Low unifying mass scales without intermediate chiral color symmetry

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The embedding of $SU(2) \times U(1) \times SU(3)$ intermediate symmetry within an $[SU(2n)]^4$ unified model is shown to be compatible with "light" embedding mass scales ($\lesssim 10^6$ GeV) if $n \geq 3$. Weak-angle values for $n \geq 3$ are shown to lie in the range $0.21 < \sin^2\theta_W < 0.25$.

The hypothesis that the known interactions are embedded in some larger unifying theory permits empirical manifestations of fundamental unity in nature.[1] Relationships between known interaction coupling constants can be obtained by embedding the known interactions within a unifying theory characterized by a single gauge coupling constant. Consequences of such unification have been explored in a number of embedding symmetries G.[2-6] Suppose symmetry breaking of G into the known interaction subgroup $[G \to SU(2) \times U(1) \times SU(3)]$ is characterized by a mass scale M.[7] All previous calculations have shown that M must be quite large ($\geqslant 10^{14}$ GeV) for the strong interactions to become appropriately strong at currently accessible energies.[2-4] Until now, lowered values of $M \approx 10^{5-7}$ GeV have been shown possible only if the known interaction symmetries are enlarged to include chiral color, in which case $G \to SU(2) \times U(1) \times SU(3)_L \times SU(3)_R$. The chiral color group is subsequently broken to conventional vector chromo dynamics at a mass scale $\lesssim m_W$, the mass scale of weak symmetry breaking.[5]

In this note, we demonstrate how lower values of M can be obtained without enlarging the known interaction subgroup. We choose $G = [SU(2n)]^4$ as the embedding symmetry and show that the hierarchy

$$G = [SU(2n)]^4 \to SU(2) \times U(1) \times SU(3) \qquad (1)$$

is compatible with low ($\lesssim 10^6$ GeV) unifying mass scales provided $n \geqslant 3$, corresponding to the existence of leptonic color degrees of freedom within the unifying symmetry.[8] We also find for the above hierarchy [Eq. (1)] that

$$\sin^2\theta_W = (3n-4)/[12(n-1)] + O(\alpha/\alpha_s)\,, \qquad (2)$$

corresponding to asymptotic ($\alpha_s \gg \alpha$) weak-angle values between 0.21 and 0.25 for $n \geqslant 3$.

Consider a gauge theory based on $G = SU(2n)_L \times SU(2n)_R \times SU(2n)'_L \times SU(2n)'_R$, where the unprimed groups correspond to flavor gauge transformations and the primed groups correspond to color gauge transformations. If $n=2$, G corresponds to $[SU(4)]^4$ unifying symmetry proposed in Ref. 5. Similarly, $n=3$ corresponds to an $[SU(6)]^4$ model in which the SU(6) color group consists of three hadronic and three leptonic colors.[8] In general, we will constrain $[SU(2n)]^4$ to have $2n$ flavors, 3 hadronic colors and $2n-3$ leptonic colors. The fermion representations of $[SU(2n)]^4$ may be written as follows (i., $\alpha = 1, \ldots, 2n$):

$$\psi_{i\alpha}^{L(R)} = \frac{1+(-)\gamma_5}{2}\begin{bmatrix} u_r^{2/3} & u_y^{2/3} & u_b^{2/3} & \nu_e^0 & L^+ & M^0\ldots \\ d_r^{-1/3} & d_y^{-1/3} & d_b^{-1/3} & e^- & L^0 & M^-\ldots \\ c_r^{2/3} & c_y^{2/3} & c_b^{2/3} & \cdot & \cdot & \cdots \\ s_r^{-1/3} & s_y^{-1/3} & s_b^{-1/3} & \cdot & \cdot & \cdots \\ t_r^{2/3} & t_y^{2/3} & t_b^{2/3} & \cdot & \cdot & \cdots \\ b_r^{-1/3} & b_y^{-1/3} & b_b^{-1/3} & \cdot & \cdot & \cdots \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdots \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdots \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdots \end{bmatrix}. \qquad (3)$$

The rows (columns) represent flavor (color) degrees of freedom. Charges are indicated by the fermion superscripts.[9] L and M represent additional lepton colors [(ν_μ, μ), (ν_τ, τ), and (ν_e, e) can be chosen to belong to the same or to different leptonic colors]. ψ^L transforms as a $(2n, 1, 2\bar{n}, 1)$ representation of G; similarly ψ^R transforms as $(1, 2n, 1, 2\bar{n})$.

