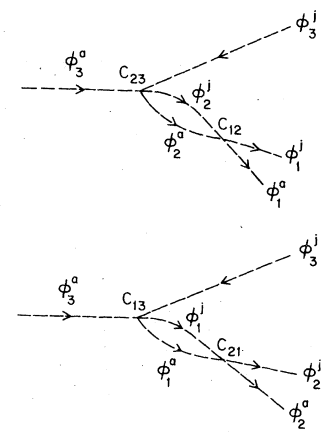

FIG. 4. Contributions to ΔB from ϕ_3 decay in the Weinberg (three-Higgs) model with $M_3 > M_2 > M_1$.

where I is the one-loop Feynman integral. Similarly, the Born amplitude for $\varphi_3^a \to \varphi_2^a + \varphi_2^j + \bar{\varphi}_3^j$ is c_{23} and will interfere with the one-loop diagram in Fig. 4(b) to give

$$r_{3\to 2} - \bar{r}_{3\to 2} \propto -4 \int d\omega\, \mathrm{Im}(c_{12}c_{23}c_{31})\, \mathrm{Im}[I_1(s+i\epsilon)]. \tag{3.5}$$

Explicit calculation verifies that

$$r_{3\to 1} + r_{3\to 2} = \bar{r}_{3\to 1} + \bar{r}_{3\to 2}$$

as guaranteed by the CPT theorem.

To calculate ΔB we evaluate the integrals over three-body phase space in (3.4) and obtain

$$\Gamma_{3\to 1} - \bar{\Gamma}_{3\to 1} = \frac{\mathrm{Im}(c_{12}c_{23}c_{31})}{64(2\pi)^4(M_3)^3}\left[(M_3)^4 - (M_2)^4 + 4M_1^2M_3^2 - 4M_1^2M_2^2 - 2(M_1^2M_2^2 + M_1^2M_3^2 + M_2^2M_3^2)\ln\left(\frac{M_3}{M_2}\right)^2\right]. \tag{3.6}$$

For simplicity we shall neglect M_1 and M_2 with respect to M_3. In this approximation the total decay rate is

$$\Gamma_3 \approx \left[|c_{13}|^2 + |c_{23}|^2 + |b_{13}|^2 + |b_{23}|^2 + O\left(b_{ij}g^2\left(\frac{M_3}{\tilde{M}}\right)^2\right) + O\left(g^4\left(\frac{M_3}{\tilde{M}}\right)^4\right)\right]\frac{M_3}{32(2\pi)^3},$$

and using $\frac{1}{2}(B_1 - B_2) = \frac{1}{12}$ we obtain

$$\Delta B \approx \left(\frac{1}{48\pi}\right)\frac{\mathrm{Im}(c_{12}c_{23}c_{31})}{|c_{13}|^2 + |c_{23}|^2 + |b_{13}|^2 + |b_{23}|^2}.$$

This can give quite a large baryon asymmetry. The upper bound is attained when $b \ll c$ so that $\Delta B \le c/96\pi$ or

$$\frac{n_B}{n_\gamma} < 10^{-4}c.$$

Quartic couplings such as c_{rs} are presumably less than 1 so as not to invalidate perturbation theory,[17] but this still gives a comfortable upper bound of 10^{-4} for the baryon asymmetry.

To summarize, the asymmetry arises from the fact that though the number of φ_3 and $\bar{\varphi}_3$'s are equal, their decay leads to an excess of φ_1 over $\bar{\varphi}_1$ and a matching excess of $\bar{\varphi}_2$ over φ_2 because of CP violation. Since both are out of equilibrium the imbalance persists when φ_1 and φ_2 decay to fermionic states with different baryon number.

IV. CONCLUSIONS

We have shown, within the framework of a given scenario for the evolution of the universe, that a satisfactory value for n_B/n_γ can be obtained and that this ratio is quite sensitive to various ingredients of gauge theories, namely how CP is introduced and how the Higgs mesons couple. In appendices A and B we discuss some other modes of CP violation.

It would be impressive if one could not only obtain the magnitude of n_B/n_γ, but predict the sign of the asymmetry to understand why the universe is made of matter instead of antimatter. We have tried, but failed to relate the sign of the asymmetry to the parameters of CP violation, as determined by K-meson decay. The problem is that the arbitrary sign of the Higgs-meson couplings prevents a direct comparison.

