Current quark masses and structure functions

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Bare quark distribution functions are constructed for valence quarks in 0^-, 1^- mesons and $(1/2)^+$, $(3/2)^+$ baryons. The integrated $\langle x^{-1}\rangle$ moments of these structure functions lead to five independent determinations of the bare or current quark mass scale, all giving $\hat{m} = (m_u + m_d)/2 \approx 62$ MeV and all consistent with the current quark mass ratio $m_s/\hat{m} \approx 5$. Furthermore, the approach also leads to a charmed current quark mass $m_c \approx 1200$ MeV.

I. INTRODUCTION

It is becoming increasingly clear that the quark masses, which appear in the chiral-SU(4)-breaking Hamiltonian density (i.e., the quark mass matrix)

$$\mathcal{H}' = \bar{q}\,\mathfrak{M}\,q = m_u\bar{u}u + m_d\bar{d}d + m_s\bar{s}s + m_c\bar{c}c\ , \tag{1}$$

play a significant role in any quark theory of strong, electromagnetic, or weak interactions. The first estimate of the quark mass ratio $m_s/\hat{m}$, where $\hat{m} = \frac{1}{2}(m_u + m_d)$ and m_s are the nonstrange and strange quark masses, based upon *simple assumptions* about the SU(3) transformation properties of the "bad" quark operators $\bar{q}\lambda_i q$, led to[1] $m_s/\hat{m} \approx 25$. This scheme also required hadron mass splittings to be linear in the quark masses so that the observed decuplet mass splitting of $\Delta m_D(\Delta S = 1) \approx 150$ MeV sets the quark mass scale of $m_s \approx 150$ MeV or $\hat{m} \approx 150/25 \approx 5$ MeV.

On the other hand, present chiral-breaking *phenomenology* suggests a quark mass ratio of[2] $m_s/\hat{m} \approx 5$–6. Also, the extracted fixed Compton pole[3] and independently threshold photoproduction do not favor a very small value of $\hat{m}$ but, very roughly,[4,5] $\hat{m} \sim m_\pi$. This approach has a theoretical basis in the context of the light-plane transformation properties of the bad quark-density operators[5-7] and current vs constituent quarks.[8]

In this paper we will work completely within the framework of quark structure functions as defined in terms of the scaling variable $x = (p_0 + p_3)^{\text{qk}}/(p_0 + p_3)^{\text{had}}$ in the infinite-momentum frame.

We argue that our approach is in the spirit of the parton model and quantum chromodynamics (QCD). Accordingly, we apply these techniques in the scaling region in order to find the *bare* or valence structure functions describing the probability of finding a quark with momentum fraction x in a hadron. The integrated $\langle x^{-1}\rangle$ moments of these *bare* structure functions will in turn set the *scale* for the *bare* (i.e., current) quark masses determined by the hadronic matrix elements of (1). This is true provided that these bare quark structure functions are normalized so as to account for all the momentum of the simplest bare quark state in the same way that the physical structure functions are normalized so as to account for all the momentum of the quarks plus sea plus Regge contributions. Consequently, these bare quark structure functions coincide with the physical structure functions for $x \to 1$, where momentum conservation requires that if the simplest quark state alone carries all the momentum, then the remaining components (sea and Regge contributions for the physical state, nothing for the bare quark state) carry no momentum. On the other hand, for $x \approx 0$ the two structure functions are radically different, since momentum conservation does not uniquely determine the distribution of momentum. Thus, in the $x \gtrsim \frac{1}{2}$ region, the bare structure functions resemble the physical structure functions, while, for $x \lesssim \frac{1}{2}$, the bare structure functions must be determined purely by theoretical argument. The fact that the physical structure function is known to account for roughly one half of the hadron's momentum is not relevant; we demand that the bare structure function account for all the hadron's momentum, and we normalize accordingly.

Such bare valence structure functions contain sufficient dynamical information to compute the mass of the composite hadron from first-order perturbation theory and the matrix elements of $\langle \text{had}|\bar{q}q|\text{had}\rangle$. The latter matrix element most naturally is proportional to the quark mass in question and an integrated bare valence structure function. Therefore, for example, quark sea distributions, which represent radiative corrections to the simplest quark distribution, must be excluded from our bare structure functions. Moreover, these functions are constructed to contain no scaling violations. Presumably they represent driving terms in QCD Bethe-Salpeter equations, which in turn generate the physical structure functions including scaling violations.

The theory is very restrictive, however, because there are *five* independent determinations of the quark mass scale $\hat{m}$ or splittings $m_s^2 - \hat{m}^2$, two determinations from the $\frac{1}{2}^+$ baryons and one each from the $0^-, 1^-$ mesons and $\frac{3}{2}^+$ baryons. In Ref. 5 it was shown that, *independent* of the integrated structure-function scale, both pseudoscalar-meson masses and baryon masses, πN σ term, and Goldberger-Treiman discrepancies lead to the *same* quark mass ratio

$$m_s/\hat{m} \approx 5\,. \tag{2}$$

Given our theoretical bare quark structure function with extremely *insensitive* integrated scales, we show here that all five bare structure-function $\langle x^{-1}\rangle$ moments lead to the *same* nonstrange-quark mass scale,

$$\hat{m} \approx 62 \text{ MeV}\,. \tag{3}$$

Since the theory is very much overdetermined but always consistent with (2) and (3), we feel less hesitant to extract the charmed-quark mass in a similar manner. Again we find consistency in the charmed-meson sector for the quark mass ratio:

$$m_c/\hat{m} \approx 19\text{–}20\,. \tag{4}$$

Then our quark mass scale (3) unambiguously predicts

$$m_c \approx 1200 \text{ MeV}\,. \tag{5}$$

The latter value is, of course, about what one would expect in order that the charmed-quark mass be somewhat less than the static mass of $\frac{1}{2}m_\psi \sim 1550$ MeV. This most natural requirement places a strong constraint on any theory of chiral-symmetry breaking—a constraint that appears to be satisfied only in our scheme.

In Sec. II we review the connection between the light-plane decomposition of the bad quark densities $\bar{q}\lambda_i q$ and demonstrate that the hadronic matrix elements of (1) contain an additional power of quark mass to leading order in the parton model. The quark mass ratio (2) is then obtained in three ways. In Sec. III we begin the discussion of bare structure functions in terms of the naive weak-binding approximation and point out its shortcomings for the $\langle x^{-1}\rangle$ moments. Then we improve upon the approximation by invoking the quark-structure-function analysis of Farrar and Jackson[9] and reinforce it via the spectator-helicity rule.[10,11] These refined structure functions are then used to extract the quark mass scale (3) in Sec. IV. In Sec. V we extend the analysis to the charmed-quark sector, deriving (4) and (5) and comparing the splitting predictions with the usual static quark mass picture. The SU(2) breaking which arises from the difference between m_u and m_d is discussed in Sec. VI.

In Sec. VII we return to the chiral limit for the masses of the pseudoscalar and vector mesons and for the octet and decuplet baryons; we attempt to relate these quantities to SU(6) splitting via a spin-spin force. We also discuss the connection between our quark masses with the presently accepted picture of the static quark masses. We then summarize our results and draw conclusions in Sec. VIII.

