Mechanisms for cosmological baryon production

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General formulas are given for the mean net baryon number produced in the decay of superheavy scalar or vector bosons. These results are used to make rough numerical estimates of the cosmological baryon abundance that would result from such decay processes in the very early universe.

I. INTRODUCTION

The universe appears to have a baryon-number density that is nonzero but small. Quantitatively, assuming that all galaxies are composed of matter rather than antimatter,[1] the ratio of the baryon-number density n_B to the dimensionless entropy density s/k of the 3 °K microwave background is of order[2] 10^{-8} to 10^{-10}. If baryon number is conserved and the expansion of the universe is essentially adiabatic, then the quantity $n_B k/s$ is a constant, which governs the whole course of cosmic evolution. Thus it is an important matter to learn why this ratio is not zero, and why, though not zero, it is so small.

Recently a number of authors[3-10] have considered the possibility that the cosmic baryon-number excess was produced by physical baryon-number-nonconserving processes, which are cosmologically insignificant at present, but may have occurred at significant rates in the very early universe. It has become clear that in order to produce an appreciable baryon excess, it is necessary not only that some reactions violate baryon-number and CP conservation, but also that these reactions occurred at a time when the expansion of the universe had already pulled the cosmic particle distributions out of the equilibrium form.

The simplest way that this can happen[11] is for an equilibrium distribution to be established[12] for some heavy "X boson" at $kT \gg m_X$, with equal numbers of X and its antiparticle $\overline{X}$, and for equilibrium then to be lost when kT drops below m_X, because the decay rates of X and $\overline{X}$ are less than the rate of expansion of the universe at that time. When the X and $\overline{X}$ finally decay, at temperatures $kT \ll m_X$ which are low enough to prevent inverse decay, the baryon-entropy ratio produced will be[8]

$$kn_B/s = 45\zeta(3)(N_X/N)\Delta B/2\pi^4 ,$$

where ΔB is the mean baryon number produced in the decay of a single X or $\overline{X}$ boson, and N_X and N are the (suitably weighted) numbers of species of X bosons and of all particles with masses $m \lesssim m_X$, respectively.

In order to calculate the crucial quantity ΔB, we need a specific theory of baryon nonconservation. A class of such theories has been provided over the last few years by the grand unified gauge models, which unite the strong with the weak and electromagnetic interactions.[13] There is as yet no one grand unified model that clearly is realized in nature, so we choose here to work in a more general theoretical framework. Our main assumption is that there is some simple grand unified gauge group, whose spontaneous breakdown at the grand unification scale leaves unbroken only the gauge groups SU(3) and SU(2)×U(1) of the observed strong and weak and electromagnetic interactions.

As recognized some time ago,[14] this general framework provides a natural explanation for the fact that baryon-nonconserving processes are so slow at ordinary energies. The masses of those gauge bosons of the grand unified group which are not associated with SU(3) or SU(2) × U(1), and in particular of the bosons which mediate baryon nonconservation, are roughly of the order of the critical energy M where the strong and weak and electromagnetic couplings merge into the single coupling of the grand gauge group. But the decrease of the strong-interaction coupling is so slow that M must be enormous, and the proton lifetime, which is proportional to M^4, must be correspondingly long. Specifically, if we fix the ratios of the SU(3) and SU(2)× U(1) couplings at M by the assumption that there is a representation of some grand unified gauge group consisting solely (or chiefly) of quark-lepton families like those already observed, and take the observed values of e and the quantum-chromodynamic scale parameter Λ as an input, then M is found[14,15] to be of order 10^{16} GeV. (The same analysis[14,15] yields a Z^0-γ mixing parameter $\sin^2\theta$ between 0.19 and 0.21, only a little lower than the present experimental value $\sin^2\theta = 0.23 \pm 0.02$.)

These considerations lead us to assume that the superheavy vector and scalar bosons that mediate baryon-nonconserving reactions have masses in the range of 10^{14} to 10^{16} GeV.[16] For vector bosons, this is probably too low to allow the production of an appreciable baryon excess. As remarked in Refs. 6 and 8, a gauge boson with mass $m_X > kT$ will have a decay rate of order $\alpha m_X N$, so that these bosons decay when $\alpha m_X N$ becomes equal to the cosmic expansion rate $H = 1.66\ (kT)^2 N^{1/2}/m_P$, where $m_P = 1.22 \times 10^{19}$ GeV. This occurs at a temperature $kT \simeq (N^{1/2}\alpha m_X m_P)^{1/2}$, which is smaller than m_X only if m_X is above a value $N^{1/2}\alpha m_P \approx 10^{17} N^{1/2}$ GeV. On the other hand, for Higgs bosons we must replace α with $G_F \bar{m}^2/4\pi$, where $\bar{m}$ is an rms quark or lepton mass; for $\bar{m} \simeq 2$ GeV, the Higgs bosons will decay at temperatures kT which are below their mass m_X provided that m_X is greater than $N^{1/2} G_F \bar{m}^2 m_P / 4\pi \approx 3 \times 10^{13} N^{1/2}$ GeV. We will consider the decays of both superheavy gauge and Higgs bosons here, but it is the Higgs-boson decays that seem most relevant for cosmological baryon production.

At energies of the order of the superheavy gauge and Higgs bosons, it is a very good approximation to neglect the spontaneous breakdown of SU(2) $\times$ U(1) to $\mathrm{U}(1)_{\mathrm{em}}$, so that particle states and interactions can be analyzed using SU(3)$\times$SU(2)$\times$U(1) as if it were unbroken. In this way, it has been possible to classify the vector and scalar bosons that can mediate baryon nonconservation in general theories.[17] This classification is reviewed in Sec. II, and SU(3)$\times$SU(2)$\times$U(1) is used to give explicit forms for the most general baryon-violating boson-fermion interactions that can arise in renormalizable theories.

In Sec. III, we use the results of Sec. II to give general results for the mean baryon excess ΔB produced per X or $\bar{X}$ boson decay. This calculation is aided by a general theorem proved in an appendix, which indicates that graphs of first order in baryon-violating interactions but of arbitrary order in baryon-conserving interactions make no contribution to ΔB. We find that in general ΔB will receive its leading contributions from the interference of tree graphs with one-loop graphs in which a boson with baryon-violating interactions is exchanged between the fermions in the final state.[18]

Finally, in Sec. IV we apply this analysis in simple cases, and obtain rough numerical estimates for ΔB and kn_B/s. Our conclusions are stated in Sec. V.

II. PARTICLE SPECIES AND INTERACTIONS

The processes of interest to us in this paper occur at enormous temperatures, very much higher than the masses of the $W^{\pm}$ and Z^0. At such temperatures, it is an excellent approximation to neglect the spontaneous breakdown of SU(2)$\times$U(1) to electromagnetic gauge invariance and treat SU(2)$\times$U(1) as well as SU(3) color as an unbroken symmetry. In this section we will describe the SU(3)$\times$SU(2)$\times$U(1) classification[17] of the particle species that will be of relevance to us, and we will give general expressions for their mutual interactions.

