

Possible structure of the Pomeron and its effects on one-particle inclusive cross sections in the central region

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It is shown that the main features of the central plateau of the one-pion inclusive proton-proton cross sections can be accounted for by a non-Regge (Pomeron) contribution $\sigma_P(s)$. The function $\sigma_P(s)$ is the solution of an integral equation of the convolution type. $\sigma_P(s)$ contains an "asymptotic part" $A + B \ln s$ (responsible for the asymptotic features of the plateau) and a nonasymptotic part Γ/\sqrt{s} which determines the approach to scaling.

I. INTRODUCTION

The properties of the one-particle inclusive cross sections $ab \rightarrow cX$ have been studied theoretically by many authors (see, e.g., Refs. 1-7). Most of these papers are based on Mueller's theorem^{8,9} plus the assumption that, in the central region, the forward amplitude $ab\bar{c} \rightarrow ab\bar{c}$ is dominated by a double Regge graph.

Assuming double Pomeron exchange, this approach gives, asymptotically, for the one-particle inclusive cross section, a zero-slope central plateau whose height is energy-independent. These features are not supported by present-energy data. Moreover, with this approach it is hard to explain why the asymptotic value of the inclusive cross section is approached from below and why the reactions in which the $a\bar{c}$ and $b\bar{c}$ channels are exotic exhibit (not "early scaling" but) the strongest s dependence of the cross section. To explain these features, one has to introduce negative couplings (which seems unnatural) and/or to introduce effectively a "Pomeron daughter" trajectory Q , which accounts for threshold effects. In all these papers the Pomeron has been treated, as usual, as a Regge pole with (approximately) unit intercept.

On the other hand, it is generally believed that the Pomeron (let alone its daughter) is not a genuine Regge pole but rather a convenient name for some complicated interaction mechanism, whose nature is unknown. It is therefore interesting to investigate whether an expression better than the simple Regge form $\sigma_P \propto s^{\alpha_P(s)-1}$ can be obtained for the Pomeron cross section, and whether such an expression would lead to a better agreement with the central-region inclusive-cross-section data. The main ideas of this approach are given in this paper and they are

applied to the reaction $ab \rightarrow cX$, where $a = b = \text{proton}$ and $c = \pi^+$. The results are in very good agreement with the experimental data.

In Sec. II our assumptions are stated and discussed, an expression is derived for the one-particle inclusive cross section $f(p_c)$, and an integral equation is obtained for the Pomeron contribution $\sigma_P(s)$ to the high-energy hadronic cross sections. In Sec. III a function $\sigma_P(s)$ is given which satisfies that integral equation within terms of $O(1/s)$ and is consistent with the Froissart bound. In Sec. IV our results are compared to the experimental data. Finally, our conclusions are briefly reviewed in Sec. V.

II. BASIC ASSUMPTIONS AND EQUATIONS OF THE MODEL

In the Mueller-Regge approach, the basic formula giving the inclusive cross section $f(p_c)$ for $ab \rightarrow cX$ is

$$f(p_c) = \sum_{ij} \beta_{ij} |s_{a\bar{c}}|^{\alpha_i(s)-1} |s_{b\bar{c}}|^{\alpha_j(s)-1}, \quad (1)$$

where α_i, α_j are Reggeon or Pomeron trajectories,

$$\begin{aligned} s_{a\bar{c}} &= (p_a - p_c)^2 \simeq -m_a \mu_c \exp(y_a - y_c), \\ s_{b\bar{c}} &= (p_b - p_c)^2 \simeq -m_b \mu_c \exp(y_b - y_c), \end{aligned} \quad (2)$$

p_i and y_i are the four-momenta and rapidities of the various particles,

$$\mu_c = (m_c^2 + p_T^2)^{1/2}, \quad (3)$$

and

$$s_{a\bar{c}} s_{b\bar{c}} / \mu_c^2 = s = m_a m_b \exp(y_a - y_b),$$

with $y_a = \ln(\sqrt{s}/m_a)$ and $y_b = -\ln(\sqrt{s}/m_b)$.