II. LIGHT-PLANE DECOMPOSITION OF QUARK OPERATORS AND THE QUARK MASS RATIO

By way of review, we recall that if $m_s = \hat{m}$, then the light-plane charges Q_i^L are identical[12] to the spacelike charges Q_i. However, for the theories in which we are interested, $m_s \neq \hat{m}$ forces $Q_i^L \neq Q_i$; as a consequence, one expects hadron states to transform, to a good approximation,[13] irreducibly with respect to the Q_i^L. On the other hand, the chiral decomposition of the semistrong Hamiltonian density

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}'\,, \tag{6a}$$

$$[Q_i^5, \mathcal{H}_0] = 0\,, \quad [Q_i^5, \mathcal{H}'] \neq 0 \tag{6b}$$

is governed by the spacelike integrals of the axial-vector currents Q_i^5, and $\mathcal{H}'$ is given by the quark mass matrix (1). The mismatch between light-plane and spacelike charges then requires the "bad" quark operators, such as $\bar{q}\lambda_i q$, to have complicated SU(3) properties, in contrast to the simple transformation properties $\bar{q}\lambda_i q \sim \lambda_i$ as assumed in GMOR (Ref. 1). For example, in a vector gauge theory, the "good" two-component quark fields φ are projected from the Dirac four-component fields q, which satisfy $(i\not\partial - \mathfrak{M} - g\not B)q = 0$; this leads to the light-plane decomposition[5-8] of (1) for $x^0 + x^3 = 0$ in the light-plane gauge $B_+ = 0$:

$$\bar{q}\mathfrak{M}q \sim \varphi^\dagger \vec{\sigma}_\perp \cdot (\vec{\nabla}_\perp + ig\vec{B}_\perp)\mathfrak{M}\nabla_-^{-1}\varphi + \varphi^\dagger \mathfrak{M}^2 \nabla_-^{-1}\varphi\,. \tag{7}$$

It is clear from (7) that the operator $\bar{q}\mathfrak{M}q$ has mixed internal-symmetry transformation properties. If the first term in (7) dominates over the second, then the quark masses must necessarily be quite small and one would be led to simple octet transformation properties and the GMOR value for the quark mass ratio $m_s/\hat{m} \approx 25$, based on the pseudoscalar mass spectrum.

From the current quark picture point of view, however, the spin-flip structure of the first term in (7) would suppress it relative to the second, nonflip term. This is similar to what is called Z-diagram suppression in the infinite-momentum frame.[14-16] As a consequence, $\hat{m}$ cannot be very small compared to typical hadronic mass scales. We shall see later how this manifests itself in our scheme. Now, suppression of Z diagrams, equivalent to suppression of the helicity-flip term in (7), leads to simple forms for matrix elements of the quark scalar density operators. In particular, for proton matrix elements, SU(3) breaking is expressed as [14-16]

$$\langle p|\bar{u}u|p\rangle = 2m_u\tilde{f}_u\,,\quad \langle p|\bar{d}d|p\rangle = 2m_d\tilde{f}_d\,, \tag{8a}$$

$$\langle p|\bar{s}s|p\rangle = 2m_s\tilde{f}_s\,, \tag{8b}$$

with hyperon matrix elements determined by SU(3) generated by the Q_i^L, e.g.,

$$\langle \Sigma^+|\bar{u}u|\Sigma^+\rangle = 2m_u\tilde{f}_u\,,\quad \langle \Sigma^+|\bar{s}s|\Sigma^+\rangle = 2m_s\tilde{f}_d\,. \tag{8c}$$

Here $\tilde{f}_{u,d}$ are simply reduced matrix elements of the scalar quark densities. We will show later how these quantities are related to $\langle x^{-1}\rangle$ moments of structure functions; for the purposes of this section, however, no such additional information is needed.

Analogously, pion matrix elements of scalar densities may be expressed as

$$\langle \pi^+|\bar{u}u|\pi^+\rangle = 2m_u\tilde{h}\,,\quad \langle \pi^+|\bar{d}d|\pi^+\rangle = 2m_d\tilde{h}\,, \tag{9a}$$

$$\langle \pi^+|\bar{s}s|\pi^+\rangle = 2m_s\tilde{h}_s\,, \tag{9b}$$

with SU(3) similarly determining the kaon matrix elements

$$\langle K^+|\bar{u}u|K^+\rangle = 2m_u\tilde{h}\,,\quad \langle K^+|\bar{s}s|K^+\rangle = 2m_s\tilde{h}\,, \tag{9c}$$

$$\langle K^+|\bar{d}d|K^+\rangle = 2m_d\tilde{h}_s\,. \tag{9d}$$

It is possible to extend this scheme to decuplet and vector-meson matrix elements of bad quark densities.

To make use of the above results, we employ the decomposition of the total Hamiltonian density into an SU(3) × SU(3)- and SU(3)-invariant part $\mathcal{H}_0$ [which includes SU(3)-invariant sea and gluon contributions] and the chiral-breaking part $\mathcal{H}'$ as given by (6):

$$\langle P_i|\mathcal{H}|P_i\rangle = 2m_i^2\,, \tag{10}$$

where (10) is any diagonal hadronic matrix element of $\mathcal{H}$ in the rest frame, and we have used covariant normalization of states.

For the quark mass ratio, consider first pseudoscalar-meson mass splitting,

$$2m_\pi^2 = 2m_{0P}^2 + 4\hat{m}^2\tilde{h} = 2m_s\tilde{h}_s\,, \tag{11a}$$

$$2m_K^2 = 2m_{0P}^2 + 2(\hat{m}^2 + m_s^2)\tilde{h} = 2\hat{m}^2\tilde{h}_s\,, \tag{11b}$$

where we shall retain the $\tilde{h}_s$ terms for the moment. The difference between (11a) and (11b) eliminates the unknown chiral-symmetric m_{0P}^2 term, leading to

$$m_K^2 - m_\pi^2 = (m_s^2 - \hat{m}^2)(\tilde{h} - \tilde{h}_s) \approx 0.23\ \mathrm{GeV}^2\,. \tag{12}$$

Also, in the soft pion limit, the $\mathcal{H}_0$ contribution vanishes and $i\partial A = [Q_5, \mathcal{H}']$ gives[1,5,17]

$$m_\pi^2 \approx \langle \pi|\mathcal{H}'|\pi\rangle_{\mathrm{soft}} \approx 2\hat{m}^2\tilde{h} + m_s^2\tilde{h}_s\,. \tag{13}$$

Comparing (13) with (11a) then forces $m_{0P}^2 \approx 0$, the Goldstone limit. Then we assume the Zweig rule $\tilde{h}_s = 0$, which follows directly from the $\pi\pi$ σ term of $\sigma_{\pi\pi} = m_\pi^2$ and (13). Then the ratio of (12) to (13) is independent of the scale $\tilde{h}$ and determines the quark mass ratio[5]

$$m_s^2/\hat{m}^2 \approx (2m_K^2/m_\pi^2) - 1\,,\quad m_s/m \approx 5\,. \tag{14}$$

Alternatively, one can consider the ratio of the helicity-*nonflip* matrix elements $\langle 0|\bar{q}\gamma_5\lambda_K q|K\rangle$ and $\langle 0|q\gamma_5\lambda_\pi q|\pi\rangle$. Since again quark mass factors as in (8) occur, in this case $m_s + \hat{m}$ and $2\hat{m}$, respectively, the quark model commutators

$$[Q_5^\pi, \mathcal{H}'] = -i\hat{m}\bar{q}\gamma_5\lambda_\pi q\,,$$

$$[Q_5^K, \mathcal{H}'] = -i\tfrac{1}{2}(m_s + \hat{m})\bar{q}\gamma_5\lambda_K q\,,$$

lead to[5,7,18]

$$m_s/\hat{m} \approx (2m_K/m_\pi) - 1 \approx 6\,. \tag{15}$$

As discussed above, it is satisfying and significant that (14) and (15) give roughly the same quark mass ratio.

In the present paper, we shall choose to work with scalar rather than pseudoscalar densities, in order to extract quark masses. As noted above, for light quarks the difference is not significant; in the charm sector, however, we will see below that a choice should be made. It behooves us to comment on this problem and to indicate why we believe our choice is the more reliable one. A comparison of "PCAC" (partially conserved axial-vector current) and "mass formula" estimates of $m_s/\hat{m}$, (15) and (14), respectively, is essentially a comparison of estimates of matrix elements of v's and u's. Numerically, these are not identical but they are close, because m_K is not too much larger than m_π. They do have the property that m_K, m_π are first order in the quark masses m_u, m_s.[19]

Higher-order terms will modify the relations—just as higher-order terms are required to ensure that $f_K > f_\pi$; the axial-vector charge operator is $\varphi^\dagger\sigma_3\varphi$ in terms of current quarks (i.e., to lowest order in quark masses), but there will be corrections. For the axial charge and other local operators, these can be estimated by the method of

Carlitz and Weyers,[20] modified to incorporate some mass-breaking effects. The method breaks down for charmed-quark–carrying hadrons—the "corrections" may well be larger than the zeroth order term. Moreover, corrections to matrix elements of nonlocal operators such as

$$\int \varphi^{+}(x)(\vec{\sigma}_{\perp}\cdot\vec{\nabla}_{\perp}+\mathfrak{M})\epsilon(x^{-}-\xi)\frac{\lambda_i}{2}\begin{pmatrix}1\\ \sigma_3\end{pmatrix} \times\varphi(\vec{x}_{\perp},x^{+},\xi)d^2x_{\perp}d\xi\,dx^{-}\,, \tag{16}$$

which come from $\bar{q}\lambda_i q$ and $\bar{q}\lambda_i\gamma_5 q$, are more uncertain, since the nonlocality in x^- will generate corrections. Thus the scalar and pseudoscalar density operators will have to be corrected in higher order; the corrections are *different* for $\bar{q}\lambda_i q$ and $\bar{q}\lambda_i\gamma_5 q$, since $\bar{q}\lambda_i q$ requires a correction arising from the ∇_-^{-1} *only*, while $q\lambda_i\gamma_5 q$ requires this and *additionally* one from the σ_3 (just as Q_5^i does).