First, there are the "ordinary" leptons and quarks. These apparently form sequences, with left-handed fermion fields

$$l_{aL} \equiv \begin{pmatrix} \nu_a \\ e_a \end{pmatrix}_L (1, 2, \tfrac{1}{2}), \quad \bar{e}_{aL}(1, 1, -1),$$

$$q_{aL} \equiv \begin{pmatrix} u_a \\ d_a \end{pmatrix}_L (3, 2, -\tfrac{1}{6}), \quad \bar{u}_{aL}(\bar{3}, 1, +\tfrac{2}{3}), \quad \bar{d}_{aL}(\bar{3}, 1, -\tfrac{1}{3}). \tag{1}$$

$$e_1 = e, \quad e_2 = \mu, \quad e_3 = \tau, \ldots$$

$$u_1 = u, \quad u_2 = c, \quad u_3 = t, \ldots$$

$$d_1 = d, \quad d_2 = s, \quad d_3 = b, \ldots .$$

In the usual notation, subscripts L and R indicate multiplication with $\frac{1}{2}(1+\gamma_5)$ and $\frac{1}{2}(1-\gamma_5)$, and the numbers in parentheses give the SU(3) multiplicity, the SU(2) multiplicity, and the value of the U(1) quantum number $Y \equiv T_3 - Q$.

The only renormalizable interactions of a vector field V^μ with a pair of fermion fields (here including antifermion fields) are of the form $V^{\mu\dagger}\psi_{1R}^T \gamma_\mu \psi_{2L}$. Therefore, we can make a complete list of all vector bosons that can couple to a pair of ordinary fermions by multiplying together all left-handed fields of leptons, quarks, antileptons, and antiquarks with all right-handed fields and adding up their SU(3)$\times$SU(2)$\times$U(1) quantum numbers. In a similar way, the only renormalizable interactions of a scalar field S with a pair of fermion fields are of the form $S^\dagger \psi_{1L}^T \psi_{2L}$ or $S^\dagger \psi_{1R}^T \psi_{2R}$, so we can catalog all scalar bosons that couple to ordinary fermions by multiplying all left-handed fields of leptons, quarks, antileptons, and antiquarks with each other, and the same for the right-handed fermion fields.

These lists of possible vector or scalar fields have an interesting feature[17] that greatly simplifies discussions of baryon nonconservation. Almost all of the scalar and vector fields that can couple to a pair of ordinary fermions couple only to channels with a single value of the baryon number and a single value of the lepton number. Such bosons can be assigned a baryon number and a lepton number in such a way that these quantities are conserved in the boson-fermion interactions. The only bosons which couple to two-fermion channels with varied baryon and/or lepton numbers are

$(3, 2, \frac{5}{6})$ vectors X_V: charges $-\frac{1}{3}, -\frac{4}{3}$,

$(3, 2, -\frac{1}{6})$ vectors X_V': charges $\frac{2}{3}, -\frac{1}{3}$,

$(3, 1, \frac{1}{3})$ scalars X_S: charge $-\frac{1}{3}$,

plus the corresponding antibosons. Using SU(3) ×SU(2)×U(1), we easily see that the coupling of these bosons to ordinary fermions must take the form

$$g_{\chi,ab}(\overline{l}_{Laj}\gamma_\mu d^c_{Rb\alpha})V^\mu_{\chi\alpha j}+\text{H.c.}, \qquad (2)$$

$$h_{\chi,ab}\epsilon_{jk}\epsilon_{\alpha\beta\gamma}(\overline{u}^c_{Ra\alpha}\gamma_\mu q_{Lb\beta j})V^\mu_{\chi\gamma k}+\text{H.c.}, \qquad (3)$$

$$j_{\chi,ab}(\overline{q}_{La\alpha j}\gamma_\mu e^c_{Rb})V^\mu_{\chi\alpha j}+\text{H.c.}, \qquad (4)$$

$$g'_{\eta,ab}(\overline{l}_{Laj}\gamma_\mu u^c_{Rb\alpha})V'^\mu_{\eta\alpha j}+\text{H.c.}, \qquad (5)$$

$$h'_{\eta,ab}\epsilon_{jk}\epsilon_{\alpha\beta\gamma}(\overline{d}^c_{Ra\alpha}\gamma_\mu q_{Lb\beta j})V'^\mu_{\eta\gamma k}+\text{H.c.}, \qquad (6)$$

$$F^{(1)}_{\xi,ab}(\overline{q}_{La\alpha j}l^c_{Lbk})S_{\xi\alpha}\epsilon_{jk}+\text{H.c.}, \qquad (7)$$

$$F^{(2)}_{\xi,ab}(\overline{u}^c_{Ra\alpha}d_{Rb\beta})S_\xi\epsilon_{\alpha\beta\gamma}+\text{H.c.}, \qquad (8)$$

$$G^{(1)}_{\xi,ab}(\overline{u}_{Ra\alpha}e^c_{Rb})S_{\xi\alpha}+\text{H.c.}, \qquad (9)$$

$$\tfrac{1}{2}G^{(2)}_{\xi,ab}(\overline{q}^c_{La\alpha j}q_{Lb\beta k})S_{\xi\gamma}\epsilon_{jk}\epsilon_{\alpha\beta\gamma}+\text{H.c.} \qquad (10)$$

In the notation used here, χ, η, and ξ label various species of X_V, X_V', and X_S bosons of each SU(3) × SU(2) × U(1) type, a and b label fermions in the sequences (1), α, β, and γ are SU(3) indices, j and k are SU(2) indices, $\epsilon_{\alpha\beta\gamma}$ and ϵ_{jk} are the totally antisymmetric SU(3) and SU(2) tensors, with $\epsilon_{123} \equiv \epsilon_{12} \equiv +1$, and c denotes the Lorentz-invariant complex conjugation of fermion fields. The anticommutativity of fermion fields yields

$$G^{(2)}_{\xi,ab}=G^{(2)}_{\xi,ba}\,. \qquad (11)$$

So far, we have made no use of grand unified gauge theories. Such theories impose relations among the various vector and scalar couplings in Eqs. (2)–(10). As an example, let us explore the consequences of the assumption that the grand unified gauge group contains SU(5)[13] as a subgroup (not necessarily less strongly broken than the rest of the group) and that the left-handed fermions in (1) fall into the representations $\underline{\overline{5}}$ and $\underline{10}$ of SU(5).

