Effective Hamiltonian for $\Delta S = 1$ weak nonleptonic decays in the six-quark model

Frederick J. Gilman and Mark B. Wise

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

(Received 18 June 1979)

Strong-interaction corrections to the nonleptonic weak-interaction Hamiltonian are calculated in the leading-logarithmic approximation using quantum chromodynamics. Starting with a six-quark theory, the W boson, t quark, b quark, and c quark are successively considered as "heavy" and the effective Hamiltonian calculated. The resulting effective Hamiltonian for strangeness-changing nonleptonic decays involves u, d, and s quarks and has possible CP-violating pieces both in the usual $(V - A) \times (V - A)$ terms and in induced, "penguin"-type terms. Numerically, the CP-violating compared to CP-conserving parts of the latter terms are close to results calculated on the basis of the lowest-order "penguin" diagram.

I. INTRODUCTION

In the standard six-quark model with charge $+\frac{2}{3}$ quarks u, c, and t and charge $-\frac{1}{3}$ quarks d, s, and b, the left-handed quarks are assigned to weak-isospin doublets and the right-handed quarks to weak-isospin singlets of the $SU(2) \otimes U(1)$ gauge group of weak and electromagnetic interactions. The mixing between quarks in doublets characterized, say, by their charge $+\frac{2}{3}$ members, is describable by three Cabibbo-type angles θ_1, θ_2, and θ_3, and by a single phase, δ, which results in CP violation. The nonleptonic weak interaction that can result in a net change in quark flavors is given to lowest order in weak interactions, and zeroth order in strong interactions, by the product of a weak current of left-handed quarks, a charged W-boson propagator, and another weak current of left-handed quarks. Neglecting the momentum-transfer dependence of the W-boson propagator, one has the usual local $(V - A) \times (V - A)$ structure of a current-current weak nonleptonic Hamiltonian.

With the introduction of strong interactions, in the form of quantum chromodynamics (QCD), things become more complicated. Consider, for example, that part of the nonleptonic Hamiltonian responsible for decay of kaons and hyperons which we write in terms of the "light" quarks u, d, and s. As the strong interactions are turned on, not only is the lowest-order $(V - A) \times (V - A)$ term involving u, d, and s quarks modified by gluon exchanges between the quarks, but there are diagrams involving virtual "heavy" quarks in loops which contribute to the strangeness-changing nonleptonic Hamiltonian. These alter the strength of the $(V - A) \times (V - A)$ terms and introduce new terms with different chiral structure, e.g., $(V - A) \times V$.

It is the purpose of this paper to calculate the effective nonleptonic Hamiltonian for strangeness-changing decays in the six-quark model. We successively consider the W boson, t quark, b quark, and c quark as very heavy, and use renormalization-group techniques to calculate (in the leading-logarithmic approximation) the resulting effective Hamiltonian remaining at each stage.

The basic techniques for carrying out such calculations have been laid out previously.[1-4] They were even applied in the four-quark model to get the effective Hamiltonian for strangeness-changing decays with the charm quark (and W boson) taken as heavy. However, there is only one Cabibbo angle in the four-quark model and no CP-violating phase. It is the CP-violating pieces of the effective nonleptonic Hamiltonian which are of special interest to us in this paper.

In a previous paper[5] we have raised the possibility that the diagram in Fig. 1 (the so-called "penguin" diagram) gives rise to a term H^{penguin} in the effective Hamiltonian that yields amplitudes for strange-particle decay with important CP-violating parts. Other analyses[3,4,6] claim that such penguin-type terms make a major contribution to the amplitudes for K decay into pions and are responsible for the $\Delta I = \frac{1}{2}$ rule. Assuming this we showed that in the six-quark model the magnitude of CP violation arising from the contribution of the matrix element of H^{penguin} to the decay amplitude is comparable to that coming from the mass matrix. It followed that the six-quark model yields predictions for the CP-violation parameters of the kaon system (in particular ϵ'/ϵ) which are distinguishable from those of the superweak model.

Two questions could be asked about the validity of using Fig. 1 to estimate the ratio of CP-violating to CP-conserving amplitudes. First is the effect of multiple soft-gluon exchanges. This has been answered in Ref. 7 where it is shown how the local four-fermion structure of the effective Hamiltonian is preserved despite the presence of
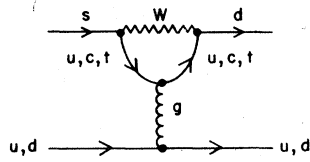
FIG. 1. "Penguin"-type diagram.

multiple soft-gluon exchanges. Essentially, owing to gauge invariance such soft-gluon effects go into corrections to the matrix elements of the local four-fermion operator H^{penguin} resulting from a calculation of the lowest-order diagram in Fig. 1. Thus, to leading order in the large masses the ratio of imaginary (CP-violating) to real (CP-conserving) parts of the $K \to 2\pi$ amplitude previously estimated by us is unchanged by the presence of multiple soft-gluon exchanges.

A second question is the effect of hard-gluon exchanges. These are expected to alter the results of our previous calculations. This paper provides a detailed answer of the amount of this change. We systematically analyze the QCD corrections to the effective Hamiltonian in leading-logarithmic approximation.

In the next section we describe the method by which the effective Hamiltonian for nonleptonic strangeness-changing decays is to be calculated in the six-quark model. Our approach is pedagogical and emphasizes the underlying assumptions and the conditions necessary for the validity of the leading-logarithmic approximation. We proceed by successively considering the W boson, t quark, b quark, and finally c quark as heavy. In Sec. III, numerical results are given. As expected, CP-violating terms appear in the resulting effective Hamiltonian, both in the old terms of $(V-A) \times (V-A)$ form and in new penguin-type terms. In the former they are quite small, but in the latter are large. The ratio of CP-violating to CP-conserving amplitudes in penguin terms is comparable to that calculated on the basis of the lowest-order diagram in Fig. 1 for a typical set of parameters. Conclusions are then drawn in Sec. IV. Many of the details concerning the matrices of anomalous dimensions and their eigenvectors and eigenvalues are relegated to an appendix.

II. DERIVATION OF THE EFFECTIVE NONLEPTONIC WEAK HAMILTONIAN

In the standard model[8] where the gauge group of weak and electromagnetic interactions is SU(2) $\otimes$ U(1), the six quarks, u, c, and t with charge $+\frac{2}{3}$ and d, s, and b, with charge $-\frac{1}{3}$, are assigned to left-handed doublets and right-handed singlets:

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L ; \begin{pmatrix} c \\ s' \end{pmatrix}_L ; \begin{pmatrix} t \\ b' \end{pmatrix}_L ;$$

$$(u)_R ; (d)_R ; (c)_R ; (s)_R ; (t)_R ; (b)_R .$$

The standard choice of quark fields is such that[9]

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix},$$

(1)

where $c_i = \cos\theta_i$, $s_i = \sin\theta_i$, $i \in \{1, 2, 3\}$. Equation (1) defines the three Cabibbo-type mixing angles θ_i and the CP-violating phase, δ. Without loss of generality the angles θ_i may be chosen to lie in the first quadrant.[10]

Weak interactions involving the charged hadronic current follow from the interaction term in the Hamiltonian density

$$\mathcal{H}_I(x) = \frac{g}{2\sqrt{2}} J_\mu^+(x) W_\mu^-(x) + \text{H.c.},$$

(2)

where W_μ^- is the charged-W-boson field, J_μ^+ is the charged weak current defined by

$$J_\mu^+(0) = \overline{u}(0)\gamma_\mu(1-\gamma_5)d'(0) + \overline{c}(0)\gamma_\mu(1-\gamma_5)s'(0)$$
$$+ \overline{t}(0)\gamma_\mu(1-\gamma_5)b'(0)$$
$$= (\overline{u}d')_{V-A} + (\overline{c}s')_{V-A} + (\overline{t}b')_{V-A},$$

(3)

and g is the gauge coupling constant of the weak SU(2) subgroup. With no strong interactions the lowest-order weak current-current interaction at zero momentum transfer is described by the effective Hamiltonian density

$$\mathcal{H}_{\text{eff}}(0) = \frac{g^2}{8M_W^2} J_\mu^+(0) J_\mu^-(0) + \text{H.c.},$$

(4)

so that the Fermi coupling $G_F/\sqrt{2} = g^2/(8M_W^2)$. In particular, the strangeness-changing piece of Eq. (4) is

$$\mathcal{H}_{\text{eff}}^{(\Delta S=1)} = \frac{G_F}{\sqrt{2}} [-c_1 s_1 c_3 (\overline{s}_\alpha u_\alpha)_{V-A} (\overline{u}_\beta d_\beta)_{V-A} + s_1 c_2 (c_1 c_2 c_3 - s_2 s_3 e^{-i\delta})(\overline{s}_\alpha c_\alpha)_{V-A} (\overline{c}_\beta d_\beta)_{V-A}$$

$$+ s_1 s_2 (c_1 s_2 c_3 + c_2 s_3 e^{-i\delta})(\overline{s}_\alpha t_\alpha)_{V-A} (\overline{t}_\beta d_\beta)_{V-A}],$$

(5)

where we have made the color indices α and β on the quarks (which when repeated are summed from 1 to 3) explicit in preparation for the inclusion of the strong interactions. It is convenient to rewrite Eq. (5) as

$$\mathcal{H}_{\text{eff}}^{(\Delta S=1)} = -\frac{G_F}{2\sqrt{2}}[A_c(O_c^{(+)}+O_c^{(-)})+A_t(O_t^{(+)}+O_t^{(-)})],$$

(6)

where

$$O_q^{(\pm)} = [(\bar{s}_\alpha u_\alpha)_{V-A}(\bar{u}_\beta d_\beta)_{V-A} \pm (\bar{s}_\alpha d_\alpha)_{V-A}(\bar{u}_\beta u_\beta)_{V-A}]$$
$$-[u \rightarrow q],$$

(7)

and

$$A_c = s_1 c_2(c_1 c_2 c_3 - s_2 s_3 e^{-i\delta}),$$ (8a)

$$A_t = s_1 s_2(c_1 s_2 c_3 + c_2 s_3 e^{-i\delta}).$$ (8b)

Normal ordering of the four-fermion operators is understood. The space-time coordinates of all operators are suppressed.

Now introduce the strong interactions in the form of quantum chromodynamics (QCD), the gauge theory based on the color SU(3) gauge group involving vector gluons interacting with quarks. The strong interactions modify the lowest-order weak effective Hamiltonian from the form in Eqs. (4) and (5). We now proceed to derive in leading-logarithmic approximation the form of the effective weak Hamiltonian in the presence of strong interactions with heavy W bosons and heavy t, b, and c quarks.