An independent estimate of the quark mass ratio can be found from the octet baryon mass splittings analogous to (11):

$$\langle N|\mathcal{H}|N\rangle = 2m_N^2 = 2m_{0B}^2 + 2\hat{m}^2(\bar{f}_u+\bar{f}_d)+2m_s^2\bar{f}_s\,. \tag{17}$$

Along with similar SU(3) splittings for the hyperons,[5] we may combine (17) to find the SU(3) average mass[21]

$$m_B^2\,|_{\mathrm{SU}(3)} = m_{0B}^2 + \tfrac{1}{3}(2m^2+m_s^2)(\bar{f}_u+\bar{f}_d+\bar{f}_s) = \tfrac{1}{8}(2m_{\Xi}^2+2m_N^2+m_{\Lambda}^2+3m_{\Sigma}^2)\approx 1.34\ \mathrm{GeV}^2\,. \tag{18}$$

Then eliminating the unknown chiral-symmetric mass m_{0B}^2 between (17) and (18), we obtain the nucleon SU(3) mass shift (the quadratic analog of 1149 − 939 = 210 MeV) in terms of quark parameters for $\bar{f}_s=0$ as

$$\Delta m_N^2 = \tfrac{1}{3}(m_s^2-\hat{m}^2)(\bar{f}_u+\bar{f}_d) = m_B^2\,|_{\mathrm{SU}(3)} - m_N^2 \approx 0.46\ \mathrm{GeV}^2\,. \tag{19}$$

To obtain the quark mass ratio from (19), we must also use the nucleon analog of the $\pi\pi$ σ term, extracted to be about 65 MeV from the most recent accurate data[22]:

$$2m_N\sigma_{\pi N} = \langle N|[Q_5^{\pi}, i\partial A^{\pi}]|N\rangle = \hat{m}\langle N|\bar{u}u+\bar{d}d|N\rangle = 2\hat{m}^2(\bar{f}_u+\bar{f}_d)\,. \tag{20}$$

The form (20), first derived in Ref. 14, has the same quark-structure-function dependence as (19) so that their ratio is independent of the quark scales

$$m_s^2/\hat{m}^2 \approx (3\Delta m_N^2/m_N\sigma_{\pi N})+1\,, \tag{21a}$$

$$m_s/\hat{m}\approx 5 \tag{21b}$$

for $\sigma_{\pi N}\approx 65$ MeV.

This result, saying that $\sigma_{\pi N}$ is a true measure of the quark mass *ratio*, has also been derived in light-plane language.[23] Moreover, the fact that (21) is consistent with the quark mass ratio as deduced from pseudoscalar-meson states, (14) and (15), argues strongly for no Z diagram or no quark helicity-flip term, assumptions leading to our initial ansatz (8) or (9). Further support for $m_s/\hat{m}\approx 5$ is obtained from the known Goldberger-Treiman discrepancy $\Delta_{\pi N}=1-m_N g_A/f_\pi g_{NN}$, which leads to[2,5,23] $m_s/\hat{m}\approx 5$ in *either* our chiral-breaking scheme or that of GMOR; no additional quark mass factors appear here because $\langle N|\bar{q}\gamma_5\lambda_\pi q|N\rangle$ is a spin-flip transition dominated by a $\vec{\sigma}_\perp\cdot\vec{\nabla}_\perp$-type operator rather than $\mathfrak{M}$ as in (7). In short, our scheme alone is consistent with *all* the data; the alternative GMOR picture requires $\sigma_{\pi N}$ and $\Delta_{\pi N}$ to be reduced, respectively, to $\sigma_{\pi N}\to 25$ MeV and $\Delta_{\pi N}\to 0.01$, rather than $\sigma_{\pi N}\sim 65$ MeV, $\Delta_{\pi N}\sim 0.06$.

The present model therefore gives a consistent description of chiral-symmetry breaking characterized by the current quark mass ratio (2), and we thus proceed to consider the current quark mass *scale*.

III. BARE QUARK STRUCTURE FUNCTIONS

The bare quark probability scales, which we were so careful to divide out in the last section, have a very simple structure. Normalizing the pion charge matrix element to the valence value of unity (one up quark in the π^+), we have[5,7,17,24]

$$\frac{1}{2E}\langle\pi^+|\bar{u}\gamma_+u|\pi^+\rangle = \int_0^1 dx\,h(x)=1\,, \tag{22a}$$

$$\langle\pi^+|\bar{u}u|\pi^+\rangle = 2m_u\bar{h}\,,\quad \bar{h}=\int_0^1 dx\,h(x)/x\,. \tag{22b}$$

The factor of $2m_u$ in (22) is a reflection of the additional quark mass factor in the dominant second term of (7), while the factor of x^{-1} in $\bar{h}$ is generated by the bilocal operator ∇_-^{-1} in (7). Similarly, for the proton we have in the parton-quark picture[5,14-16]

$$\frac{1}{2E}\langle p|\bar{u}\gamma_+u|p\rangle = \int_0^1 dx\,u(x)=2\,, \tag{23a}$$

$$\langle p|\bar{u}u|p\rangle = 2m_u\bar{f}_u\,,\quad \bar{f}_u=\int_0^1 dx\,u(x)/x\,, \tag{23b}$$

while for the Δ^{++} decuplet matrix elements

$$\frac{1}{2E}\langle\Delta^{++}|\bar{u}\gamma_+u|\Delta^{++}\rangle = \int_0^1 dx\,f_D(x)=3\,, \tag{24a}$$

$$\langle\Delta^{++}|\bar{u}u|\Delta^{++}\rangle = 2m_u\bar{f}_D\,,\quad \bar{f}_D=\int_0^1 dx\,f_D(x)/x\,. \tag{24b}$$

The 1^--vector mesons have structure functions as defined for the pions (22), but with $h \to h_V$.

If $\tilde{h}$, $\tilde{h}_V$, $\tilde{f}_u$, $\tilde{f}_D$ are to be finite, it is clear that the quark probability distributions $h(x)$, etc., must *vanish* as $x \to 0$. This tells us again that we are dealing with the bare structure functions which do vanish as $x \to 0$ once the sea and gluon contributions are subtracted out. The bare structure functions are then normalized to account for all of the hadron's momentum. Thus, for our bare structure functions we demand that

$$2\langle x\rangle_u + \langle x\rangle_d = \int_0^1 dx\, x[u(x)+d(x)] = 1\,, \tag{25a}$$

$$\tilde{h}_s = \tilde{h}_s^V = \tilde{f}_s = 0\,, \tag{25b}$$

where the down structure function in (25a) is normalized to $\int_0^1 dx\, d(x) = 1$ with

$$\langle p|\bar{d}d|p\rangle = 2m_d\tilde{f}_d\,, \quad \tilde{f}_d = \int_0^1 dx\, d(x)/x\,. \tag{26}$$