Let us denote the single gauge coupling constant of G as g_G.[10] The interaction of fermions with the gauge-boson multiplets W_L^I, W_R^J, V_L^K, and V_R^M ($I, J, K, M = 1, \ldots, 4n^2-1$) of $SU(2n)_L$, $SU(2n)_R$, $SU(2n)'_L$, and $SU(2n)'_R$, respectively, is given by

$$\mathcal{L}_I = ig_G(\bar{\psi}^L_{i\alpha}\gamma\tfrac{1}{2}\lambda^I_{ij}\psi^L_{j\alpha}W^I_L + \bar{\psi}^R_{i\alpha}\gamma\tfrac{1}{2}\lambda^J_{ij}\psi^R_{j\alpha}W^J_R - \bar{\psi}^L_{i\alpha}\gamma\tfrac{1}{2}\lambda^K_{B\alpha}\psi^L_{i\beta}V^K_L - \bar{\psi}^R_{i\alpha}\gamma\tfrac{1}{2}\lambda^M_{\beta\alpha}\psi^R_{i\beta}V^M_R), \qquad (4)$$

where λ^I are the $4n^2-1$ λ matrices of SU($2n$). The fermion charge assignments of Eq. (3) determine the structure of the photon, which is seen to acquire components W^F_L, W^F_R, V^C_L, and V^C_R from the gauge boson multiplets of each SU($2n$) group. The photon field A can therefore be written as

$$A = (e/g_G)[\sqrt{n}\,(W^F_L + W^F_R) - [(3n-4)/3]^{1/2}(V^C_L + V^C_R)]. \qquad (5)$$

$[e^{-2} = (4n - \frac{8}{3})g_G^{-2}]$, where the gauge bosons $W^F_{L,R}$ couple to the generating matrix given by

$$\left(\frac{\lambda^F}{2}\right)_{ij} = \frac{1}{2\sqrt{n}}\begin{bmatrix} +1 & & & & & & & & 0 \\ & -1 & & & & & & & \\ & & +1 & & & & & & \\ & & & -1 & & & & & \\ & & & & \cdot & & & & \\ & & & & & \cdot & & & \\ & & & & & & \cdot & & \\ & & & & & & & +1 & \\ 0 & & & & & & & & -1 \end{bmatrix} \qquad (6)$$

and the gauge bosons $V^C_{L,R}$ couple to the generating matrix

$$\left(\frac{\lambda^C}{2}\right)_{\alpha\beta} = \frac{1}{[12(3n-4)]^{1/2}}\begin{bmatrix} +1 & & & & & & & & & & 0 \\ & +1 & & & & & & & & & \\ & & +1 & & & & & & & & \\ & & & -3 & & & & & & & \\ & & & & +3 & & & & & & \\ & & & & & -3 & & & & & \\ & & & & & & \cdot & & & & \\ & & & & & & & \cdot & & & \\ & & & & & & & & \cdot & & \\ & & & & & & & & & +3 & \\ 0 & & & & & & & & & & -3 \end{bmatrix}. \qquad (7)$$

Note that λ^F differentiates between up and down members of embedded SU(2) doublets, and λ^C differentiates between three hadronic colors and ($2n-3$) leptonic colors.

Embedding of the known interaction subgroup SU(2) × U(1) × SU(3) within $[\mathrm{SU}(2n)]^4$ is straightforward. The canonical gauge boson W^F_L corresponds to W^3_L, the neutral member of the SU(2) gauge-boson multiplet. The remainder of the photon [Eq. (5)] corresponds to the U(1) gauge boson B:

$$B = [\sqrt{n}\,W^F_R - [(3n-4)/3]^{1/2}(V^C_L + V^C_R)]/[(9n-8)/3]^{1/2}. \qquad (8)$$

To see these correspondences, consider the fermion current coupling to W^3_L and B in the SU(2) × U(1) theory:

$$\mathcal{L}_I(W^3_L, B) = ig\,W^3_{L\mu}\left[\bar{\nu}\gamma^\mu\frac{(1+\gamma_5)}{4}\nu - \bar{e}\gamma^\mu\frac{(1+\gamma_5)}{4}e + \bar{u}\gamma^\mu\frac{(1+\gamma_5)}{4}u - \bar{d}\gamma^\mu\frac{(1+\gamma_5)}{4}d + \dots\right]$$
$$+ ig'B_\mu\left[\bar{\nu}\gamma^\mu\frac{(-1-\gamma_5)}{4}\nu - \bar{e}\gamma^\mu\frac{(3-\gamma_5)}{4}e + \bar{u}\gamma^\mu\frac{(5-3\gamma_5)}{12}u - \bar{d}\gamma^\mu\frac{(1-3\gamma_5)}{12}d + \dots\right], \qquad (9)$$