First, we take the W boson as much heavier than any other mass scale in the problem and consider the S-matrix elements of the weak interaction between low-momentum hadron states composed of light quarks and differing in strangeness by one unit. This is just the calculation performed in Ref. 11. Using the operator-product expansion[12] (noting that the operators $O_c^{(\pm)}$ and $O_t^{(\pm)}$ are multiplicatively renormalized and do not mix with other operators at the one-loop level), we have that to leading order in the heavy W-boson mass

$$-\frac{i}{2}\int d^4x \langle|T(\mathcal{H}_I(x),\mathcal{H}_I(0))|\rangle = -\frac{G_F}{2\sqrt{2}}\left[A_c^{(+)}\left(\frac{M_W}{\mu},g\right)\langle|O_c^{(+)}(0)|\rangle + A_c^{(-)}\left(\frac{M_W}{\mu},g\right)\langle|O_c^{(-)}(0)|\rangle\right.$$

$$\left.+A_t^{(+)}\left(\frac{M_W}{\mu},g\right)\langle|O_t^{(+)}(0)|\rangle + A_t^{(-)}\left(\frac{M_W}{\mu},g\right)\langle|O_t^{(-)}(0)|\rangle\right],$$

(9)

where μ is the renormalization point of the strong interactions. The matrix elements of the right-hand side are to be evaluated to all orders in the strong interactions and to zeroth order in the weak interactions.

The Wilson coefficients $A_t^{(\pm)}(M_W/\mu,g)$ and $A_c^{(\pm)}(M_W/\mu,g)$ depend on the choice of renormalization scheme. Of course, the renormalized operators $O_t^{(\pm)}$ and $O_c^{(\pm)}$ also depend on the renormalization scheme in such a way that physical quantities are rendered scheme independent. We use the mass-independent minimal-subtraction scheme[13] where the renormalization-group equations[14] are

$$\left(\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}-\gamma^{(\pm)}(g)\right)A_q^{(\pm)}\left(\frac{M_W}{\mu},g\right)=0.$$

(10)

The $\gamma^{(\pm)}$ characterize the anomalous dimension of the operators $O_q^{(\pm)}$ with $q=c$ or t. The function $\beta(g)$ has the perturbation expansion[15]

$$\beta(g) = -(33-2N_f)\frac{g^3}{48\pi^2}+O(g^5),$$ (11)

where N_f (which equals 6 here) is the number of quark flavors. A standard one-loop calculation[11]

shows that $\gamma^{(\pm)}(g)$ has the perturbation expansion

$$\gamma^{(+)}(g) = \frac{g^2}{4\pi^2}+O(g^4),$$ (12a)

$$\gamma^{(-)}(g) = -\frac{g^2}{2\pi^2}+O(g^4).$$ (12b)

With the running coupling constant $\bar{g}(y,g)$ defined by

$$\ln y = \int_g^{\bar{g}(y,g)}\frac{dx}{\beta(x)}$$

(13)

and $\bar{g}(1,g)=g$, Eq. (10) has the solution

$$A_q^{(\pm)}\left(\frac{M_W}{\mu},g\right) = \exp\left(\int_g^{\bar{g}(M_W/\mu,g)} -\frac{\gamma^{(\pm)}(x)}{\beta(x)}dx\right)$$

$$\times A_q^{(\pm)}\left(1,\bar{g}\left(\frac{M_W}{\mu},g\right)\right).$$

(14)

In a leading-logarithmic calculation the coefficients $A_q^{(\pm)}(1,\bar{g}(M_W/\mu,g))$ can be replaced by their free-field values A_q given in Eq. (8) because the running fine-structure constant $\alpha=\bar{g}^2/4\pi$ is small at the mass scale of the W and because the value of their first dependent variable being unity implies no other large logarithms can be generated by higher-order strong interactions. Using Eqs.

(11) and (12)

$$-\frac{\gamma^{(\pm)}(x)}{\beta(x)} = \frac{2a^{(\pm)}}{x} + \text{terms finite at } x = 0, \qquad (15)$$

with

$$a^{(+)} = \frac{6}{33 - 2N_f}, \qquad (16a)$$

$$a^{(-)} = \frac{-12}{33 - 2N_f}. \qquad (16b)$$

Choosing μ above the onset of scaling, Eq. (15) may be substituted back into Eq. (14) to obtain the result:[16]

$$A_q^{(\pm)}\left(\frac{M_W}{\mu}, g\right) = \left[\frac{\bar{g}^2(M_W/\mu, g)}{\bar{g}^2(1, g)}\right]^{a^{(\pm)}} A_q$$

$$= \left[\frac{\alpha(M_W^2)}{\alpha(\mu^2)}\right]^{a^{(\pm)}} A_q. \qquad (17)$$

At this stage our effective weak Hamiltonian density is

$$\mathcal{H}_{\text{eff}}^{(\Delta S = 1)} = -\frac{G_F}{2\sqrt{2}}\left\{\left[\frac{\alpha(M_W^2)}{\alpha(\mu^2)}\right]^{a^{(+)}}(A_c O_c^{(+)} + A_t O_t^{(+)})\right.$$

$$\left. + \left[\frac{\alpha(M_W^2)}{\alpha(\mu^2)}\right]^{a^{(-)}}(A_c O_c^{(-)} + A_t O_t^{(-)})\right\}. \qquad (18)$$

The matrix elements of the above effective weak Hamiltonian density are to be evaluated to all orders in the strong interactions and to zeroth order in the weak interactions. Note that $\mathcal{H}_{\text{eff}}$ does not explicitly involve the W-boson field. We want to derive an effective Hamiltonian without explicit dependence on the heavy W-boson, t-quark, b-quark, and c-quark fields. Equation (18) is the first step towards this goal.

We now proceed to consider the t quark as very heavy and eliminate it from explicitly appearing in our effective weak Hamiltonian for strangeness-changing processes. What happens to the operators $O_c^{(\pm)}$ and $O_t^{(\pm)}$ is different, and we consider the more complicated case of $O_t^{(\pm)}$ first.

We assume that m_t is much greater than all other quark masses, the momenta of the external states, and the renormalization point mass, μ. The work of Appelquist and Carrazone[17] tells us that to order $1/m_t^2$ all the dependence of amplitudes on the heavy t-quark mass can be absorbed into renormalization effects and hence into a redefinition of the coupling constant, mass parameters, and scale of operators. This suggests the following factorization:

$$\langle|O_t^{(\pm)}|\rangle = \sum_i B_i^{(\pm)}\left(\frac{m_t}{\mu}, g\right)\langle|O_i|\rangle' + O\left(\frac{1}{m_t^2}\right), \qquad (19)$$

where the primed matrix elements are evaluated to all orders in an effective theory of strong interactions[18] with five quark flavors, coupling $g'(m_t/\mu, g)$ and mass parameters $m_u', m_d', \ldots, m_b'$. Thus,

$$\langle|O_i|\rangle' = \langle|O_i|\rangle(g', \mu, m_u', \ldots, m_b').$$

To carry out the expansion of Eq. (19) in leading-log approximation we find that six linearly independent operators O_i are sufficient. We choose them as follows:

$$O_1 = (\bar{s}_\alpha d_\alpha)_{V-A}(\bar{u}_\beta u_\beta)_{V-A},$$

$$O_2 = (\bar{s}_\alpha d_\beta)_{V-A}(\bar{u}_\beta u_\alpha)_{V-A},$$

$$O_3 = (\bar{s}_\alpha d_\alpha)_{V-A}[(\bar{u}_\beta u_\beta)_{V-A} + \cdots + (\bar{b}_\beta b_\beta)_{V-A}], \qquad (20)$$

$$O_4 = (\bar{s}_\alpha d_\beta)_{V-A}[(\bar{u}_\beta u_\alpha)_{V-A} + \cdots + (\bar{b}_\beta b_\alpha)_{V-A}],$$

$$O_5 = (\bar{s}_\alpha d_\alpha)_{V-A}[(\bar{u}_\beta u_\beta)_{V+A} + \cdots + (\bar{b}_\beta b_\beta)_{V+A}],$$

$$O_6 = (\bar{s}_\alpha d_\beta)_{V-A}[(\bar{u}_\beta u_\alpha)_{V+A} + \cdots + (\bar{b}_\beta b_\alpha)_{V+A}].$$

These operators are sufficient since they close under renormalization at the one-loop level. The operators O_1 and O_2 already occur to zeroth order in strong interactions: We read off from Eq. (7) that

$$B_1^{(\pm)} \equiv B_1^{(\pm)}(1, 0) = \pm 1,$$

$$B_2^{(\pm)} \equiv B_2^{(\pm)}(1, 0) = +1. \qquad (21a)$$

The operators O_3, O_4, O_5, and O_6 are generated by the strong interactions through penguin-type diagrams, so that in free-field theory

$$B_3^{(\pm)} = B_4^{(\pm)} = B_5^{(\pm)} = B_6^{(\pm)} = 0. \qquad (21b)$$

However, the operators O_i are not multiplicatively renormalized at the one-loop level, i.e., they mix among themselves. As shown in the appendix, the renormalization-group equation their coefficients $B_i^{(\pm)}(m_t/\mu, g)$ satisfy is

$$\sum_j \left[\left(\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} + \gamma_t(g)m_t\frac{\partial}{\partial m_t} + \gamma^{(\pm)}(g)\right)\delta_{ij}\right.$$

$$\left. - \gamma_{ij}'^T(g')\right]B_j^{(\pm)}\left(\frac{m_t}{\mu}, g\right) = 0. \qquad (22)$$

Here γ'^T is the transpose of the anomalous-dimension matrix of the operators O_i in the effective theory of strong interactions with five quarks and coupling g'. It is the eigenvectors of γ'^T that correspond to operators which are multiplicatively renormalized. We write the coefficient functions $\tilde{B}_i^{(\pm)}(m_t/\mu, g)$ of these multiplicatively renormalized operators as

$$\tilde{B}_i^{(\pm)}\left(\frac{m_t}{\mu}, g\right) = \sum_j V^{-1}{}_{ij}B_j^{(\pm)}\left(\frac{m_t}{\mu}, g\right), \qquad (23)$$

and denote the corresponding eigenvalues of

γ'^T by γ'_i. The matrix γ' is found in the Appendix along with its eigenvalues and the matrix V. For the $\tilde{B}_i^{(\pm)}(m_t/\mu,g)$, the renormalization-group equation corresponding to Eq. (22) is

$$\left(\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}+\gamma_t(g)m_t\frac{\partial}{\partial m_t}+\gamma^{(\pm)}(g)-\gamma'_i(g')\right)$$
$$\times\tilde{B}_i^{(\pm)}\left(\frac{m_t}{\mu},g\right)=0. \quad (24)$$

The solution to this equation may be found with the aid of the running coupling constant $\bar{g}(y,g)$ defined by[1]

$$\ln y=\int_g^{\bar{g}(y,g)}\left(\frac{1-\gamma_t(x)}{\beta(x)}\right)dx, \quad (25)$$

with $\bar{g}(1,g)=g$. Note that this is not the usual definition of the running coupling constant [Eq. (13)], but the integrand in Eq. (25) for small x has the same leading behavior given by $1/\beta(x)$ as the integrand in Eq. (13). Setting $y=m_t/\mu$, it is now easily shown that the solution of Eq. (24) is

$$\tilde{B}_i^{(\pm)}\left(\frac{m_t}{\mu},g\right)=\exp\left(\int_g^{\bar{g}(m_t/\mu,g)}\frac{\gamma^{(\pm)}(x)}{\beta(x)}dx\right)$$
$$\times\exp\left(\int_{g'(m_t/\mu,g)}^{g'(1,\bar{g})}\frac{-\gamma'_i(x)}{\beta'(x)}dx\right)\tilde{B}_i^{(\pm)}(1,\bar{g}). \quad (26)$$

