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Spin effects in large-transverse-momentum exclusive scattering processes

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Nucleon-nucleon scattering at large momentum transfer is analyzed within the framework of quantum-chromodynamic quark and gluon interactions. The spin dependence of the hadronic amplitude is found to be particularly sensitive to the underlying dynamical mechanisms. Detailed discussions of the quark-interchange and Landshoff pinch-singularity contributions are given for large-angle pp, np, and $p\bar{p}$ elastic scattering. A possible explanation is given for the large spin-spin correlations measured by Crabb *et al*. We also define a new SU(2) symmetry group, H spin, which generalizes conventional helicity, and is an exact symmetry of the quark-interchange process.

I. INTRODUCTION

One of the most interesting areas of possible application of quantum chromodynamics (QCD) and asymptotic freedom is the domain of exclusive processes at large momentum transfer, such as hadron scattering at large t and u, and elastic form factors at large t. Those applications are interesting because the form of the hadronic amplitude in this region depends in detail on the interactions of the quark and gluon constituents at short distances as well as the properties of the bound-state wave function which allow the final-state hadrons to be reformed at large momentum transfer.

Part of the motivation for this work has come from the striking spin correlation in pp scattering recently measured at Argonne by Crabb *et al.*[1] for polarized protons ($p_{1ab}=11.75$ GeV) scattering on a polarized target. A remarkable result is that at the largest momentum transfer ($p_T^2=5.09$ GeV^2, $\theta_{c.m.}=90°$), one finds that it is ~4 times more likely for protons to scatter when their spins are both parallel and normal to the scattering plane than when they are antiparallel:

$$\frac{d\sigma/dt(\uparrow\uparrow)}{d\sigma/dt(\uparrow\downarrow)}=3.9^{+1.5}_{-1.0}\,. \tag{1.1}$$

This result is particularly interesting since it occurs in the same momentum-transfer regime where the dimensional-counting scaling law for fixed-angle scattering appears to describe the data. For example, the recent measurements of Stone *et al.*[2] show that at 90°

$$\frac{d\sigma}{dt}(np\to np)\propto s^{-10.40\pm0.34}$$

and (1.2)

$$\frac{d\sigma}{dt}(pp\to pp)\propto s^{-9.81\pm0.05}$$

for $10<s<22.4$ GeV^2. This is good agreement with the counting rule[3]

$$\frac{d\sigma}{dt}(A+B\to C+D)=\frac{1}{s^{n-2}}f(t/s)\,, \tag{1.3}$$

where $n=n_A+n_B+n_C+n_D$ is the total number of initial and final constituent fields ($n=12$ here). An overall fit to all available pp data[4] for $|t|, |u|>2.3$ GeV^2 gives $s^{-9.7\pm0.5}f(\theta_{c.m.})$. Equation (1.3) is a consequence of scale invariance of the underlying constituent interactions, and is not inconsistent with the whole range of exclusive scattering measurements, including meson-baryon scattering, meson photoproduction, and elastic form factors. In QCD, the dimensional-counting rules hold asymptotically for Feynman diagrams involving the minimum number of off-shell quark and gluon exchanges. Logarithmic corrections can arise from the running coupling constant and higher-order gluon-exchange diagrams.[3] In addition, as first discussed by Landshoff,[5] there are potentially important "pinch-singularity diagrams" contributing to elastic hadron-hadron scattering which involve succession of nearly-on-shell quark-quark scattering amplitudes. The "Sudakov" form factors associated with these amplitudes in fact lead to an asymptotic damping[6] of such contributions although, as we shall discuss in Sec. III, they may be playing an important phenomenological role in the subasymptotic region.

The consistency of the data with the predicted

$s^{-10}f(\theta_{c.m.})$ behavior implies that a description of the spin dependence of pp scattering at large momentum transfer should be possible at the quark and gluon level. Moreover, asymptotic freedom implies that the basic subprocesses responsible for the large momentum transfer can be calculated in terms of perturbative diagrams.

As we shall show in this paper, it is difficult to find any *single* simple quark-gluon mechanism which can give a spin correlation larger than 2 for $d\sigma/dt(\uparrow\uparrow)/d\sigma/dt(\uparrow\downarrow)$. However, it is possible that the interference between two competing amplitudes may well describe the data. In addition to the fixed-$\theta_{c.m.}$ power-law behavior and spin dependence there are other phenomenological parameters of exclusive scattering that can discriminate between different dynamical mechanisms:

(a) the form of the angular dependence of $f(t/s)$ in (1.3),

(b) the flavor, isospin, and crossing dependence of the amplitude, as obtained from ratios such as $d\sigma/dt(np\to np)/d\sigma/dt(pp\to pp)$ and $d\sigma/dt(\bar{p}p\to\bar{p}p)/d\sigma/dt(pp\to pp)$.

The spin dependence of nucleon-nucleon scattering is particularly sensitive to the detailed form of the theory since it depends on the way that spin information is transferred from the nucleon to its constituents as well as the spin couplings at the quark and gluon level. In this paper we shall explore the implications of these phenomenological constraints of fixed-angle scattering for perturbative QCD.

The outline of this paper is as follows. In the next section we make some remarks concerning a sum rule for the polarization asymmetries in proton-proton scattering and the sensitivity of elastic scattering to spin-dependent effects. In Sec. III we review the general features of specific QCD mechanisms for exclusive scattering: gluon exchange, quark exchange (or interchange),[7] and the Landshoff triple-qq-scattering pinch contribution. Specific predictions for spin correlations are given and compared with the data, and predictions for np and $\bar{p}p$ scattering are also given. Section IV is devoted to the quark-exchange model and the idea of H spin. The latter is a generalization of helicity and is an exact symmetry group for the quark-interchange amplitude.

II. SPIN EFFECTS IN NUCLEON-NUCLEON SCATTERING

It is interesting to observe that, independent of dynamics, there are always significant spin correlations in the elastic scattering of two identical fermions at $\theta_{c.m.}=90°$. It is well known that time-reversal and parity invariance only allow five independent proton-proton spin amplitudes. For the 90° scattering of identical particles, all amplitudes involving a single helicity flip [e.g., $M(++,+-)$] vanish. As we show in the Appendix this implies a sum rule for the polarization asymmetries:

$$A_{nn}-A_{ll}-A_{ss}=1\,. \tag{2.1}$$

By definition

$$A_{ll}=\frac{(d\sigma/dt)(\uparrow\uparrow)+(d\sigma/dt)(\downarrow\downarrow)-(d\sigma/dt)(\uparrow\downarrow)-(d\sigma/dt)(\downarrow\uparrow)}{(d\sigma/dt)(\uparrow\uparrow)+(d\sigma/dt)(\downarrow\downarrow)+(d\sigma/dt)(\uparrow\downarrow)+(d\sigma/dt)(\downarrow\uparrow)}\,, \tag{2.2}$$

where $d\sigma/dt(\uparrow\uparrow)$ is the elastic cross section with initial spins both polarized along the beam (z) direction, $S_z^a=S_z^b=+\frac{1}{2}$. Similarly, the spin asymmetry A_{nn} refers to initial spins polarized along the normal ($\hat{n}=\hat{x}$) to the scattering plane, and A_{ss} refers to spins polarized (sideways) in the plane (parallel to $\hat{y}$). In each case, the final spins are summed over. (Notice that A_{ll} would have an overall minus sign if we had used c.m. helicities instead of S_z.)