We now make the following assumptions:

(I) We substitute the simple expression $s_i^{\alpha_P(s)-1}$ of the Pomeron contribution to the cross sections

by a general expression $\sigma_P(s_i)$. Since the mechanism producing the Pomeron contribution is unknown, σ_P is, at this stage, an unknown function, which will be approximately determined below. We shall assume,¹⁰ however, that the same unknown mechanism is effective in all the Pomeron-exchange procedures and hence the same function $\sigma_P(s_i)$ (apart from a multiplicative constant) appears in all these cases.

(II) The role of the Mandelstam variable s , for the channels $a\bar{c}$ and $b\bar{c}$, is played (not by $s_{a\bar{c}}$ and $s_{b\bar{c}}$ but) by

$$\begin{aligned} s_1 &= \frac{|s_{a\bar{c}}|}{\alpha\mu_c} s_0 = s_0 \exp \left[\ln \left(\frac{\sqrt{s}}{\alpha\mu_c} \right) - y_c \right], \\ s_2 &= \frac{|s_{b\bar{c}}|}{\alpha\mu_c} s_0 = s_0 \exp \left[\ln \left(\frac{\sqrt{s}}{\alpha\mu_c} \right) + y_c \right], \end{aligned} \quad (4)$$

where $s_0 = 1 \text{ GeV}^2$ and α is a dimensionless constant which determines the energy scale. (As will be seen in Sec. IV, α is 1.7 for π^+ and 1.4 for π^- .)

Then Eq. (1) can be generalized to

$$\begin{aligned} f(p_c) &= [g_{PP} \sigma_P(s_1/s_0) \sigma_P(s_2/s_0) \\ &\quad + g_{PR} \sigma_P(s_1/s_0) \sigma_R(s_2/s_0) \\ &\quad + g_{RP} \sigma_R(s_1/s_0) \sigma_P(s_2/s_0) \\ &\quad + g_{RR} \sigma_R(s_1/s_0) \sigma_R(s_2/s_0)] \Psi(\mu_c), \end{aligned} \quad (5)$$

where $\sigma_P(s_i/s_0)$ and $\sigma_R(s_i/s_0)$ are the Pomeron and Reggeon cross sections and where $\Psi(\mu_c)$ is a function which depends strongly on p_T . For $\Psi(\mu_c)$ one can use the phenomenological ansatz of Ref. 11, viz.,

$$\Psi(\mu_c) = \exp(b\mu_c), \quad (6)$$

where $b = -7.1 \text{ GeV}^{-1}$ for π^+ and $b = -7.2 \text{ GeV}^{-2}$ for π^- .¹²

The functions $\sigma_P(s_i/s_0)$ and $\sigma_R(s_i/s_0)$ reduce to simple expressions for large values of s_i . These functions, however, as they stand in Eq. (5), may (and in fact, as will be seen below, do) contain also nonasymptotic terms which become important when $|y_c|$ is not very small.

Let us now consider the interval

$$-\frac{\sqrt{s}}{\alpha\mu_c} \leq y_c \leq \frac{\sqrt{s}}{\alpha\mu_c}, \quad (7)$$

which corresponds to $-0.6 < x < 0.6$ for π^+ and $-0.7 < x < 0.7$ for π^- , x being the Feynman variable. Since the end points of that interval correspond to $s_1 = s_0$ and $s_2 = s_0$, the inclusive cross section (5) can be nonzero only within that interval. This can be looked upon as a threshold effect. In fact, if one requires $\sigma_P(s_i/s_0)$ and $\sigma_R(s_i/s_0)$ to vanish at and below some threshold (which is certainly not smaller than s_0), then $f(p_c)$ must also vanish for

values of s_1 and s_2 smaller or equal to that threshold. Hence, $f(p_c)$ must be zero out of the interval (7). Moreover, in the case where $c = \pi^\pm$, in which no diffraction peak appears and most pions are produced at or near the central region, one may assume that the majority of pions are produced via the interaction mechanism that leads to Eq. (5) and neglect the pions which are not produced via that mechanism. Hence we make the following assumption:

(III) Practically all the pions are produced within the interval (7), through double Regge and/or Pomeron exchange.