β' is the β function in the effective theory with five quarks and coupling g'. This β function has the perturbation expansion

$$\beta'(g')=-(33-2N_f)\frac{g'^3}{48\pi^2}+O(g'^5) \quad (27)$$

with $N_f=5$, and we write

$$-\frac{\gamma'_i(x)}{\beta'(x)}=\frac{2a'_i}{x}+\text{finite terms at }x=0. \quad (28)$$

Choosing μ as before, above the onset of scaling, we may use Eqs. (15) and (28) to get[16]

$$\tilde{B}_i^{(\pm)}\left(\frac{m_t}{\mu},g\right)=\left[\frac{\alpha(m_t^2)}{\alpha(\mu^2)}\right]^{-a^{(\pm)}}\left[\frac{\alpha(m_t^2)}{\alpha'(\mu^2)}\right]^{a'_i}\tilde{B}_i^{(\pm)}(1,\bar{g}). \quad (29)$$

We have used $g'(1,\bar{g})\approx\bar{g}(m_t/\mu,g)$, which is valid in a leading-log calculation since the running fine-structure constant is small at the t-quark mass. Finally, using the linear relationship between the eigenvectors $\tilde{B}_i$ and the B_i we have

$$B_k^{(\pm)}\left(\frac{m_t}{\mu},g\right)=\left[\frac{\alpha(m_t^2)}{\alpha(\mu^2)}\right]^{-a^{(\pm)}}\sum_{i,j}V_{kj}\left[\frac{\alpha(m_t^2)}{\alpha'(\mu^2)}\right]^{a'_j}$$
$$\times V^{-1}{}_{ji}B_i^{(\pm)}(1,\bar{g}). \quad (30)$$

Notice that the factor $[\alpha(m_t^2)/\alpha(\mu^2)]^{-a^{(\pm)}}$ out in front of the summation in Eq. (30) combines with the earlier factor $[\alpha(M_W^2)/\alpha(\mu^2)]^{a^{(\pm)}}$ in Eq. (16) to give $[\alpha(M_W^2)/\alpha(m_t^2)]^{a^{(\pm)}}$. In leading-log approximation the coefficients $B_i^{(\pm)}(1,\bar{g})$ can be replaced by their free-field values as given in Eq. (21), since no large logarithms can be generated from QCD loop integrals with the first argument of $B_i^{(\pm)}(m_t/\mu,g)$ set equal to unity and because we assume the running fine-structure constant is small at the t-quark mass.

The case of the operators $O_c^{(\pm)}$ is much simpler. The charm quark field which appears explicitly in these operators is of course not directly affected at this stage of considering the t quark as very heavy and the $O_c^{(\pm)}$ are just multiplicatively renormalized:

$$\langle|O_c^{(\pm)}|\rangle=B^{(\pm)}\left(\frac{m_t}{\mu},g\right)\langle|O_c^{(\pm)}|\rangle'. \quad (31)$$

Note that the matrix elements on the right-hand side are again to be evaluated in the effective five-quark theory with coupling $g'(m_t/\mu,g)$. The coefficients $B^{(\pm)}(m_t/\mu,g)$ satisfy

$$\left(\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}+\gamma_t(g)m_t\frac{\partial}{\partial m_t}+\gamma^{(\pm)}(g)-\gamma'^{(\pm)}(g')\right)$$
$$\times B^{(\pm)}\left(\frac{m_t}{\mu},g\right)=0. \quad (32)$$

The anomalous dimension $\gamma'^{(\pm)}(g')$ is that of $O_c^{(\pm)}$ and is a function of the coupling g' in the effective five-quark theory, while $\gamma^{(\pm)}(g)$ depends on g, the coupling in the six-quark theory.

Solving Eq. (32) in the same manner as Eq. (24) gives

$$B^{(\pm)}\left(\frac{m_t}{\mu},g\right)=\exp\left(\int_g^{\bar{g}(m_t/\mu,g)}\frac{\gamma^{(\pm)}(x)}{\beta(x)}dx\right)$$
$$\times\exp\left(\int_{g'}^{g'(1,\bar{g})}-\frac{\gamma'^{(\pm)}(x)}{\beta'(x)}dx\right)B^{(\pm)}(1,\bar{g})$$
$$=\left[\frac{\alpha(m_t^2)}{\alpha(\mu^2)}\right]^{-a^{(\pm)}}\left[\frac{\alpha(m_t^2)}{\alpha'(\mu^2)}\right]^{a'^{(\pm)}}B^{(\pm)}(1,\bar{g}). \quad (33)$$

In leading-logarithmic approximation $B^{(\pm)}(1,\bar{g}(m_t/\mu,g))$ can be replaced by its free-field value of $+1$.

Our effective weak Hamiltonian density is now free of explicit dependence on the heavy t-quark field and has the form

$$\mathcal{H}_{\text{eff}}^{(\Delta S=1)} = -\frac{G_F}{2\sqrt{2}}\left\{ \left(\frac{\alpha(m_t^2)}{\alpha'(\mu^2)}\right)^{a'(+)} \left(\frac{\alpha(M_W^2)}{\alpha(m_t^2)}\right)^{a(+)} A_c O_c^{(+)} + \left(\frac{\alpha(m_t^2)}{\alpha'(\mu^2)}\right)^{a'(-)} \left(\frac{\alpha(M_W^2)}{\alpha(m_t^2)}\right)^{a(-)} A_c O_c^{(-)} \right.$$

$$+ \sum_k \left[\sum_{i,j} V_{kj} \left(\frac{\alpha(m_t^2)}{\alpha'(\mu^2)}\right)^{a_j} V^{-1}{}_{ji} B_i^{(+)}\right] \left(\frac{\alpha(M_W^2)}{\alpha(m_t^2)}\right)^{a(+)} A_t O_k$$

$$+ \left. \sum_k \left[\sum_{i,j} V_{kj} \left(\frac{\alpha(m_t^2)}{\alpha'(\mu^2)}\right)^{a_j} V^{-1}{}_{ji} B_i^{(-)}\right] \left(\frac{\alpha(M_W^2)}{\alpha(m_t^2)}\right)^{a(-)} A_t O_k \right\}. \quad (34)$$

All operators on the right-hand side are to have their matrix elements evaluated in the effective theory with five quarks, coupling $g'(m_t/\mu, g)$ and masses $m_u', m_d', \ldots, m_b'$.

The next step of considering the b quark as very heavy is similar to what was just accomplished for the t quark, with the addition of some indices. This time the matrix elements of the operators O_i of Eq. (20) evaluated in the effective five-quark theory are to be expressed in terms of matrix elements of

$$P_1 = (\bar{S}_\alpha d_\alpha)_{V-A}(\bar{u}_\beta u_\beta)_{V-A},$$

$$P_2 = (\bar{S}_\alpha d_\beta)_{V-A}(\bar{u}_\beta u_\alpha)_{V-A},$$

$$P_3 = (\bar{S}_\alpha d_\alpha)_{V-A}[(\bar{u}_\beta u_\beta)_{V-A} + \cdots + (\bar{C}_\beta C_\beta)_{V-A}],$$

$$P_4 = (\bar{S}_\alpha d_\beta)_{V-A}[(\bar{u}_\beta u_\alpha)_{V-A} + \cdots + (\bar{C}_\beta C_\alpha)_{V-A}], \quad (35)$$

$$P_5 = (\bar{S}_\alpha d_\alpha)_{V-A}[(\bar{u}_\beta u_\beta)_{V+A} + \cdots + (\bar{C}_\beta C_\beta)_{V+A}],$$

$$P_6 = (\bar{S}_\alpha d_\beta)_{V-A}[(\bar{u}_\beta u_\alpha)_{V+A} + \cdots + (\bar{C}_\beta C_\alpha)_{V+A}],$$

evaluated in an effective theory with four-quark flavors (u, d, s, and c). The coupling and masses in the effective four-quark theory are denoted by $g''(m_b'/\mu, g')$ and $m_u'', \ldots, m_c''$, respectively. To leading order we may write

$$\langle |O_k|\rangle' = \sum_n C_k^n\left(\frac{m_b'}{\mu}, g'\right)\langle P_n\rangle'', \quad (36)$$

where the prime (double prime) denotes evaluation in the effective five (four) quark theory. The $C_k^n(m_b'/\mu, g')$ can be shown to obey an equation of the form

$$\sum_{k,n}\left[\left(\mu\frac{\partial}{\partial\mu} + \beta'(g')\frac{\partial}{\partial g'} + \gamma_b' m_b'\frac{\partial}{\partial m_b'}\right)\delta_{jk}\delta_{mn}\right.$$

$$\left. + \gamma_{jk}'(g')\delta_{mn} - \delta_{jk}\gamma_{mn}''^T(g'')\right]C_k^n\left(\frac{m_b'}{\mu}, g'\right) = 0, \quad (37)$$

with γ' and γ'' being anomalous-dimension matrices of the operators $O_1, \ldots, O_6$ and $P_1, \ldots, P_6$, respectively.

Defining the linear combinations of coefficient functions

$$\tilde{C}_k^n\left(\frac{m_b'}{\mu}, g'\right) = \sum_l W^{-1}{}_{nl}C_k^l\left(\frac{m_b'}{\mu}, g'\right) \quad (38)$$

as corresponding to operators which are multiplicatively renormalized, i.e., do not mix with other operators, the renormalization-group equations diagonalize into the form

$$\left(\mu\frac{\partial}{\partial\mu} + \beta(g')\frac{\partial}{\partial g'} + \gamma_b' m_b'\frac{\partial}{\partial m_b'} + \gamma_k'(g') - \gamma_n''(g'')\right)$$

$$\times \sum_j \tilde{C}_j^n\left(\frac{m_b'}{\mu}, g'\right)V_{jk} = 0. \quad (39)$$

The matrices W and γ'' together with the eigenvalues of the latter are found in the Appendix.

With the aid of a new running coupling defined by

$$\ln y = \int_{g'}^{\bar{g}'(y, g')}\frac{1 - \gamma_b'(x)}{\beta'(x)}dx, \quad (40)$$

these equations may be solved very analogously to Eq. (24). We leave out some of the details and skip to the solution in the leading-logarithmic approximation:

$$C_k^n\left(\frac{m_b'}{\mu}, g'\right) = \sum_{i,l}\left\{\sum_j V_{ij}\left[\frac{\alpha'(m_b'^2)}{\alpha'(\mu^2)}\right]^{-a_j} V^{-1}{}_{jk}\right\}\left\{\sum_m W_{nm}\left[\frac{\alpha'(m_b'^2)}{\alpha''(\mu^2)}\right]^{a_m''} W^{-1}{}_{ml}\right\} C_i^l\left(1, \bar{g}'\left(\frac{m_b'}{\mu}, g'\right)\right). \quad (41)$$

For reasons stated before, in a leading-logarithm calculation the coefficients $C_i^l(1, \bar{g}')$ can be replaced by their free-field values:

$$C_i^l \equiv C_i^l(1, 0) = \delta_{il}. \quad (42)$$

The operators $O_c^{(\pm)}$ are multiplicatively renormalized and the expansion of their matrix elements gives results like those in Eq. (33) with appropriate changes.