X_V. The X_V bosons couple to fermion pairs forming the SU(5) representations $\underline{\overline{5}}\times\underline{5}$ in Eqs. (2) and $\underline{\overline{10}}\times\underline{10}$ in Eqs. (3) and (4). Thus these bosons must belong to the SU(5) representations $\underline{24}$ or $\underline{75}$. If they all belong to the $\underline{24}$ representations, then the $\underline{10}\times\underline{\overline{10}}$ couplings are related by

$$h_{\chi,ab}=j_{\chi,ab}. \qquad (12)$$

Further, if there is only one species of X_V bosons, which forms part of the multiplet of SU(5) gauge bosons [as is the case in grand unified theories[13] based on SU(5) and SO(10)], then the couplings are further constrained by

$$g_{ab}=-\tfrac{1}{2}h_{ab}=-\tfrac{1}{2}j_{ab}=g_0\delta_{ab}. \qquad (13)$$

X_V'. The X_V' bosons couple to fermion pairs forming the SU(5) representations $\underline{5}\times\underline{10}$, so they must belong to the SU(5) representations $\underline{\overline{10}}$ or $\underline{40}$. If they all belong to the $\underline{\overline{10}}$ representation, then their couplings are constrained by

$$g'_{\eta,ab}=-h'_{\eta,ab}. \qquad (14)$$

Further, if the grand unified gauge group contains SO(10) (Ref. 13) as a subgroup, and if there is only one species of X_V' bosons, which forms part of the multiplet of SO(10) gauge bosons, then

$$g'_{ab}=-h'_{ab}=g_0\delta_{ab}. \qquad (15)$$

Of course, in an SU(5) theory, there is no X_V'.

X_S. The X_S bosons couple to fermion pairs forming the SU(5) representations $\underline{5}\times\underline{10}$ in Eqs. (7) and (8), and $\underline{10}\times\underline{10}$ in Eqs. (9) and (10). Hence these bosons must belong to the SU(5) representations $\underline{\overline{5}}$, $\underline{45}$, $\underline{50}$. If they belong solely to the $\underline{5}$ representation, then their couplings are related by

$$F^{(1)}_{\xi,ab}=F^{(2)}_{\xi,ab}\equiv F_{\xi,ab}, \qquad (16)$$

$$G^{(1)}_{\xi,ab}=-G^{(2)}_{\xi,ab}\equiv G_{\xi,ab}\,. \qquad (17)$$

III. BARYON PRODUCTION IN BOSON DECAY

We want to calculate the mean baryon number produced in the decays of one of the X_V, X_V', or X_S bosons and the corresponding antibosons. Each of the bosons X_V, X_V', and X_S has decay modes of the type $X\to QL$ and $X\to\overline{Q}\overline{Q}$, where Q denotes an arbitrary quark and L denotes an arbitrary lepton; the antibosons have decay modes $\overline{X}\to\overline{Q}\overline{L}$ and $\overline{X}\to QQ$. The branching ratios for $X\to QL$, $X\to\overline{Q}\overline{Q}$, $\overline{X}\to\overline{Q}\overline{L}$, and $\overline{X}\to QQ$ will be denoted r, $1-r$, $\overline{r}$, and $1-\overline{r}$, respectively. The mean net baryon number produced in X and $\overline{X}$ decay is then

$$\Delta B=\tfrac{1}{2}[\tfrac{1}{3}r-\tfrac{2}{3}(1-r)-\tfrac{1}{3}\overline{r}+\tfrac{2}{3}(1-\overline{r})]=\tfrac{1}{2}(r-\overline{r}). \qquad (18)$$

Hence our task is to calculate the difference in the branching ratios for boson and antiboson decay.

In carrying out this calculation, we are guided by the theorem proved in the Appendix, which shows that $r-\overline{r}$ can receive no contribution from graphs which are of first order in baryon-violating interactions, even if the graphs involve an arbitrary number of baryon-conserving interactions.[19] We therefore calculate the decay amplitudes for $X_V\to QL$, $X_V'\to QL$, and $X_S\to QL$, including both tree graphs and the one-loop graphs in which a X_V, X_V', or X_S boson is exchanged between the final fermions. The relevant Feynman diagrams are shown in Figs. 1 and 2. A straightforward calculation gives the $X\to QL$ decay amplitudes (in the notation of Sec. II)

$$A(V_{\chi\alpha j}\to l_{La j}+d_{Rb\alpha})=(g_\chi)_{ab}-\sum_\eta (g'_\eta h_\chi h'^\dagger_\eta)_{ab}I_{VV}(m_\eta/m_\chi)-\sum_\xi (F_\xi^{(2)\dagger}h_\chi F_\xi^{(1)})_{ba}I_{VS}(m_\xi/m_\chi), \tag{19}$$

$$A(V_{\chi\alpha j}\to q_{La\alpha j}+e_{Rb})=(j_\chi)_{ab}+\sum_{\chi'}(h^\dagger_{\chi'}h_\chi j_{\chi'})_{ab}I_{VV}(m_{\chi'}/m_\chi)+\sum_\xi (G_\xi^{(2)\dagger}h_\chi^T G_\xi^{(1)})_{ab}I_{VS}(m_\xi/m_\chi), \tag{20}$$

$$A(V'_{\eta\alpha j}\to l_{La j}+u_{Rb\alpha})=(g'_\eta)_{ab}-\sum_\chi (g_\chi h'_\eta h^\dagger_\eta)_{ab}I_{VV}(m_\chi/m_\eta)+\sum_\xi (F_\xi^{(2)*}h'_\eta F_\xi^{(1)})_{ba}I_{VS}(m_\xi/m_\eta), \tag{21}$$

$$A(S_{\xi\alpha}\to q_{La\alpha j}+l_{Lbk})=\epsilon_{jk}\Big[F^{(1)}_{\xi,ab}-\sum_\chi (h^\dagger_\chi F_\xi^{(2)}g^T_\chi)_{ab}I_{SV}(m_\chi/m_\xi)+\sum_\eta (h'^\dagger_\eta F_\xi^{(2)T}g'^T_\eta)_{ab}I_{SV}(m_\eta/m_\xi)$$
$$-\sum_{\xi'}(G_{\xi'}^{(2)\dagger}G_\xi^{(2)}F_{\xi'}^{(1)})_{ab}I_{SS}(m_{\xi'}/m_\xi)\Big], \tag{22}$$

$$A(S_{\xi\alpha}\to u_{Ra\alpha}+e_{Rb})=G^{(1)}_{\xi,ab}+2\sum (h^*_\chi G_\xi^{(2)}j_\chi)_{ab}I_{SV}(m_\chi/m_\xi)-\sum_{\xi'}(F_{\xi'}^{(2)*}F_\xi^{(2)T}G_{\xi'}^{(1)})_{ab}I_{SS}(m_{\xi'}/m_\xi). \tag{23}$$

