Where is the η_c?

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Incorporating asymptotic freedom into the simple Coulomb plus linear effective quark-antiquark potential, we predict the masses of the $^1S_0$ partners of J/ψ and ψ', η_c and η_c', to be $m_{\eta_c} \approx 3.01$ GeV and $m_{\eta_c'} \approx 3.63$ GeV. Further implications of asymptotic freedom are considered, particularly the quark-mass dependence of the various level spacings.

In a previous paper[1] we investigated the charmonium and Υ spectra with the simple Coulomb plus linear effective potential. The parameters of the potential were determined by states other than the $^1S_0$ partners η_c and η_c' of ψ and ψ'. The main predictions were as follows:

(i) The hyperfine splittings are small and therefore η_c and η_c' are *not* to be identified with $X(2830)$[2,3] and $\chi(3450)$,[4] respectively.

(ii) The $M1$ transition rates are well below the available experimental upper limits.

Preliminary analysis of recent experiments at SLAC of a fraction of the full data sample find no substantial evidence for the states $X(2830)$ (Ref. 5) and $\chi(3450)$,[6] lending support to our predictions and at the same time raising the question: Where are the η_c and η_c'?

In this paper we impose the important constraints due to asymptotic freedom, not taken into account in Ref. 1, in order to provide the most reliable estimates of the masses of η_c and η_c' within the framework of effective potentials. Experimental confirmation of these estimates will give quantitative support for the whole approach. The effect of asymptotic freedom is most important at small r, where it softens the singularity at the origin. This in turn reduces the matrix element $\langle \nabla^2 V \rangle$ that directly controls the hyperfine splitting ($\vec{S}_1 \cdot \vec{S}_2$ term). The ψ-η_c mass splitting is therefore sensitive to asymptotic freedom.

The potential we find most suitable is a simple interpolation form, which gives the correct behavior suggested by asymptotic freedom and confinement in the small- and large-r limits.

We take

$$V(r) = -\left(\frac{4}{3}\right)\frac{\alpha(r)}{r} + (g_S + g_V)r + V_0\,, \qquad (1)$$

where

$$\alpha(r) = \left(\frac{4\pi}{9}\right)\frac{a}{\ln(b + 1/\Lambda^2 e^{2\gamma} r^2)}$$

and Λ has been set equal to 2.5 fm^{-1}. We have found the following values of the parameters for an overall best fit[7]:

$$a = 0.5\,,\quad g_S = g_V = 0.6 \text{ GeV/fm}\,,$$
$$b = 6.0\,,\quad V_0 = -1.0 \text{ GeV}\,,$$
$$M_c = 1.9 \text{ GeV}/c^2\,,\quad M_b = 5.24 \text{ GeV}/c^2\,.$$

The form for $\alpha(r)$ is suggested by a Fourier transform of the asymptotic-freedom result for $\alpha_s(q^2)$ which, for large momentum transfers, behaves as ($n_f = 3$)

$$\alpha_s(q^2) \to \left(\frac{12\pi}{27}\right)\frac{1}{\ln(-q^2/\Lambda^2)}\,,\quad -q^2 >> \Lambda^2\,. \qquad (2)$$

Hence $\alpha(r)$ has the following small-distance behavior[8]:

$$\alpha(r) \to \frac{4\pi}{9}\frac{1}{\ln(1/\Lambda^2 e^{2\gamma} r^2)}\,,\quad r << \Lambda^2 e^{2\gamma} \qquad (3)$$

where γ is Euler's constant.

The form of $\alpha(r)$ in Eq. (1) was obtained from Eq. (3) by moving the pole[9] from $1/(\Lambda^2 e^{2\gamma} r^2) = 1$ to $(1-b)$, corresponding to an unphysical value of r. The resulting effective potential, Eq. (1), has the correct behavior demanded by asymptotic freedom for small r and goes smoothly into a mixture of scalar and vector confining linear potentials. We do not use strictly the values of the parameters as given by quantum chromodynamics (QCD) because the calculation involves the entire range of r, not just small r; hence the parameters a and b, but we demand that a and b be of order unity.

We now apply the standard[10] relativistic corrections to $V(r)$ as given in Ref. 1. This gives the spin-independent corrections

$$V_{SI} = \frac{2}{3M_Q^2}\frac{1}{r^3}[-2\alpha(r) + 2r\alpha'(r) - r^2\alpha''(r)]\vec{r}\cdot\vec{\nabla} - \frac{\vec{p}^4}{4M_Q^3} + \frac{2}{3M_Q^2}\frac{1}{r}[2\alpha(r) - r\alpha'(r)]\vec{\nabla}^2 - \frac{1}{3M_Q^2}\frac{1}{r}\left(2\alpha''(r) + r\frac{\alpha'''(r)}{2}\right)$$
$$+ \frac{4}{3M_Q^2}\frac{1}{r^3}[\alpha(r) - r\alpha'(r)]l(l+1) + \frac{1}{M_Q^2}\frac{1}{r}(g_V - g_S)[l(l+1) + 2]$$

TABLE I. Spectrum of the charmonium system in GeV.