Our effective Hamiltonian now takes the following form at the four-quark level:

$$\mathcal{H}_{\text{eff}}^{(\Delta S=1)} = -\frac{G_F}{2\sqrt{2}} \left\{ \left(\frac{\alpha'(m_b'^2)}{\alpha''(\mu^2)} \right)^{a''(+)} \left(\frac{\alpha(m_t^2)}{\alpha'(m_b'^2)} \right)^{a'(+)} \left(\frac{\alpha(M_W^2)}{\alpha(m_t^2)} \right)^{a(+)} A_c O_c^{(+)} \right.$$

$$+ \left(\frac{\alpha'(m_b'^2)}{\alpha''(\mu^2)} \right)^{a''(-)} \left(\frac{\alpha(m_t^2)}{\alpha'(m_b'^2)} \right)^{a'(-)} \left(\frac{\alpha(M_W^2)}{\alpha(m_t^2)} \right)^{a(-)} A_c O_c^{(-)}$$

$$+ \sum_{k,n} \left[\sum_{l,m} W_{nm} \left(\frac{\alpha'(m_b'^2)}{\alpha''(\mu^2)} \right)^{a_m''} W^{-1}_{ml} C_k^l \right]$$

$$\left. \times \left[\sum_{i,j} V_{kj} \left(\frac{\alpha(m_t^2)}{\alpha'(m_b'^2)} \right)^{a_j'} V^{-1}_{ji} \left(B_i^{(+)} \left(\frac{\alpha(M_W^2)}{\alpha(m_t^2)} \right)^{a(+)} A_t + B_i^{(-)} \left(\frac{\alpha(M_W^2)}{\alpha(m_t^2)} \right)^{a(-)} A_t \right) \right] P_n \right\}. \quad (43)$$

The final step of considering the charm quark as very heavy is more questionable from the phenomenological viewpoint. It also involves a technical point which is easy to miss. When we proceed to expand the matrix elements of the operators $P_1, \ldots, P_6$ evaluated in the effective four-quark theory in terms of matrix elements of operators evaluated in an effective three-quark theory, it is natural to define[2]

$$Q_1 = (\bar{s}_\alpha d_\alpha)_{V-A} (\bar{u}_\beta u_\beta)_{V-A},$$

$$Q_2 = (\bar{s}_\alpha d_\beta)_{V-A} (\bar{u}_\beta u_\alpha)_{V-A},$$

$$Q_3 = (\bar{s}_\alpha d_\alpha)_{V-A} [(\bar{u}_\beta u_\beta)_{V-A} + (\bar{d}_\beta d_\beta)_{V-A} + (\bar{s}_\beta s_\beta)_{V-A}],$$

$$Q_4 = (\bar{s}_\alpha d_\beta)_{V-A} [(\bar{u}_\beta u_\alpha)_{V-A} + (\bar{d}_\beta d_\alpha)_{V-A} + (\bar{s}_\beta s_\alpha)_{V-A}],$$

$$Q_5 = (\bar{s}_\alpha d_\alpha)_{V-A} [(\bar{u}_\beta u_\beta)_{V+A} + (\bar{d}_\beta d_\beta)_{V+A} + (\bar{s}_\beta s_\beta)_{V+A}],$$

$$Q_6 = (\bar{s}_\alpha d_\beta)_{V-A} [(\bar{u}_\beta u_\alpha)_{V+A} + (\bar{d}_\beta d_\alpha)_{V+A} + (\bar{s}_\beta s_\alpha)_{V+A}].$$

$$(44)$$

These operators close under renormalization at the one-loop level, but they are linearly dependent:

$$Q_4 = -Q_1 + Q_2 + Q_3. \quad (45)$$

Hence we must then use only five operators,[19] which we choose as Q_1, Q_2, Q_3, Q_5, and Q_6.

Expressing matrix elements of the operators evaluated in the effective four-quark theory in terms of matrix elements of operators evaluated in the effective three-quark theory, we write

$$\langle |P_n| \rangle'' = \sum_{r=1,2,3,5,6} D_n^r \left(\frac{m_c''}{\mu}, g'' \right) \langle |Q_r| \rangle''', \quad (46)$$

with g''' and m_u''', m_d''', m_s''' representing the coupling constant and quark masses in the effective three-quark theory. The linear combinations

$$\tilde{D}_n^r \left(\frac{m_c''}{\mu}, g'' \right) = \sum_s X^{-1}_{rs} D_n^s \left(\frac{m_c''}{\mu}, g'' \right) \quad (47)$$

are the coefficients of multiplicatively renormalized operators. The diagonalized renormalization-group equations are

$$\left[\mu \frac{\partial}{\partial \mu} + \beta''(g'') \frac{\partial}{\partial g''} + \gamma_c'' m_c'' \frac{\partial}{\partial m_c''} + \gamma_n''(g'') - \gamma_r'''(g''') \right] \sum_m \tilde{D}_m^r \left(\frac{m_c''}{\mu}, g'' \right) W_{mn} = 0, \quad (48)$$

and have the solution in leading-logarithmic approximation after reexpressing the $\tilde{D}$'s in terms of D's,

$$D_l^r \left(\frac{m_c''}{\mu}, g'' \right) = \sum_{n,p} \left\{ \sum_m W_{nm} \left[\frac{\alpha''(m_c''^2)}{\alpha''(\mu^2)} \right]^{-a_m''} W^{-1}_{ml} \right\} \left\{ \sum_q X_{rq} \left[\frac{\alpha''(m_c''^2)}{\alpha'''(\mu^2)} \right]^{a_q'''} X^{-1}_{qp} \right\} D_n^p(1, \bar{g}''). \quad (49)$$

In leading-logarithm approximation the $D_n^p(1, \bar{g}'')$ can be replaced by their free-field values, D_n^p. These are δ_{np}, except when $n=4$, in which case $D_4^1 = -1$, $D_4^2 = 1$, $D_4^3 = 1$, and $D_4^5 = D_4^6 = 0$.

Because we are considering the charm quark as heavy, the operators $O_c^{(\pm)}$ are no longer just multiplicatively renormalized at the one-loop level and we must also expand

$$\langle |O_c^{(\pm)}| \rangle'' = \sum_r D_r^{(\pm)} \left(\frac{m_c''}{\mu}, g'' \right) \langle |Q_r| \rangle'''. \quad (50)$$

The renormalization-group equations obeyed by the $D_r^{(\pm)}(m_c''/\mu, g'')$ are

$$\sum_r \left[\left(\mu \frac{\partial}{\partial \mu} + \beta''(g'') \frac{\partial}{\partial g''} + \gamma_c''(g'') m_c'' \frac{\partial}{\partial m_c''} + \gamma''^{(\pm)}(g'') \right) \delta_{pr} - \gamma_{pr}'''^T(g''') \right] D_r^{(\pm)} \left(\frac{m_c''}{\mu}, g'' \right) = 0. \quad (51)$$

The coefficients corresponding to multiplicatively renormalized operators are just as in Eq. (47), and the solution to Eq. (51) with the usual approximations is

$$D_r^{(\pm)} \left(\frac{m_c''}{\mu}, g'' \right) = \left[\frac{\alpha''(m_c''^2)}{\alpha''(\mu^2)} \right]^{-a'''(\pm)} \sum_{p,q} X_{rq} \left[\frac{\alpha''(m_c''^2)}{\alpha'''(\mu^2)} \right]^{a_q'''} X^{-1}_{qp} D_p^{(\pm)}(1, \bar{g}''). \quad (52)$$

The free-field values, $D_p^{(\pm)} \equiv D_p^{(\pm)}(1, 0)$, are $D_1^{(\pm)} = \pm 1$, $D_2^{(\pm)} = +1$, and all others zero.

We are finally ready to collect all our results and write the previously advertised effective Hamiltonian in the "light" three-quark sector. It is the following sum of Wilson coefficients times local four-fermion operators which do not explicitly involve the heavy W-boson, top, bottom, and charm quark fields:

$$
\begin{aligned}
\mathcal{K}^{(\Delta S=1)} = -\frac{G_F}{2\sqrt{2}} \Bigg\{ &\sum_r \left[\sum_{p,q} X_{rq} \left(\frac{\alpha''(m_c''^2)}{\alpha'''(\mu^2)} \right)^{a_q'''} X^{-1}{}_{qp} D_p^{(+)} \right] \left(\frac{\alpha'(m_b'^2)}{\alpha''(m_c''^2)} \right)^{a''(+)} \left(\frac{\alpha(m_t^2)}{\alpha'(m_b'^2)} \right)^{a'(+)} \left(\frac{\alpha(M_W^2)}{\alpha(m_t^2)} \right)^{a(+)} A_c Q_r \\
&+ \sum_r \left[\sum_{p,q} X_{rq} \left(\frac{\alpha''(m_c''^2)}{\alpha'''(\mu^2)} \right)^{a_q'''} X^{-1}{}_{qp} D_p^{(+)} \right] \left(\frac{\alpha'(m_b'^2)}{\alpha''(m_c''^2)} \right)^{a''(-)} \left(\frac{\alpha(m_t^2)}{\alpha'(m_b'^2)} \right)^{a'(-)} \left(\frac{\alpha(M_W^2)}{\alpha(m_t^2)} \right)^{a(-)} A_c Q_r \\
&+ \sum_{k,n,r} \left[\sum_{p,q} X_{rq} \left(\frac{\alpha''(m_c''^2)}{\alpha'''(\mu^2)} \right)^{a_q'''} X^{-1}{}_{qp} D_n^p \right] \left[\sum_{l,m} W_{nm} \left(\frac{\alpha'(m_b'^2)}{\alpha''(m_c''^2)} \right)^{a_m''} W^{-1}{}_{ml} C_k^l \right] \\
&\times \left[\sum_{i,j} V_{kj} \left(\frac{\alpha(m_t^2)}{\alpha'(m_b'^2)} \right)^{a_j'} V^{-1}{}_{ji} \left(B_i^{(+)} \left(\frac{\alpha(M_W^2)}{\alpha(m_t^2)} \right)^{a(+)} A_t + B_i^{(-)} \left(\frac{\alpha(M_W^2)}{\alpha(m_t^2)} \right)^{a(-)} A_t \right) \right] Q_r \Bigg\}.
\end{aligned}
$$

(53)

All summations are from 1 through 6, except those over p, q, and r which run through 1, 2, 3, 5, and 6.

III. NUMERICAL RESULTS FOR THE EFFECTIVE NONLEPTONIC HAMILTONIAN

We are now in a position to perform the arithmetic operations made explicit in Eq. (53) and to examine the resulting Wilson coefficients of the operators Q_1, Q_2, Q_3, Q_5, and Q_6 in the effective Hamiltonian for nonleptonic, strangeness-changing interactions. Since the matrices, V, W, and X, as given in the appendix, are composed of irrational numbers and since various fractional powers of $\alpha(M^2)$ with $M^2 = M_W^2$, m_t^2, etc. are rampant, quantitatively rather little is transparent about these coefficients in general. We then are forced to proceed by choosing a parametrization for $\alpha(M^2)$ and values for the W and quark masses, substituting in Eq. (53), and reading off the coefficients of the Q_i for that particular set of choices.

Moreover, our outlook is basically qualitative. We have calculated the QCD effects in the leading-logarithmic approximation. While we have some confidence that at the first step M_W is a large-enough mass for this to be a credible procedure, by the last step of considering m_c a heavy mass we have used this approximation beyond the region where it can be reasonably justified.