The sum rule implies that for 90° pp scattering, we cannot have simultaneously $A_{nn}=A_{ll}=A_{ss}=0$; i.e., there must always be some spin asymmetry. In a model where the basic interactions are independent of the spin direction such as the constituent interchange model,[7] we have

$$A_{nn}=-A_{ll}=-A_{ss}=\tfrac{1}{3}\,, \tag{2.3}$$

i.e., $d\sigma/dt(\uparrow\uparrow)/d\sigma/dt(\uparrow\downarrow)=2$ for spins normal to the plane. Thus particle identity induces a significant spin asymmetry.

In a perturbative QCD model one generally expects that the double helicity-flip amplitude $M(++,--)$ is negligible at high energies. If we assume this is the case then

$$A_{nn}=-A_{ss}\quad\text{(all angles)}\,, \tag{2.4}$$

and the sum rule becomes

$$2A_{nn}-A_{ll}=1\quad(\theta_{c.m.}=90°)\,. \tag{2.5}$$

It is thus very important that measurements of A_{nn} and A_{ll} both be made at 90° at the same energy; any deviation from $2A_{nn}-A_{ll}-1=0$ would imply a significant contribution from the double-helicity-flip amplitude and would tend to rule out a simple perturbative explanation of the data. For reference, in the case of 90° electron-electron scattering [QED in Born approximation ($m_e^2/s\to 0$)], one has $A_{nn}=-A_{ss}=\frac{1}{9}$ and $A_{ll}=-\frac{7}{9}$.

Because of coherence and particle identity, large-angle pp scattering is a sensitive test of spin effects. In contrast, in typical inclusive reactions, any spin correlation which is important at the quark-gluon level quickly becomes diluted when the hadronic wave function is taken into account; the net polarization of quark in a nucleon with a valence wave function $|n\rangle = |q\uparrow q\uparrow q\downarrow\rangle$ is $\frac{1}{3}$.

For example, consider the simplest QCD model where the nucleon-nucleon inclusive cross section for the production of large-p_t jets can be computed from an incoherent sum of quark-quark cross sections. The spin asymmetry of the n-n cross section is then given by

$$A(N_1, N_2) = \int_0^1 dx_1 \int_0^1 dx_2 \sum_{\alpha,\beta} \Delta G_{q_\alpha/N_1}(x_1) \times \Delta G_{q_\beta/N_2}(x_2) A(q_\alpha, q_\beta)\,, \tag{2.6}$$

where $A(q_\alpha, q_\beta)$ is the quark-quark spin asymmetry at $\hat{s} = x_1 x_2 s$, $\hat{t} = x_1 t$, $\hat{u} = x_2 u$, and

$$\Delta G_{q_\alpha/N}(x) = \frac{dN}{dx}(q_\alpha\uparrow/N\uparrow) - \frac{dN}{dx}(q_\alpha\downarrow/N\uparrow) \tag{2.7}$$

gives the net number of quarks of flavor α and light-cone fraction x aligned with the nucleon spin. In the case of the proton, the quark valence wave function is completely determined by isospin and color symmetry, and one has for a spin-up proton

$$N_{u\uparrow} = \tfrac{5}{3}\,, \quad N_{u\downarrow} = \tfrac{1}{3}\,, \quad N_{d\uparrow} = \tfrac{1}{3}\,, \quad N_{d\downarrow} = \tfrac{2}{3}\,. \tag{2.8}$$

Thus, averaging over x_1 and x_2,

$$\overline{A}(p, p) = A_{q_\alpha = q_\beta}(\tfrac{17}{81}) + A_{q_\alpha \neq q_\beta}(-\tfrac{8}{81})\,, \tag{2.9}$$

where we have distinguished the cases of equal- and unequal-flavor quark-quark scattering. For gluon exchange in QCD the longitudinal spin asymmetry (averaged over color) is maximal at 90°

$$\begin{aligned} A^{ll}_{q_\alpha = q_\beta} &= -\tfrac{5}{11} \quad (\theta_{\text{c.m.}} = 90°)\,, \\ A^{ll}_{q_\alpha \neq q_\beta} &= -\tfrac{3}{5} \quad (\theta_{\text{c.m.}} = 90°)\,, \end{aligned} \tag{2.10}$$

i.e.,

$$|A_{ll}(pp)| \leq 0.036\,. \tag{2.11}$$

The above estimate is clearly quite crude, and can be circumvented if the distributions for spin-up and spin-down quarks do not have the same x dependence. For example, it has been suggested that a quark with $x \sim 1$ will have the same spin as the parent nucleon.[8] Calculations have been done with selected quark distributions which do give spin asymmetries in inelastic scattering several times larger than those estimated above.[9]

III. LARGE-ANGLE SCATTERING MECHANISMS AND SPIN

The spin-dependence observed by Crabb *et al.*[1] is so striking that a new look at the possible mechanisms of large-angle scattering is certainly required. In this chapter we shall review some basic mechanisms and discuss their consequences for spin correlations.

A. Quark-quark scattering

At first sight, the most obvious mechanism which can transfer large amounts of momentum between colliding hadrons is the $qq \to qq$ scattering in impulse approximation. That is, one quark from one hadron scatters from another quark in another hadron, after which the quarks must share the transferred momentum with the other quarks in their respective hadrons if the hadrons are not to break up. It is easy to see that

$$\frac{d\sigma}{dt}(pp \to pp) = C \frac{d\sigma}{dt}(qq \to qq) F_p^4(t) + (u\text{-channel exchange})\,, \tag{3.1}$$

where $F_p(t)$ is the form factor and $C \leq 81$ is a factor which counts the number of coherent diagrams. For the experiment of Crabb *et al.*,[1] $p_{\text{lab}} = 11.75$ GeV, $\theta_{\text{c.m.}} = 90°$ ($s = 23.8$ GeV^2, $t = -10.1$ GeV^2),

$$\frac{d\sigma^{\text{expt}}}{dt}(pp \to pp) \cong 1 \times 10^{-5} \ \frac{\text{mb}}{\text{GeV}^2}\,;$$

thus we need

$$\frac{d\sigma}{dt}(qq \to qq) \geq 100 \text{ mb/GeV}^2\,. \tag{3.2}$$

This is many orders of magnitude too large to be understood in QCD; e.g., single-gluon exchange (t channel) gives

$$\frac{d\sigma}{dt}(qq \to qq) = \frac{2}{9}\,\alpha_s^2(t)\,\frac{4\pi}{t^2} \simeq 10^{-3} \text{ mb/GeV}^2\,. \tag{3.3}$$

In fact, this is an overestimate since single-gluon exchange between singlets vanishes. Even if this estimate could be circumvented, the angular distribution predicted by (3.1) is incompatible with the data; in particular, vector gluon exchange implies that the Regge behavior will stay close to $\alpha_{\text{eff}}(t) \sim 1$ for all t, whereas the data indicate that $\alpha_{\text{eff}}(t) \lesssim -1$ at large t.[10]

Although it is possible that the $qq \to qq$ hard-scattering subprocess could be important in high-p_T jet production experiments, it is unlikely that it plays any significant role in elastic scattering.