There are no more than the above (five) bare structure functions in the SU(3) extension of the $0^-, 1^-$ meson and $\frac{1}{2}^+, \frac{3}{2}^+$ baryon matrix elements because, e.g.,

$$\langle\Sigma^+|\bar{u}u|\Sigma^+\rangle = 2m_u\tilde{f}_u\,, \quad \langle\Sigma^+|\bar{s}s|\Sigma^+\rangle = 2m_s\tilde{f}_d\,, \tag{27a}$$

$$\langle\pi^+|\bar{d}d|\pi^+\rangle = 2m_d\tilde{h}\,, \quad \langle K^+|\bar{s}s|K^+\rangle = 2m_s\tilde{h}\,, \tag{27b}$$

$$\langle\Sigma^{*+}|\bar{u}u|\Sigma^{*+}\rangle = \tfrac{2}{3}2m_u\tilde{f}_D\,, \quad \langle\Xi^{*0}|\bar{u}u|\Xi^{*0}\rangle = \tfrac{1}{3}2m_u\tilde{f}_D\,, \tag{27c}$$

$$\langle\Sigma^{*+}|\bar{u}u|\Sigma^{*+}\rangle = \tfrac{2}{3}2m_u\tilde{f}_D\,, \quad \langle\Xi^{*0}|\bar{u}u|\Xi^{*0}\rangle = \tfrac{1}{3}2m_u\tilde{f}_D\,. \tag{27d}$$

The latter equations (27d) are a consistent statement of the bare valence picture with SU(3) broken only in the quark mass factors in (23b), (27a), (27b), etc., and not in the bare structure-function integral.

We next construct an initial (naive) dynamical model for the bare structure functions in the *weak-binding approximation*. Here we assume that the bound valence quarks in the hadrons are free, with each quark carrying an equal share of the hadron momentum:

$$h(x) = \delta(x-\tfrac{1}{2})\,, \quad h_V(x) = \delta(x-\tfrac{1}{2})\,, \tag{28a}$$

$$u(x) = 2\delta(x-\tfrac{1}{3})\,, \quad d(x) = \delta(x-\tfrac{1}{3})\,, \tag{28b}$$

$$f_D(x) = 3\delta(x-\tfrac{1}{3})\,. \tag{28c}$$

While this is certainly a very crude model, it will turn out that its integrated $\langle x^{-1}\rangle$ moments

$$\tilde{h} = \tilde{h}_V = 2\,, \quad \tilde{f}_u = 6 = 2\tilde{f}_d\,, \quad \tilde{f}_D = 9 \tag{29}$$

are surprisingly close to those derived from more sophisticated approaches. By construction, (28) forces the valence quarks to carry all of the momentum as in (25). A major flaw in this model is that the SU(6) structure of (29) and (28) leads to $m_\Sigma = m_\Lambda$. That is, in terms of the baryon semistrong d/f ratio, the formalism (22)–(27) to (10) leads to[3,5]

$$d/f = 1 - 2(\tilde{f}_d/\tilde{f}_u) = -\frac{3}{5}\left(\frac{3m_\Sigma^2 - m_N^2 - m_\Lambda^2 - m_\Xi^2}{m_\Xi^2 - m_N^2}\right) \approx -0.28\,, \tag{30a}$$

$$\tilde{f}_d/\tilde{f}_u \approx 0.64\,, \tag{30b}$$

and not $\tilde{f}_d/\tilde{f}_u = 0.5$ as in (29).

To improve upon the weak-binding approximation we next invoke the vector-gluon analysis near $x = 1$,[9,10] whereby the valence quarks are free but exchange a single gluon for the case of the pion, as shown in Fig. 1.

The external photon probes the hadron's structure function in the scaling region. For large Q^2 and $x \approx 1$, the quark which couples to the electromagnetic current necessarily has a very large invariant mass p^2, even in the scaling limit[25] $[p^2 \sim m^2(1-x)^{-1}]$, where m is a characteristic mass or $p_\perp$ scale (for the virtual quark). Under the usual assumption that the quarks, roughly speaking, equally share the energy of the hadron, each of the internal lines marked with an x in Fig. 1 have a large virtual mass of order $(Q^2)^{1/2}$; therefore, it is reasonable to expect that the effective quark-gluon couplings in Fig. 1 are small, so that one may use lowest-order perturbation theory. Similarly, the lowest-order graphs, such as the one of Fig. 2, dominate the proton structure function $u(x)$ as $x \to 1$ and $Q^2 \to \infty$.

Direct computation of the Feynman graphs of Figs. 1 and 2 then leads to[9,10,26]

$$h(x) \sim (1-x)^2 \text{ as } x \to 1\,, \tag{31a}$$

$$u(x) \sim (1-x)^3 \text{ as } x \to 1\,. \tag{31b}$$

The fact that the struck quark in Fig. 2 has the same spin direction as the proton follows from subtle arguments[27] concerning the polarizations of all the particles. This result together with (31a)

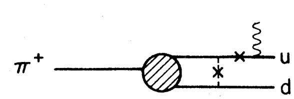

FIG. 1. Single-gluon exchange (dashed line) between valence u and d quarks in the π^+. The external photon (wavy line) is virtual, with $-k^2 = Q^2 \to \infty$. Quark propagators marked with an x have large invariant masses. The shaded confinement wave functions are presumed to keep the emerging quarks at low invariant mass.

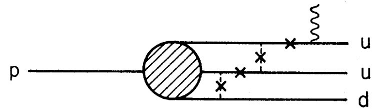

FIG. 2. Gluon-exchange graph for the proton analogous to Fig. 1.

is most simply expressed by the "spectator[10]-helicity[11] rule"

$$G_{a|A}(x) \sim (1-x)^{2n_s-1+2|\lambda_A-\lambda_a|} \text{ as } x \to 1\,, \tag{32}$$

where n_s is the number of spectator quarks in hadron A when quark a acquires all the momentum and λ_A, λ_a are the respective helicities in the infinite-momentum frame.

The rule says that it becomes hard to force one of the quarks to have all of the momentum of the hadron at $x=1$, and it becomes progressively more difficult the greater the number of spectator quarks we must stop to do it, since $\sum x_i = 1$. The specific exponent is a consequence of QCD; the $(1-x)^3$ dependence near $x=1$ for the nucleon agrees well with data[28] on $F_2^{ep}(x)$ and $F_2^{en}(x)$, as does (31a) for the pion structure function[29] near $x=1$.

This powerful rule (32) not only allows us to deduce immediately that $h_V(x) \sim (1-x)^2$, $d(x) \sim (1-x)^3$, and $f_D(x) \sim (1-x)^3$ near $x \to 1$, but it also enables us to construct the $x \to 0$ behavior of the five *bare* structure functions. In this case, momentum conservation requires (25), i.e., $x + x' = 1$, where x' represents the momentum fraction of the spectator quarks in the hadron in question. We can therefore extend (32) to

$$G_{a|A}(x) \sim x^{2n_s'-1+2|\lambda_A-\lambda_a'|} \text{ as } x \to 0\,, \tag{33}$$

where n_s' and λ_a' represent the number of spectators and helicity of the quark (or diquark pair) with momentum fraction x'. For the case of the 0^- and 1^- mesons, quark-antiquark symmetry requires $h(x)$ and $h_V(x)$ to be symmetric in x and $x' = 1-x$ so that, in fact, $h(x)$ and $h_V(x) \to x^2$ as $x \to 0$. The decuplet bare structure function is only slightly more complicated with $1-x' = x$ representing a diquark quark of spin-1 with momentum fraction x. Then with $n_s = 1$ and $2|\lambda_A - \lambda_a| = 1$ we apply (32) near $x' \to 1$ or $x \to 0$ to find $f_D(x) \to x^2$ as $x \to 0$. Knowledge of the end-point behavior plus normalization conditions, which reflect the constraint that the simplest bare quark state carry all of the hadron's momentum, essentially pins down these three bare structure functions to be

$$h(x) = G_{u/\pi^+}(x) = G_{\bar{d}/\pi^+}(x) = 30x^2(1-x)^2\,, \tag{34a}$$

$$h_V(x) = 30x^2(1-x)^2\,, \tag{34b}$$

$$f_D(x) = 504x^2(1-x)^5\,. \tag{34c}$$

While determination of the leading $x \to 0$ dependence of the proton bare structure functions with $n_s = 1$ and $2|\lambda_A - \lambda_a| = 1$ gives the same x^2 behavior as in (34), the complete x dependence is more complicated for two reasons. First, following Ref. 9, because the proton quark spins are not all aligned as in the 0^-, 1^-, and $\frac{3}{2}^+$ cases, we must fold in the QCD color-statistics factor

