PHYSICAL REVIEW D VOLUME 20, NUMBER 9 1 NOVEMBER 1979

# Energy estimates of cosmic-ray events

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(Received 12 January 1979)

We propose new methods for estimating the energy of the incident particles in high-energy cosmic-ray collisions. We demonstrate their validity in accelerator experiments.

Ever since the discovery of cosmic rays by Victor F. Hess in 1912, studies of the interactions of extremely-high-energy cosmic rays have been a unique source of information on particle and nuclear interactions at energies far beyond the energies imparted to particles by the most powerful laboratory acclerators. Unfortunately, the energies of the very energetic cosmic-ray particles cannot be measured directly. They have to be deduced from observed properties of the secondaries produced in their interactions with atomic nuclei. Various methods for estimating the energies have been proposed in the past[1] before detailed studies of particle-nucleus interactions in the energy range up to 400 GeV were performed at laboratory accelerators.[2] In this note we show that two alternative general pictures of particle-nucleus interactions which emerged from these studies lead to the same estimate which is a modification of the Castagnoli method.[1] We also modify the $p_T$ method, where a constant value of the average transverse momentum, $\langle p_T\rangle = 0.35$ GeV, of the produced charged particles is assumed. We test our modified methods in accelerator experiments with known beam energies and we demonstrate their validity.

## ENERGY ESTIMATES FOR *pp* COLLISIONS

For a high-energy particle the pseudorapidity variable $\eta \equiv -\ln[\tan(\theta/2)] \approx \ln(2P_L/P_T)$, where $\theta$, $P_L$, and $P_T$ are respectively the angle and the parallel and the perpendicular components of the momentum of the produced particles with respect to the incident momentum, is related to its rapidity $y = \frac{1}{2}\ln[(E+P_L)/(E-P_L)] = \ln[(E+P_L)/m_T]$ through $\eta \approx y + \ln(m_T/P_T)$, where $m_T = (m^2 + P_T{}^2)^{1/2}$. $E$ and $m$ are, respectively, the energy and mass of the produced particle. From high-energy accelerator data we know that the $P_T$ distribution of produced particles falls rapidly with $P_T$ and since most of the produced particles have very large lab energies, their average rapidity and pseudorapidity satisfy $\langle\eta\rangle \approx \langle y\rangle + \langle\ln(m_T/P_T)\rangle$. Since the relative abundance of different masses and the small-$P_T$ behavior of most of the produced particles are approximately energy independent, consequently $\Delta \equiv \langle\eta\rangle - \langle y\rangle$ is energy independent. For *pp* collisions the $y$ distribution of the final particles has to be symmetric around $y_{\text{c.m.}}$, the rapidity of the center of mass, i.e., $\langle y\rangle = y_{\text{c.m.}} = \frac{1}{2}\ln(2E/m_p)$, where $m_p$ is the proton mass, and thus

$$\langle\eta\rangle = 0.5\ln E + a\,, \tag{1}$$

where $a \equiv -0.5\ln(m_p/2) + \Delta$. $\Delta$ can be determined from a single experiment at a fixed incident energy, e.g., from the measurement of $dn/d\eta$ at 400 GeV by the Alma Ata–Gatchina–Moscow–Tashkent collaboration $\Delta \approx 0.45$ (Ref. 3) and Eq. (1) can then be written as

$$E = 0.4(m_p/2)e^{2\langle\eta\rangle}\,. \tag{2}$$

We note that the energy estimate of Castagnoli *et al.*[1] can be written as

$$E = (m_p/2)e^{2\langle\eta'\rangle}\,, \tag{3}$$

where $\eta' \equiv -\ln(\frac{1}{2}\tan\theta)$. It can be derived from Eq. (1) if one neglects the difference between rapidity and pseudorapidity ($\Delta = 0$) and if one makes the approximation $\tan(\theta/2) \approx \frac{1}{2}\tan\theta$, i.e., $\eta \approx \eta'$. From Eqs. (2) and (3) we thus conclude that for *pp* collisions the Castagnoli method overestimates the incident energy by a factor of 2.5.