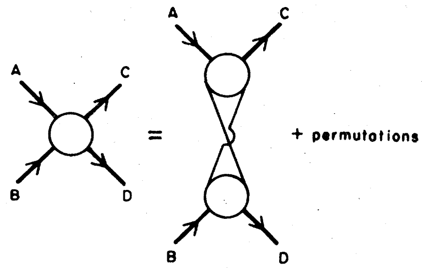

FIG. 1. Generic diagram for the constituent-interchange model.

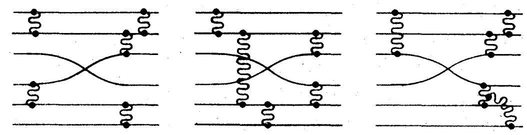

FIG. 2. Some examples of graphs that can contribute to quark interchange. Each graph is the lowest order that will allow quark interchange and equal sharing of momentum among the quarks in each nucleon. Many more such graphs could be drawn.

B. Quark interchange

In addition to quark scattering via gluon-exchange processes, QCD also predicts that hadrons can scatter at large momentum transfer by quark exchange or interchange.[7] This is the leading QCD contribution to large-angle Compton scattering. Although single-gluon exchange is forbidden in lowest order, quark interchange is not, and one can easily see that quark interchange always occurs in leading order. Witten[11] has shown that for $N_{\text{color}} \to \infty$ with $N\alpha_s$ fixed, quark interchange is rigorously the leading dynamical mechanism in baryon-baryon scattering.

The cross section of the singlet quark-interchange amplitude has the characteristic form[7]

$$\frac{d\sigma}{dt}(AB \to CD) = \frac{d\sigma}{dt}(Aq \to Cq)\, F_{BD}{}^2(t) + (\text{permutations}) . \tag{3.4}$$

where $(d\sigma/dt)(Aq \to Cq)$ is the amplitude for q-hadron scattering at the reduced kinematics (see Fig. 1). For $pp \to pp$ scattering we take

$$\frac{d\sigma}{dt}(pq \to pq) \propto \frac{1}{s^2} F_p{}^2(u) , \tag{3.5}$$

which has the correct power dependence predicted by QCD, and corresponds to $j=0$ behavior in the (diquark) u channel. Thus

$$\frac{d\sigma}{dt}(pp \to pp) = C\, \frac{F_p{}^2(t) F_p{}^2(u)}{s^2} , \tag{3.6}$$

where C counts the number of coherent diagrams, spin states etc. Although the amplitude is reduced by $\frac{1}{3}$ for color, there are an enormous number of distinct coherent QCD diagrams which contribute to the quark-interchange amplitude (see Fig. 2), where we include diagrams where gluons are exchanged between hadrons, as well as different flavor exchanges. Thus it is conceivable that the quark-interchange diagrams are sufficiently large as to account for the observed cross sections, although a reliable calculation of the normalization has not been given. Equation (3.6) gives at large s and fixed $\theta_{\text{c.m.}}$,

$$\frac{d\sigma}{dt} = \frac{1}{s^{10}} f(\theta_{\text{c.m.}}) , \quad f(\theta_{\text{c.m.}}) \sim \left(\frac{1}{1-\cos^2\theta}\right)^4 . \tag{3.7}$$

The best power-law fit to the $pp \to pp$ data[4] gives $s^{-9.7\pm0.5}$ over a large range of angles and energies and $s^{-10.4}$ for $np \to np$.[2] The angular dependence predicted by (3.7) is compatible with the data for $|t|, |u| \gtrsim 4$ GeV^2. Equation (3.6) predicts asymptotic Regge behavior $M_{pp\to pp} \sim u^{\alpha(t)}\beta(t)$, with $\alpha(t) \to -2$ at $t \to -\infty$, and $\beta(t) \to F(t) \sim c/t^2$, which is compatible[10] with the data. Thus the quark-interchange amplitude appears to be the dominant QCD mechanism, and is roughly compatible with the features of large-angle pp data.

As shown in Sec. IV, the amplitudes for quark interchange can be readily calculated in terms of their spin and isospin properties (see Table I). We have at 90°

$$\frac{(d\sigma/dt)(np \to np)}{(d\sigma/dt)(pp \to pp)} = \frac{1^2 + (\frac{47}{31})^2 + (\frac{16}{31})^2}{2^2 + 1^2 + 1^2} = 0.594 \tag{3.8}$$

and

$$A_{nn}(pp \to pp) = \tfrac{1}{3} , \qquad A_{nn}(np \to np) = -0.439 . \tag{3.9}$$

The first result can be compared with the measurement of Stone *et al.*[2] at 90°,

$$\frac{(d\sigma/dt)(np \to np)}{(d\sigma/dt)(pp \to pp)} = 0.34 \pm 0.05 \tag{3.10}$$

for $10 < s < 25$ GeV^2. At the highest energy measured, the ratio is 0.50 ± 0.22, so the prediction 0.594 is not ruled out as a high-energy limit. The $\bar{p}p/pp$ cross-section ratio at large angles is also consistent with crossing the interchange ampli-

TABLE I. Helicity amplitudes $M(\lambda_c\lambda_d, \lambda_a\lambda_b)$ for nucleon-nucleon scattering in the constituent-interchange model.

A. $pp \to pp$

$M(++,++)=\frac{1}{9}[31f(s,t)+31f(s,u)]$

$M(+-,+-)=\frac{1}{9}[14f(s,t)+17f(s,u)]$

$M(-+,+-)=\frac{1}{9}[17f(s,t)+14f(s,u)]$

B. $np \to np$

$M(++,++)=\frac{1}{9}[14f(s,t)+17f(s,u)]$

$M(+-,+-)=\frac{1}{9}[22f(s,t)+25f(s,u)]$

$M(-+,+-)=\frac{1}{9}[-8f(s,t)-8f(s,u)]$

C. $\bar{p}p \to \bar{p}p$

$M(+-,+-)=\frac{1}{9}[31f(u,t)+31f(u,s)]$

$M(++,++)=\frac{1}{9}[17f(u,t)+14f(u,s)]$

$M(-+,+-)=\frac{1}{9}[14f(u,t)+17f(u,s)]$

D. Predictions for asymmetries at 90° ($u=t\cong -\frac{1}{2}s$)

$A_{nn}(pp)=-A_{ll}(pp)=-A_{ss}(pp)=\frac{1}{3}$

$A_{nn}(np)=-A_{ll}(np)=-A_{ss}(np)=-0.439$

tude.[7,10]

The biggest failure of the interchange mechanism is in the spin correlation. For all angles we predict from Table I

$$A_{nn}=\frac{1}{3}\,\frac{1-(\frac{3}{31})^2\chi^2}{1+\frac{1}{3}(\frac{3}{31})^2\chi^2}\,, \qquad (3.11)$$

where

$$\chi=\frac{f(\theta)-f(\pi-\theta)}{f(\theta)+f(\pi-\theta)}\,.$$

Thus A_{nn} is predicted to be within 2% of $\frac{1}{3}$ even when $\chi=1$ [$\chi=0$ for the form in Eq. (3.6)]. The data clearly indicate that A_{nn} is not a constant near $\frac{1}{3}$.

Our expectation, then, is that there is an additional amplitude which strongly interferes with the quark-interchange contributions at Argonne energies; most plausibly, the quark-interchange contribution is dominant at asymptotic t and u, and the interfering amplitude is most important at low t and u. As we shall discuss below, the behavior of A_{ll} and A_{ss} in the interference region can play an important role in sorting out the possible subasymptotic contributions.