Here I_{VV}, I_{VS}, I_{SV}, and I_{SS} are the Feynman integrals for vector exchange in vector decay, scalar exchange in vector decay, vector exchange in scalar decay, and scalar exchange in scalar decay, respectively. For the corresponding antiparticle processes, we may simply replace all coupling constants with their complex conjugates. By taking the difference of the absolute-value squares of the amplitudes (19)–(23) for particles and antiparticles, summing over fermion labels a and b [and, for (22), j and k as well], and dividing by the corresponding sums that appear in the tree approximation for the total rate, one obtains the difference in branching ratios

$$r(V_\chi\to QL)-r(\overline{V}_\chi\to\overline{QL})=4[\mathrm{Tr}(g^\dagger_\chi g_\chi)+2\,\mathrm{Tr}(h^\dagger_\chi h_\chi)+\mathrm{Tr}(j^\dagger_\chi j_\chi)]^{-1}$$
$$\times\Big[\sum_\eta \mathrm{Im}\,\mathrm{Tr}(g^\dagger_\chi g'_\eta h_\chi h'^\dagger_\eta)\,\mathrm{Im}I_{VV}(m_\eta/m_\chi)+\sum_\xi \mathrm{Im}\,\mathrm{Tr}(g^*_\chi F_\xi^{(2)\dagger}h_\chi F_\xi^{(1)})\,\mathrm{Im}I_{VS}(m_\xi/m_\chi)$$
$$-\sum_{\chi'}\mathrm{Im}\,\mathrm{Tr}(j^\dagger_\chi h^\dagger_{\chi'}h_\chi j_{\chi'})\,\mathrm{Im}I_{VV}(m_{\chi'}/m_\chi)-\sum_\xi \mathrm{Im}\,\mathrm{Tr}(j^\dagger_\chi G_\xi^{(2)\dagger}h^T_\chi G_\chi^{(1)})\,\mathrm{Im}I_{VS}(m_\xi/m_\chi)\Big], \tag{24}$$

$$r(V'_\eta\to QL)-r(\overline{V}'_\eta\to\overline{QL})=4[\mathrm{Tr}(g'^\dagger_\eta g'_\eta)+2\,\mathrm{Tr}(h'^\dagger_\eta h'_\eta)]^{-1}$$
$$\times\Big[\sum_\chi \mathrm{Im}\,\mathrm{Tr}(g'^\dagger_\eta g_\chi h'_\eta h^\dagger_\chi)\,\mathrm{Im}I_{VV}(m_\chi/m_\eta)-\sum_\xi \mathrm{Im}\,\mathrm{Tr}(g'^*_\eta F_\xi^{(2)*}h'_\eta F_\xi^{(1)})\,\mathrm{Im}I_{VS}(m_\xi/m_\eta)\Big], \tag{25}$$

$$r(S_\xi\to QL)-r(\overline{S}_\xi\to\overline{QL})=4[2\,\mathrm{Tr}(F_\xi^{(1)\dagger}F_\xi^{(1)})+2\,\mathrm{Tr}(F_\xi^{(2)\dagger}F_\xi^{(2)})+\mathrm{Tr}(G_\xi^{(1)\dagger}G_\xi^{(1)})+2\,\mathrm{Tr}(G_\xi^{(2)\dagger}G_\xi^{(2)})]^{-1}$$
$$\times\Big[2\sum_\chi \mathrm{Im}\,\mathrm{Tr}(F_\xi^{(1)\dagger}h^\dagger_\chi F_\xi^{(2)}g^T_\chi)\,\mathrm{Im}I_{SV}(m_\chi/m_\xi)-2\sum_\eta \mathrm{Im}\,\mathrm{Tr}(F_\xi^{(1)\dagger}h'_\eta F_\xi^{(2)T}g'^T_\eta)\mathrm{Im}I_{SV}(m_\eta/m_\xi)$$
$$+2\sum_\xi \mathrm{Im}\,\mathrm{Tr}(F_\xi^{(1)\dagger}G_{\xi'}^{(2)\dagger}G_\xi^{(2)}F_{\xi'}^{(1)})\mathrm{Im}I_{SS}(m_{\xi'}/m_\xi)$$
$$-2\sum_\chi \mathrm{Im}\,\mathrm{Tr}(G_\xi^{(1)\dagger}h^*_\chi G_\xi^{(2)}j_\chi)\,\mathrm{Im}I_{SV}(m_\chi/m_\xi)$$
$$+\sum_{\xi'}\mathrm{Im}\,\mathrm{Tr}(G'^{(1)\dagger}F_{\xi'}^{(2)*}F_\xi^{(2)T}G_{\xi'}^{(1)})\,\mathrm{Im}I_{SS}(m_{\xi'}/m_\xi)\Big]. \tag{26}$$

The imaginary parts of the integrals I_{VV}, I_{VS}, etc., are easily calculated; we give here only the results for scalar and vector exchange in scalar boson decay:

$$\mathrm{Im}I_{SS}(\rho)=-\frac{1}{16\pi}\,[1-\rho^2\ln(1+1/\rho^2)], \tag{27}$$

$$\mathrm{Im}I_{VS}(\rho)=-\frac{1}{8\pi}\ln(1+1/\rho^2), \tag{28}$$

where ρ is the ratio of the masses of the exchanged boson and the decaying boson.

We see that in general the branching-ratio difference $r-\bar{r}$ can receive nonzero contributions from

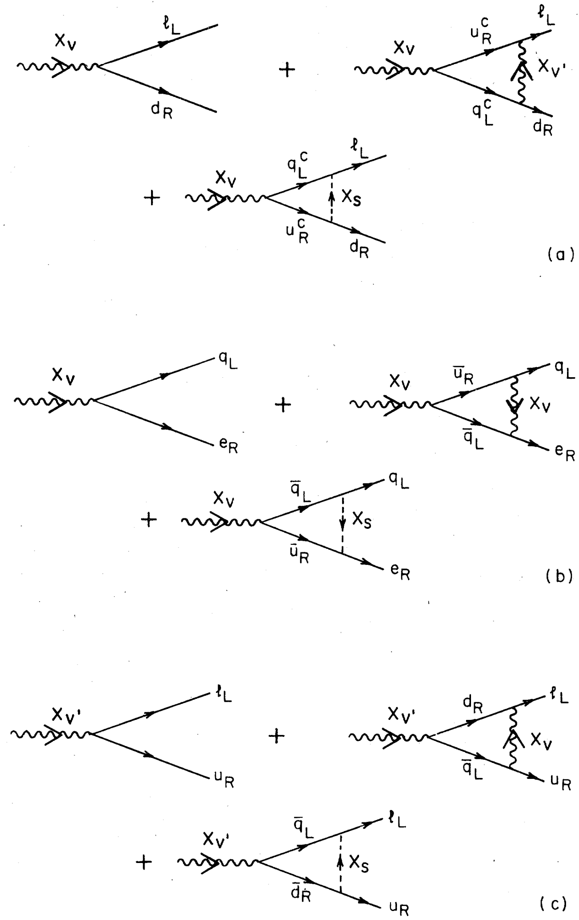

FIG. 1. Feynman diagrams for the decay of X_V and X_V' bosons into quark plus lepton. In the notation used here, l_L, e_R, q_L, d_R, u_R stand for generic quarks and leptons distinguished by their SU(3) ×SU(2) ×U(1) transformation properties, as described in Sec. II.