On the positive side, what is carried out here is well defined and systematic. The degree of accuracy is obviously no worse than any of the earlier calculations[3,4] which involve only the "heavy" charm quark (and W boson) in the leading-logarithmic approximation. Not only is the accuracy of the calculation expected to be better for the b and t quarks, but their effect was not taken into account previously. With regard to CP violation they play a dominant role as we shall see

presently.

To investigate the effective nonleptonic Hamiltonian numerically we first of all need to decide on the running QCD fine-structure constant $\alpha(Q^2)$, the values of the heavy quark masses, and μ^2 or alternatively $\alpha(\mu^2)$. In the leading-logarithmic approximation

$$
\alpha(Q^2) = \frac{12\pi}{33 - 2N_f} \frac{1}{\ln(Q^2/\Lambda^2)},
$$

(54)

where we take $\Lambda^2 = 0.1$ GeV^2, a value consistent with recent data when QCD is used to parametrize the breakdown of scale invariance in deep-inelastic neutrino scattering.[20] When the leading-logarithmic approximation is valid, the calculation is insensitive to the precise value of Λ. The number of quark flavors is $N_f = 6$ for the fine-structure constant we have called $\alpha(Q^2)$, while $\alpha'(Q^2)$, $\alpha''(Q^2)$, and $\alpha'''(Q^2)$ have $N_f = 5$, 4, and 3, respectively, as they pertain to effective theories with those corresponding numbers of quark flavors.

We take m_c to be 1.5 GeV and m_b to be 4.5 GeV on the basis of ψ and Υ spectroscopy.[21] The t-quark mass is unknown at this time, and we use values of 15 and 30 GeV to get an idea of the sensitivity of the results to this quantity. For M_W we take the value 85 GeV, consistent with the value obtained within $SU(2) \otimes U(1)$, given the recent measurements[22] of $\sin^2\theta_W$. In evaluating Eq. (53) we do not differentiate between m_b' and m_b, m_c'' and m_c, etc., again consistent with our leading-logarithm approximation philosophy.

Finally a value is required for $\alpha(\mu^2)$ [or more exactly $\alpha'''(\mu^2)$]. We want to choose μ to be a typical "light" hadron mass scale or inverse size, where $\alpha(\mu^2)$ is of order unity. We let $\alpha(\mu^2) = 0.75$, 1.0, and 1.25 to check the variation of the result-

ing effective nonleptonic Hamiltonian to this choice. In fact, the values of S-matrix elements of the weak interaction cannot depend on the choice of the renormalization point μ, or equivalently $\alpha(\mu^2)$. The matrix elements of the four-fermion operators, Q_i, also have an implicit μ dependence which exactly compensates that of their coefficients which we have calculated. We are left to make a choice of μ, hopefully close to the typical light hadron mass scale of the problem, so that "hard"-gluon effects are contained as much as possible in the Wilson coefficients and not the matrix elements of Q_i, but high enough that their calculation in leading-logarithm approximation makes some sense.[23]

In terms of the operators, Q_1, Q_2, Q_3, Q_5, and Q_6 defined previously in Eq. (44), the nonleptonic Hamiltonian involving u, d, and s quark fields has the form

$$\mathcal{H}_{\text{eff}}^{(\Delta S=1)} = -\frac{G_F}{\sqrt{2}}s_1c_1c_3[(-0.87+0.036\tau)Q_1$$

$$+(1.51-0.036\tau)Q_2$$

$$+(-0.021-0.012\tau)Q_3$$

$$+(0.011+0.007\tau)Q_5$$

$$+(-0.047-0.072\tau)Q_6],$$

(55)

when $m_t = 15$ GeV and $\alpha(\mu^2) = 1$ and where

$$\tau = s_2^2 + s_2 c_2 s_3 e^{-i\delta}/c_1 c_3,$$ 　(56)

along with the other masses specified previously. Values of the coefficients for all six cases corresponding to $\alpha(\mu^2) = 0.75$, 1.0, and 1.25 and $m_t = 15$ GeV and 30 GeV are found in Table I.

Referring back to Eq. (5), we see that before accounting for the effects of QCD, the coefficients of the usual four-fermion operator Q_1, as well as the "penguin"-induced operators Q_3, Q_5, and Q_6 were all zero. In the sector involving u, d, s quarks the strangeness-changing weak Hamiltonian

then just involves Q_2 with unit coefficient. Thus the presence of strong-interaction QCD corrections has brought in the operators Q_1, Q_3, Q_5, and Q_6, changed the coefficient of Q_2, and given all coefficients an imaginary (CP-violating) part through the quantity τ, which enters through penguin-type diagrams involving a heavy-quark loop.

The portion of the nonleptonic Hamiltonian involving only the operators Q_1 and Q_2 is the traditionally calculated $(V-A)\times(V-A)$ four-fermion piece with neglect of all penguin-type effects. The sum of coefficients of Q_1 and Q_2 is proportional to the coefficient of an operator transforming purely as $I = \frac{3}{2}$, which cannot mix under strong-interaction renormalization with penguin contributions which are pure $I = \frac{1}{2}$. As a consequence, one simple check of the calculation is to note that the quantity τ, arising from penguin contributions, always has the same magnitude and opposite sign in its contribution to the coefficients of Q_1 and Q_2.

The combination of operators $Q_2 - Q_1$ transforms purely as $I = \frac{1}{2}$, while the combination $Q_1 + Q_2$ has an $I = \frac{3}{2}$ piece. The ratio of coefficients of $Q_2 - Q_1$ and $Q_2 + Q_1$ is a measure of $\Delta I = \frac{1}{2}$ or octet enhancement by QCD, as first calculated in Ref. 11. Our inclusion of penguin operators and their mixing makes little numerical difference for the coefficients of Q_1 and Q_2. Slightly more important in comparison with earlier work is our taking into account not only the heavy W boson, but each heavy quark successively in computing the leading-logarithmic QCD effects. As a result the earlier $[\alpha(M_W^2)/\alpha(\mu^2)]^{a(\pm)}$ is replaced by

$$[\alpha(M_W^2)/\alpha(m_t^2)]^{a(\pm)}[\alpha(m_t^2)/\alpha'(m_b'^2)]^{a'(\pm)}$$

$$\times[\alpha(m_b'^2)/\alpha''(m_c''^2)]^{a''(\pm)}[\alpha''(m_c''^2)/\alpha'''(\mu^2)]^{a'''(\pm)},$$

even if all penguin effects are neglected. Numerically the coefficient of $Q_2 - Q_1$ is enhanced by a factor of 2 to 3 and that of $Q_2 + Q_1$ suppressed by 0.6 to 0.7 for our choice of masses. In agreement

TABLE I. Coefficients of the operators Q_1, Q_2, Q_3, Q_5, and Q_6 defined in Eq. (46) in the effective Hamiltonian, $\mathcal{H}_{\text{eff}} = (-G_F s_1 c_1 s_3/\sqrt{2})(\sum_i C_i Q_i)$, for strangeness-changing, nonleptonic weak decays; $\tau \equiv s_2^2 + s_2 c_2 s_3 e^{-i\delta}/c_1 c_3$.

Parameters	C_1	C_2	C_3	C_5	C_6
$\alpha(\mu^2) = 0.75$,　$m_t = 15$ GeV	$-0.72 + 0.035\tau$	$+1.40 - 0.035\tau$	$-0.013 - 0.015\tau$	$+0.007 + 0.008\tau$	$-0.025 - 0.059\tau$
$\alpha(\mu^2) = 1.00$,　$m_t = 15$ GeV	$-0.87 + 0.036\tau$	$+1.51 - 0.036\tau$	$-0.021 - 0.012\tau$	$+0.011 + 0.007\tau$	$-0.047 - 0.072\tau$
$\alpha(\mu^2) = 1.25$,　$m_t = 15$ GeV	$-1.00 + 0.036\tau$	$+1.61 - 0.036\tau$	$-0.028 - 0.010\tau$	$+0.015 + 0.006\tau$	$-0.069 - 0.085\tau$
$\alpha(\mu^2) = 0.75$,　$m_t = 30$ GeV	$-0.71 + 0.042\tau$	$+1.39 - 0.042\tau$	$-0.013 - 0.017\tau$	$+0.007 + 0.009\tau$	$-0.025 - 0.076\tau$
$\alpha(\mu^2) = 1.00$,　$m_t = 30$ GeV	$-0.86 + 0.043\tau$	$+1.50 - 0.043\tau$	$-0.021 - 0.013\tau$	$+0.011 + 0.008\tau$	$-0.047 - 0.093\tau$
$\alpha(\mu^2) = 1.25$,　$m_t = 30$ GeV	$-0.99 + 0.043\tau$	$+1.60 - 0.043\tau$	$-0.027 - 0.011\tau$	$+0.014 + 0.007\tau$	$-0.068 - 0.109\tau$

with all earlier results this is in the correct direction, but much too small to explain the high degree of accuracy of the $\Delta I = \frac{1}{2}$ rule in nonleptonic decays of strange particles.

The penguin terms Q_3, Q_5, and Q_6 transform as purely $I = \frac{1}{2}$ on the other hand. Our calculation indicates their coefficients are smaller than those of Q_1 and Q_2, typically by an order of magnitude for Q_6. However, arguments can be made that the $(V - A) \times (V + A)$ structure of Q_6 leads to enhanced matrix elements,[24] by one order of magnitude or more, when the nonleptonic decays involve pions in the final state. Rather extensive analyses of strange baryon and meson decays seems to support the hypothesis that the matrix elements of the operator Q_6 make major contributions to such decays and can qualitatively account for the success of the $\Delta I = \frac{1}{2}$ rule.[3,4,6,25]

As already noted, through strong-interaction effects each operator in the effective Hamiltonian has a coefficient with an imaginary as well as real part. This imaginary part, which in each case enters through Imτ and is then proportional to $s_2 c_2 s_3 \sin\delta$, leads to CP violation in decay amplitudes.

This is in addition to CP-violating effects which occur in the mass matrix in the six-quark model. We recall that for the K^0-$\bar{K}^0$ system, calculation of the contribution to the mass matrix given in Fig. 2 leads to[26]

$$\epsilon_m \equiv \frac{\mathrm{Im} M_{12}}{\mathrm{Re} M_{12}} = 2s_2 c_2 s_3 \sin\delta \, P(\theta_2, \eta) \tag{57}$$

with

$$P(\theta_2, \eta) = \frac{s_2{}^2 \left(1 + \dfrac{\eta \ln\eta}{1 - \eta}\right) - c_2{}^2 \left(\eta + \dfrac{\eta \ln\eta}{1 - \eta}\right)}{c_2{}^4 \eta + s_2{}^4 - 2s_2{}^2 c_2{}^2 \left(\dfrac{\eta \ln\eta}{1 - \eta}\right)}, \tag{58}$$

when s_1 and s_3 are considered as small quantities. Here $\eta = m_c{}^2/m_t{}^2$ and M_{12} is the element of the K^0-$\bar{K}^0$ mass matrix defined by[27]

$$M_{12} = \langle K^0 | H_W | \bar{K}^0 \rangle + \sum_n \frac{\langle K^0 | H_W | n \rangle \langle n | H_W | \bar{K}^0 \rangle}{m_{K^0} - m_n} + \cdots. \tag{59}$$
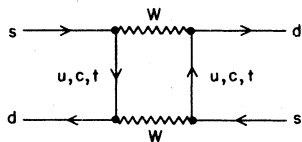

FIG. 2. Diagram contributing to the K^0-$\bar{K}^0$ mass matrix.