These results for the quark-interchange model have also been obtained by Farrar, Gottlieb, Sivers, and Thomas,[12] who also consider the possibility that nonperturbative effects (quark-quark scattering via instantons) can explain the data.

C. The Landshoff contribution: Triple-gluon exchange

As Landshoff has discussed,[5] there are potentially large contributions to nucleon-nucleon scattering which can arise from three successive nearly on-shell quark-quark scatterings, each through the angle $\hat{\theta}=\theta_{\rm c.m.}$. The Landshoff amplitude has the form

$$M_{pp}=\left(\frac{i}{(stu\mu^2)^{1/2}}\right)^2 M_{qq}{}^3\,, \qquad (3.12)$$

where μ^2 is a hadronic scale size, and the factor i arises from integration in the Glauber-type nearly real intermediate states. A very complete calculation of the Landshoff amplitude for triple-gluon exchange in QCD has been given by Farrar and Wu.[5] (See also Ref. 6)

The most crucial prediction of the three-gluon-exchange mechanism is the angular distribution. Because of the vector exchange, the effective Regge behavior is again fixed at $\alpha(t)\cong 1$, in contradiction to the large-angle data. It may be, following Donnachie and Landshoff,[5] that these contributions play an important role for very large s, where the CERN ISR cross section is reasonably consistent with the predicted form $d\sigma/dt \to c/t^8$ ($s \gg |t|$). If we fit the Landshoff cross section to the final ISR cross section ($s>800$ GeV^2, $4<|t|<12$ GeV^2), then this vector-exchange contribution extrapolates to a cross section at least 10^3 times smaller than the $s=40$ GeV^2 cross section near 90°, in addition to having an incompatible angular distribution. We also note that there is an additional suppression of the $qq\to qq$ near-on-shell amplitude due to gluon corrections to the $q\bar{q}g$ vertices not included in the Born approximation estimate.[6] Asymptotically these "quark-form-factor" corrections yield an asymptotic power behavior in QCD which fails faster at fixed angle than the quark-interchange contribution. For completeness we give the prediction of the Landshoff three-gluon-exchange contribution in Table II for the $np\to np$ and $pp\to pp$ spin amplitudes. As an illustration of the types of interference patterns possible, if the Landshoff amplitude is $L(\theta_{\rm c.m.})$ and the quark interchange amplitude is $Q(\theta_{\rm c.m.})$, then at 90° the $pp\to pp$ spin correlation is

$$A_{nn}=\frac{2(Q-\frac{3}{8}L)^2}{(2Q-L)^2+2(Q-\frac{3}{8}L)^2}\,. \qquad (3.13)$$

Thus if $L\to 2Q$, one can obtain a maximal spin correlation $A_{nn}\to 1$, $r_{nn}\equiv d\sigma/dt(\uparrow\uparrow)/d\sigma/dt(\uparrow\downarrow)\to\infty$. The Landshoff contribution alone gives $A_{nn}(90°)=0.22$, $r_{nn}=1.56$. We also note that if we choose $L/Q=1.491$ to give $A_{nn}(pp)=0.6$ to agree with the Argonne data at $p_{\rm lab}=11.75$ GeV/c, then one pre-

TABLE II. Helicity amplitudes resulting from applying the Landshoff mechanism to triple gluon exchange and triple $J^P=0^+$, $I=0$ meson exchange. Results for gluon exchange at all angles are in Farrar and Wu, Ref. 5. $P(z)$ is defined in the text.

A. Triple gluon exchange at 90°

(a) $pp \to pp$

$M(++,++)=L$

$M(+-,+-)=\frac{3}{8}L$

$M(-+,+-)=\frac{3}{8}L$

(b) $np \to np$

$M(++,++)=\frac{1}{2}L$

$M(+-,+-)=\frac{3}{8}L$

$M(-+,+-)=0$

(c) Predictions for asymmetries at 90°

$A_{nn}(pp)=-A_{ss}(pp)=0.22 \quad A_{ll}(pp)=-0.56$

$A_{nn}(np)=-A_{ss}(np)=0 \quad A_{ll}(np)=-0.36$

B. Triple σ exchange (any angle)

(a) $pp \to pp$

$M(--,++)=P(z)+P(-z)$

$M(-+,+-)=P(z)$

$M(+-,+-)=P(-z)$

(b) $np \to np$

$M(--,++)=P(z)$

$M(-+,+-)=P(z)$

dicts

$$\frac{(d\sigma/dt)(np\to np)}{(d\sigma/dt)(pp\to pp)}=1.925\,, \tag{3.14}$$

which is incompatible with the Stone *et al.*[2] result. Given the above difficulties, especially the problem with the angular shape, we conclude that the three-gluon-exchange Landshoff contribution is not playing an important role in the fixed-angle scattering data.

D. Meson-exchange contributions

Even though the coupling constant of a pion to a nucleon is large ($g^2/4\pi \cong 14$), the contribution of single-pion exchange to large-angle scattering is small in the fixed-angle region:

$$\frac{d\sigma}{dt} \simeq \frac{\pi}{s^2}\left(\frac{g_{\pi NN}{}^2}{4\pi}\right)^2 F_p{}^4(t)\left(\frac{t}{t-m_\pi{}^2}\right)^2$$

$$\simeq 10^{-10}\ \text{mb/GeV}^2\,, \tag{3.15}$$

compared to the data $\simeq 10^{-5}$ mb/GeV^2, at $s\simeq 24$ GeV^2, $t \simeq -10.4$ GeV^2.

However, the Landshoff three-gluon-exchange mechanism suggests another possibility. In the Landshoff amplitude, each $qq\to qq$ amplitude transfers $\frac{1}{3}$ of the exchanged momentum, i.e., $\hat{t}\cong 1/9t$, $\hat{s}\cong 1/9s$. Even for $|t|\sim 10$ GeV^2, $\hat{t}$ is still reasonably small, and only relatively low-energy $qq\to qq$ kinematics are involved. Thus, rather than perturbative QCD, we should consider a more conventional description of the $qq\to qq$ amplitude as far as the Landshoff contributions are concerned.

It is clear from dispersion theory, that the $qq\to qq$ amplitude receives contribution from t-channel meson exchange, π, σ, ρ, ω, A, etc.; in addition to more complicated cut contributions. The complete analysis of the Landshoff diagram, which requires three $M_{qq\to qq}$ amplitudes, is thus very complex. We know from our previous analysis that the contribution of three elementary vector exchanges gives $\alpha_{\text{eff}}(t)\cong 1$, and an angular dependence which is difficult to reconcile with the observed large-angle data.

We thus turn our attention to the scalar- and pseudoscalar-meson-exchange contribution [or alternatively, Reggeon exchange with $\alpha_{\text{eff}}(t)\sim 0$]. The coupling of a π or a σ to a quark can be normalized if we assume impulse approximation

$$g_{\pi qq}\left\langle N\downarrow\right|\sum \bar{q}\gamma_5\vec{\tau}q\left|N\uparrow\right\rangle = g_{\pi NN}\bar{u}_N\gamma_5\vec{\tau}u_N\,. \tag{3.16}$$

We then find

$$\frac{g_{\pi uu}{}^2}{4\pi}=\left(\frac{3}{5}\right)^2\frac{g_{\pi NN}{}^2}{4\pi}\cong 5\,, \tag{3.17}$$

i.e., a reasonably large coupling constant. The same remarks can be made for the σ. We have not attempted to normalize absolutely the Landshoff contribution from these contributions, but because of the small momentum transfer involved, it is possible that multiple-meson-exchange contributions can make a significant contribution to the large-angle amplitude, at least for moderate values of t.