## ENERGY ESTIMATES FOR PARTICLE-NUCLEUS COLLISIONS

Let us first consider collective models for particle-nucleus interactions. The collective tube models[4] (CTM), for instance, assume that the incident particle interacts collectively with a tube of $\nu$ nucleons that it encounters on its path through the nucleus and that the $p$-tube collision resembles an elementary *pp* collision. The rapidity of the $p$-tube c.m. is given by $y_{\text{c.m.}} = 0.5\ln(2E/\nu m_p)$ and therefore $\langle y\rangle = 0.5\ln(2E/m_p) - 0.5\ln\nu$, i.e.,

$$\langle\eta\rangle_\nu = 0.5\ln E + a_\nu\,, \tag{4}$$

where now $a_\nu \equiv -0.5\ln(\nu m_p/2) + \langle\ln(m_T/p_T)\rangle$ and the average is carried over all $p$-tube charged products. Unfortunately there is no direct way to de-

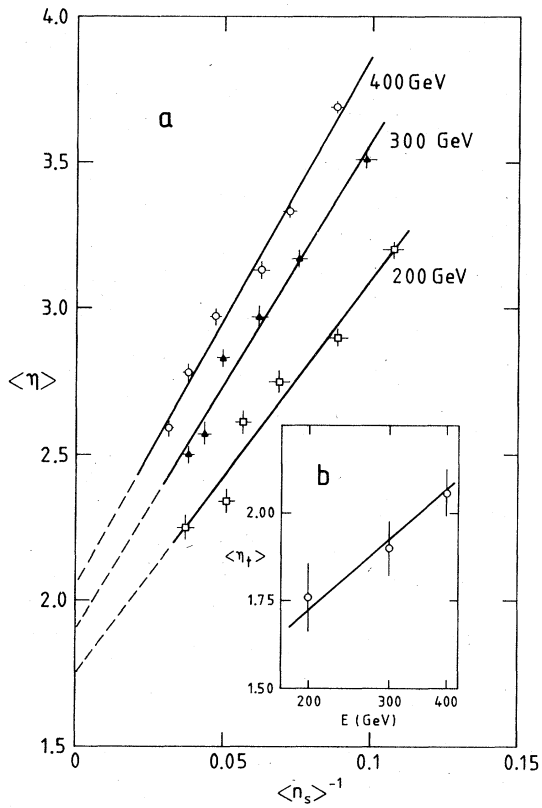


FIG. 1. (a) The average pseudorapidity as a function of $\langle n_s\rangle^{-1}$ at different energies. The straight lines are best fits to the experimental data (Ref. 8). (b) $\langle\eta\rangle_t$ values as a function of $\ln E$ obtained from the straight line fits in (a). The solid line is $0.5\ln E$+const.

termine $\nu$. However, if we assume that $N_h$, the number of heavy-track particles[5] in $p$-emulsion collisions, is a "measure" of $\nu$, we can write

$$\langle\eta\rangle_{N_h}=0.5\ \ln E+a_{N_h}\,, \tag{5}$$

where the subscript $N_h$ indicates that $\langle\eta\rangle$ and $a$ depend on $N_h$.

The independent-particle models also lead to Eq. (5): Consider for instance the independent-particle-fragmentation (IPF) model[6] where the incident particle excites all the $\nu$ nucleons that it collides with on its straight path through the nucleus and then fragments when it emerges at the back of the nucleus. Such a picture leads to the prediction

$$dn/dy=\nu dn_t/dy+dn_p/dy\,, \tag{6}$$

where $n_p$ and $n_t$ are the number of fragments associated with the projectile and target nucleon, respectively. Integration of Eq. (6) with $\langle n_t\rangle=\langle n_p\rangle$ gives

$$\langle n\rangle_{pA}=(1+\langle\nu\rangle)\langle n\rangle_{pN}/2\,, \tag{7}$$

where $\langle n\rangle_{pA}$ and $\langle n\rangle_{pN}$ are, respectively, the average multiplicities in particle-nucleus and in particle-nucleon collisions at the same incident energy and $\langle\nu\rangle$ is given by $\langle\nu\rangle=A\sigma_{pN}/\sigma_{pA}$ where the cross sections are the total inelastic ones.[7] Let $\langle y_p\rangle$ and $\langle y_t\rangle$ be the average rapidities of the projectile and the target nucleon fragments, respectively. For $pp$ collisions $\langle y_t\rangle$ and $\langle y_p\rangle$ should be symmetrically located around $y_{\text{c.m.}}$ and thus $\langle y_p\rangle=2y^{pp}_{\text{c.m.}}-\langle y_t\rangle$. With $\langle n_t\rangle=\langle n_p\rangle$, the average rapidity in a particle-nucleus reaction can be written

$$\langle y\rangle_{pA}=\frac{\nu\langle y_t\rangle+\langle y_p\rangle}{\nu+1}=\frac{(\nu-1)\langle y_t\rangle+2y^{pp}_{\text{c.m.}}}{\nu+1}\,. \tag{8}$$