We now write the energy sum rule

$$\sqrt{s} \sigma_{\text{inel}}(ab) = \sum_c \pi \int E_c f(p_c) dp_T^2 dy_c. \quad (8)$$

Let us consider the contribution to the right-hand side of (8) of events in which some species of particles c (e.g., π^+) are produced in the interval (7). Let λ_c be the fraction of this contribution over $\sqrt{s} \sigma_{\text{inel}}$. In other words, let us define a parameter λ_c by the equation

$$\lambda_c = \frac{\pi}{\sigma_{\text{inel}}} \int \frac{E_c}{\sqrt{s}} f(p_c) dy_c dp_T^2, \quad (9)$$

where the integral with respect to y_c is taken in the interval (7). We now make the following assumption:

(IV) The parameter λ_c is a constant (s -independent.)

The validity of this assumption has been checked for $c = \pi^+$ and $c = \pi^-$, and for $s = 24, 48, 543, 936, 1989, 2809, \text{ and } 3969 \text{ GeV}^2$. For each value of s , the integral with respect to y_c has been calculated by numerical integration of the experimental data of Refs. 13–15, with $\alpha = 1.7$ for π^+ and $\alpha = 1.4$ for π^- . The integration with respect to p_T has been performed by the technique of Ref. 16, using as input the data for $p_T = 0.4 \text{ GeV}/c$, since for this value of p_T accurate data exist for the above seven energies. The results are shown in Fig. 1 and they justify our assumption that λ_c is s -independent, at least for $c = \pi^\pm$.

We now introduce the expression (5) of $f(p_c)$ into Eq. (9). To simplify our formulas, we introduce the notation

$$F(\mu_c) \equiv \pi \int dy_c \frac{E_c}{\sqrt{s}} [f(p_c)/\Psi(\mu_c)], \quad (10)$$

$$\langle F(\mu_c) \rangle \equiv \frac{1}{\Phi} \int d\mu_c^2 F(\mu_c) \Psi(\mu_c), \quad (11)$$

$$\Phi \equiv \int d\mu_c^2 \Psi(\mu_c). \quad (12)$$

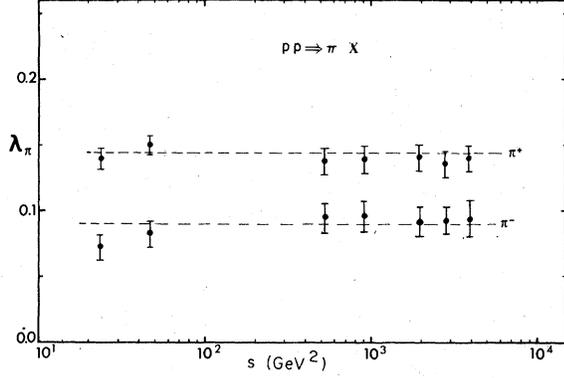


FIG. 1. The s dependence of the parameter λ_c defined in text for $c=\pi^\pm$. Data from Refs. 13-15.

Then, Eq. (9) becomes

$$\lambda_c \sigma_{\text{inel}} = \Phi \langle F(\mu_c) \rangle \quad \text{or} \quad \lambda_c \sigma_{\text{inel}} \simeq \Phi F(\langle \mu_c \rangle), \quad (13)$$

where $\langle \mu_c \rangle = \int d\mu_c^2 \Psi(\mu_c) \mu_c$ and where the integrals

$$\begin{aligned} \sigma_P \left(\frac{\langle \mu_c \rangle^2 \alpha^2}{s_0} e^S \right) = \int_0^S d\omega e^{-\omega} [G_{PP} \sigma_P(e^\omega) \sigma_P(e^{S-\omega}) + G_{PR} \sigma_P(e^\omega) \sigma_0 e^{-(S-\omega)/2} \\ + G_{RP} \sigma_P(e^{S-\omega}) \sigma_0 e^{-\omega/2} + G_{RR} \sigma_0^2 e^{-S/2}] + O(1/s), \end{aligned} \quad (14)$$

where G_{PP} , G_{PR} , G_{RP} , and G_{RR} are constants, whereas

$$S \equiv \ln[s/(\alpha \langle \mu_c \rangle)^2], \quad \omega \equiv s/2 - y_c$$