The most striking characteristic of the multiple π- or σ-exchange contribution is the presence of the spin-flip amplitude $M(--,++)$. This implies $A_{nn}\neq -A_{ss}$ and the breakdown of the 90° identity $A_{ll}=2A_{nn}-1$. Thus we re-emphasize the importance of measurements of A_{ll} and A_{nn} at the same large-angle $pp\to pp$ kinematics.

At intermediate ranges, σ exchange will dominate π exchange because of its larger coupling. The contribution to quark-quark scattering from σ exchange has the form

$$M_{qq}(--,++)\propto \frac{\hat{t}}{\hat{t}-m_\sigma{}^2}\,g^2(\hat{t}) \tag{3.18}$$

for massless quarks, where $g^2(\hat{t})$ represents the corrections to the vertices. We shall assume a monopole form, $g^2(\hat{t})=g_0^2/(1-\hat{t}/M^2)$ with $M^2\simeq 0.47$ GeV^2,[13] or $9M^2\simeq 4$ GeV as a typical illustration. We are using the σ to approximate the forces in the scalar-isoscalar channel and experience here[14] seems to indicate that a low mass, $m_\sigma\simeq 400$ MeV, is best. This is roughly the same size as the constituent quark mass, or the kinetic energy of the quark within its confined state, and we will be consistent if we neglect it.

Then, using Landshoff formula (3.12), the t-channel contribution is

$$M_{pp}=\frac{k}{stu}(1-t/9M^2)^{-3}\equiv P(z)\,, \tag{3.19}$$

where k is a constant and $z=\cos\theta_{\text{c.m.}}$. Note that the intermediate states include Δ and N^* excitations. It is straightforward to compute all of the helicity amplitudes for the three-pion exchange. If we consider this contribution, together with the quark-interchange amplitudes, then for $pp\to pp$,

$$\begin{aligned} M(++,++)&=Q(z)+Q(-z)\,,\\ M(--,++)&=P(z)+P(-z)\,,\\ M(+-,+-)&=\tfrac{14}{31}Q(z)+\tfrac{17}{31}Q(-z)+P(-z)\,,\\ M(-+,+-)&=\tfrac{17}{31}Q(z)+\tfrac{14}{31}Q(-z)+P(z)\,, \end{aligned} \tag{3.20}$$

where $P(z)$ is given by (3.19) and we shall take $Q(z)=Q(-z)\propto F(t)F(u)=(1-t/0.71\text{ GeV}^2)^{-2}(1-u/0.71\text{ GeV}^2)^{-2}$ for the quark-interchange contribution. We then have

$$A_{nn}(pp\to pp)=\frac{4Q(P+\tilde{P})+2(Q+\tilde{P})(Q+P)}{4Q^2+(P+\tilde{P})^2+(Q+P)^2+(Q+\tilde{P})^2}\,, \tag{3.21a}$$

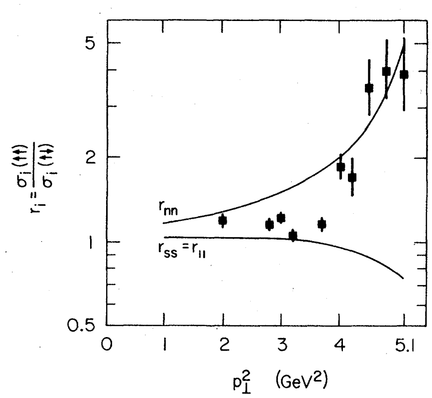

FIG. 3. Spin ratios for the illustrative model, which has triple-σ exchange amplitudes interfering with quark interchange amplitudes. The data are r_{nn} from Ref. 1. For this model, $A_{ll}=A_{ss}$.

and

$$A_{ll}(pp\to pp)=A_{ss}(pp\to pp)=\frac{-2(Q-P)(Q-\tilde{P})}{4Q^2+(P+P)+(Q+P)^2+(Q+P)^2}\,, \tag{3.21b}$$

where $\tilde{P}=P(-z)$. The triple-σ-exchange or quark-interchange contributions alone each give (at 90°) $A_{nn}=\frac{1}{3}$ and $A_{ll}=-\frac{1}{3}$, but together they can interfere to produce a striking polarization correlation (see Fig. 3). The relative magnitude of $P/Q=4.11$ at $s=23.9$ GeV^2, $\theta_{\text{c.m.}}=90°$ was chosen to give $A_{nn}=0.69$, $r_{nn}=5.45$. The corresponding prediction for A_{ll} is also given in Fig. 3. At the above kinematics, $A_{ll}=-0.56$. The particular model that we have here gives $A_{ll}=A_{ss}$ at all angles.

We can also predict the parameters of the $np\to np$ cross section, although this aspect of the model is less reliable since the contributions of π in addition to σ exchange will lead to a complicated isospin structure. However, the triple σ exchange does give $\sigma(np)/\sigma(pp)=\frac{1}{3}$, in agreement with Stone *et al.*[2]

The above model is, of course, oversimplified and is given for illustrative purposes. One easy change to make is to modify the energy dependence of the σ-exchange amplitudes by treating the exchanged particles as Regge poles. This would introduce a factor $\{1+\exp[i\pi\alpha(\hat{t})]\}$ into each of the three exchanges. The energy dependence is then quite different. For example, if we have a trajectory which passes through zero near the value of $\hat{t}$ that is correct for 90° scattering at $p_{\text{lab}}=11.75$ GeV/c, and if the trajectory has unit slope, then $\alpha(\hat{t})$ changes by 0.36 unit when we drop to $p_{\text{lab}}=8$ GeV/c ($s\simeq 17$ GeV^2), and the real part of the $\{1+\exp[i\pi\alpha(t)]\}^3$ factor has only 40% of its maximum value, thus reducing the interference with the real quark-interchange amplitude.

Although the above calculation of quark interchange plus triple σ exchange is too simple, it possesses the general features which seem to be required to understand the data. The dominant feature of the model is the presence of two interfering amplitudes which are roughly equal at $s\simeq 20$ to 30 GeV^2, with different energy dependences and roughly similar broad angular dependency at large scales. The likely candidate for the amplitude which is dominant at high energies is the quark-interchange contribution. There is much more uncertainty about the dominant low-energy contribution, but triple-meson exchange seems to be a reasonable possibility. A more ambitious calculation will require consideration of a general combination of spin exchanges, phases, and absolute normalization, but the presence of scalar or

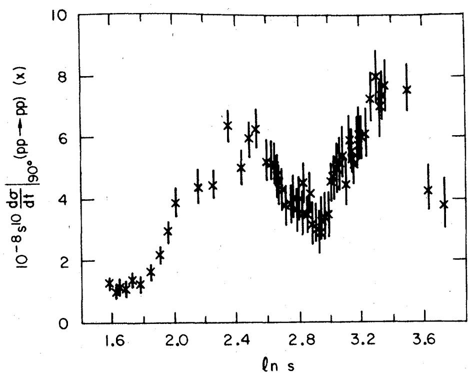

FIG. 4. Proton-proton scattering cross section at 90° multiplied by s^{10}. From Sivers *et al.* (Ref. 7) and Ref. 15. The units are mb GeV^8.

pseudoscalar exchanges predicts a large $M(--,++)$ amplitude; comparisons of the A_{ll} and A_{nn} spin correlations will clearly be central for unraveling this question.