ACKNOWLEDGMENTS

We would like to thank Professor A. Zee for encouragement and helpful discussions. The work of S. M. B. and G. S. was supported in part by the U. S. Department of Energy under Contract No. EY-76-C-02-3071, that of H. A. W. by the National Science Foundation.

APPENDIX A: RELAXING THE CONSTRAINT OF FLAVOR CONSERVATION

In the previous examples we have allowed one Higgs meson to couple to $\bar{\psi}\chi$ and only one Higgs meson (either the same or different) to couple to $\psi C\psi$ in order that all neutral Higgs-meson exchanges would automatically conserve strangeness, charm, and other quark flavors.[18] If, however, we relax this condition it may still be possible that Yukawa couplings such as

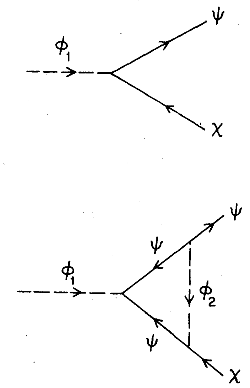

FIG. 5. A simple tree and one-loop diagram whose interference does give $\Delta E \neq 0$ when flavor conservation is abandoned.

$$\mathcal{L}_{\text{Yuk}} = \bar{\psi}_{m,\alpha\beta}\chi_n^\alpha[(f_1^\dagger)_{mn}\varphi_1^\beta + (g_2^\dagger)_{mn}\varphi_2^\beta] + \epsilon_{\alpha\beta\mu\nu\lambda}\psi_m^{\alpha\beta}C\psi_n^{\mu\nu}[(g_1)_{mn}\varphi_1^\lambda + (f_2)_{mn}\varphi_2^\lambda] + \text{H.c.}$$

only violate flavor conservation in an acceptably small way. [Note that (3.1) corresponds to $g_1 = g_2 = 0$.]

In such a model it may be possible to obtain a reasonable baryon asymmetry. For example, the tree diagram in Fig. 5 is proportional to $f_1^\dagger$ and the one-loop diagram to $f_2^\dagger g_1 g_2^\dagger$ so that summing over fermions gives

$$\Delta B(\varphi_1 \text{ decay}) \approx \frac{\text{Im Tr}(f_2^\dagger g_1 g_2^\dagger f_1)}{16\pi[\text{Tr}(f_1^\dagger f_1) + \text{Tr}(g_1^\dagger g_1)]} .$$

For large Yukawa couplings $f \approx g \approx 10^{-2}$ this might give an asymmetry as large as required by (1.1). (Of course, larger Yukawa couplings could result from hitherto unobserved heavy quarks.)

APPENDIX B: SPONTANEOUS *CP* VIOLATION AT LARGE *T*

Up until now all CP violation has come from explicit violation in the Lagrangian via complex couplings. It is also possible for CP to be an invariance of the Lagrangian that is broken spontaneously by the vacuum expectation values $\langle\varphi\rangle \approx 100$ GeV. Usually at $kT \gg 100$ GeV there is no spontaneous symmetry breaking (i.e., $\langle\varphi\rangle = 0$), however, it is possible to arrange the signs of certain quartic couplings so that the spontaneous symmetry breaking, and also the CP violation, persists at arbitrarily high temperature with $\langle\varphi\rangle \propto kT$.[19] This scheme requires a minimum of three Higgs fields: Two acquire vacuum expectation values with different phases and the third has some quartic couplings negative so that the vacuum expectation values of the radiatively corrected potential will grow with kT.

Finally the CP noninvariance arises because the physical (mass eigenstate) Higgs fields are complex linear combinations of the original fields. Consequently the physical Higgs particles have complex Yukawa couplings to the fermions and the situation effectively reduces to that of Appendix A. The baryon asymmetry is also similar,

$$\Delta B \approx O\left(\frac{f^2}{16\pi}\right)\left(R\frac{kT}{M_X}\right)^4 ,$$

except for the suppression factor due to the Higgs mixing. For such decays kT is only slightly smaller than M_X but R is a model-dependent ratio of Higgs couplings.

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[12]If M_X is too small both the direct and the inverse decay rates will exceed the expansion rate during some temperature interval. During this period X and $\bar{X}$ will remain in thermal equilibrium. When they finally depart from equilibrium there will be too few of them left to produce a significant asymmetry.

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