$$\langle n_{u\uparrow}\rangle / \langle n_{d\uparrow}\rangle = 5\,, \tag{35}$$

which says that there are five times as many up quarks as down quarks with spins pointing in the direction of the proton. This factor (35) in turn predicts that $\nu W_2^p = \frac{3}{7}$ at $x=1$, in reasonable agreement with experiment.[9] Furthermore, the 5:1 ratio is seen[30] in the inclusive cross section for $p + p \to \pi^\pm + \text{anything}$, near $x=1$. Second, we must break the SU(6)-symmetry relation $\tilde{f}_u/\tilde{f}_d = 2$, if we are to avoid the SU(6) prediction $m_\Sigma = m_\Lambda$ as in (29). These two requirements can only be satisfied if we keep not only the leading behavior near $x=1$, $(1-x)^3$, but also the next to the leading behavior as dictated by the spectator-helicity rule (32), $(1-x)^5$. The QCD color-statistics factor of these latter terms with quark spin in the opposite direction to that of the proton (so that $2|\lambda_A - \lambda_a| = 2$) is

$$\langle n_{u\downarrow}\rangle / \langle n_{d\downarrow}\rangle = \tfrac{1}{2}\,. \tag{36}$$

The combination of (35) and (36) corresponds to a total of $\frac{5}{3} + \frac{1}{3} = 2$ up quarks and $\frac{1}{3} + \frac{2}{3} = 1$ down quarks in the proton, as required.

Finally, demanding the normalizations $\int u = 2$ and $\int d = 1$, our first attempt to construct the proton bare structure functions (denoted by the subscript zero) leads to

$$u_0(x) = 20x^2[5(1-x)^3 + \tfrac{14}{5}(1-x)^5]\,, \tag{37a}$$

$$d_0(x) = 20x^2[(1-x)^3 + \tfrac{28}{5}(1-x)^5] \tag{37b}$$

and the desired ratio $\tilde{f}_{d0}/\tilde{f}_{u0} \approx 0.6$ according to (30). However, as remarked earlier, the exact bare structure functions must satisfy the momentum conservation condition (25). It turns out that in fact Eqs. (37) almost meet this requirement, giving instead $\sum \langle x_i \rangle = 1.19$. This small 19% discrepancy and $\tilde{f}_{d0}/\tilde{f}_{u0} \approx 0.6$ provide *a posteriori* justification for our entire bare structure scheme. To eliminate even this discrepancy we need only weight (37) by a smooth function of x which does not vanish at $x=0$ or $x=1$, while satisfying (25) and not altering $\tilde{f}_d/\tilde{f}_u$ to any great degree. We find that

$$u(x) = 49(1-\tfrac{1}{2}x)^4 x^2[5(1-x)^3 + 2.3(1-x)^5]\,, \tag{38a}$$

$$d(x) = 49(1-\tfrac{1}{2}x)^4 x^2[(1-x)^3 + 4.6(1-x)^5] \tag{38b}$$

is satisfactory, with the function $(1-\frac{1}{2}x)^4$ pushing the peak of (37a) down from $\lesssim 0.40$ closer to ~ 0.35, while only altering the ratio $\tilde{f}_d/\tilde{f}_u$ slightly to 0.57.

Finally, we are prepared to calculate the integrated $\langle x^{-1}\rangle$ moments of the five bare structure functions (34) and (38). Due to the x^2 behavior as $x\to 0$, these integrals are all finite as demanded if the theory is to make any sense. We obtain

$$\tilde{h}=\tilde{h}=\tfrac{5}{2}\,,\quad \tilde{f}_u\approx 7.9\,,\quad \tilde{f}_d\approx 4.5\,,\quad \tilde{f}_D=12 \tag{39}$$

and note that the scales are very close to the weak-binding results in (29), suggesting that these scales are somewhat *insensitive* to the dynamical model employed, providing that one invokes correct kinematics and valence charge normalization. This gives us added confidence that the quark masses obtained in the next section are almost model-independent results.

IV. CURRENT QUARK MASS SCALE

Given the five integrated structure-function moments in (39), we can determine five independent estimates of the current quark mass scale $\hat{m}$ or splitting $m_s^2-\hat{m}^2$. Since we know from Sec. II that $m_s/\hat{m}\approx 5$, it is clear that we will be able to obtain five values of $\hat{m}$. If the theory is meaningful, then all five determinations should be about the same.

For the 0^- mesons, we repeat the analysis for the pion which led to (14), again employing $\langle\pi|H_0|\pi\rangle\approx 0$, but this time we do not divide out the scale:

$$\langle\pi|H|\pi\rangle=2m_\pi^2=2\hat{m}^2\tilde{h}+2\hat{m}^2\tilde{h} \tag{40}$$

and for $\tilde{h}=\frac{5}{2}$ from (39),

$$\hat{m}=m_\pi/\sqrt{2}\,\tilde{h}=m_\pi/\sqrt{5}\approx 62\text{ MeV}\,. \tag{41}$$

For the 1^- mesons there is no Goldstone theorem but the analogs of (40); dividing out the factor of 2,

$$m_\rho^2=m_{0V}^2+2\hat{m}^2\tilde{h}_V\,, \tag{42a}$$

$$m_{K^*}^2=m_{0V}^2+(m_s^2+\hat{m}^2)\tilde{h}_V \tag{42b}$$

gives for $\tilde{h}_V=\frac{5}{2}$ the splitting

$$m_s^2-\hat{m}^2=(m_{K^*}^2-m_\rho^2)/\tilde{h}_V\approx 4.8m_\pi^2\,, \tag{43}$$

corresponding again to $\hat{m}\approx 62$ MeV for $m_s/\hat{m}\approx 5$. The fact that (41) is exactly reproduced is, of course, an SU(6) statement $m_{K^*}^2-m_\rho^2=m_K^2-m_\pi^2$, which is a consequence of any quark model, as is the equal splitting law $m_\varphi^2-m_{K^*}^2=m_{K^*}^2-m_\rho^2$ with $m_\rho^2=m_\omega^2$ and φ pure $\bar{s}s$. We note, however, that although $m_s\neq\hat{m}$ breaks SU(6), the SU(6) equality $\tilde{h}=\tilde{h}_V$ is still maintained and this is consistent with meson spectroscopy.

Proceeding on to the baryon sector, we note that the two $\frac{1}{2}^+$ scales $\tilde{f}_u\approx 7.9$ and $\tilde{f}_d\approx 4.5$ are only approximate results which give $\tilde{f}_d/\tilde{f}_u\approx 0.57$ rather than the experimental value 0.64. It is then preferable to combine $\tilde{f}_u$ and $\tilde{f}_d$ to obtain a pure nonstrange-quark estimate of $\hat{m}$ from the nucleon matrix elements of the σ term,[14] found in (20), giving in this case for $\sigma_N\approx 65$ MeV and $\tilde{f}_u+\tilde{f}_d\approx 12.4$

$$m_N\sigma_N=\hat{m}^2(\tilde{f}_u+\tilde{f}_d)\,,\quad \hat{m}\approx 70\text{ MeV}\,. \tag{44}$$

A second, octet-breaking estimate from the $\frac{1}{2}^+$ baryons picks out the sensitive combination $\tilde{f}_u-\tilde{f}_d$ via the baryon matrix elements (17) in order to eliminate the m_{0B}^2 contributions[3]

$$\begin{aligned} m_B^2|_8&=(m_s^2-\hat{m}^2)(\tilde{f}_u-\tilde{f}_d)\\ &=\tfrac{1}{10}(8m_\Xi^2-2m_N^2+3m_\Lambda^2-9m_\Sigma^2)\\ &\approx 0.31\text{ GeV}^2\approx 16m_\pi^2\,. \end{aligned} \tag{45}$$

Then for our estimate $\tilde{f}_u-\tilde{f}_d\approx 3.4$ we obtain the current quark splitting

$$m_s^2-\hat{m}^2=m_B^2|_8/(\tilde{f}_u-\tilde{f}_d)\approx 4.7m_\pi^2\,, \tag{46}$$

which is remarkably close to the 1^- result (43). Or equivalently for $m_s/\hat{m}\approx 5$, Eq. (46) implies $\hat{m}\approx 61$ MeV, only slightly below (41) and (44). In terms of the Σ-Λ mass difference, (17) leads to $m_\Sigma^2=m_N^2+(m_s^2-\hat{m}^2)\tilde{f}_d$, which is slightly lower than the actual Σ mass; correspondingly, m_Λ is slightly higher than experiment, both results following because (30b) is not exactly satisfied by (39).