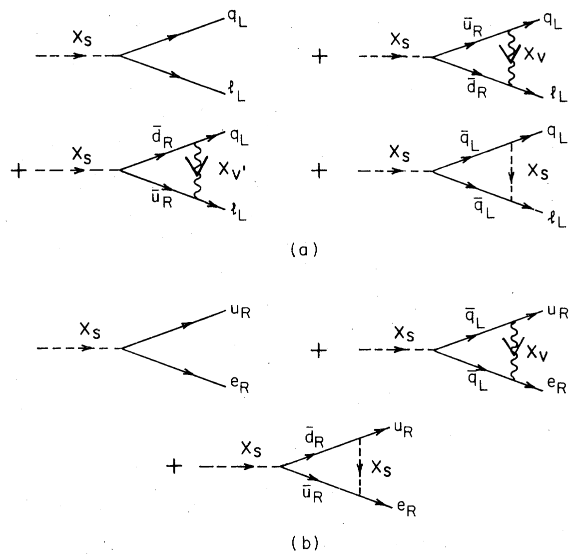

FIG. 2. Feynman diagrams for the decay of X_S bosons into quark plus lepton. Notation same as in Fig. 1.

exchange of an X_S, X_V, or X_V' boson in the decay of any of the X_S, X_V, or X_V' bosons. However, there are a number of special cases in which the branching-ratio difference cancels for an individual boson or for some set of bosons. First, note that the value of $r-\bar{r}$ for any given species of boson receives no contribution from the exchange of the same species of boson. [The traces $\mathrm{Tr}(j_\chi^\dagger h_\chi^\dagger h_\chi j_{\chi'})$, $\mathrm{Tr}F_\xi^{(1)\dagger}G_\xi^{(2)\dagger}G_\xi^{(2)}F_{\xi'}^{(1)})$, and $\mathrm{Tr}(G_\xi^{(1)\dagger}F_{\xi'}^{(2)*}F_\xi^{(2)T}G_{\xi'}^{(1)})$ are real if $\chi=\chi'$ or $\xi=\xi'$, respectively.] Hence there is no baryon production unless there are at least two species of X bosons. More generally, if some set of bosons had equal masses, spins, and lifetimes, then in calculating the cosmological baryon production we would have to add up the branching-ratio differences $r-\bar{r}$ for each of these species; inspection of Eqs. (24)–(26) shows that this sum would vanish because the exchange of an X_1 boson in X_2-boson decay would be canceled by the exchange of an X_2 boson in X_1-boson decay. Hence there is no baryon production unless some of the species of X bosons have different masses, spins, and/or lifetimes. (Of course, there is in any case no reason to expect equal masses and lifetimes for different X-boson species.) Finally, if we suppose that there is a grand unified gauge group which contains SU(5) at least as a subgroup, that the $(3, 2, -\frac{5}{6})$ X_V bosons belong to the gauge multiplet of SU(5), and that the $(3, 2, -\frac{1}{6})$ X_V' bosons are either absent or part of the gauge multiplet of SO(10), then the couplings will be constrained by Eqs. (12)–(17), and almost all of the traces appearing in Eqs. (24)–(26) will be real. The only remaining complex traces in this case are the SS terms in Eq. (26), which give

$$r(S_\xi \to QL) - r(\bar{S}_\xi \to \overline{Q}\,\overline{L})$$
$$= +4[4\,\mathrm{Tr}(F_\xi^\dagger F_\xi) + 3\,\mathrm{Tr}(G_\xi^\dagger G_\xi)]^{-1} \times \sum_{\xi'} \mathrm{Im}\,\mathrm{Tr}(F_\xi^\dagger G_{\xi'}^\dagger G_\xi F_{\xi'})\,\mathrm{Im}\,I_{SS}(m_{\xi'}/m_\xi). \qquad (29)$$

In accordance with our previous remarks, we see that this would vanish if there were just a single species[25] of X_S boson, and would vanish when summed over ξ if there were any number of X_S bosons, all with equal masses and lifetimes. However, (29) indicates that baryon production is to be expected in X_S-boson decay in even the simplest grand unified gauge theories, provided there are at least two species[20] of X_S bosons with different masses or lifetimes. This is reassuring, for as discussed in Sec. I, it is chiefly the decay of the X_S bosons that is expected to yield an appreciable baryon excess.

IV. NUMERICAL ESTIMATES

We will now use the general results of the previous section to make a rough numerical estimate of the baryon abundance that is likely to be produced cosmologically in specific models.

As shown in Ref. 8, the delayed decay of superheavy bosons and antibosons, at temperatures sufficiently far below their mass, will produce a cosmic baryon-entropy ratio

$$kn_B/s = 0.28(N_X/N)\Delta B\,, \tag{30}$$

where N_X is the number of helicity states of all such bosons and antibosons, N is the number of helicity states of all lighter particles (including a factor of $\frac{7}{8}$ for fermions), and ΔB is the mean net baryon production per boson or antiboson decay. The numerical factor in (30) is a ratio of integrals over blackbody distributions, given analytically by $45\zeta(3)/4\pi^4$. The number N_X is unknown, but N is at least 100, so it seems reasonable to take the ratio in the range

$$N_X/N \approx 10^{-2} \text{ to } 10^{-1}. \tag{31}$$

The quantity ΔB is given by Eq. (18) as

$$\Delta B = \tfrac{1}{2}(r - \bar{r})_{\rm av}, \tag{32}$$

where r and $\bar{r}$ are the branching ratios for the quark-lepton and antiquark-antilepton modes of the bosons and antibosons, respectively.

In estimating this branching ratio difference, let us first consider the contribution of X_S-boson exchange in X_S-boson decay. From inspection of either Eq. (26) or (29) and Eq. (32), we may infer that the net baryon production per X_S- or $\bar{X}_S$-boson decay in this case is

$$(\Delta B)_{SS} \approx \Gamma^2 \epsilon\, ({\rm Im} I_{SS})_{\rm av}\,, \tag{33}$$

where Γ is a typical value of the Yukawa couplings $F_\xi^{(n)}$ and $G_\xi^{(n)}$ and ϵ is a phase angle characterizing the average strength of CP violation in the interaction of X_S bosons with fermions, or in the X_S-boson propagator.