State	Spectroscopic notation	Central potential	+spin-independent corrections	+spin-dependent corrections	Experimental (Ref. 16)
η_c	$^1S_0$	3.145	3.076	3.013	
ψ	$^3S_1$	3.145	3.076	3.097	3.097 ± 0.002
χ	$^3P_0$	3.583	3.512	3.416	3.413 ± 0.005
χ	$^3P_1$	3.583	3.512	3.487	3.508 ± 0.004
χ	$^3P_2$	3.583	3.512	3.556	3.554 ± 0.005
χ	$^1P_1$	3.583	3.512	3.495	
η_c'	2^1S_0	3.801	3.673	3.634	
ψ'	2^3S_1	3.801	3.673	3.686	3.686 ± 0.003
	$^3D_1$	3.909	3.836	3.776	3.772 ± 0.006

and the spin-dependent corrections

$$V_{SD} = \frac{1}{9M_Q^2}\frac{1}{r^3}[3\alpha(r) - 3r\alpha'(r) + r^2\alpha''(r)]S_{12} + \frac{1}{12M_Q^2}\frac{1}{r}g_V S_{12} - \frac{8}{9M_Q^2}\frac{1}{r}\alpha''(r)\vec{S}_1\cdot\vec{S}_2 + \frac{4}{3M_Q^2}\frac{1}{r}g_V\vec{S}_1\cdot\vec{S}_2$$
$$+\frac{2}{M_Q^2}\frac{1}{r^3}[\alpha(r) - r\alpha'(r)]\vec{L}\cdot\vec{S} + \frac{1}{2M_Q^2}\frac{1}{r}(3g_V - g_S)\vec{L}\cdot\vec{S}\,.$$

The levels which were used to arrive at the parameters of the potential are $\psi(3095)$, $\psi'(3686)$, $\chi(3415)$, $\chi(3555)$, $\psi(3772)$, and $\Upsilon(9.46)$ to set the heavy-quark mass at $M_b = 5.24$. The results are displayed in Tables I and II.

We can make the following observations: The η_c is predicted to be within $\lesssim 90$ MeV of ψ and the η_c' within $\lesssim 50$ MeV of ψ'. These are the best estimates within the effective potential framework and can be taken as upper limits of the ψ-η_c and ψ'-η_c' splittings for the following reasons: The matrix element $\langle\nabla^2 V\rangle$ is considerably reduced by asymptotic freedom as compared with the value obtained with fixed α_s (it can also be reduced in fixed α_s

TABLE II. Spectrum of the Υ system in GeV.

State	Spectroscopic notation	Central potential	+spin-independent corrections	+spin-dependent corrections	Experimental (Ref. 17)
	$^1S_0$	9.488	9.459	9.438	
Υ	$^3S_1$	9.488	9.459	9.466	9.46 ± 0.01
	$^3P_0$	9.878	9.853	9.821	
	$^3P_1$	9.878	9.853	9.845	
	$^3P_2$	9.878	9.853	9.868	
	$^1P_1$	9.878	9.853	9.849	
	2^1S_0	10.041	9.998	9.988	
Υ'	2^3S_1	10.041	9.998	10.002	10.02 ± 0.02
	1^3D_1	10.146	10.124	10.106	
	2^3P_0	10.282	10.245	10.221	
	2^3P_1	10.282	10.245	10.238	
	2^3P_2	10.282	10.245	10.255	
	3^1S_0	10.419	10.367	10.359	
Υ''	3^3S_1	10.419	10.367	10.369	10.38 ± 0.04
	2^3D_1	10.487	10.452	10.437	
	3^3P_0	10.610	10.562	10.542	
	3^3P_1	10.610	10.562	10.556	
	3^3P_2	10.610	10.562	10.571	
	4^1S_0	10.733	10.671	10.664	
Υ'''	4^3S_1	10.733	10.671	10.673	