Finally, for the $\frac{3}{2}^+$ decuplet baryons, the quadratic mass formula (17) is known to be well satisfied by experiment and the Okubo equal-splitting law. We average over the slight deviations by computing the pure octet-splitting part via $m_{\Sigma^*}^2=m_\Delta^2+\frac{1}{3}(m_s^2-\hat{m}^2)\tilde{f}_D$, etc., leading to[4]

$$\begin{aligned} m_D^2|_8&=\tfrac{1}{3}(m_s^2-\hat{m}^2)\tilde{f}_D=\tfrac{1}{5}(m_\Omega^2+m_{\Xi^*}^2-2m_\Delta^2)\\ &\approx 0.42\text{ GeV}^2\approx 21.6m_\pi^2\,. \end{aligned} \tag{47}$$

Then for $\tilde{f}_D=12$, Eq. (47) predicts the splitting

$$m_s^2-\hat{m}^2\approx 5.4m_\pi^2\,, \tag{48}$$

only slightly higher than the 1^- and $\frac{1}{2}^+$ octet splittings. In terms of a quark mass scale with $m_s/\hat{m}\approx 5$, this discrepancy is even smaller than that of (48), $\hat{m}\sim 65$ MeV.

Thus, all five estimates appear to be consistent with the ratio $m_s/\hat{m}\approx 5$ and the scale $\hat{m}\approx 62$ MeV. It would be nice, of course, to extract this current quark mass scale directly from experiment. Two such possibilities have been considered.[3,5] The fixed-pole residue from Compton scattering

$$C^{\gamma p}\equiv\int_0^\infty\frac{dx}{x^2}\tilde{F}_2^{ep}(x)\approx\tfrac{4}{9}\tilde{f}_u+\tfrac{1}{9}\tilde{f}_d \tag{49}$$

includes unknown contributions for $x > 1$ outside the scaling region but they are thought to be small. However, even the extracted value[4] $C^{\gamma p} \approx 1$ may be altered by the currently measured rising cross section at high energies. Secondly, the photoproduction multipole amplitude at threshold can, in principle, determine the nonstrange current quark mass. A rough estimate[4,5] yields $\hat{m} \sim m_\pi$, but a more detailed covariant treatment[31] leads to values of $\hat{m}$ varying from $\sim -m_\pi$ to $\sim 2m_\pi$, depending on the data employed. Clearly, more accurate low-energy photoproduction data are needed to obtain a reliable value of $\hat{m}$ in this way.

V. CHIRAL-BREAKING ANALYSIS IN THE CHARMED-QUARK SECTOR

Having found consistency for $m_s/\hat{m}$ and $\hat{m}$ in all types of meson and baryon chiral-SU(3)-breaking configurations, we proceed to extend the theory to the SU(4) charmed-quark sector. Rather than leading to a whole host of new chiral-breaking parameters, the SU(4) sector involves only the ratio $m_c/\hat{m}$ and scale m_c, the latter two, of course, linked to $\hat{m}$ as determined in the last section.

Consider first the observed pseudoscalar-meson spectrum[32]

$$\begin{aligned} &m_\pi \approx 138 \text{ MeV}, \\ &m_K \approx 496 \text{ MeV}, \quad m_K^2 \approx 12.9 m_\pi^2 \\ &m_D \approx 1866 \text{ MeV}, \quad m_D^2 \approx 183 m_\pi^2 \\ &m_F = 2030 \pm 60 \text{ MeV}, \quad m_F^2 \approx 216 m_\pi^2 . \end{aligned} \tag{50}$$

In the current-quark model we may continue to take bare valence quark distributions so that the SU(4) extension of (22b), (27b), and (27d) is, for $|D^+\rangle = |\bar{d}c\rangle$, and $|F^+\rangle = |\bar{s}c\rangle$,

$$\langle D^+|\bar{d}d|D^+\rangle = 2m_d\tilde{h}, \quad \langle D^+|\bar{c}c|D^+\rangle = 2m_c\tilde{h}, \tag{51a}$$

$$\langle F^+|\bar{s}s|F^+\rangle = 2m_s\tilde{h}, \quad \langle F^+|\bar{c}c|F^+\rangle = 2m_c\tilde{h}, \tag{51b}$$

$$\begin{aligned}\langle D^+|\bar{u}u|D^+\rangle &= \langle D^+|\bar{s}s|D^+\rangle \\ &= \langle F^+|\bar{u}u|F^+\rangle = \langle F^+|\bar{d}d|F^+\rangle = 0 .\end{aligned} \tag{51c}$$

The charmed-hadron matrix elements of $H = H_0 + H'$ combined with (51) and the Goldstone condition then leads to

$$m_D^2 = (m_c^2 + \hat{m}^2)\tilde{h}, \tag{52a}$$

$$m_F^2 = (m_c^2 + m_s^2)\tilde{h}. \tag{52b}$$

Dividing (52) by $m_\pi^2 = 2\hat{m}^2\tilde{h}$ results in quark mass ratios similar to (14),

$$m_c^2/\hat{m}^2 = 2(m_D^2/m_\pi^2) - 1, \quad m_c/\hat{m} \approx 19 \tag{53a}$$

$$\frac{m_c^2 + m_s^2}{\hat{m}} \approx \frac{2m_F^2}{m_\pi^2}, \quad \frac{m_s}{\hat{m}} \approx 5, \quad \frac{m_c}{\hat{m}} \approx 20. \tag{53b}$$

It is certainly significant that (53a) and (53b) are in almost perfect agreement.

To reinforce this result we also work out the nonflip pseudoscalar-meson transitions in the quark model:

$$\langle 0|\partial\cdot A_\pi|\pi\rangle = f_\pi m_\pi^2 = -\hat{m}\langle 0|q\gamma_5\lambda_\pi q\,|\pi\rangle, \tag{54a}$$

$$\langle 0|\partial\cdot A_K|K\rangle = f_\pi m_K^2 = -\tfrac{1}{2}(m_s + \hat{m})\langle 0|\bar{q}\gamma_5\lambda_K q|K\rangle, \tag{54b}$$

$$\langle 0|\partial\cdot A_D|D\rangle = f_D m_D^2 = -\tfrac{1}{2}(m_c + \hat{m})\langle 0|\bar{q}\gamma_5\lambda_D q|D\rangle, \tag{54c}$$

$$\langle 0|\partial\cdot A_F|F\rangle = f_F m_F^2 = -\tfrac{1}{2}(m_c + m_s)\langle 0|\bar{q}\gamma_5\lambda_F q|F\rangle. \tag{54d}$$

Scaling the matrix elements on the right-hand side of (54) by the appropriate quark mass combination in our parton-quark-model scheme, as in the second term of (7) or as in (22)–(27) and (51), the ratio of (54b) to (54a) produces (15), while the ratio of (54c) or (54d) to (54a) predicts for $f_\pi \approx f_K \approx f_D \approx f_F$

$$m_c/\hat{m} \approx 2(m_D/m_\pi) - 1 \approx 26, \tag{55a}$$

$$\frac{m_c + m_s}{\hat{m}} \approx \frac{2m_F}{m_\pi}, \quad \frac{m_s}{\hat{m}} \approx 6, \quad \frac{m_c}{\hat{m}} \approx 23. \tag{55b}$$

Both (55a) and (55b) are reasonably consistent with one another and with (53). Just as in the nonstrange sector, we shall continue to prefer the mass-formula method of determining the current quark masses, (52) and (53), over the PCAC method, (54) and (55). For matrix elements involving (heavy) charmed quarks, an early indication that large corrections are necessary can be found in the paper of Buccella *et al.*[33]

These results for quark mass ratios involving m_c are in good agreement with some recent work. Ong[34] has shown that the discrepancy between the Weinberg[35] sum rules and the leptonic decays of J/ψ is resolved by the inclusion of effects of quark masses, along lines suggested by Bernard *et al.*[36]; he finds $m_c/m_s \approx 3.9$, in good agreement with our value of 4. Moreover, Fritzsch[37] has obtained a relation between the Cabibbo angle and the quark mass ratios:

$$\sin\theta = \frac{(m_d m_c)^{1/2} - (m_u m_s)^{1/2}}{[(m_c + m_u)(m_d + m_s)]^{1/2}}. \tag{56}$$

Without questioning the validity of his derivation, we would like to note that our derived mass ratios give $\theta \approx 13°$, quite close to the experimental value.