and, hence, $s_1/s_0 = e^\omega$, $s_2/s_0 = e^{S-\omega}$, and

$$E_c/\sqrt{s} = (e^{-\omega} + e^{-S+\omega})/2\alpha. \quad (15)$$

In writing (14) we have used the fact that $G_{RP} = G_{PR}$ (because $a=b$ =proton), and hence the integrand is symmetric with respect to the substitution $\omega \rightarrow S - \omega$, and the two integrals corresponding to the two terms of (15) can be reduced to one. The constants G of (14) are related to the constants g of (5) by

$$\begin{aligned} G_i = \frac{\pi\Phi}{\lambda_c \alpha} g_i \quad \text{for } i=PP, PR, RP, \\ G_{RR} = \frac{\pi\Phi}{\lambda_c \alpha} g_{RR} - \frac{C_R}{\sigma_0^2 \alpha \langle \mu_c \rangle}. \end{aligned} \quad (16)$$

At first sight, use of the asymptotic form $\sigma_R \sim 1/\sqrt{s_i}$ in Eq. (14) may seem unjustified, since the integral (14) contains contributions also from small s_i values. In fact, in that integral s_i can become as small as some effective threshold value s_T , for which σ_R vanishes [i.e., $s_i \geq s_T$ where $\sigma_R(s_T) = 0$]. However, the low-energy effects can be charged

with respect to μ_c^2 are taken from m_c^2 to ∞ . The last (approximate) equality of (13) can be justified by expanding $F(\mu_c)$, in the right-hand side of Eq. (11), in a power series around the point $\mu_c = \langle \mu_c \rangle$. For $c = \pi^+$ one finds $\langle \mu_c \rangle = 0.35$ GeV and

$$\langle F(\mu_c) \rangle = F(\langle \mu_c \rangle) - 0.017 F''(\langle \mu_c \rangle) + \dots \simeq F(\langle \mu_c \rangle).$$

A similar result holds also for π^- .

Now let $\alpha_R = \frac{1}{2}$ be the intercept of an effective Regge trajectory exchanged in the reactions $a\bar{c} \rightarrow a\bar{c}$ and $b\bar{c} \rightarrow b\bar{c}$. For large s , the Reggeon cross section $\sigma_R(s/s_0)$ of Eq. (5) is known to be $\sigma_R(s) \simeq (s/s_0)^{-1/2} \sigma_0$, where $\sigma_0 = 1$ mb is a constant introduced to make all the constants g of Eq. (5) dimensionless. Then, at high energies, the function $\sigma_{\text{inel}}(s)$ of Eq. (13) can be written as $\sigma_{\text{inel}} = \sigma_P + C_R/(s/s_0)^{1/2} + O(1/s)$, where C_R is a constant and σ_P is the same function as in Eq. (5). Substituting this expression in (13) and using the definition (10) of the function F , one finds the following integral equation for the unknown function σ_P :

upon the $O(1/s)$ terms of (14). E.g., one can use for σ_R the ansatz

$$\sigma_R(s) = \left(\frac{s}{s_0} \right)^{-1/2} \left[1 - \left(\frac{s}{s_T} \right)^{-1/2} \right] \sigma_0, \quad (17a)$$

which vanishes at $s = s_T$ and differs by terms of $O(1/s)$ from the asymptotic expression

$$\sigma_R(s) \simeq \left(\frac{s}{s_0} \right)^{-1/2} \sigma_0. \quad (17b)$$

Using (17a), one would end up with the same integral equation [within terms of $O(1/s)$] as the equation (14) found above, with the form (17b).

III. APPROXIMATE SOLUTION OF THE INTEGRAL EQUATION

The integral equation (14) can, in principle, be solved numerically for the unknown function σ_P . However, conciseness, as well as a more immediate applicability in phenomenological analyses and perhaps a deeper physical insight, can be gained if a relatively simple approximate closed functional form can be suggested, instead of the expression $\sigma_P \propto s^{\alpha_P - 1}$ used in the past. It is therefore interesting to investigate whether an approximate solution of (14) can be found in a closed form

which would satisfy the Froissart bound and would be consistent with the high-energy experimental data.