When $\delta = 0$ and there is no CP violation we define the real decay amplitude $A_0^{(\delta=0)}$ for $K^0 \to \pi\pi$ ($I = 0$) by

$$\langle 2\pi(I = 0) | H_W^{(\delta=0)} | K^0 \rangle = A_0^{(\delta=0)} e^{i\delta_0}, \tag{60}$$

where δ_0 is the $I = 0$ strong-interaction $\pi\pi$ phase shift. A similar definition applies to the amplitude $A_2^{(\delta=0)}$ for $K^0 \to \pi\pi$ ($I = 2$).

When $s_2 c_2 s_3 \sin\delta \neq 0$ and CP is violated, an inspection of the coefficients of the operators Q_1 and Q_2 immediately shows that the ratio of their imaginary to real parts is $\sim 10^{-2} s_2 c_2 s_3 \sin\delta$. This is not true for the penguin-type operators Q_3, Q_5, and Q_6 where the corresponding ratio is $\sim s_2 c_2 s_3 \sin\delta$. If these later operators contribute at all significantly to K^0 decay, clearly they will yield the largest CP-violating effects in these amplitudes. We recall in particular that matrix elements of Q_6 are supposed to be especially large and important in weak nonleptonic decays like $K^0 \to \pi\pi$.

Let f be the fraction[24] of the $K^0 \to \pi\pi$ ($I = 0$) amplitude arising from the penguin-type operator Q_6. Then the total amplitude for $K^0 \to \pi\pi$ ($I = 0$) when $\delta \neq 0$ is to a good approximation,

$$A_0 \approx A_0^{(\delta=0)} + if A_0^{(\delta=0)} \mathrm{Im} C_6 / \mathrm{Re} C_6, \tag{61}$$

where C_6 is the coefficient of Q_6 in the effective nonleptonic Hamiltonian. Defining

$$\xi = f \mathrm{Im} C_6 / \mathrm{Re} C_6, \tag{62}$$

we have

$$A_0 \approx A_0^{(\delta=0)} e^{i\xi}, \tag{63}$$

since ξ is small.

The standard convention that A_0 is real[27] may be accomplished by redefining the phases of the K^0 and $\bar{K}^0$ states:

$$|K^0\rangle \to e^{-i\xi} |K^0\rangle,$$

$$|\bar{K}^0\rangle \to e^{+i\xi} |\bar{K}^0\rangle.$$

At the same time

$$\frac{\mathrm{Im} M_{12}}{\mathrm{Re} M_{12}} \to \frac{\mathrm{Im} M_{12}}{\mathrm{Re} M_{12}} + 2\xi \equiv \epsilon_m + 2\xi.$$

In standard notation[27] the CP-violation parameter

$$\epsilon = i \frac{\mathrm{Im} \Gamma_{12} + i \mathrm{Im} M_{12}}{(\Gamma_S - \Gamma_L)/2 + i(m_S - m_L)}. \tag{64}$$

Experimentally[28] $\frac{1}{2}(\Gamma_S - \Gamma_L) \approx -(m_S - m_L)$. Within the convention A_0 real Im$\Gamma_{12}/\mathrm{Im} M_{12}$ can be neglected.[27,29] Using $2\mathrm{Re} M_{12} = m_S - m_L$, we have

$$\epsilon \approx \frac{1}{2\sqrt{2}} e^{i\pi/4} (\epsilon_m + 2\xi). \tag{65}$$

The phase angle $\pi/4$ in Eq. (65), which originates

TABLE II. Values of the quantity ξ, which leads to CP violation in $K^0 \to \pi\pi$ decay amplitudes; ϵ_m, the contribution to CP violation from the mass matrix calculated with $\theta_2 = 15°$; and the resulting ratio of CP-violation parameters, ϵ'/ϵ for $\theta_2 = 15°$ and $f = 0.75$.

Parameters		$\xi/f s_2 c_2 s_3 \sin\delta$	$\epsilon_m/s_2 c_2 s_3 \sin\delta$	ϵ'/ϵ
$\alpha(\mu^2) = 0.75$	$m_t = 15$ GeV	$-(0.423 + s_2^2)^{-1}$	10.4	1/50
$\alpha(\mu^2) = 1.00$	$m_t = 15$ GeV	$-(0.651 + s_2^2)^{-1}$	10.4	1/80
$\alpha(\mu^2) = 1.25$	$m_t = 15$ GeV	$-(0.811 + s_2^2)^{-1}$	10.4	1/100
$\alpha(\mu^2) = 0.75$	$m_t = 30$ GeV	$-(0.326 + s_2^2)^{-1}$	18.2	1/80
$\alpha(\mu^2) = 1.00$	$m_t = 30$ GeV	$-(0.505 + s_2^2)^{-1}$	18.2	1/120
$\alpha(\mu^2) = 1.25$	$m_t = 30$ GeV	$-(0.624 + s_2^2)^{-1}$	18.2	1/150

in the K_L and K_S mass and width values, has the precise value[29] $43.8° \pm 0.2°$, just as in the superweak model.[30]

The other CP-violation parameter is

$$\epsilon' \approx \frac{i}{\sqrt{2}} e^{i(\delta_2 - \delta_0)} \frac{\text{Im} A_2}{A_0}. \tag{66}$$

CP violation from the penguin-type operator Q_6 (with $I = \frac{1}{2}$) cannot enter the amplitude A_2 which involves a $\Delta I = \frac{3}{2}$ transition. However, the redefinition of K^0 and $\overline{K}^0$ phases to make A_0 real gives A_2 a phase $e^{-i\xi}$. The experimental $\pi\pi$ phase shifts δ_0 and δ_2 together with $A_2/A_0 \approx +\frac{1}{20}$ yields

$$\epsilon' \approx \frac{1}{20\sqrt{2}} e^{i\pi/4} (-\xi). \tag{67}$$

The experimental value[29] of the phase angle, which we have approximated by $\pi/4$ in Eq. (67), is $37° \pm 6°$. Combining Eqs. (65) and (67) gives

$$\frac{\epsilon'}{\epsilon} \approx \frac{1}{20} \left(\frac{-2\xi}{\epsilon_m + 2\xi} \right). \tag{68}$$

Values of the parameter ξ, which enters the CP-violating parameters ϵ and ϵ', are given in Table II for the different choices of m_t and $\alpha(\mu^2)$ discussed previously. Also, in Table II is the usual contribution to CP violation from the mass matrix, ϵ_m, calculated with $\theta_2 = 15°$.

Although obtained in a very different manner, the results in our earlier paper and those calculated here for ξ are quite comparable quantitatively. However, since in our earlier paper we calculated the ratio of imaginary to real parts of the single lowest-order penguin diagram, while here we have done an all-orders leading-logarithmic calculation, there is no obvious direct comparison or simple approximation in which the latter results should go over into the former. Nevertheless, the agreement not only in sign but also roughly in magnitude for ξ is gratifying and lends additional support to our earlier conclusions[5] on CP viola-

tion in the six-quark model.

The parameter ϵ_m in Table II is calculated to zeroeth order in QCD. The QCD radiative corrections to $\text{Re} M_{12}$ have been calculated[2,31] in the four-quark model using the leading-logarithmic approximation and were found to be negligible. In view of this, it is perhaps not unreasonable to assume that QCD radiative corrections to ϵ_m are also small. In what follows we shall make this assumption.

The parameter $\xi/(s_2 c_2 s_3 \sin\delta)$ is always negative and of order unity. As such, 2ξ is comparable in magnitude and opposite in sign to ϵ_m, leading to comparable contributions from decay amplitudes (2ξ) and the mass matrix (ϵ_m) to the CP-violation parameter ϵ. However, the phase δ may be freely adjusted to fit the experimental magnitude and sign[10] of ϵ and no quantitative test of the contribution from ϵ_m or 2ξ is possible from ϵ alone. But in ϵ'/ϵ the common factor $s_2 c_2 s_3 \sin\delta$ cancels out and predictions dependent on only $2\xi/\epsilon_m$ follow. In Table II values of ϵ'/ϵ are given for the different choices of m_t and $\alpha(\mu^2)$ discussed previously with $\theta_2 = 15°$ and $f = 0.75$. The sign of ϵ'/ϵ is positive and Table II indicates that values of 0.7×10^{-2} to 2×10^{-2} are typical for ϵ'/ϵ.

IV. CONCLUSIONS

In this paper we have derived the effective Hamiltonian for strangeness-changing nonleptonic decays in the six-quark model. The QCD corrections were calculated to all orders in the strong coupling in leading-logarithmic approximation by successively considering the W boson, t quark, b quark, and c quark as heavy and removing them from appearing explicitly in the effective Hamiltonian. At the last stage we remain with an effective Hamiltonian which is a sum of local four-fermion operators involving u, d, and s quark

TABLE III. Same as Table I but with $\Lambda^2 = 0.01$ GeV^2.

Parameters		C_1	C_2	C_3	C_5	C_6
$\alpha(\mu^2) = 0.75,$	$m_t = 15$ GeV	$-0.77 + 0.021\tau$	$+1.43 - 0.021\tau$	$-0.026 - 0.006\tau$	$+0.013 + 0.004\tau$	$-0.065 - 0.045\tau$
$\alpha(\mu^2) = 1.00,$	$m_t = 15$ GeV	$-0.93 + 0.021\tau$	$+1.55 - 0.021\tau$	$-0.032 - 0.005\tau$	$+0.017 + 0.003\tau$	$-0.097 - 0.055\tau$
$\alpha(\mu^2) = 1.25,$	$m_t = 15$ GeV	$-1.06 + 0.021\tau$	$+1.65 - 0.021\tau$	$-0.037 - 0.003\tau$	$+0.020 + 0.002\tau$	$-0.128 - 0.065\tau$
$\alpha(\mu^2) = 0.75,$	$m_t = 30$ GeV	$-0.76 + 0.026\tau$	$+1.42 - 0.026\tau$	$-0.025 - 0.008\tau$	$+0.013 + 0.005\tau$	$-0.065 - 0.060\tau$
$\alpha(\mu^2) = 1.00,$	$m_t = 30$ GeV	$-0.92 + 0.027\tau$	$+1.54 - 0.027\tau$	$-0.032 - 0.006\tau$	$+0.017 + 0.004\tau$	$-0.097 - 0.075\tau$
$\alpha(\mu^2) = 1.25,$	$m_t = 30$ GeV	$-1.05 + 0.027\tau$	$+1.65 - 0.027\tau$	$-0.037 - 0.004\tau$	$+0.020 + 0.003\tau$	$-0.127 - 0.088\tau$

fields times their corresponding Wilson coefficients.

Our calculation follows a well defined and systematic path to the effective nonleptonic Hamiltonian. While the interaction corresponding to the penguin diagram in Fig. 1 may be incorporated by hand into an extra term in an effective Hamiltonian,[5,6] then one does not know how to take into account higher-order QCD effects correctly.[32] In fact, as this paper has examined in detail, the penguin-type terms in the weak nonleptonic Hamiltonian originate at the same level as do the QCD corrections to the usual $(V - A) \times (V - A)$ four-fermion terms and the two kinds of operators even mix with each other.