The 1^- vector-meson sector also has a bearing on the charmed current quark mass. The mass formulas analogous to (42) are

$$m_{D^*}^2 = m_{0V}^2 + (m_c^2 + \hat{m}^2)\tilde{h}_V, \tag{57a}$$

$$m_{F^*}^2 = m_{0V}^2 + (m_c^2 + m_s^2)\tilde{h}_V. \tag{57b}$$

While the F^* mass $\sim$2140 MeV has large uncertainties, the D^* mass of 2007 MeV is accurately known.[32] Given (57) and $\tilde{h}_V = \tilde{h}$, the amazing accuracy of the corresponding meson SU(6) mass relations

$$m_{D^*}^2 - m_D^2 = m_{K^*}^2 - m_K^2 = m_\rho^2 - m_\pi^2 \qquad (58)$$

argues very strongly in favor of our quadratic-mass-formula, current-quark-model approach. Put another way, the magnitude of the charmed- to nonstrange-quark mass splittings based upon the parton structure-function scales (39), i.e., $\tilde{h} = \tilde{h}_V = \frac{5}{2}$, leads to

$$m_c^2 - \hat{m}^2 = \begin{cases} (m_D^2 - m_\pi^2)/\tilde{h} \approx 1.38 \text{ GeV}^2, & (59a) \\ (m_{D^*}^2 - m_\rho^2)/\tilde{h}_V \approx 1.37 \text{ GeV}^2. & (59b) \end{cases}$$

The consistency between the 0^- and 1^- charmed sectors is reassuring. We therefore take seriously the implied quark mass scale

$$m_c = (m_c/\hat{m})\hat{m} \approx 19 \times 62 \text{ MeV} \approx 1200 \text{ MeV}, \qquad (60)$$

with an uncertainty of about 100 MeV due to the uncertainties in the determination of the ratio $m_c/\hat{m}$ and the scale. The value implied by (59), $m_c \approx 1170$ MeV, falls within this range. With hindsight, Eq. (60) is about what one might expect for the charmed *current* quark mass scale. That is, just as our derived current quark masses $\hat{m} \approx 62$ MeV and $m_s \approx 310$ MeV are somewhat lighter than the *static* quark masses[38] of $\hat{m}^{\text{st}} \approx \frac{1}{2} m_\rho \approx 350$ MeV and $m_s^{\text{st}} \approx \frac{1}{2} m_\varphi \approx 550$ MeV, we should expect that m_c as given by (60) would be lighter than the static value of $m_c^{\text{st}} \approx \frac{1}{2} m_\psi \approx 1550$ MeV. *We regard this derived pattern and (60) as one of the major results of this analysis.*

Before leaving the meson sector, it is well to point out why the neutral isoscalar 0^- and 1^- states, $\eta, \eta', \eta_c, \omega, \varphi, \psi$, have been left out of our chiral-breaking analysis up to this point. This is because they can mix with the neutral gluons, thus destroying the simple quadratic mass formulas from which our results are derived.[38,39] The U(1) problem[40] for the η, η' states is an example of the complications that can arise. Clearly, the neutral isoscalar mesons must be incorporated into the theory, but we defer discussion of this problem to future work.

Lastly, we turn to a description of chiral breaking in the charmed-baryon sector. Unfortunately, the charmed-baryon masses are not yet well determined, although it is suggested from neutrino events at BNL and CERN,[41] $\bar{\Lambda}$ photoproduction at Fermilab,[42] and inclusive hadron-to-lepton cross-section ratios at SLAC[43] that the low-lying charmed baryons are in the 2.3 GeV region. The static quark mass picture[38] certainly suggests this region, but as we shall show, so does the current quark picture.

TABLE I. The masses of the charmed baryons (in MeV). Masses in the column labeled DGG are from Ref. 38; masses in the column labeled FS are results of the present work, normalized to the same value of the C_0 mass for comparison. Notation is that of Ref. 44.

	DGG	FS	Quark content
C_0	2200	2200	$c[ud]$
C_1	2360	2700	cuu, $c\{ud\}$, cdd
A	2420	2310	$c[su]$, $c[sd]$
S	2510	2780	$c\{su\}$, $c\{sd\}$
T	2680	2750	css
$X_{u,d}$	3550	3490	ccu, ccd
X_s	3730	3570	ccs
C_1^*	2420	2810	cuu, $c\{ud\}$, cdd
S^*	2560	2890	$c\{su\}$, $c\{sd\}$
T^*	2720	2860	css
$X_{u,d}^*$	3610	3570	ccu, ccd
X_s^*	3770	3650	ccs
θ	4810	4300	ccc

Consider, for example, the C_1 isovector charmed baryons[44] with quark content $C_1^{++}(cuu)$, $C_1^+(c\{ud\})$, and $C_1^0(cdd)$. Ignoring the SU(2) splitting, our current-quark scheme gives

$$m_{C_1}^2 = m_N^2 + (m_c^2 - \hat{m}^2)\tilde{f}_d, \qquad (61a)$$

$$\frac{m_{C_1}^2 - m_N^2}{m_\Sigma^2 - m_N^2} = \frac{(m_c/\hat{m})^2 - 1}{(m_s/m)^2 = 1}. \qquad (61b)$$

Given $m_c/\hat{m} \approx 19$ from (53a) and $m_s/\hat{m} \approx 5$, Eq. (61b) predicts that $m_{C_1} \approx 2.97$ GeV, while $\tilde{f}_d \approx 4.5$ and (59) predict from (61a) that $m_{C_1} \approx 2.65$ GeV. We see that there is a 300 MeV uncertainty in our predictions due to our imprecise knowledge of m_c^2, $m_s/\hat{m}$, $\tilde{f}_u$, and $\tilde{f}_d$. Nevertheless, the *pattern* of the spectrum of charmed baryons can be obtained simply from our estimates of these parameters together with mass formulas such as (61), which are analogous to formulas of Gaillard *et al.*[44] In Table I we present our results for the charmed-baryon masses normalized to the same value for the C_0 mass so as to facilitate comparison. There is a qualitative distinction in the pattern of symmetry breaking between our prediction and that of Ref. 38. However, owing to the uncertainty in the various parameters (we have used $\tilde{f}_u = 7.9$ and $\tilde{f}_d = 4.5$ to obtain the values given in the table) a quantitative comparison is not possible.

VI. MASS DIFFERENCE OF u AND d QUARKS

Now that we have determined the quark masses m_c, m_s, and $\hat{m}$, for completeness we should fix the u and d quark masses separately, rather than only their average mass $\hat{m}$. To estimate the SU(2) quark mass splitting, we may use the SU(3) × SU(3) Dashen theorem[45] for pseudoscalar electromagnetic mass splittings or, alternatively, the fitted $\mathcal{H}_{JJ}$ matrix elements for baryon electromagnetic mass splittings (these two estimates are, in fact, consistent[5]).