Since, in deriving Eq. (14), we have omitted terms of $O(1/s)$, we require that the approximate solution should satisfy (14) within terms of order $(1/s)$. A relatively simple ansatz, suggested by the behavior of high-energy cross sections, which satisfies all the above requirements is

$$\sigma_P(s/s_0) = A + B \ln(s/s_0) + \frac{\Gamma}{(s/s_0)^{1/2}} + O(1/s). \quad (18)$$

Here A and B have the known fixed values observed from high-energy cross sections (e.g. from Ref. 17), viz., $A \approx 23.3$ mb and $B \approx 2.5$ mb for $a=b$ =proton. The constant Γ is not directly observable from high-energy cross sections because in practice the third term of (18) is confused with Regge terms of the same order of magnitude. An "effective order of magnitude" of Γ can be estimated by requiring that σ_P should vanish at some reasonable low-energy threshold (e.g., near $s=4m_p^2$

$\approx 4 \text{ GeV}^2$ for $a=b$ =proton) i.e., that the effective value of Γ should also simulate the effect of the terms of $O(1/s)$, which have been omitted. Then $\Gamma \approx -45$ mb.

An ansatz even simpler than (18) which is also suggested by the high-energy data, viz., the ansatz $\sigma_P = A + B \ln(s/s_0)$, cannot reproduce that threshold behavior. Moreover, it leads to unacceptable behavior for $f(p_c)$ and will not be further considered.

If one requires that the ansatz (18) satisfy the integral equation (14) within terms of $O(1/s)$, one obtains three consistency equations. The fastest way to obtain these equations is by taking Laplace transforms (LT's) of both sides of (14). If one uses the variable S instead of s and denotes by τ the conjugate variable of S , then the LT of the Pomeron cross section (18) is

$$\bar{\sigma}(\tau) = \frac{A}{\tau} + \frac{B}{\tau^2} + \frac{\Gamma}{\tau + 1/2}. \quad (19)$$