We should remark that while the spin-averaged pp elastic cross section does show the s^{-10} behavior predicted by the constituent-interchange model there are oscillations about this behavior.[15] A plot of $s^{10}\, d\sigma/dt(90°)$ versus s shows maxima a factor of 2–3 above the minima is shown in Fig. 4. One minimum is at $s \simeq 19$ GeV^2 and the following maximum is at $s \simeq 26$ GeV^2. The experiment[1] which prompted this investigation is at $s = 23.8$ GeV^2. We might guess that the peak of the oscillation is connected to the same interfering process which gives the large asymmetry. Then the asymmetry will rise further with a small increase in energy; the results of asymmetry measurements at $p_{1ab} = 12.75$ GeV/c ($s = 25.7$ GeV^2) will be quite interesting.

IV. THE CONSTITUENT-INTERCHANGE MODEL AND *H* SPIN

We have concluded that quark exchange, or quark interchange, is the dominant process for nucleon-nucleon elastic scattering at high energies and large angles. In this section, we will draw out the predictions of this model for the scattering of polarized nucleons. Figure 5 gives a picture of the process, with the momenta and helicities labeled.

There are two ways that we shall obtain our results. The first is by straightforward counting of the ways and probabilities of exchanging quarks of a given flavor and helicity. The second method is more elegant and can be used to derive additional results, and relies upon a symmetry of quark exchange which is an SU(2) based on helicity rather than ordinary spin. It is called *H* spin and is defined in detail below.

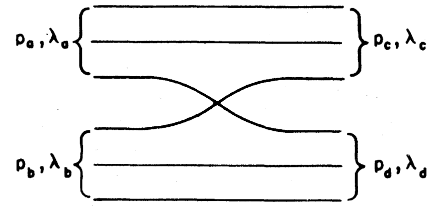

FIG. 5. Generic quark-interchange graph with momenta and helicities labeled.

A. Quark counting

At the outset, it is useful to remember that the nucleon consists of three valence quarks, whose wave functions are completely symmetric in space, and antisymmetric in color. The spin-flavor (or spin-isospin) part of the wave function is symmetric and for a proton is given by[16]

$$\begin{aligned}\sqrt{18}\,p_\uparrow = {} & 2u^r_\uparrow u^y_\uparrow d^b_\downarrow + 2u^r_\uparrow d^y_\downarrow u^b_\uparrow + 2d^r_\downarrow u^y_\uparrow u^b_\uparrow \\ & -u^r_\uparrow u^y_\downarrow d^b_\uparrow - u^r_\uparrow d^y_\uparrow u^b_\downarrow - d^r_\uparrow u^y_\uparrow u^b_\downarrow \\ & -u^r_\downarrow u^y_\uparrow d^b_\uparrow - u^r_\downarrow d^y_\uparrow u^b_\uparrow - d^r_\uparrow u^y_\downarrow u^b_\uparrow\,. \end{aligned} \tag{4.1}$$

The arrows refer to spin up or down along a stated direction. We can get the neutron wave function by changing $d \to u$ and $u \to -d$, and we can get the opposite spin or helicity states by appropriately changing the spins or helicities of each quark. The elastic scattering amplitudes will of course contain factors due to color and to recombining the quarks into the proper spatial wave function. But since these factors do not depend on either the flavor or helicity of the nucleons we shall ignore them. We shall also ignore exchanges of nonvalence quarks: Their contribution to the scattering amplitude falls by a higher power[3] of s than the contribution due to valence quarks, and will be small at high energy.

Let us begin by scattering two positive helicity protons upon each other. Since the elementary process in QCD is gluon exchange, none of the quarks can flip helicity, and the final protons must both have positive helicity. Note also that the two exchanged quarks must have identical flavor and helicity. Then by considering the overlap of the exchanged quark state with the final protons, one can see that the amplitude is

$$\langle ++ \,|\, T \,|\, ++\rangle = (N_{u^+}{}^2 + N_{u^-}{}^2 + N_{d^+}{}^2 + N_{d^-}{}^2) f(s,t)\,, \tag{4.2}$$

where $f(s,t)$ comes from the color and space part of the wave functions, and N_{u^+} is the number of up quarks with positive helicity in a positive-helicity proton etc., and

$$N_{u+}=\tfrac{5}{3}, \quad N_{d+}=\tfrac{1}{3}, \qquad N_{u-}=\tfrac{1}{3}, \quad N_{d-}=\tfrac{2}{3}. \tag{4.3}$$

Thus,

$$\langle ++|T|++\rangle=\tfrac{1}{9}\times 31\times f(s,t). \tag{4.4}$$

For proton-proton scattering, one must add the amplitude obtained when (p_c, λ_c) and (p_d, λ_d) are interchanged in Fig. 5, and after so doing we have given the result in Table I.

Let us also consider explicitly the case where one proton has positive and one proton has negative helicity; $\lambda_a, \lambda_b=+,-$. When the upper and lower protons in Fig. 5 maintain their helicity ($\lambda_c, \lambda_d=+,-$), then the interchanged quarks must again have identical flavor and helicity, and one can obtain

$$\langle +-|T|+-\rangle=(2N_{u+}N_{u-}+2N_{d+}N_{d-})f(s,t) = \tfrac{1}{9}\times 14\times f(s,t). \tag{4.5}$$

On the other hand, if the upper and lower protons each flip helicity ($\lambda_c\lambda_d=-+$), then the interchanged quarks still must have identical flavor, and the quark that flows from proton a to proton d must have positive helicity, while the other exchanged quark must have negative helicity. One gets, explicitly,

$$\langle -+|T|+-\rangle=\tfrac{1}{9}\times 17\times f(s,t). \tag{4.6}$$

After including the interchange $c\leftrightarrow d$ for identical protons, we obtain the results in Table I. Similar results were obtained independently in Ref. 12.

Since the quarks cannot flip their helicity, the remaining two independent amplitudes, $M(-+,++)$ and $M(--,++)$ are both zero.

Having gained experience with the protons, the case of np elastic scattering is straightforward, with the results given in Table I.

B. *H* spin

In certain scattering problems it is convenient to generalize the concept of helicity to include a full SU(2) group with helicity-flip operators in addition to the conventional helicity. We call this SU(2) algebra H spin, and we define it as follows: The z component or longitudinal component of H spin for a particle is its helicity.

$$H_z=H_l=\tfrac{1}{2}\vec{\sigma}\cdot\hat{p}. \tag{4.7a}$$

To define the helicity-flip operators unambiguously there must be a preferred direction normal to the momenta of the particles. In two-body scattering problems this direction is just the normal to the scattering plane, denoted by the unit vector $\hat{n}$. Thus we define

$$H_x=H_n=\tfrac{1}{2}\vec{\sigma}\cdot\hat{n}. \tag{4.7b}$$

The third or "sideways" component of H spin is then uniquely defined by the commutation rules to give

$$H_y=H_s=\tfrac{1}{2}\vec{\sigma}\cdot\hat{p}\times\hat{n}. \tag{4.7c}$$

The x component of H spin is seen to be identical to the corresponding component of ordinary spin. However, the y and z components are different, since the direction of the axis depends upon the momentum of the particle and is different for different particles and for the initial and final states of the same particle. Furthermore, for both H_z and H_y the configuration described as "parallel" or "antiparallel" spin for a pair of particles moving in opposite directions in the center-of-mass system are reversed from the case of ordinary spin. Thus for these components, parallel ordinary spin means antiparallel H spin and vice versa.