where g and g' are the SU(2) and U(1) coupling constants, respectively. The electromagnetic current is easily seen to couple to

$$A = e(W^3_L/g + B/g'), \qquad (10)$$

in which case the usual relations for e and θ_W are obtained[11]:

$$e^{-2}=g^{-2}+g'^{-2}, \quad \tan\theta_W=g'/g\,. \tag{11}$$

The couplings of fermions [Eq. (3)] to W_L^F and B [Eq. (9)] gauge bosons of the $[\mathrm{SU}(2n)]^4$ unified theory are obtained from Eqs. (4), (6), and (7) $(\psi_{i\alpha}\equiv\psi^L_{i\alpha}+\psi^R_{i\alpha})$:

$$\mathcal{L}_I(W_L^F,B)=ig_G\,W^F_{L\mu}(\bar\psi^L_{i\alpha}\gamma^\mu\tfrac{1}{2}\lambda^F_{ij}\psi^L_{i\alpha})+\frac{ig_GB_\mu}{[(9n-8)/3]^{1/2}}\times\{\sqrt{n}\;\bar\psi^R_{i\alpha}\gamma^\mu\tfrac{1}{2}\lambda^F_{ij}\psi^R_{j\alpha}+[(3n-4)/3]^{1/2}\,\bar\psi_{i\alpha}\gamma^\mu\tfrac{1}{2}\lambda^C_{\alpha\beta}\psi_{i\beta}\}\,. \tag{12}$$

Comparison of Eqs. (10) and (12) shows that for the $\mathrm{SU}(2)\times\mathrm{U}(1)$ theory to be embedded within $[\mathrm{SU}(2n)]^4$,

$$g=g_G/\sqrt{n}\,, \tag{13}$$

$$g'=g_G/[(9n-8)/3]^{1/2}\,. \tag{14}$$

Finally, we consider the correspondence between g_G and the chromodynamic coupling constant g_3. Chromodynamic $\mathrm{SU}(3)_C$ is embedded within $\mathrm{SU}(2n)'_L\times\mathrm{SU}(2n)'_R$. For example, the V^3 gluon of $\mathrm{SU}(3)_C$ corresponds to the linear combination $(V_L{}^3+V_R{}^3)/\sqrt{2}$ of $\mathrm{SU}(2n)'_L$ and $\mathrm{SU}(2n)'_R$ gauge bosons. Comparison of the currents coupled to V^3 of the embedded theory and $V_{L,R}{}^3$ of the embedding theory shows that

$$g_3=g_G/\sqrt{2}\,. \tag{15}$$

Equations (13)–(15) relate values of the known interaction coupling constants to the $\mathrm{SU}(2n)$ gauge coupling constant provided $G=[\mathrm{SU}(2n)]^4$ symmetry is unbroken. These relations are applicable only for scales of momenta μ much than M. If $\mu\ll M$, the embedding symmetry G is no longer manifest. The effective coupling constants g_3, g, and g' (or g_3, g_2, and g_1, to switch to an obvious notation) obey renormalization-group equations of the form[12] $\mu\partial g_i/\partial\mu=\beta(g_i)$ $(i=1,2,3)$ subject to the boundary conditions described by Eqs. (13)–(15) at $\mu=M$.

Standard decoupling theorem arguments[2-6,13] lead to the following expressions for the effective known interaction coupling constants when $\mu\ll M$:

$$g_3{}^{-2}(\mu)=2[g_G{}^{-2}(M)+2b_l\ln M/\mu]+2b_3\ln M/\mu\,, \tag{16a}$$

$$g^{-2}(\mu)=n[g_G{}^{-2}(M)+2b_l\ln M/\mu]+2b_2\ln M/\mu\,, \tag{16b}$$

$$g'^{-2}(\mu)=[(9n-8)/3][g_G{}^{-2}(M)+2b_l\ln M/\mu]\,. \tag{16c}$$

When $\mu=M$, the boundary conditions of Eqs. (13)–(15) are explicitly satisfied. The $2b_l\ln M/\mu$ term signifies the contribution of entirely light (mass $\lesssim m_W$) multiplets of G to the rescaling of g_G in the various subgroup sectors. The fermion multiplets [Eq. (3)] are assumed to be in this class of light multiplets. Entirely heavy (mass $\gtrsim M$) multiplets of G contribute $O(\mu^2/M^2)$ terms to $\beta(g_i)$,[13] which are disregarded in Eq. (16). The only significant differential contributions to $\beta(g_i)$ are assumed to arise through the gauge multiplet of G,[2] which contains decoupled SU(3), SU(2), and U(1) multiplets of light gauge bosons in addition to the remaining massive gauge bosons corresponding to broken symmetries.[14] The differential contributions of SU(3) and SU(2) submultiplets to the magnitudes of g_3 and g_2, respectively, correspond to the $2b_3\ln M/\mu$ and $2b_2\ln M/\mu$ terms in Eqs. (16a) and (16b), where $b_3=-3[11/48\pi^2]$ and $b_2=-2[11/48\pi^2]$ (Ref. 12) [there is no contribution to $\beta(g_i)$ from the "U(1) submultiplet"].