By the convolution theorem, the LT of the right-

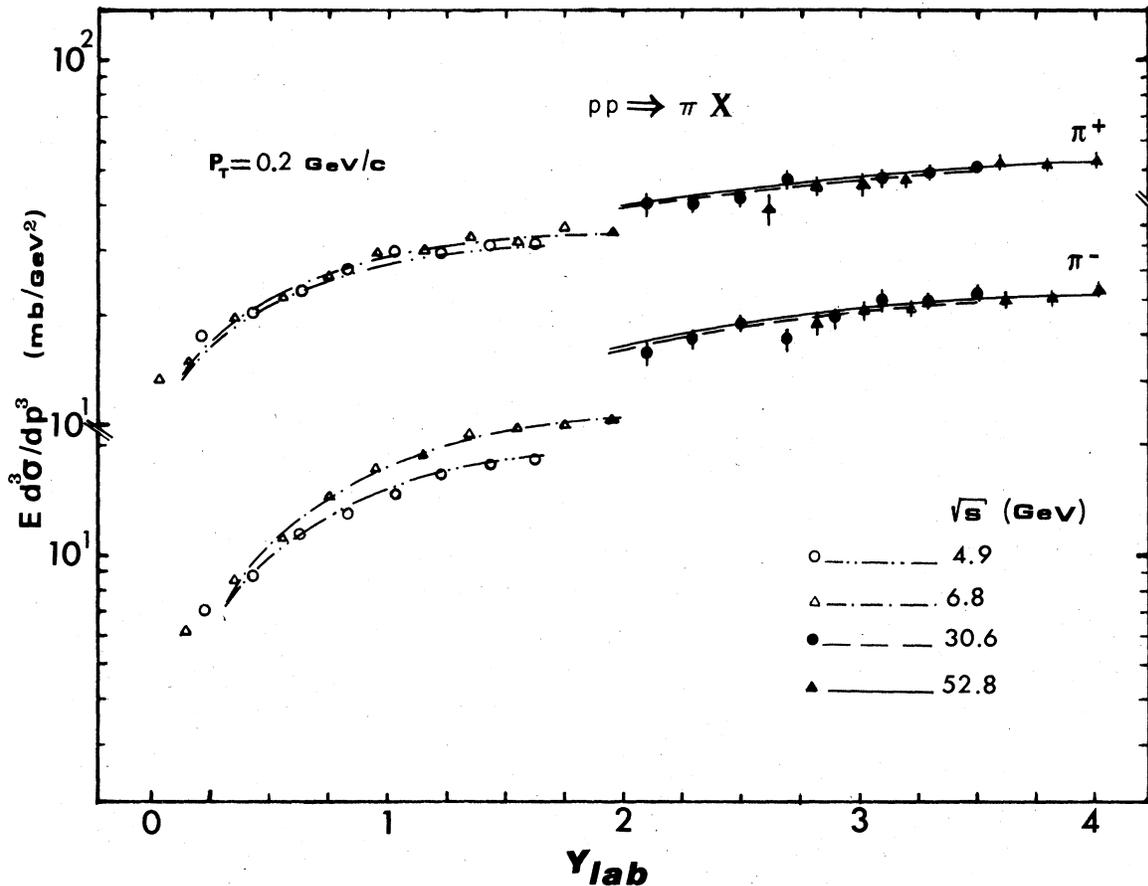


FIG. 2. A fit of the inclusive $pp \rightarrow \pi^\pm X$ cross sections calculated from formula (5) with σ_P and σ_R given by (18) and (17a) and with $\alpha(\pi^+) = 1.7$, $s_T(\pi^+) = 1 \text{ GeV}^2$, $\alpha(\pi^-) = 1.4$, $s_T(\pi^-) = 1.9 \text{ GeV}^2$. Data from Refs. 13-15.

hand side of (14) is

$$G_{PP}\bar{\sigma}(\tau+1)\bar{\sigma}(\tau) + G_{PR}\sigma_0\bar{\sigma}(\tau+1)\frac{1}{\tau+\frac{1}{2}} + G_{RP}\sigma_0\bar{\sigma}(\tau)\frac{1}{\tau+\frac{3}{2}} + G_{RR}\sigma_0^2\frac{1}{(\tau+\frac{1}{2})(\tau+\frac{3}{2})}. \quad (20)$$

Using (19) and (20), equating the coefficients of $1/\tau$, $1/\tau^2$, and $1/(\tau+\frac{1}{2})$ on both sides of the LT of (14), and omitting terms proportional to $1/(\tau+k)$ with $k \geq 1$ [since they correspond to terms of $O(1/s^k)$ with $k \geq 1$], one finds the consistency equations

$$1 = G_{PP}(A + B + \frac{2}{3}\Gamma), \quad (21a)$$

$$G_{PP}(A + 2B + \frac{4}{3}\Gamma) = -\ln M_c, \quad (21b)$$

$$G_{PP}\Gamma_1(2A + 4B + \Gamma_1) + G_{RR} - \frac{G_{PR}^2}{G_{PP}} = \Gamma/\sqrt{M_c}, \quad (21c)$$

where

$$\Gamma_1 = \Gamma + G_{PR}/G_{PP}, \quad M_c = \alpha^2 \langle \mu_c \rangle^2 / s_0. \quad (22)$$

Thus, the function (18) satisfies the integral equation (14) within terms of $O(1/s)$, provided the constants A , B , Γ , G_{PP} , G_{PR} , G_{RR} , and α satisfy Eq. (21).