To estimate Γ, we will assume that the X_S bosons interact about as strongly with any fermion as do the $(1, 2, -\frac{1}{2})$ doublets (ϕ^+, ϕ^0), whose vacuum expectation values give masses to the quarks, leptons, $W^\pm$, and Z^0. That is,

$$\Gamma \approx \overline{m} G_F^{\,1/2}, \tag{34}$$

where $\overline{m}$ is the rms value of quark and lepton masses and G_F is the Fermi coupling constant. [For instance, if the scalar bosons formed just a single SU(5) quintet, consisting of one $(3, 1, \frac{1}{3})$ X_S boson plus one $(1, 2, -\frac{1}{2})$ ϕ doublet, then we would have Yukawa couplings $F^{(1)} = F^{(2)} = m_E 2^{1/4} G_F^{\,1/2} = m_D 2^{1/4} G_F^{\,1/2}$ and $G^{(1)} = -G^{(2)} = m_U 2^{1/4} G_F^{\,1/2}$, where m_E, m_D, and m_U are the mass matrices of the leptons and quarks of e type, d type, and u type, respectively. Of course, in this particularly simple case, CP could not be violated in X_S-boson interactions.] In estimating $\overline{m}$, we must keep in mind that the values of quark masses at very high energies are likely to be less than their "observed" values at ordinary energies by a factor of order 3 to 4.[15] Taking the b and t quark masses (at ordinary energies) as $m_b = 4.75$ GeV and $m_t = 10$ to 20 GeV, we find

$$\overline{m} \approx 1.1 \text{ to } 2.5 \text{ GeV}. \tag{35}$$

Equation (35) then gives

$$\Gamma^2 \approx 10^{-5} \text{ to } 10^{-4}. \tag{36}$$

In estimating the average value of the integral ${\rm Im} I_{SS}$, we must take into account the exchange of each X_S boson in the decay of each other. Equation (26) shows that the exchange of X_{S1} in the decay of X_{S2} makes a contribution to the branching-ratio difference $r - \bar{r}$ which is of opposite sign to the contribution of X_{S2} exchange in the decay of X_{S1}. If one X_S boson is somewhat heavier than all the others, then the dominant contribution to $r - \bar{r}$ comes from the exchange of the lighter bosons in the decay of the heavier one, and $({\rm Im} I_{SS})_{\rm av} \simeq {\rm Im} I_{SS}(0) = -\frac{1}{32}\pi$. If several of the heavier X_S bosons are of comparable mass, then some cancellation will occur, but there is no special reason to expect complete cancellation. As a reasonable lower bound on ${\rm Im} I_{SS}$ we will take $\frac{1}{4}$ of its value ${\rm Im} I_{SS}(1) = 0.19/16\pi$ for equal mass. This gives

$$|{\rm Im} I_{SS}| \simeq 10^{-3} \text{ to } 10^{-2}. \tag{37}$$

To estimate ϵ, we must rely on what we know of CP violation at ordinary energies. The violation of CP can be either intrinsic or spontaneous, and in either case, it can operate through gauge boson exchange,[22] Higgs boson exchange,[21] or both. In the case of gauge boson exchange, the CP violation can be traced to phases in the quark mass matrix, which appear in the quark-$W^\pm$ interaction after the quark fields are redefined to make the quark mass matrix real and diagonal. The phases in the quark mass matrix would not contribute to observed violations of CP if there were just four quarks,[22] so since we do not know the strength of the mixing of the b and t quarks with the four lighter quarks, all that we can deduce from the observed strength of the CP violation in K_L^0 decay is that the phases in the quark mass matrix would have to be in the range of 10^{-2} to 1 rad.[23] These phases would have to arise from an intrinsic CP violation in the coupling of scalar fields to quarks or from a CP violation in the scalar field vacuum expectation values, due either to an intrinsic CP violation in the scalar self-interaction or to a spontaneous breakdown of CP invariance. On the other hand, if the CP violation at ordinary energies is due to Higgs boson exchange, then these effects would be naturally suppressed relative to ordinary weak interactions by factors $(m_{\text{quark}}/m_{\text{Higgs}})^2$, so the phases in the Higgs boson exchange would have to be close to 1 rad.[21] These phases can arise from an intrinsic CP violation in the coupling of scalar fields to quarks or from a CP violation in the scalar propagator, due to either an intrinsic CP violation in the self-coupling of scalar fields or a spontaneous breakdown of CP invariance.

What does this tell us about CP violation in X_S-boson interactions? If the CP violation at ordinary energies is intrinsic, then we expect a similar CP violation in the couplings of X_S bosons to quarks and leptons and in the X_S-boson propagators, so that

$$|\epsilon| \approx 10^{-2} \text{ to } 1. \tag{38}$$

On the other hand, if the CP violation at ordinary energies arises spontaneously in the breakdown of SU(2)×U(1) to U(1), then we would expect this CP violation to disappear at temperatures above about 300 GeV.[24] However, whether the CP violation at ordinary energies is intrinsic or spontaneous, it is possible that there is an entirely different CP violation in X_S interactions, due to a spontaneous breaking of CP in the breakdown of the grand unified gauge group to SU(3) × SU(2) × U(1). We know nothing about the magnitude of such a CP violation, and in lieu of better information we will take (38) as our estimate of ϵ.

If we now use Eqs. (36)–(38) in Eq. (33), we find a mean net baryon number produced in X or $\bar{X}$ decay:

$$|\Delta B| \approx 10^{-10} \text{ to } 10^{-6}. \tag{39}$$

With Eqs. (30) and (31), this gives a baryon-entropy ratio

$$|kn_B/s| \approx 10^{-13} \text{ to } 10^{-7}. \tag{40}$$

Now let us consider the contribution to ΔB of X_V or X_V' exchange in X_S decay. We assume now that the grand unified gauge group is sufficiently complicated so that the 1st, 2nd, and 4th traces in the numerator of Eq. (26) are not all automatically real. From Eqs. (26) and (32), we have

$$\Delta B_{SV} \approx g^2 \epsilon' (\text{Im} I_{SV})_{\text{av}}, \tag{41}$$

where g is a typical value of the vector-boson coupling constants g_χ, h_χ, j_χ, g_η', or h_η' and ϵ' in a phase characterizing the CP violation in the coupling of vector or scalar bosons to quarks and leptons or in the vector-boson propagator. In any kind of grand unified theory, we expect $g^2/4\pi$ to be comparable with (though somewhat larger than[14,15]) the fine-structure constant α, so

$$g^2 \approx 10^{-1}. \tag{42}$$

It is difficult to estimate ϵ' because the possible CP violation in the X_V or X_V' couplings or propagators has no direct analog at experimentally accessible energies. However, ϵ' can, like ϵ, receive contributions from CP violation in the coupling of X_S bosons to fermions, so we shall take for $|\epsilon'|$ the same estimate as for $|\epsilon|$

$$|\epsilon| \approx 10^{-2} \text{ to } 1. \tag{43}$$

For the average value of $\text{Im} I_{SV}$, we take a rounded estimate

$$|(\text{Im} I_{SV})_{\text{av}}| \approx 10^{-3} \text{ to } 10^{-1} \tag{44}$$

corresponding to a ratio of vector- to scalar-boson masses in the range 0.3 to 6 in Eq. (28).

The mean baryon excess from X_V or X_V' exchange in X_S-boson decay is now given by (41)–(44) as

$$|(\Delta B)_{SV}| \approx 10^{-6} \text{ to } 10^{-2}. \tag{45}$$

Hence, in theories with a sufficiently complicated group structure, we expect a baryon-entropy ratio given by Eqs. (45) and (30) as

$$|kn_B/s| \approx 10^{-9} \text{ to } 10^{-3}. \tag{46}$$

The baryon production ΔB associated with exchange of a scalar or vector boson in X_V or X_V' decay may be estimated as roughly comparable to the value of ΔB for exchange of the same boson in X_S decay. We will not go into this in detail here, as X_S decay seems more promising than X_V or X_V' decay as a mechanism for cosmological baryon production.