In the resulting Hamiltonian there are five linearly independent four-fermion local operators. Two of these are the usual $(V - A) \times (V - A)$ operators, but with coefficients that have been changed by QCD effects. Numerical evaluation gives an enhancement of the combination transforming as $I = \frac{1}{2}$, but only by a factor of 2 to 3. As already concluded by others,[11] this is in the right direction, but is inadequate in magnitude to explain the success of the $\Delta I = \frac{1}{2}$ rule for strange-particle nonleptonic weak decays. The other three operators are penguinlike, purely $I = \frac{1}{2}$, and arise through QCD diagrams involving heavy quark loops. Although their coefficients turn out to be small upon numerical evaluation, it is arguable[3,4]

that they have enhanced matrix elements for weak decays of kaons and hyperons. If important portions of such amplitudes come from these penguin-like operators, an explanation of the $\Delta I = \frac{1}{2}$ rule is then possible.

The QCD corrections result in imaginary, CP-violating parts to the coefficients of all five operators. For the penguinlike operators the imaginary part of their coefficients is about the same magnitude as their real part times $s_2 c_2 s_3 \sin\delta$. Assuming these operators make a dominant contribution to the $K \to \pi\pi$ $(I = 0)$ decay amplitude results in comparable contributions to CP violation in the K^0-$\bar{K}^0$ system from the mass matrix and the decay amplitude itself. Both these contributions are proportional to $s_2 c_2 s_3 \sin\delta$, the magnitude and sign of which may be fixed to give the observed value of the CP-violation parameter ϵ.

However, in the quantity ϵ'/ϵ the factor $s_2 c_2 s_3 \sin\delta$ cancels out and we predict real values ranging from $+0.7 \times 10^{-2}$ to $+2 \times 10^{-2}$ from our calculation and choice of parameters. The smaller values correspond to larger values of m_t or $\alpha(\mu^2)$. Using a larger value of θ_2 or a smaller value of Λ can also give smaller values of ϵ'/ϵ. For example, with[33] $\Lambda^2 = 0.01$ GeV^2 but the other parameters chosen as before we find (see Tables III and IV) values of ϵ'/ϵ ranging from about $+0.3 \times 10^{-2}$ to $+0.5 \times 10^{-2}$.

The present experimental value is ϵ'/ϵ

TABLE IV. Same as Table II but with $\Lambda^2 = 0.01$ GeV^2.

Parameters		$\xi/f s_2 c_2 s_3 \sin\delta$	$\epsilon_m/s_2 c_2 s_3 \sin\delta$	ϵ'/ϵ
$\alpha(\mu^2) = 0.75,$	$m_t = 15$ GeV	$-(1.461 + s_2^2)^{-1}$	10.4	1/190
$\alpha(\mu^2) = 1,$	$m_t = 15$ GeV	$-(1.760 + s_2^2)^{-1}$	10.4	1/230
$\alpha(\mu^2) = 1.25,$	$m_t = 15$ GeV	$-(1.963 + s_2^2)^{-1}$	10.4	1/260
$\alpha(\mu^2) = 0.75,$	$m_t = 30$ GeV	$-(1.076 + s_2^2)^{-1}$	18.2	1/260
$\alpha(\mu^2) = 1,$	$m_t = 30$ GeV	$-(1.295 + s_2^2)^{-1}$	18.2	1/310
$\alpha(\mu^2) = 1.25,$	$m_t = 30$ GeV	$-(1.442 + s_2^2)^{-1}$	18.2	1/350

$= -0.003 \pm 0.014$, but experiments now planned[34] should be capable of measuring or limiting ϵ'/ϵ to the level of a fraction of a percent. As such they might be capable of distinguishing the six-quark model[35] with important penguinlike contributions to $K \to 2\pi$ decay from the superweak model,[30] where $\epsilon' = 0$, as explanations of the violation of CP invariance.

ACKNOWLEDGMENTS

APPENDIX

In this section we outline the derivation of the equations and give numerical results for the quantities which appear in Sec. II. In Sec. II a rather fundamental role was played by the renormalization group Eqs. (22), (32), (37), and (48). To get Eq. (22), for example, one merely applies $\mu\, d/d\mu$ to both sides of Eq. (19) using

$$\mu \frac{d}{d\mu}\langle|\,O_t^{(\pm)}\,|\rangle$$

$$= \left(\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} + \sum_q \gamma_q(g)m_q\frac{\partial}{\partial m_q}\right)\langle|\,O_t^{(\pm)}\,|\rangle$$

$$= -\gamma^{(\pm)}(g)\langle|\,O_t^{(\pm)}\,|\rangle, \tag{A1}$$

$$\mu\frac{d}{d\mu}\langle|\,O_i\,|\rangle'$$

$$= \left(\mu\frac{\partial}{\partial\mu} + \beta'(g')\frac{\partial}{\partial g'} + \sum_q \gamma_q'(g')m_q'\frac{\partial}{\partial m_q'}\right)\langle|\,O_i\,|\rangle'$$

$$= -\sum_j \gamma'_{ij}(g')\langle|\,O_j\,|\rangle', \tag{A2}$$

and

$$\mu\frac{d}{d\mu}B_j^{(\pm)}\left(\frac{m_t}{\mu},g\right)$$

$$= \left(\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} + \gamma_t(g)m_t\frac{\partial}{\partial m_t}\right)B_j^{(\pm)}\left(\frac{m_t}{\mu},g\right). \tag{A3}$$

In Eqs. (A1) and (A3) the partial derivative with respect to μ is at constant g and m_q, where $q \in \{u, d, \ldots, t\}$, while in Eq. (A2) it is at constant g' and m_q', where $q \in \{u, d, \ldots, b\}$.

The $\gamma^{(\pm)}(g)$ and the matrix $\gamma'_{kj}(g')$ arise because the operators $O_t^{(\pm)}$ and O_i are local four-fermion operators and require renormalization. The renormalization of the operators $O_q^{(\pm)}$ at the one-
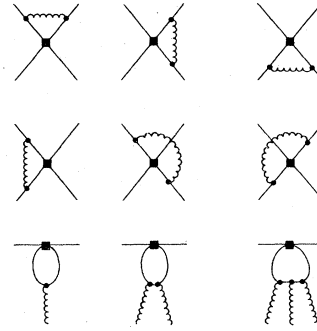

FIG. 3. Diagrams entering the calculation of the renormalization of the local four-fermion operators (represented by the square box) through QCD effects.

loop level was considered in Ref. 11 where it was shown that the $\gamma^{(\pm)}(g)$ are given by Eq. (12). From Eq. (16), with $N_f = 6$, it follows that

$$a^{(+)} = \tfrac{6}{21}, \qquad a^{(-)} = -\tfrac{12}{21}. \tag{A4}$$

At the one-loop level the operators O_j undergo a renormalization

$$O_j^0 = \sum_k Z_{jk}O_k, \tag{A5}$$

where a superscript "0" denotes a bare unrenormalized quantity. Z_{jk} is the matrix renormalization which arises because of the composite nature of the local four-fermion operators O_j. The matrix $\gamma'_{ij}(g')$ is defined by

$$\gamma'_{ij}(g') = \sum_k Z^{-1}{}_{ik}\mu\frac{d}{d\mu}Z_{kj}. \tag{A6}$$

Note that the Z_{jk} are a function of the coupling g' since the renormalization of the operators O_j is calculated in the effective five-quark theory with that coupling. A straightforward calculation of the "infinite part" of the one-particle-irreducible diagrams in Fig. 3, using Landau gauge, gives

$$\gamma'_{ij}(g') = \frac{g'^2}{8\pi^2}\begin{bmatrix} -1 & 3 & 0 & 0 & 0 & 0 \\ 3 & -1 & -\frac{1}{9} & \frac{1}{3} & -\frac{1}{9} & \frac{1}{3} \\ 0 & 0 & -\frac{11}{9} & \frac{11}{3} & -\frac{2}{9} & \frac{2}{3} \\ 0 & 0 & \frac{22}{9} & \frac{2}{3} & -\frac{5}{9} & \frac{5}{3} \\ 0 & 0 & 0 & 0 & 0 & -3 \\ 0 & 0 & -\frac{5}{9} & \frac{5}{3} & -\frac{5}{9} & -\frac{19}{3} \end{bmatrix} + O(g'^4). \tag{A7}$$

In the calculation of the renormalization of the local four-fermion operators, O_j, the masses of the light up, down, and strange quarks was set to zero. If this was not done the operators O_j would close under renormalization at the one-loop level.

At the two-loop level a transition color magnetic moment term must be added. However, the presence of such an operator does not alter the Wilson coefficients of the local four-fermion operators, O_j, from their value calculated with the light quark masses set to zero. The transition color magnetic moment operator itself is explicitly proportional to a light quark mass yielding small matrix elements. Also the Wilson coefficient of the magnetic moment

operator is expected to be small. These facts justify our approximation of setting the u, d, and s quark masses to zero.

The matrix $\gamma_{ij}'^T(g')$ can be diagonalized by the transformation

$$\sum_{k,l} V^{-1}{}_{ii}\gamma_{lk}'^T(g')V_{kj} = \delta_{ij}\gamma_j'(g'), \tag{A8}$$

where

$$V_{kj} = \begin{bmatrix} 0 & -0.69483 & 0 & 0 & 0.70576 & 0 \\ 0 & 0.69483 & 0 & 0 & 0.70576 & 0 \\ 0.15042 & 0.23161 & -1.253 & 0.16684 & -0.10082 & 0.42681 \\ -0.2089 & -0.23161 & 1.0843 & 0.081196 & -0.10082 & 0.82414 \\ 0.032942 & 0 & 0.10426 & 0.93924 & 0 & -0.3322 \\ 0.61688 & 0 & 0.21323 & -0.34513 & 0 & 0.28045 \end{bmatrix} \tag{A9}$$

and

$$\gamma_j'(g') = \frac{g'^2}{8\pi^2} \begin{bmatrix} -6.8954 \\ -4 \\ -3.2429 \\ 1.1166 \\ 2 \\ 3.1327 \end{bmatrix} + O(g'^4). \tag{A10}$$

Combining (A10) with the perturbative expansion of $\beta'(g')$ in Eq. (27) yields the a_j' of Eq. (28):

$$a_j' = \begin{bmatrix} -0.8994 \\ -\frac{12}{23} \\ -0.42299 \\ 0.14564 \\ \frac{6}{23} \\ 0.40861 \end{bmatrix}. \tag{A11}$$

Note that $a_2'=a'^{(-)}$ and $a_4'=a'^{(+)}$, where

$$-\frac{\gamma'^{(\pm)}(x)}{\beta'(x)} = \frac{2a'^{(\pm)}}{x} + \text{terms finite at } x=0 \tag{A12}$$

and

$$\gamma'^{(+)}(g') = \frac{g'^2}{4\pi^2} + O(g'^4), \tag{A13a}$$

$$\gamma'^{(-)}(g') = -\frac{g'^2}{2\pi^2} + O(g'^4). \tag{A13b}$$

The case where the quark is treated as very heavy is similar to the above and we simply state results:

$$\gamma_{mn}''(g'') = \frac{g''^2}{8\pi^2} \begin{bmatrix} -1 & 3 & 0 & 0 & 0 & 0 \\ 3 & -1 & -\frac{1}{9} & \frac{1}{3} & -\frac{1}{9} & \frac{1}{3} \\ 0 & 0 & -\frac{11}{9} & \frac{11}{3} & -\frac{2}{9} & \frac{2}{3} \\ 0 & 0 & \frac{23}{9} & \frac{1}{3} & -\frac{4}{9} & \frac{4}{3} \\ 0 & 0 & 0 & 0 & 1 & -3 \\ 0 & 0 & -\frac{4}{9} & \frac{4}{3} & -\frac{4}{9} & -\frac{20}{3} \end{bmatrix} + O(g''^4). \tag{A14}$$