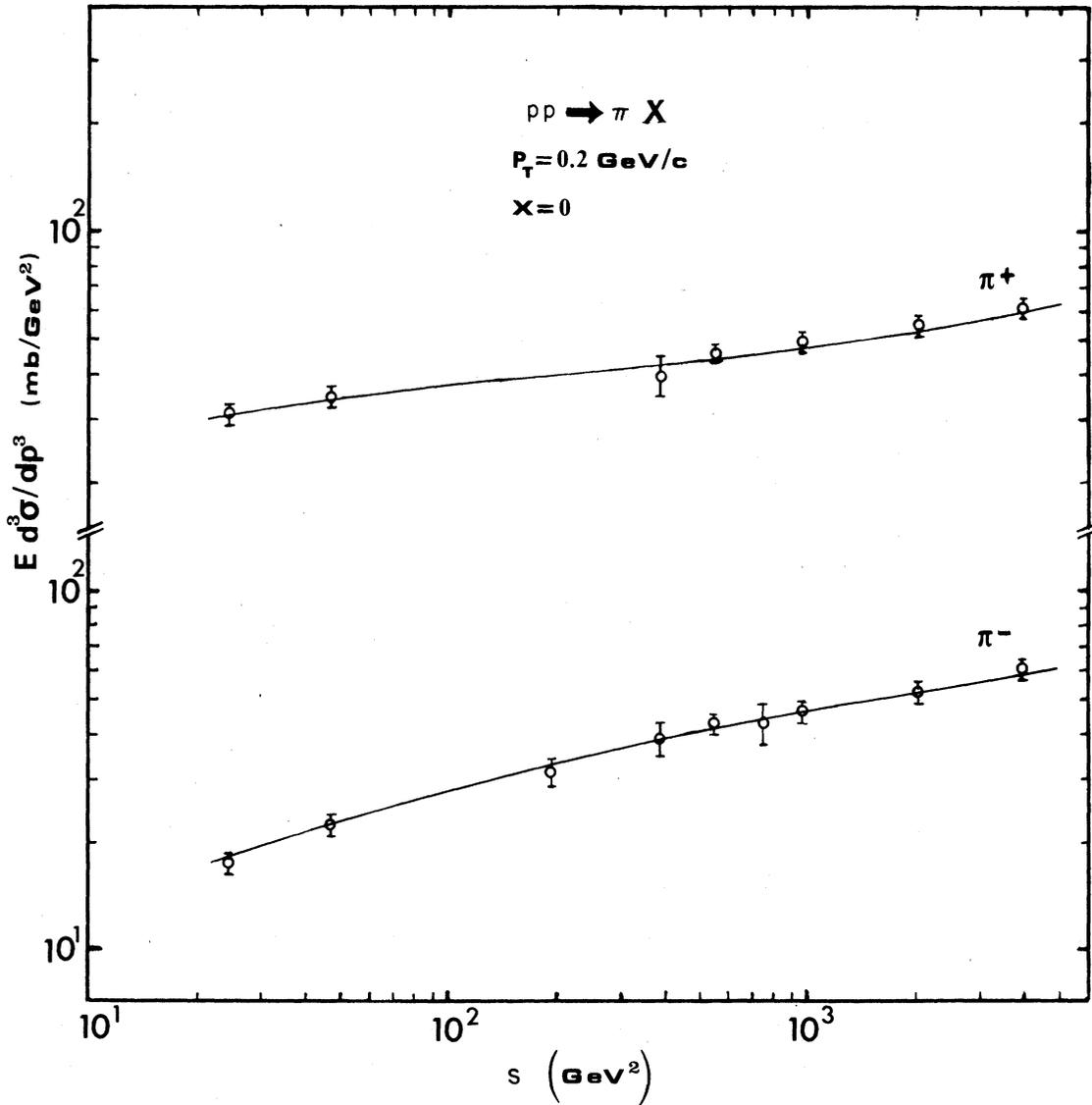


FIG. 3. The energy dependence of the inclusive $pp \rightarrow \pi^+ X$ cross section calculated from formula (5) at $y_{c.m.} = 0$, with σ_P and σ_R given by (18) and (17a) and with $\alpha(\pi^+) = 1.7$, $s_T(\pi^+) = 1 \text{ GeV}^2$, $\alpha(\pi^-) = 1.4$, $s_T(\pi^-) = 1.9 \text{ GeV}^2$. Data from Refs. 18 and 19.