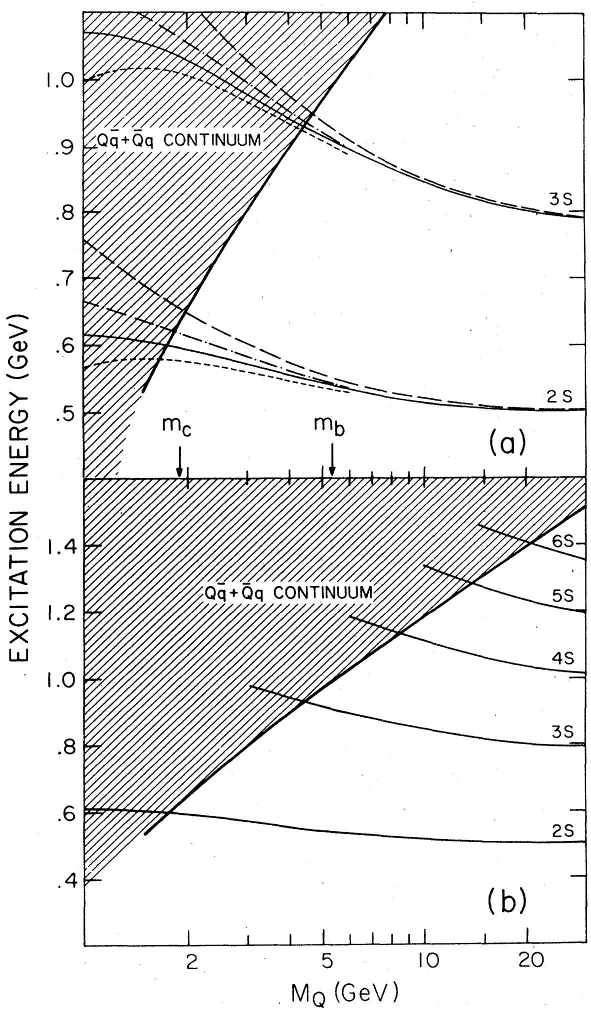

FIG. 1. (a) 2S and 3S excitation energies above the 1S level as a function of quark mass. The potential used is given by Eq. (1) with parameters $b=6.0$, $a=0.5$, $V_0=-1.0$, and $(g_S+g_V)=1.2$. The effects of spin-independent corrections are shown for different values of g_V: ----- $g_V=1.2$, ——— $g_V=0.6$, and — • — • $g_V=0$. The central potential (no relativistic corrections) result is depicted by ——— ——— ———. The effects of spin-dependent corrections are not included. The $Q\bar{q}+\bar{Q}q$ continuum is shown. (b) The nS energies above the 1S level as a function of quark mass. The potential used is given in Eq. (1) with parameters as given in text. The effects of spin-dependent corrections are not included. The $Q\bar{q}+\bar{Q}q$ continuum is shown.

treatments by application of the Van Royen–Weisskopf formula as was done in Ref. 1). The effects due to higher-order relativistic corrections and inelastic channels can only further reduce this value.[11] Perhaps more important is the fact that the confining potential, $\sim(g_S+g_V)r$, should not be taken to the origin. If the confining potential represents the energy due to the bubble (bag), separating the dense instanton phase from the dilute phase,[12] then $V(r)$ should be linear only at large r, when the bag is sufficiently elongated. At small r the confining part of the force is expected to vanish. Since a potential that is flatter at the origin would lead to a smaller matrix element $\langle\nabla^2 V\rangle$, and since the ψ-η_c splitting is proportional to $\langle\nabla^2 V\rangle$, this again can only lead to smaller hyperfine splittings.

The near quark-mass independence of level spacings is displayed in Fig. 1. In this formulation of the effective potential this equality comes about by three conspiring effects. First, the combination of Coulomb and linear potentials would give a flat region, in fact a minimum, in the excitation energies at quark masses M_Q, where states change from being dominated by the confining potential to being dominated by the Coulomb potential.[13] Second, the spin-independent relativistic corrections suppress the increase in the spacings, notably 2S-1S, as displayed in Fig. 1. These two effects result in the rough equality $\psi'-\psi\approx\Upsilon'-\Upsilon$. The third effect, asymptotic freedom, becomes increasingly important at large M_Q. The consequence of replacing the Coulomb constant α_s by the running coupling $\alpha(r)$ as suggested by asymptotic freedom is to suppress the increase in nS-1S spacings almost completely.[14] With the suggestion of m_t at roughly 15 GeV,[15] we find the 2S-1S splitting for that system to be about 25 MeV smaller than the $\Upsilon'-\Upsilon$ splitting. In Fig. 1, we display at what quark mass the various S levels fall below $Q\bar{q}+\bar{Q}q$ continuum. Thus we find the 3S level in Υ to be below threshold as well as the 5S level for $m_t=15$ GeV. The $2\,^3D_1$ level in Υ [not shown in Fig. 1(b)] should be about 40 MeV above threshold.

We find that the charmonium and Υ systems can be well described by an effective potential which incorporates the effects of asymptotic freedom. Asymptotic freedom is important in decreasing the ψ-η_c splitting by reducing the matrix element $\langle\nabla^2 V\rangle$. We predict reliable upper limits of 90 and 55 MeV for the ψ-η_c and ψ'-η_c' splittings, respectively. Splittings this small suggest that these states will best be found in hadronic production.

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