From Eq. (6) one can easily show that if $N_h$ is a measure of $\nu$ then for a fixed incident energy

$$\langle\eta\rangle_{N_h}=\langle\eta_t\rangle+\text{const}/\langle n_s\rangle_{N_h}\,, \tag{9}$$

where $\langle n_s\rangle_{N_h}$ is the average number of shower particles produced in stars with $N_h$ heavy tracks, and the constant depends on energy but not on $N_h$. In Fig. 1(a) we have plotted $\langle\eta\rangle$ as a function of $\langle n_s\rangle_{N_h}^{-1}$ for 200, 300, and 400 GeV $p$-emulsion experiments.[8] We find that indeed $\langle\eta\rangle$ for different $N_h$ bins lie on straight lines. In Fig. 1(b) we have plotted $\langle\eta_t\rangle$ values obtained from the intercepts of the lines in Fig. 1(a) as a function of $\ln E$. We find that $\langle\eta_t\rangle$ can be well represented by a straight line with the slope 0.5. With $\langle y_t\rangle=0.5\ln E+\text{const}$, Eq.

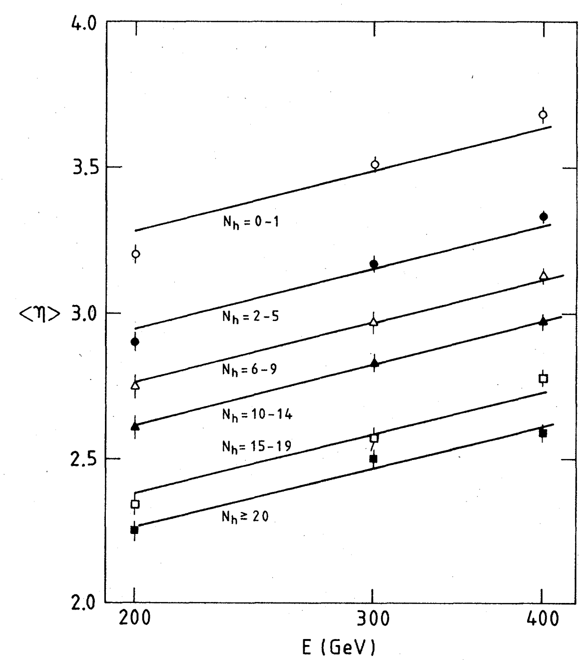


FIG. 2. The average pseudorapidity for different $N_h$ bins as a function of $\ln E$. The straight lines are best fits of the form $0.5\ln E+a$ to the experimental data (Ref. 8).

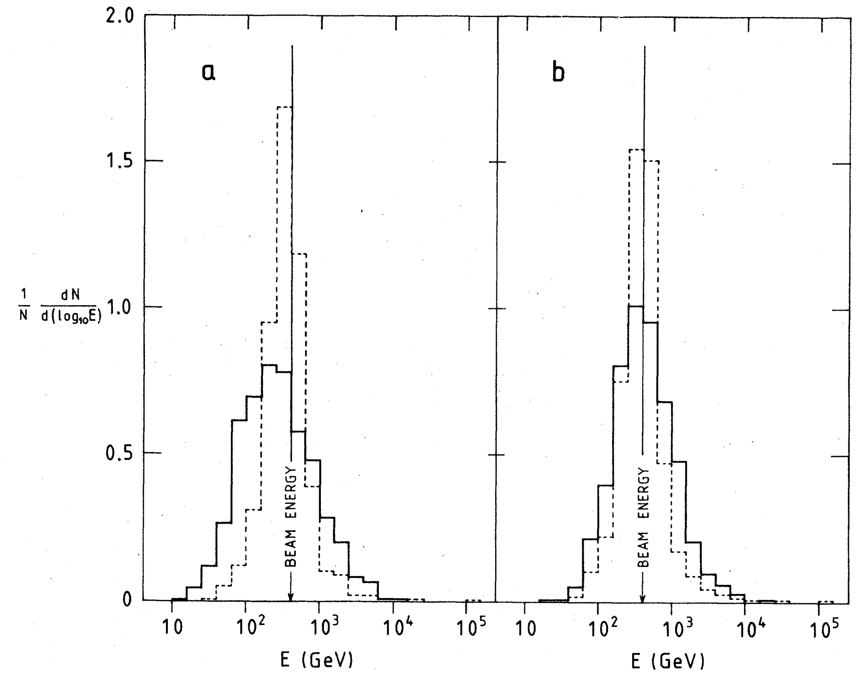


FIG. 3. The incident proton energies estimated for a sample of 400-GeV proton-emulsion collisions (Ref. 8). (a) Solid histogram: Eq. (3), dashed: Eq. (12). (b) Solid histogram: Eq. (11), dashed: Eq. (13).