IV. COMPARISON WITH EXPERIMENT

The values of the Pomeron constants A , B , Γ have been given in the previous section. Now, requiring that the expression $\sigma_{\text{inel}} = \sigma_P + C_R/(s/s_0)^{1/2}$ should approximate the observed behavior of $\sigma_{\text{inel}}(pp)$, one can estimate that C_R is of the order of 100 mb. (However, all the subsequent fits of this paper are not sensitive to the value of C_R .) Also, from Fig. 1, one has immediately $\lambda_{\pi^+} = 0.14$, $\lambda_{\pi^-} = 0.09$ and from Eq. (12), for the assumed form of Ψ , one finds $\Phi_{\pi^+} = 0.0294 \text{ GeV}^2$, $\Phi_{\pi^-} = 0.0284 \text{ GeV}^2$, $\langle \mu_{\pi^+} \rangle = 0.348 \text{ GeV}$, and $\langle \mu_{\pi^-} \rangle = 0.0347 \text{ GeV}$. Then, once some value is assumed for α , everything is fixed: The constraint equations (21) can be solved for G_{PP} , G_{PR} , and G_{RR} and then Eqs. (16) can be used to find g_{PP} , g_{PR} , and g_{RR} . Once the constants g are determined, the inclusive cross section $f(p_c)$ can be calculated from (5) with σ_P given by (18) and σ_R given by (17a) or (17b). If (17b) is used, then the energy dependence of the central ($x=0$) point and the slope of the central plateau can be fitted, but the shape of the edges of the plateau cannot be reproduced, since (17b) has not an acceptable low-energy (threshold) behavior. If (17a) is used, then also the shape of the edges of the plateau is very well reproduced, provided the "threshold parameter" s_T is appropriately adjusted. Thus, for each c -particle species, there are two free parameters (α and s_T) with which one fits the energy dependence of $f(p_c)$ at $x=y_c=0$ and the shape of the plateau, including the edges. The results shown in Figs. 2 and 3 are obtained for $\alpha(\pi^+) = 1.7$, $s_T(\pi^+) = 1 \text{ GeV}^2$, and $\alpha(\pi^-) = 1.4$, $s_T(\pi^-) = 1.9 \text{ GeV}^2$. The agreement with the experimental data is very good throughout the range $24 \text{ GeV}^2 < s < 3000 \text{ GeV}^2$.

Moreover, it is easy to see that (18) implies the following welcome properties for $f(p_c)$:

- (a) The plateau height increases, even asymptotically.
- (b) On either side of the central point, $f(p_c)$ decreases like y_c^2 (even asymptotically).
- (c) Scaling is approached from below, like $1/s^{1/4}$.
- (d) If the channels $a\bar{c}$ and $b\bar{c}$ were exotic, one

would have $g_{RP} = g_{PR} = g_{RR} = 0$.

Then the s dependence of $f(p_c)$ would be the strongest, because the coefficient of $1/s^{1/4}$ (which is proportional to $\Gamma_1 = \Gamma + G_{PR}/G_{PP} < 0$ with $\Gamma \approx -45$) is absolutely larger if $G_{PR} = 0$.

V. CONCLUSIONS

The central idea of this paper is that the Pomeron (being a more complicated entity than a simple Regge pole) may have a structure which cannot be adequately reproduced by the classical parametrization $\sigma_P(s) \propto s^{\alpha_P} t^{-1}$.

This possibility has been investigated by deriving an integral equation for the Pomeron cross section $\sigma_P(s)$, and then writing an approximate solution which satisfies this equation within terms of $O(1/s)$. This solution strongly suggests the existence of negative nonasymptotic terms *within* the Pomeron. [It is significant that the simple form $\sigma_P(s) \sim A + B \ln s$, suggested by the asymptotic data, does *not* satisfy the integral equation, for reasonable values of the parameters, unless such negative nonasymptotic terms are included.]

It is conceivable that these terms, if they exist, are not directly observable in σ_{tot} (because their effect is confused with that of Regge terms of the same order of magnitude), but they may be detectable indirectly, through their effect on inclusive cross sections.

On the other hand, if such terms are included in the expression of $\sigma_P(s)$, then not only is the integral equation satisfied within terms of $O(1/s)$ but also (at least for $pp \rightarrow \pi^+ X$) the slope of the central plateau and the energy dependence of its height are reproduced quite well throughout the region $25 < s < 3000 \text{ GeV}^2$, with only one free parameter α . If threshold effects in the Reggeon cross section are taken into account (which involves the introduction of an additional "threshold parameter" s_T), then also the observed shape of the edges of the plateau can be reproduced. Moreover, essentially all the observed central-region features [see points (a)–(d) of Sec. IV], which are very hard to explain with the traditional Pomeron parametrization, are here obtained as natural consequences.

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