V. CONCLUSIONS

We have seen that the delayed decay of a black-body distribution of X_S bosons at temperatures below their mass may be expected to produce a baryon-entropy ratio at least of order 10^{-13} to 10^{-7}, provided that there are enough species of X_S bosons. In sufficiently complicated theories, baryon number can also be produced in X_V or X_V' exchange processes, yielding a larger baryon-entropy ratio, of order 10^{-9} to 10^{-3}.

These ranges of possible baryon-entropy ratios overlap the values $kn_B/s \approx 10^{-10}$ to 10^{-8} that are allowed by astronomical observations.[2] However, the range of theoretical values is clearly far too broad for us to be able to conclude that X-boson decay really is the source of the observed cosmic abundance of baryons. In the absence of a specific grand unified theory, all that we can conclude now is that X-boson decay is a plausible mechanism for cosmological baryon production.

Let us mention one last point: We have made no attempt here to predict the *sign* of the baryon excess produced cosmologically. Of course, whatever kinds of particles survive the early universe would inevitably be called "matter," not "antimatter." The only real question is whether "matter," as defined by CP-violating cosmological baryon production processes, is the same as "matter," as defined by the observed CP violations in K_L decay.

It is not impossible that this question could some day be answered. For instance, phases in the interaction of scalar fields with quarks can contribute to the CP violation in both X-boson decay and K_L^0 decay. (Recall that these phases produce phases in the quark mass matrix, which produce phases in the interaction of $W^{\pm}$ bosons with quarks of definite mass,[22] which can contribute to CP violation in K_L^0 decay.) If such phases furnish the dominant contribution to CP violation in both K_L^0 and X decay, and if some grand gauge group relates the phases in the couplings of $(1,2,-\frac{1}{2})$ ϕ doublets and $(3,1,\frac{1}{3})$ X_S bosons to quarks, then it might be possible to relate the sign of the CP violation in K_L^0 decay and X_S-boson decay, provided we can learn how to calculate K_L^0-decay amplitudes despite the complication of strong interactions. But this must clearly wait until we have in hand a specific grand unified gauge theory.

Note added in proof. (1) After this paper was completed, we received a report by S. Barr, G. Segrè, and H. A. Weldon, which deals in a similar way with the problem of calculating the cosmological baryon production. The topics dealt with in these papers are also discussed by P. Cox and A. Yildiz, Harvard Report No. HUTP-79/A019 (unpublished). (2) There are two additional kinds of boson which can have baryon-nonconserving interactions with pairs of ordinary fermions and/or antifermions. They are an SU(3)-triplet SU(2)-singlet scalar X_S' with charge $-\frac{4}{3}$, which can decay into the channels $d_R e_R$ and $\bar{u}_R \bar{u}_R'$, and an SU(3)-triplet SU(2)-triplet scalar X_S'', which can decay into the channels $q_L l_L$ and $\bar{q}_L \bar{q}_L'$. These cannot contribute to nucleon decay (because Fermi statistics require their two-quark decay channels to consist of quarks from different generations), and they were omitted in Ref. 8, note (1). The existence of these bosons would provide additional mechanisms for cosmological baryon production: interference between the Born approximation and X_S' or X_S'' exchange in X_S decay, and interference between the Born approximation and X_S exchange in X_S' or X_S'' decay. Our numerical estimates in Sec. IV apply also to these contributions.

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APPENDIX

This appendix will consider baryon-violating decays in the approximation that the decay amplitude is calculated to first order in the baryon-violating interaction H', but to all orders in other interactions. It will be shown that in this approximation, TCP invariance requires that the rate for decay of a particle X into all final states with a given value B of the baryon number equals the rate for the corresponding decay of the antiparticle $\bar{X}$ into all states with baryon number $-B$.[19] As discussed in the text, this theorem indicates that in calculations of cosmological baryon production, we must consider graphs which are at least of second order in the baryon-violating interactions.

To first order in the baryon-violating interaction H', the decay amplitude for a baryon-violating decay of a particle X to some final state f may be written

$$A(X\to f)=(\psi_f^{\text{out}},H'\psi_X),$$

where ψ_f^{out} and ψ_X are eigenstates of the baryon-conserving part of the Hamiltonian, with outgoing-wave boundary conditions in ψ_f^{out}. (Since ψ_X is a one-particle state, there is no distinction between ψ_X^{out} and ψ_X^{in}.) According to TCP invariance, the amplitude for the corresponding antiparticle decay process is

$$A(\bar{X}\to\bar{f})\equiv(\psi_{\bar{f}}^{\text{out}},H'\psi_{\bar{X}})=(\psi_X,H'\psi_f^{\text{in}})$$

with bars denoting the TCP conjugates of the various states. Inserting a complete set of "out" states gives then

$$\begin{aligned}A(\bar{X}\to\bar{f})&=\sum_g(\psi_X,H'\psi_g^{\text{out}})(\psi_g^{\text{out}},\psi_f^{\text{in}})\\&=\sum_g A(X\to g)^*S_{gf}^0,\end{aligned}$$

where S^0 is the S matrix in the absence of the baryon-violating interaction H'. The total rate for $\bar{X}$ decay into all states $\bar{f}$ with a given value $-B$ for the baryon number $B(\bar{f})$ is then

$$\bar{\Gamma}(-B)=\sum_{\bar{f}:B(f)=-B}\rho_{\bar{f}}|A(\bar{X}\to\bar{f})|^2$$
$$=\sum A(X\to g)^*A(X\to h)\sum_{f:B(f)=B}\rho_{\bar{f}}S^0_{gf}S^{0*}_{gh},$$

where $\rho_{\bar{f}}$ is a phase-space factor. Now *TCP* further tells us that all masses are equal in the corresponding processes $X\to f$ and $\bar{X}\to\bar{f}$, so the phase-space factors are equal:

$$\rho_{\bar{f}}=\rho_f\,.$$

Also, S^0 is unitary in the space of states with a given baryon number, so

$$\sum_{f:B(f)=B}\rho_f S^0_{gf}S^{0*}_{hf}=\begin{cases}\rho_g\delta_{gh}: B(g)=B,\\ 0\quad: B(g)\neq B,\end{cases}$$

and therefore

$$\bar{\Gamma}(-B)=\sum_{g:B(g)=B}|A(X\to g)|^2\rho_g\,.$$

But this is the total rate $\Gamma(+B)$ for X decay with final states with baryon number $+B$, so $\bar{\Gamma}(-B)$ equals $\Gamma(+B)$ in this approximation, as was to be proved.

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[1]For a discussion of the evidence regarding the possible cosmological abundance of antimatter, see G. Steigman, Annu. Rev. Astron. Astrophys. 14, 339 (1976).