$\gamma_{mn}''^T(g'')$ is diagonalized by the transformation

$$\sum_{l,k} W^{-1}{}_{ni}\gamma_{lk}''^T(g'')W_{km} = \delta_{nm}\gamma_n''(g''), \tag{A15}$$

where

$$W_{km} = \begin{bmatrix} 0 & 0.67552 & 0 & 0 & 0.70598 & 0 \\ 0 & -0.67552 & 0 & 0 & 0.70598 & 0 \\ -0.13011 & -0.33776 & -1.2092 & 0.14075 & -0.11766 & 0.47246 \\ 0.18274 & 0.33776 & 1.1043 & 0.067129 & -0.11766 & 0.80199 \\ -0.02959 & 0 & 0.064119 & 0.96326 & 0 & -0.30023 \\ -0.65316 & 0 & 0.14969 & -0.34859 & 0 & 0.23908 \end{bmatrix}, \tag{A16}$$

and

$$\gamma_n''(g'') = \frac{g''^2}{8\pi^2} \begin{bmatrix} -7.0428 \\ -4 \\ -3.501 \\ 1.0974 \\ 2 \\ 2.8909 \end{bmatrix} + O(g''^4). \qquad \text{(A17)}$$

It follows from (A17) and the perturbative expansion of $\beta''(g'')$ that

$$a_n'' = \begin{bmatrix} -0.845\,14 \\ -\frac{12}{25} \\ -0.420\,12 \\ 0.131\,69 \\ \frac{6}{25} \\ 0.346\,91 \end{bmatrix}. \qquad \text{(A18)}$$

$$X_{np} = \begin{bmatrix} 0.168\,66 & -0.714\,36 & 0.052\,633 & 0.848\,53 & 0.690\,88 \\ -0.168\,66 & 0.714\,36 & -0.052\,633 & 0.565\,69 & -0.690\,88 \\ -0.050\,165 & -0.030\,949 & -0.165\,52 & -0.282\,84 & -1.1481 \\ 0.028\,133 & 0.018\,728 & -1.0044 & 0 & 0.232\,29 \\ 0.783\,61 & 0.049\,722 & 0.357\,26 & 0 & -0.174\,86 \end{bmatrix} \qquad \text{(A21)}$$

and

$$\gamma_q'''(g''') = \frac{g'''^2}{8\pi^2} \begin{bmatrix} -7.2221 \\ -3.7559 \\ 1.0761 \\ 2 \\ 2.6797 \end{bmatrix} + O(g'''^4). \qquad \text{(A22)}$$

Note that these eigenvalues check with those of Ref. 3 where the effective Hamiltonian for strangeness-changing nonleptonic decays was calculated

Again $a_2'' = a''^{(-)}$ and $a_4'' = a''^{(+)}$.

When the heavy-charm-quark expansion is performed only the five operators Q_1, Q_2, Q_3, Q_5, and Q_6 defined in Eq. (44) are required. We find that

$$\gamma_{pr}'''(g''') = \frac{g'''^2}{8\pi^2} \begin{bmatrix} -1 & 3 & 0 & 0 & 0 \\ \frac{8}{3} & -\frac{2}{3} & \frac{2}{9} & -\frac{1}{9} & \frac{1}{3} \\ -\frac{11}{3} & \frac{11}{3} & \frac{22}{9} & -\frac{2}{9} & \frac{2}{3} \\ 0 & 0 & 0 & 1 & -3 \\ -1 & 1 & \frac{2}{3} & -\frac{1}{3} & -7 \end{bmatrix} + O(g'''^4). \qquad \text{(A19)}$$

The matrix $\gamma_{pr}'''^T(g''')$ is diagonalized by the transformation

$$\sum_{p,r} X^{-1}{}_{np} \gamma_{pr}'''^T(g''') X_{rq} = \delta_{nq} \gamma_q'''(g'''), \qquad \text{(A20)}$$

where

in the four-quark model using a different operator basis. The fourth eigenvalue corresponds to the multiplicatively renormalized isospin-$\frac{3}{2}$ operator $3Q_1 + 2Q_2 - Q_3$. Finally

$$a_q''' = \begin{bmatrix} -0.802\,46 \\ -0.417\,32 \\ 0.119\,57 \\ \frac{6}{27} \\ 0.297\,74 \end{bmatrix}. \qquad \text{(A23)}$$

[1] E. Witten, Nucl. Phys. <u>B104</u>, 445 (1976).

[2] E. Witten, Nucl. Phys. <u>B122</u>, 109 (1977).

[3] M. A. Shifman et al., Zh. Eksp. Teor. Fiz. Pis'ma Red <u>22</u>, 123 (1975) [JETP Lett. <u>22</u>, 55 (1975)]; M. A. Shifman et al., Nucl. Phys. <u>B120</u>, 316 (1977).

[4] M. A. Shifman et al., ITEP Reports Nos. ITEP-63 and ITEP-64, 1976 (unpublished).

[5] F. J. Gilman and M. B. Wise, Phys. Lett. <u>83B</u>, 83 (1979).

[6] M. K. Gaillard, in *Weak Interactions—Present and Future*, proceedings of the SLAC Summer Institute on Particle Physiscs, 1978, edited by M. C. Zipf, (SLAC, Stanford, 1978), p. 397.

[7] M. B. Wise and E. Witten, Phys. Rev. D <u>20</u>, 1216 (1979).

[8] S. Weinberg, Phys. Rev. Lett. <u>19</u>, 1264 (1967); A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium No. 8)*, edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367.

[9] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. <u>49</u>,

652 (1973).

[10]The signs of the quark fields may be chosen so that θ_1, θ_2, and θ_3 (but not necessarily δ) lie in the first quadrant and all sines and cosines of them are positive. In Ref. 5 the measured phase of the CP-violation parameter ϵ was used to show that δ lies in the upper half plane.

[11]M. K. Gaillard and B. W. Lee, Phys. Rev. Lett. **33**, 108 (1974); G. Altarelli and L. Maiani, Phys. Lett. **52B**, 351 (1974).

[12]K. G. Wilson, Phys. Rev. **179**, 1499 (1969).

[13]S. Weinberg, Phys. Rev. D **8**, 3497 (1973); G. 't Hooft, Nucl. Phys. **B61**, 455 (1973).

[14]M. Gell-Mann and F. Low, Phys. Rev. **95**, 1300 (1954); E. C. G. Stueckelberg and A. Peterman, Helv. Phys. Acta **26**, 499 (1953); C. G. Callan, Phys. Rev. D **2**, 1541 (1970); K. Symanzik, Commun. Math. Phys. **18**, 227 (1970).

[15]D. J. Gross and F. Wilczek, Phys. Rev. Lett. **30**, 1343 (1973); H. D. Politzer, *ibid.* **30**, 1346 (1973).

[16]The running fine-structure constants are defined by $\alpha(M^2) = \bar{g}(M/\mu, g)^2/4\pi$ so that $\alpha(\mu^2) = g^2/4\pi$. Similarly $\alpha' = \bar{g}'^2/4\pi$, $\alpha'' = \bar{g}''^2/4\pi$, and $\alpha''' = \bar{g}'''^2/4\pi$.

[17]T. Appelquist and J. Carrazone, Phys. Rev. D **11**, 2856 (1975).

[18]A detailed discussion of what is meant by an effective theory of strong interactions can be found in Secs. 2 and 3 of Ref. 1.

[19]The local four-fermion operators used in Sec. 5 of Ref. 2 are linearly dependent.

[20]J. G. H. DeGroot *et al.*, Phys. Lett. **82B**, 292 (1979); **82B**, 456 (1979); Z. Phys. **C1**, 142 (1979).

[21]See the review of J. D. Jackson, C. Quigg, and J. L. Rosner, in *Proceedings of the 19th International Conference on High Energy Physics, Tokyo, 1978*, edited by S. Homma, K. Kawaguchi, and H. Miyazawa (Phys. Soc. Japan, Tokyo, 1979), p. 391.

[22]See the compilation of C. Baltay, in *Proceedings of the 19th International Conference on High Energy Physics, Tokyo, 1978*, edited by S. Homma, M. Kawaguchi, and H. Miyazawa (Phys. Soc. Japan, Tokyo, 1979), p. 882.

[23]This point has been emphasized in Ref. 11 and in J. Ellis *et al.*, Nucl. Phys. **B100**, 313 (1975).

[24]The matrix elements of Q_5 also have the chiral structure $(V-A)\times(V+A)$ which leads to enhanced matrix elements. However, the arrangement of color indices is such that the matrix elements of Q_5 are suppressed by a factor of $\frac{1}{3}$ compared to those of Q_6.

[25]J. Finjord, Phys. Lett. **76B**, 116 (1978).

[26]J. Ellis *et al.*, Nucl. Phys. **B109**, 213 (1976).

[27]We adopt the conventions of T. D. Lee and C. S. Wu [Annu. Rev. Nucl. Sci. **16**, (1966)], except that we label K^0 eigenstates by S and L.

[28]Particle Data Group, Phys. Lett. **75B**, 1 (1978).

[29]K. Kleinknecht, in *Proceedings of the XVII International Conference on High Energy Physics, London, 1974*, edited by J. R. Smith (Rutherford Laboratory, Chilton, Didcot, Berkshire, England, 1974), p. III-23.

[30]L. Wolfenstein, Phys. Rev. Lett. **13**, 562 (1964).

[31]A. I. Vainshtein *et al.*, Yad, Fiz. **23**, 1024 (1976) [Sov. J. Nucl. Phys. **23**, 540 (1976)].

[32]These higher-order QCD corrections were neglected in Ref. 5. Since the lowest-order penguin terms themselves arise as QCD corrections, it is not possible to include higher-order QCD effects in the leading-logarithm approximation in the manner suggested in Ref. 6.

[33]Such a "small" value of Λ is suggested by a comparison of QCD sum rules with experimental data on e^+e^- annihilation. See, S. I. Eidelman *et al.*, Phys. Lett. **82B**, 278 (1979); M. A. Shifman *et al.*, Nucl. Phys. **B147**, 385 (1979).

[34]R. Bernstein *et al.*, Fermilab experiment E-617 and B. Winstein, private communication.

[35]Our conclusions are based on the SU(2)$\otimes$U(1) gauge theory with six quarks and the minimal Higgs sector. It is possible to add extra Higgs so that CP violation also occurs in the Higgs sector. See, for example, T. D. Lee, Phys. Rev. D **8**, 1226 (1973); Phys. Rep. **9C**, 143 (1974); S. Weinberg, Phys. Rev. Lett. **37**, 657 (1976); P. Sikivie, Phys. Lett. **65B**, 141 (1976).