Perhaps the cleanest way to indicate the result is to concentrate on the $m_{\Sigma^+} - m_{\Sigma^-} = -7.9$ MeV splitting, since $\mathcal{H}_{JJ}$ contributes almost nothing to this *difference*. In our scheme, we may then write

$$m_{\Sigma^+}^2 - m_{\Sigma^-}^2 = (m_u^2 - m_d^2)\tilde{f}_u = -0.019 \text{ GeV}^2 . \quad (62)$$

Then for $f_u = 7.9$ and $\hat{m} = 62$ MeV, Eq. (62) leads to

$$\begin{aligned} &m_d - m_u \approx 19 \text{ MeV}, \quad m_d/m_u \approx 1.37, \\ &m_u \approx 52 \text{ MeV}, \quad m_d \approx 72 \text{ MeV}. \end{aligned} \quad (63)$$

On the other hand, the SU(2) splitting may be extracted from meson mass data. The K-meson electromagnetic mass difference arises from two sources,

$$\begin{aligned} m_{K^+}^2 - m_{K^0}^2 &= (m_{K^+}^2 - m_{K^0}^2)_{JJ} + (m_{K^+}^2 - m_{K^0}^2)_{ud} \\ &= -0.0040 \text{ GeV}^2 , \end{aligned} \quad (64)$$

where the second term is induced by the u-d quark mass difference

$$(m_{K^+}^2 - m_{K^0}^2)_{ud} = (m_u^2 - m_d^2)\tilde{h} . \quad (65)$$

The first term is given by Dashen's theorem,

$$(m_{K^+}^2 - m_{K^0}^2)_{JJ} = (m_{\pi^+}^2 - m_{\pi^0}^2) = +0.0013 \text{ GeV}^2 . \quad (66)$$

Therefore, for $\tilde{h} = 2.5$ and $\hat{m} = 62$ MeV,

$$\begin{aligned} &m_d - m_u \approx 17 \text{ MeV}, \quad m_d/m_u \approx 1.32, \\ &m_u \approx 53 \text{ MeV}, \quad m_d \approx 71 \text{ MeV}, \end{aligned} \quad (67)$$

in good agreement with (63), which was obtained from baryon mass data alone.

VII. CHIRAL-SYMMETRIC MASSES AND SU(6) SPLITTING OF $\mathcal{H}_0$

Now that we know the scale of all of the current quark masses, we may return to the general chiral decomposition (34) and identify the various *chiral-symmetric* masses, m_{0P}^2, m_{0V}^2, m_{0B}^2, and m_{0D}^2 which are the hadronic matrix elements of $\mathcal{H}_0$ within a given SU(6) multiplet. We already know from a comparison of (11a) and (13) that the Goldstone, pseudoscalar mass is

$$m_{0P}^2 = 0 . \quad (68a)$$

Likewise, $2m_\rho^2 = 2m_{0V}^2 + 4\hat{m}^2\tilde{h}_V$ so that

$$m_{0V}^2 = m_\rho^2 - 2\hat{m}^2\tilde{h}_V \approx 31m_\pi^2 - m_\pi^2 \approx 30m_\pi^2 . \quad (68b)$$

For the octet baryons, (42) gives

$$m_{0B}^2 = m_N^2 - \hat{m}^2(\tilde{f}_u + \tilde{f}_d) \approx 46m_\pi^2 - 3m_\pi^2 \approx 43m_\pi^2 , \quad (68c)$$

corresponding to $m_{0B} \approx 910$ MeV, while for the decuplet baryons

$$m_{0D}^2 = m_\Delta^2 - \hat{m}^2\tilde{f}_D \approx 78m_\pi^2 - 3m_\pi^2 \approx 75m_\pi^2 . \quad (68d)$$

We observe that $m_{0V}^2 - m_{0P}^2 \approx m_{0D}^2 - m_{0B}^2 \approx 30m_\pi^2$, which suggests that the chiral-symmetric SU(6) splitting between the meson multiplets, and also between the baryon multiplets, may have the same origin, such as a spin-spin term in $\mathcal{H}_0$. That this is reasonable was suggested some time ago[46] in a study of the phenomenology of the current-constituent quark transformation for the vector-gluon model. The Hamiltonian for the vector-gluon model is easily written; formulated in terms of null-plane operators, the interaction piece of the Hamiltonian has a term linear in the transverse components of the gluon field $\vec{A}_\perp$, a term quadratic in $\vec{A}_\perp$, and a "Coulomb" term in which $\vec{A}_\perp$ does not appear.[47] In order to exhibit the SU(6) properties of these terms, one must first use the Melosh transform so as to express them as functions of constituent quark operators. One finds that the "Coulomb" piece of the interaction gives rise to a contribution of the form

$$\begin{aligned} &\int d\xi\, d^2x_\perp dx^- \varphi^\dagger(x)\vec{\sigma}_\perp \cdot \vec{\nabla}_\perp \varphi(x)|x^- - \xi| \\ &\quad \times \varphi^\dagger(\vec{x}_\perp, \xi)\vec{\sigma}_\perp \cdot \vec{\nabla}_\perp \varphi(\vec{x}_\perp, \vec{\xi}) , \end{aligned} \quad (69)$$

which is *not* invariant under SU(6) but in fact transforms as a sum of terms which occur in the product $\underline{35} \times \underline{35}$. It is SU(3) invariant, however, and thus can give rise to the kind of mass splitting described above.

The mechanism suggested above is analogous to that used by De Rújula *et al.*[38] to construct a spin-spin force between *static* quarks. Their static quark masses, $\hat{m}(\text{static}) \approx \frac{1}{2}m_\rho \approx \frac{1}{3}m_\Delta \approx 350$ MeV and $m_s(\text{static}) \approx \frac{1}{2}m_\varphi \approx \frac{1}{3}m_\Omega \approx 550$ MeV, are derived by linking the Δ-N splitting with the Σ-Λ splitting. For our current quark scheme, the Δ-N splitting is associated with the chiral-symmetric part of $\mathcal{H}$, while the Σ-Λ splitting depends upon the SU(6) breaking of $\tilde{f}_d/\tilde{f}_u$ from 0.5 to 0.6 according to (30). It would be a quite convincing test of our scheme to show that a spin-spin force, such as that contained in (69), in fact leads to the prescribed SU(6) splittings (68). We are currently examining this possibility.

VIII. CONCLUSION

We have shown that in the present current-quark model a consistent picture emerges for the theory of chiral-symmetry breaking and the phenomenology of hadron spectroscopy if the current quark masses (i.e., the masses appearing in the QCD Lagrangian density) are $\hat{m} \approx 62$ MeV, $m_s \approx 5\hat{m} \approx 310$ MeV, and $m_c \approx 1200$ MeV. The quark mass ratio $m_s/\hat{m}$ is determined in three different ways (0^- masses, $\sigma_{\pi N}$, Goldberger-Treiman discrepancy) and the quark mass scale is set in five independent ways from the $\langle x^{-1}\rangle$ bare structure-function moments. The ratio $m_c/\hat{m}$ is found from the 0^--meson masses, and is consistent with the 1^- masses.

Our current-quark picture does not contradict, but rather complements the static-quark picture.[38] The latter approach is best suited to explain the 1^- vector mesons and $\frac{3}{2}^+$ baryons; it does not describe the 0^- mesons well, especially the light-mass pion. On the other hand, our chiral-breaking approach is designed to explain, first and foremost, the pion; however, we have seen that it also gives a respectable description of all of the other 0^- and 1^-, $\frac{1}{2}^+$ and $\frac{3}{2}^+$ states, light and heavy. Our approach, however, is not consistent with the competing GMOR chiral-breaking model,[1] with current quark masses[48] $\hat{m} \approx 5$ MeV, $m_s \approx 25\hat{m} \approx 150$ MeV, and $m_c \approx 1500$ MeV. This latter theory is based solely on the ratio $m_s/\hat{m}$ as found from the 0^- masses, assuming the first, spin-flip term in (7) to have simple SU(3) transformation properties and to dominate the quark mass matrix. In defense of our current-quark approach to chiral-symmetry breaking, we note that quark models not directly related to the mass ratio $m_s/\hat{m}$ (such as the MIT bag model, mass-breaking scheme[49] or the one-loop calculation of quark masses in QCD[50]) appear to generate nonstrange quark masses of the order of 40–120 MeV, roughly consistent with our scale of 62 MeV. Moreover, as mentioned above, recent analysis[34] resolving the discrepancy between the Weinberg sum rules[35] and the leptonic decays of J/ψ results in a current quark mass ratio $m_c/m_s \approx 3.9$; a similar result follows[51] from arguments based on asymptotic freedom. All these estimates are consistent with our results, but differ drastically from the GMOR values.[52]

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