[2]In this estimate, we take a range of values for the baryonic mass density of the universe which would give a deceleration parameter q_0 between 0.02 (corresponding to a low estimate of the mass density actually observed in galaxies) and 2 (corresponding to the upper bound on nonlinearity in the red-shift–distance relation), with a Hubble constant taken as 50 (km/sec)/Mpc.

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[8]S. Weinberg, Phys. Rev. Lett. 42, 850 (1979).

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[10]Implications of cosmological baryon production for the nature of cosmological inhomogeneities are discussed by M. S. Turner and D. N. Schramm, Nature 279, 303 (1979); M. S. Turner, *ibid.* (to be published); C. J. Hogan, *ibid.* (to be published); R. W. Brown and F. W. Stecker, NASA Report No. TM 80291 (unpublished); J. J. Aly, Institute of Astronomy, Cambridge, U. K. report (unpublished); W. H. Press and E. T. Vishniac, Harvard-Smithsonian Center for Astrophysics Reports Nos. 1147 and 1148 (unpublished); J. D. Barrow, Oxford Dept. of Astrophysics report (unpublished).

[11]This is the scenario adopted in Ref. 8. It is also treated in Sec. II of Ref. 6, and in the second article of Ref. 4. One of us (S.W.) owes much to conversations with F. Wilczek on this matter.

[12]A mechanism for establishing an initial thermal equilibrium distribution of scalar bosons at very early times was discussed in Ref. 8, Note (3). In this article we will simply assume that an equilibrium distribution of X bosons (with equal numbers of X and $\bar{X}$) was established before the temperature dropped below the X-boson mass, but we will not rely on any particular picture of how this came about.

[13]The first specific grand unified gauge model was that of J. C. Pati and A. Salam, Phys. Rev. D 8, 1240 (1973); 10, 275 (1974). They noted that baryon nonconservation occurs naturally in their model and similar models because leptons and quarks appear in the same gauge multiplet. In the present work, we adopt a somewhat different view of the strong interactions from that of Pati and Salam: We assume that the only colored gauge bosons with masses below the grand unification mass scale are the eight gluons of quantum chromodyanmics. The simplest grand unified gauge model of this type is the SU(5) model of H. Georgi and S. L. Glashow [Phys. Rev. Lett. 32, 483 (1974)]; other leading models of this type include the SO(10) model of H. Georgi [in *Particles and Fields—1974* proceedings of the 1974 meeting of the Division of Particles and Fields of the American Physical Society, Williamsburg, edited by Carl Carlson (AIP, N.Y., 1975)]; H. Fritzsch and P. Minkowski [Ann. Phys. (N.Y.) 93, 193 (1975)], H. Georgi and D. V. Nanopoulos [Phys. Lett. 82B, 392 (1979); Nucl. Phys. B155, 52 (1979); and the models based on exceptional groups by F. Gürsey, P. Ramond, and P. Sikivie [Phys. Lett. 60B, 177 (1975)], F. Gürsey and P. Sikivie [Phys. Rev. Lett. 36, 775 (1976)], P. Ramond [Nucl. Phys. B110, 214 (1976)], etc.

[14]H. Georgi, H. R. Quinn, and S. Weinberg, Phys. Rev. Lett. 33, 451 (1974).

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[16]In recent calculations that take into account mass-dependent terms and two-loop corrections in the renormalization-group equations, it was found that the superheavy gauge boson masses are 50–100 times less than the nominal grand unified mass scale calculated in Refs. 14 and 15; see T. J. Goldman and D. A. Ross, Phys. Lett. 84B, 208 (1979); D. Ross, Nucl. Phys. B140, 1 (1978); W. Marciano, Phys. Rev. D 20, 274 (1979).

[17]Reference 8, Note (1). Also see S. Weinberg in *Proceedings of the Einstein Centennial Symposium at Jerusalem*, 1979 (unpublished).

[18]There are other one-loop graphs, in which a virtual X boson is exchanged between the initial X boson and one of the final fermions. These graphs are not con-

sidered here because in the decay of the lighter X bosons they have no absorptive part, and hence cannot contribute to the branching ratio differences calculated in Sec. III.

[19]This is a special case of a theorem mentioned by S. Weinberg, Phys. Rev. 110, 782 (1958).

[20]Note that in an SO(10) theory, the scalar bosons that can couple to fermions would have to belong to the 10, 120, or 126 representations. With only one scalar multiplet, such a theory would yield the unacceptable result that the mass matrices of the charge $+\frac{2}{3}$ and charge $-\frac{1}{3}$ quarks would have to be equal (for the case of 10) or proportional (for the case of 120 or 126), so that in addition to having wrong quark mass relations, such a theory would have no Cabibbo mixing. Grand unified theories with a single scalar multiplet also lead to incorrect quark-lepton mass relations. These results are avoided if there are several 10's (see, e.g., Georgi and Nanopoulos, Ref. 13), in which case there are also several X_S bosons. In addition, if the observed CP violation at ordinary energies is due to exchange of Higgs bosons (as in Ref. 21), then there must be several $(1, 2, -\frac{1}{2})$ doublets (ϕ^+, ϕ^0), in which case we also expect several species of X_S boson. We emphasize, however, that the converse need not hold: For instance, if there were just two ϕ doublets and two X_S bosons, then CP would not be violated at ordinary energies by Higgs-boson exchange, but it could be violated in X_S boson decay by X_S-boson exchange. Thus, even if it were found that CP violation at ordinary energies is due to the exchange of $W^\pm$ and not Higgs bosons, we could not conclude that the X_S-boson exchange terms in (26) or (29) must vanish.

[21]T. D. Lee, Phys. Rev. D 8, 1226 (1973); Phys. Rep. 9C, 143 (1974); S. Weinberg, Phys. Rev. Lett. 37, 657 (1976).

[22]M. Kobayashi and K. Maskawa, Prog. Theor. Phys. 49, 652 (1973); L. Maiani, Phys. Lett. 68B, 183 (1976); S. Pakvasa and and H. Sugawara, Phys. Rev. D 14, 305 (1976); J. Ellis, M. K. Gaillard and D. V. Nanopoulos, Nucl. Phys. B109, 213 (1976).

[23]J. Ellis, M. K. Gaillard, D. V. Nanopoulos, and S. Rudaz, Nucl. Phys. B131, 285 (1977).

[24]D. A. Kirzhnits and A. D. Linde, Phys. Lett. 42B, 471 (1972); S. Weinberg, Phys. Rev. D 9, 3357 (1974); L. Dolan and R. Jackiw, *ibid*. 9, 3320 (1974); C. Bernard, *ibid*. 9, 3312 (1974); J. Iliopoulos and N Papanicolaou, Nucl. Phys. B111, 209 (1976). It is not, however, inevitable that spontaneously broken CP invariance must be restored at high temperature; see R. M. Mohapatra and G. Senjanović, Phys. Rev. Lett. 42, 1651 (1979), and references quoted therein.

[25]The calculations of Ref. 7 show that baryon production can occur with just a single species of X_S boson, but in a higher order of perturbation theory.