Substitution of Eqs. (16) into (11) leads to the following relations between $\alpha_s/\alpha(\equiv g_3{}^2/e^2)$, $\sin^2\theta_W$ and the embedding mass scale M:

$$1-(2n-\tfrac{4}{3})(\alpha/\alpha_s)=e^2[g^{-2}+g'^{-2}-(2n-4/3)g_3{}^{-2}]=[11\alpha(n-1)/\pi]\ln M/\mu\,, \tag{17}$$

$$\sin^2\theta_W=e^2/g^2=4\pi\alpha\{n[g_G{}^{-2}(M)+2b_l\ln M/\mu]+2b_2\ln M/\mu\}=\frac{3n-4}{12(n-1)}+\left(\frac{\alpha}{\alpha_5}\right)\frac{9n-8}{18(n-1)}\,. \tag{18}$$

The last line of Eq. (18) is obtained by using Eqs. (16a) and (17) to eliminate g_G, b_l, and $\ln M/\mu$ from the previous line.

The weak-angle expression of Eq. (18) is most amusing. If $n\geq 3$ and $\alpha_s\gg\alpha$, then $\frac{5}{24}<\sin^2\theta_W<\frac{1}{4}$, a range consistent with most present empirical evidence. If $n=2$,

$$\sin^2\theta_W=\tfrac{1}{6}+5\alpha/9\alpha_s\,, \tag{19}$$

a result obtained previously for embedding within "single-lepton-color" models[3] as well as within the SU(5) model.[2]

The lowered unification mass scale promised at the beginning of this paper is realized through rearrangement of Eq. (17):

$$\frac{\alpha_s}{\alpha}=\frac{2n-\frac{4}{3}}{1-(11\alpha/\pi)(n-1)\ln M/\mu}\,. \tag{20}$$

If $n=2$, Eq. (2) is identical to expressions obtained for SU(5) and for single-lepton-color embedding theories.[2,3] If we require α_s to be greater than 0.1 at physical mass scales $[\mu\approx 10\text{ GeV}]^2$, then $M\gtrsim 10^{15}$ GeV. However, when n is increased from 2 to 3, the coefficient of $\ln M/\mu$ doubles and the numerator of Eq. (20) also increases. For $n=3$, $\alpha_s(\mu=10\text{ GeV})\geq 0.1$ provided $M \gtrsim 3\times 10^6$ GeV. Higher values of n allow even lower values of M/μ to effect ratios of $\alpha_s/\alpha \gg 1$.

To summarize, $[SU(2n)]^4$ is a simple extension of $[SU(4)]^4$ for which moderate embedding mass scales can be realized without enlarging the known interaction symmetry group. In particular, chiral color is shown to be unnecessary for the generation of sufficiently strong chromodynamics.

It has been pointed out elsewhere that a low unifying mass scale is required for integer-quark-charge theories.[6] There must be a symmetry-breaking mass scale of order 10^5 GeV for integer-charge quarks to be sufficiently unstable to explain their present nonobservation, and that mass scale must correspond to the ultimate unifying mass scale if $\sin^2\theta_W$ is to be appreciably below $\frac{3}{8}$.[6] Hence $[SU(2n)]^4$ with $n\geq 3$ may become a candidate for embedding symmetry in integer-quark-charge models; the range of weak-angle magnitudes obtained is especially attractive.

Nevertheless, there exists no evidence for leptonic color at present, and $G=[SU(2n)]^4$ unified symmetry can only be regarded as speculative.[15] We have used the example of $[SU(2n)]^4$ unifying symmetry in order to demonstrate how physical weak-angle values and moderate unification mass scales can be obtained without resorting to enlarging the conventional known interaction symmetry group.

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[14] If scalar field multiplets of $[SU(2n)]^4$ are mixtures of light and superheavy particles, Eqs. (16) will no longer be strictly decoupled. However, β-function dependence on scalar field multiplets is quite weak compared with the term arising from the gauge multiplet (Ref. 12).

[15] Additional leptons within the fermion multiplets of $SU(2n)^4$ must be assumed heavy.