(8) lead to Eqs. (4) and (5), i.e., both CTM and IPF models lead to the same estimate

$$E = C_{N_h}(m_p/2)e^{2\langle\eta\rangle N_h}. \tag{10}$$

Equation (10) predicts that $\langle\eta\rangle_{N_h}$ as a function of $\ln E$ fall on parallel straight lines with a universal slope 0.5. In Fig. 2 we plot $\langle\eta\rangle_{N_h}$ as a function of $\ln E$ for different $N_h$ bins from 200, 300, and 400 GeV $p$-emulsion experiments.[8] Within statistical errors the points do fall on parallel lines with the slope 0.5. $C_{N_h}$ is approximately given by $C_{N_h} \approx 0.5 + 0.17N_h$ and thus

$$E = (0.5 + 0.17N_h)(m_p/2)e^{2\langle\eta\rangle}. \tag{11}$$

We note that for stars with $N_h = 3$, Castagnoli's estimate as represented by Eq. (3) accidentally yields estimated energies similar to those obtained from Eq. (11), but for stars with $N_h$ values which are considerably different from 3 the two estimates differ considerably.

The incident energy may also be determined if one assumes that the produced particles are emitted with $\langle p_T\rangle = 0.35$ GeV/$c$ and that on the average the neutral secondaries are about $\frac{1}{2}$ of the charged particles. With these assumptions

$$E \approx 0.53 \sum_i (\sin\theta_i)^{-1}\ \text{GeV}. \tag{12}$$

$\langle p_T\rangle$ for nuclear targets is considerably larger than for hydrogen targets.[2] Experimentally we find for 200, 300, and 400 GeV proton-emulsion data[8] that

$$E \approx 0.62 \sum_i (\sin\theta_i)^{-1}\ \text{GeV}. \tag{13}$$

Figure 3(a) shows the distributions of estimated energies that were obtained from Eqs. (3) and (12) for a sample of proton-emulsion collisions at 400 GeV.[8] Figure 3(b) presents the distributions obtained from the modified methods as given by Eqs. (11) and (13). The distributions obtained from the modified methods are centered around the correct energy, while Eq. (3) yields a broader distribution which is not centered around 400 GeV.

## ENERGY ESTIMATES FOR NUCLEUS-NUCLEUS COLLISIONS

The CTM visualizes these collisions as a sum of incoherent tube-tube collisions that take place in the intersection region of the colliding nuclei.[9] Tube collisions are regarded as elementary-particle collisions and consequently if $\nu_p$ and $\nu_t$ are respectively the number of nucleons in the incident tube and in the target tube, the rapidity of the tube-tube c.m. is given by $y_{\text{c.m.}} = 0.5\ln(2E_n/m_p) - 0.5\ln(\nu_t/\nu_p)$, where $E_n$ is the energy per nucleon of the incoming nucleus. When we sum over the different tube collisions, we get

$$\langle y\rangle \approx 0.5\ln(2E_n/m_p) - 0.5\ln(\langle\nu_t\rangle/\langle\nu_p\rangle), \qquad (14)$$

i.e., the CTM energy estimates for a nucleus-nucleus collision and for a particle-nucleus collision are the same if $\nu^* \equiv \langle\nu_t\rangle/\langle\nu_p\rangle = \nu$.