Let us observe that the constituent-interchange model is H-spin invariant. In a model with quark-interchange and quark helicity conservation, the transition amplitude from initial hadron states A_i and B_i to final hadron states A_f and B_f can be written as

$$\langle A_f B_f|T|A_i B_i\rangle = \sum_{\alpha,\beta}\langle A_f|q_\alpha^\dagger q_\beta|A_i\rangle\langle B_f|q_\beta^\dagger q_\alpha|B_i\rangle f(s,t), \tag{4.8}$$

where $f(s,t)$ is independent of spin and flavor quantum numbers, $q_\alpha^\dagger$ and q_α denote the creation and destruction operators for a quark with quantum numbers α, and α includes flavor and helicity. The H-spin conservation can be made manifest by writing out just the helicity sum,

$$(q_+^\dagger q_+)_A(q_+^\dagger q_+)_B+(q_+^\dagger q_-)_A(q_-^\dagger q_+)_B + (q_-^\dagger q_+)_A(q_+^\dagger q_-)_B+(q_-^\dagger q_-)_A(q_-^\dagger q_-)_B, \tag{4.9}$$

and noting that it contains only terms which are products of two H-spin scalars or scalar products of two H-spin vectors, $(\vec{\sigma})_A\cdot(\vec{\sigma})_B$. In terms of operators acting on the helicity indices of the quarks, we could write T as

$$T=\tfrac{1}{2}\sum_{i\in A}\sum_{j\in B}[1_i 1_j+\vec{\sigma}_i\cdot\vec{\sigma}_j]f(s,t), \tag{4.10}$$

and if we restore the sum on flavor (isospin),

$$T=\tfrac{1}{4}\sum_{i\in A}\sum_{j\in B}[1_i 1_j+\vec{\tau}_i\cdot\vec{\tau}_j][1_i 1_j+\vec{\sigma}_i\cdot\vec{\sigma}_j]f(s,t). \tag{4.11}$$

The operators act on quark i in nucleon A and quark j in nucleon B; the unit operators will give the number of quarks. There is thus a full $SU(4)_H \simeq SU(2)_H \times SU(2)_I$ symmetry in this case. This symmetry property leads to the following theorem: All predictions of the model are identical in the three commonly used bases which correspond to H_z, H_x, and H_y diagonal, respectively. This follows because the three bases are obtained from one another by 90° H-spin rotations which leave the amplitude invariant. However, one must be careful in using the H_z and H_x bases because of the way the direction of H spin is defined.

Let us apply the theorem to the quantities A_{nn}, A_{ll}, and A_{ss} for the asymmetries in the total elastic cross sections,

$$A_{ii} = [\sigma(\uparrow\uparrow) - \sigma(\uparrow\downarrow)]/[\sigma(\uparrow\uparrow) + \sigma(\uparrow\downarrow)], \qquad (4.12)$$

where $\sigma(\uparrow\uparrow)$ and $\sigma(\uparrow\downarrow)$ denote the cross sections for parallel and antiparallel *spins* and i is either n, l, or s. Then because of the reversal of parallel and antiparallel for H_y and H_z, our symmetry argument immediately gives the result

$$A_{nn} = -A_{ll} = -A_{ss}. \qquad (4.13)$$

At 90°, where the absence of the double-flip amplitude leads to the sum rule,

$$A_{nn} - A_{ll} - A_{ss} = 1, \qquad (4.14)$$

we obtain the result

$$A_{nn}(90°) = -A_{ll}(90°) = -A_{ss}(90°) = \tfrac{1}{3}. \qquad (4.15)$$

This general result follows only from the H-spin invariance of the amplitude of the constituent-exchange model. It is independent of the wave functions used for the proton. In particular, there is no assumption of SU(6) for the proton spin-isospin wavefunction. Thus if there is disagreement with experiment at 90°, then the constituent-exchange model with helicity conservation is in trouble, and it cannot be saved by SU(6) breaking in the wave functions. Some other mechanism must be present which violates H-spin conservation in the amplitudes.

The proton-proton matrix elements can be fairly easily calculated using the H-spin formalism. If there is no helicity exchange, then only the first and last terms of Eq. (4.9), summed over flavor, contribute and we can write down directly,

$$\langle p_+p_+|T|p_+p_+\rangle = [N_{u+}^2 + N_{u-}^2 + N_{d+}^2 + N_{d-}^2] f(s,t), \qquad (4.16a)$$

$$\langle p_+p_-|T|p_+p_-\rangle = [2N_{u+}N_{u-} + 2N_{d+}N_{d-}] f(s,t). \qquad (4.16b)$$

Where there is a helicity flip at each vertex, we use the Wigner-Eckart theorem,

$$\begin{aligned}\langle p_-p_+|T|p_+p_-\rangle &= \tfrac{1}{2}\langle p_-|\sigma_-|p_+\rangle\langle p_+|\sigma_+|p_-\rangle f(s,t)\\ &= \langle p_+|\sigma_z|p_+\rangle\langle p_+|\sigma_z|p_+\rangle f(s,t)\\ &= [(N_{u+}-N_{u-})^2 + (N_{d+}-N_{d-})^2] f(s,t),\end{aligned} \qquad (4.17)$$

where σ acts on the quark constituents, $\sigma_\pm = (\sigma_x \pm i\sigma_y)/\sqrt{2}$, and a sum on flavor is implied in the middle steps.

The neutron can also be included by purely algebraic techniques by noting that the transition amplitude is also isospin invariant, so that we have a $SU(2)_I \times SU(2)_H$ symmetry. [In fact, the bilinear products $q_\alpha^\dagger q_\beta$ generates a full $SU(4)_H$ symmetry which can be exploited to make predictions for Δ production.] If there is no flavor interchange between the nucleons, we have

$$\begin{aligned}\langle n_+p_+|T|n_+p_+\rangle &= [2N_{u+}N_{d+} + 2N_{u-}N_{d-}] f(s,t)\\ &= \tfrac{14}{9} f(s,t),\end{aligned} \qquad (4.18a)$$

$$\begin{aligned}\langle n_+p_-|T|n_+p_-\rangle &= [2N_{u+}N_{d-} + 2N_{u-}N_{d+}] f(s,t)\\ &= \tfrac{22}{9} f(s,t),\end{aligned} \qquad (4.18b)$$

$$\begin{aligned}\langle n_-p_+|T|n_+p_-\rangle &= 2(N_{u+}-N_{u-})(N_{d+}-N_{d-}) f(s,t)\\ &= -\tfrac{8}{9} f(s,t).\end{aligned} \qquad (4.18c)$$

Thus, all matrix elements are reduced to linear combinations of expectation values of matrix elements of number operators for the four quark states, $u+$, $u-$, $d+$, and $d-$. All the number operators above are for a positive-helicity proton; we use isospin symmetry to relate them to the neutron number operators.

When there is flavor exchange between the nucleons, then

$$\begin{aligned}\langle p_+n_+|T|n_+p_+\rangle &= \tfrac{1}{2}\langle p_+|\tau_+|n_+\rangle\langle n_+|\tau_-|p_+\rangle f(s,t)\\ &= \langle p_+|\tau_z|p_+\rangle\langle p_+|\tau_z|p_+\rangle f(s,t)\\ &= [(N_{u+}-N_{d+})^2 + (N_{u-}-N_{d-})^2] f(s,t)\\ &= \tfrac{17}{9} f(s,t),\end{aligned} \qquad (4.19a)$$

and similarly,

$$\begin{aligned}\langle p_+n_-|T|n_+p_-\rangle &= 2(N_{u+}-N_{d+})(N_{u-}-N_{d-}) f(s,t)\\ &= -\tfrac{8}{9} f(s,t),\end{aligned} \qquad (4.19b)$$

$$\begin{aligned}\langle p_-n_+|T|n_+p_-\rangle &= (N_{u+}-N_{u-}-N_{d+}+N_{d-})^2 f(s,t)\\ &= \tfrac{25}{9} f(s,t).\end{aligned} \qquad (4.19c)$$

The results given above in terms of the number operators depend only on being able to factor the spin-isospin wave function of the nucleon from the color and space wave functions, while the numerical results depend on the specific form of the wave function given earlier.

We might remark that the scattering operator can be recast into a form where the matrices operate directly on the nucleons rather than on the quarks as in Eq. (4.11). It is

$$T=[\tfrac{9}{4}+\tfrac{1}{4}\vec{\sigma}_1\cdot\vec{\sigma}_2+\tfrac{1}{4}\vec{\tau}_1\cdot\vec{\tau}_2+\tfrac{1}{4}K\vec{\sigma}_1\cdot\vec{\sigma}_2\vec{\tau}_1\cdot\vec{\tau}_2], \quad (4.20)$$

where $K=(N_{u+}-N_{u-}-N_{d+}+N_{d-})^2$ and for the case of SU(6) symmetry $K=\frac{25}{9}$.

Note added in proof. The value $A_{nn}(np\to np)$ ~ -0.2, reported by D. G. Crabb *et al.*, [Phys. Rev. Lett. 43, 983 (1979)] at 6 GeV/c and $p_\perp^2=1$ (GeV/c)^2 is in qualitative agreement with the constituent-interchange-model (CIM) prediction, Eq. (3.9), of -0.439 for 90° and higher momenta. The sign is particularly significant, since positive and negative signs for $A_{nn}(pp\to pp)$ and $A_{nn}(np\to np)$, respectively, are characteristic of the CIM at any scattering angle, as can be seen from Table I. Exchange of quarks with the same flavor and parallel spins gives the dominant contribution, and the probability of finding two quarks of the same flavor with parallel spins is greater for parallel nucleon spins in the pp system and for antiparallel spins in the np system.

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APPENDIX A: *NN* SCATTERING AMPLITUDES

We define the scattering amplitudes using the Jacob-Wick[17] phase conventions. Without using parity, time reversal, or identical-particle symmetry, there are sixteen independent helicity amplitudes, $M(\lambda_c\lambda_d,\lambda_a\lambda_b)(\theta,\phi)$. Parity cuts this number in half,

$$M(-\lambda_c,-\lambda_d,-\lambda_a,-\lambda_b)(\theta,\phi)=\eta M(\lambda_c\lambda_d,\lambda_a\lambda_b)(\theta,\pi-\phi), \quad (A1)$$

where η is unity for nucleons, and is

$$\eta=\frac{\eta_c\eta_d}{\eta_a\eta_b}(-1)^{s_c+s_d-s_a-s_b},$$

in general, where η_i is the intrinsic parity and s_i the spin of particle i. The ϕ dependence may be removed by choosing a particular value and using[17]

$$M(\lambda_c\lambda_d,\lambda_a\lambda_b)(\theta,\phi)=\frac{1}{p}\sum_J(J+\tfrac{1}{2})\langle\lambda_c\lambda_d|T^J|\lambda_a\lambda_b\rangle e^{i(\lambda-\mu)\phi}d^J_{\lambda\mu}(\theta), \quad (A2)$$

where $\lambda=\lambda_a-\lambda_b$ and $\mu=\lambda_c-\lambda_d$. We will choose $\phi=\pi/2$ (scattering in the y-z plane) so that the pairs of amplitudes related by parity are related with positive sign.

Parity alone is sufficient to show that all the single-flip *spin* amplitudes are zero.[18]

Time-reversal invariance will reduce the number of independent amplitudes to six, using

$$M(\lambda_a\lambda_b,\lambda_c\lambda_d)(\theta,\phi)=M(\lambda_c\lambda_d,\lambda_a\lambda_b)(\theta,\pi-\phi). \quad (A3)$$

Finally, if we have identical particles, then there is one more independent relation,

$$M(++,+-)=M(++,-+) \quad (A4)$$

so that there are only five independent helicity amplitudes. Also, for identical particles scattering at 90°, the last-named amplitude must be zero. We choose our independent amplitudes to be

$$\begin{aligned}&M(++,++),\\&M(+-,+-),\\&M(-+,+-),\\&M(++,+-),\\&M(--,++).\end{aligned} \quad (A5)$$

If the elementary interactions conserve quark helicity, then the last two listed amplitudes are zero at all angles.

The asymmetries defined in the text can be expressed in terms of the helicity amplitudes by

$$\begin{aligned}D\times A_{nn}&=2\,\mathrm{Re}\,M^*(++,++)M(--,++)\\&\quad+2\,\mathrm{Re}\,M^*(+-,+-)M(-+,+-)\\&\quad+4|M(++,+-)|^2,\\D\times A_{ll}&=-|M(++,++)|^2-|M(--,++)|^2\\&\quad+|M(+-,+-)|^2+|M(-+,+-)|^2,\\D\times A_{ss}&=2\,\mathrm{Re}\,M^*(++,++)M(--,++)\\&\quad-2\,\mathrm{Re}\,M^*(+-,+-)M(-+,+-),\end{aligned} \quad (A6)$$

where

$$\begin{aligned}D=&|M(++,++)|^2+|M(--,++)|^2+|M(+-,+-)|^2\\&+|M(-+,+-)|^2+4|M(++,+-)|^2.\end{aligned}$$

For scattering of identical particles at 90°, we may use $M(+-,+-)=M(-+,+-)$ to show that

$$A_{nn}-A_{ll}-A_{ss}=1\,. \tag{A7}$$

We shall also quote a relation for the "analyzing power," A. This is defined for scattering an unpolarized beam on a target polarized normal to the scattering plane.

$$A=\frac{d\sigma(\uparrow)-d\sigma(\downarrow)}{d\sigma(\uparrow)+d\sigma(\downarrow)}=\frac{2}{D}\,\mathrm{Re}M^*(++,+-)\times\{M(++,++)+M(--,++)+M(+-,+-)+M(-+,+-)\}\,. \tag{A8}$$

The above formula is valid at all angles, and shows that if the underlying process is helicity conserving, then the analyzing power is always zero.

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