The IPF model visualizes nucleus-nucleus collision as a sum of independent particle-particle collisions. If $N_p$ and $N_t$ are the number of participant nucleons of the projectile and the target respectively, then[10]

$$dn/dy = N_t dn_t/dy + N_p dn_p/dy, \qquad (15)$$

$$\langle n\rangle_{A_1A_2} = (\langle N_t\rangle + \langle N_p\rangle)\langle n\rangle_{pp}/2, \qquad (16)$$

and

$$\langle y\rangle = \frac{N_t\langle y_t\rangle + N_p\langle y_p\rangle}{N_t + N_p} = \frac{(N_t/N_p)\langle y_t\rangle + \langle y_p\rangle}{N_t/N_p + 1}. \qquad (17)$$

Since $N_t/N_p = \langle\nu_t\rangle/\langle\nu_p\rangle \equiv \nu^*$, Eq. (17) is identical to Eq. (8) if $\nu^* = \nu$ and the estimates of $E_n$ of the IPF model for nucleus-nucleus reactions and for particle-nucleus reactions are basically the same if $\nu^* = \nu$.

Since the CTM expression is easier to handle, we will use it to estimate the energies, but we will bear in mind that the IPF model will give approximately the same estimate. From Eq. (14) we arrive at the energy estimate

$$\langle\eta\rangle = 0.5\ln E_n - 0.5\ln(\nu^* m_p/2) + \langle\ln(m_T/P_T)\rangle. \qquad (18)$$

No accelerator data are available on high-energy nucleus-nucleus collisions so $\langle\ln(m_T/P_T)\rangle$ has to be evaluated theoretically. If we assume that the shower particles contain mainly pions produced in the collision and protons from the colliding nuclei, i.e., $n_s = n_s^\pi + n_s^p$ then from known $p_T$ distributions for pions and protons we obtain the estimate

$$\langle\ln(m_T/P_T)\rangle \approx (0.2n_s^\pi + 1.6n_s^p)/n_s = 0.2 + 1.4n_s^p/n_s,$$

and consequently an estimate of $E_n$ is given by

$$E_n \approx 0.67\nu^*(m_p/2)\exp(2\langle\eta\rangle - 2.8n_s^p); \qquad (19)$$

$\nu^*$ for peripheral collisions is approximately 1 and for central collisions it is approximately $(A_t/A_p)^{1/3}$, where $A_t$ and $A_p$ are the atomic numbers of the two nuclei. $n_s^p$ can be estimated from the difference between the initial charges and the charges carried by heavy-track particles. Unfortunately at present formula (19) cannot be tested experimentally since no accelerator beams of high-energy ions are currently available.

## ACKNOWLEDGMENTS

One of us, A. Dar, gratefully acknowledges the financial support of Nordita, Copenhagen. The research has been sponsored by the Swedish Natural Science Research Council which is gratefully acknowledged.

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[1]See for instance C. Castagnoli *et al.*, Nuovo Cimento 10, 1239 (1953); G. Cocconi, Phys. Rev. 111, 1699 (1958); M. Schein *et al.*, *ibid.* 116, 1238 (1958); E. Lohrman *et al.*, *ibid.* 122, 672 (1961); F. Brisbout *et al.* (ICEF collaboration), Nuovo Cimento Suppl. 1, 1039 (1963); B. Hildebrand and R. Silberberg, *ibid.* 1, 1118 (1963); R. E. Gibbs *et al.*, Phys. Rev. D 10, 783 (1974) and references therein.

[2]See, for instance, *Multiple Production on Nuclei at Very High Energies*, edited by G. Bellini *et al.* (ICTP Press, Trieste, 1977) and references therein.

[3]Alma Ata–Gatchina–Moscow–Tashkent collaboration, Report No. HEPI 67-78, Alma Ata, 1978 (unpublished).

[4]G. Berlad *et al.*, Phys. Rev. D 13, 161 (1976); Y. Afek *et al.*, Ref. 2 pp. 591–669 and references therein.

[5]Shower particles are singly charged particles with $v/c > 0.7$. Heavy-track particles are charged particles with $v/c < 0.7$.

[6]A. Dar and J. Vary, Phys. Rev. D 6, 2412 (1972).

[7]Equations (6) and (7) are consistent with the measurements of Ref. 8 and of W. Busza *et al.*, Phys. Rev. 34, 836 (1975); C. Halliwell *et al.*, Phys. Rev. Lett. 39, 1499 (1977); J. E. Elias *et al.*, *ibid.* 41, 285 (1978).

[8]I. Otterlund *et al.*, Nucl. Phys. 142B, 445 (1978) and references therein.

[9]Y. Afek *et al.*, Nuovo Cimento 43, 485 (1978) and Ref. 2, pp. 591–669.

[10]We neglect here attenuation due to repeated collisions. For experimental tests of formulas (15) and (16) see for instance I. Otterlund and E. Stenlund, Lund University Report No. LUIP 7806, 1978 (unpublished).