Unitarity and gluonic corrections to the pion form factor around the ϕ peak

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The transition $\phi \rightarrow \pi^+\pi^-$, which is forbidden by the Okubo-Zweig-Iizuka rule and by isospin conservation, is discussed in the context of unitarity and quantum chromodynamics. The analysis reveals that basic aspects of these theoretical contexts may be clearly tested by a reasonably accurate measurement of $\sigma(e^+e^- \rightarrow \pi^+\pi^-)$ around the ϕ peak.

I. INTRODUCTION

The validity of the Okubo-Zweig-Iizuka (OZI) rule is an attractive and firmly established feature of strong-interactions dynamics.[1] It is only an approximate rule, presenting small but definite violations in all the cases where it can be applied. For these reasons, the origin of such a systematic violation has been discussed by many authors, and different mechanisms accounting for this effect have been proposed. These may be classified into two main categories. First, the systematic occurrence of OZI-forbidden transitions has been traditionally attributed to unitarity effects proceeding through physical intermediate states. In principle, these effects are well understood and can be evaluated in the framework of more or less sophisticated models.[2,3,4] In practice, the results turn out to be often inconsistent and the whole situation is not entirely satisfactory.[4] With the advent of quantum chromodynamics (QCD), a second kind of OZI-violating effects has appeared, namely, those involving quark-antiquark annihilation diagrams.[5] In this case, the transition between two OZI-disconnected states proceeds through virtual, multigluon intermediate states giving additional and interesting contributions to the unitarity corrections.

In a recent paper, Renard[6] has investigated the characteristics and relative importance of the two types of (*a priori*) superimposed contributions in a particularly interesting case. It concerns e^+e^- annihilation channels into an odd number of pions, and one has to assume the dominance of a series of radial excitations of the ideally mixed ω-ϕ system. The gluonic contributions are predicted to dominate when connecting the lowest-mass ω^*-ϕ^* resonances, while unitarity effects (showing a completely different shape) will become relevant for ϕ^* masses above 1.5 GeV. Unfortunately, the identification and even the existence of these ideal ω^*-ϕ^* systems may be doubtful. For this reason, we have turned our attention to related processes where similar questions can be more easily investigated from both theoretical and experimental points of view.

In the present paper we will be mainly concerned with the cross section $\sigma(e^+e^- \rightarrow \pi^+\pi^-)$ near the ϕ-resonance region or, equivalently, with the pion form factor $F_\pi(s)$ at $s \sim m_\phi^2$. Its measurement appears as a reasonably simple task and has already been performed by two experimental groups.[7,8] The accuracy of their data, however, is still insufficient for our main purposes, but it seems realistic to hope that this situation will be greatly improved during the next years with the forthcoming generation of e^+e^- machines. From a theoretical point of view, the process $e^+e^- \rightarrow \gamma \rightarrow \phi \rightarrow \pi^+\pi^-$ is also particularly interesting. It involves a final decay which is forbidden by the OZI rule and by G-parity conservation. This double forbiddenness and the kinematical fact that m_ϕ is just above the threshold of the allowed strong decays $\phi \rightarrow K^+K^-$, $K^0\bar{K}^0$ will play a central role in the following discussion.

Most of the previous work on this and related subjects has been performed[2,3,6,9,10] in the context of rather sophisticated models: resonance mixing, dispersive treatment, dual amplitudes, etc. On the contrary, the present analysis will be based on simple vector-meson-dominance (VMD) ideas, which are reasonably accurate for our purposes and do not require the introduction of free or unknown parameters. In this context, $F_\pi(s)$ at $s \sim m_\phi^2$ is essentially given by the ρ-pole contribution $F_\pi^\rho(s)$ and the correction arising from the presence of the ϕ peak. One has

$$F_\pi^{\rho+\phi}(s) = F_\pi^\rho(s)\left(1 + \frac{f_\rho m_\phi^2}{f_\phi m_\rho^2}\,\frac{T_{\phi\rho}(s)}{m_\phi^2 - s - i m_\phi \Gamma_\phi}\right), \quad (1)$$

where $e m_{\rho,\phi}^2/f_{\rho,\phi}$ are the $\rho\gamma, \phi\gamma$ couplings and the amplitude $T_{\phi\rho}(s)$ accounts for the doubly forbidden $\phi \rightarrow \rho$ conversion. $F_\pi^\rho(s)$ may simply be given by the Gounaris-Sakurai[11] formula, which represents the dominant contribution to $F_\pi(s)$. It leads to excellent agreement with the data for a wide range of energies,[12] thus justifying the adequacy of a pole model at $s \sim m_\phi^2$. The last term in Eq. (1) accounts for the presence of the ϕ meson and gives a non-negligible correction to $F_\pi^\rho(s)$ only at $s \simeq m_\phi^2$. It contains the expression in which we are directly interested, namely, the amplitude $T_{\phi\rho}(s)$.

The evaluation of the imaginary and real parts of this amplitude taking into account the different contributions of intermediate states will be performed in Secs. II and III, respectively. As a result, one obtains a reasonably accurate and well-understood prediction for the unitarity effects expected in $F_\pi(s)$ around the ϕ-meson peak. In Sec. IV, we present a rough estimate of the dominant contribution from the point of view of QCD, which turns out to be far from negligible. The conclusions are briefly summarized in Sec. V.

II. ABSORPTIVE PART OF THE AMPLITUDE

Let us first consider the absorptive part of $T_{\phi\rho}(s)$. According to Eq.(1) and working at $s=m_\phi^2$, the unitarity corrections due to the ϕ resonance are dominated by $\mathrm{Im}\, T_{\phi\rho}(m_\phi^2)$, i.e.,

$$\frac{F_\pi^{\rho+\phi}(m_\phi^2)}{F_\pi^\rho(m_\phi^2)} \simeq 1 - \frac{m_\phi^2}{m_\rho^2}\frac{f_\rho}{f_\phi}\frac{\mathrm{Im}T_{\phi\rho}(m_\phi^2)}{m_\phi\Gamma_\phi} . \qquad (2)$$

From a theoretical point of view, $\mathrm{Im}T_{\phi\rho}(m_\phi^2)$ and the ratio (2) can be evaluated on rather solid grounds. There are only three main contributions, originated by the three dominant intermediate states, $i=K^+K^-, K^0\bar{K}^0, \eta\gamma$, appearing in the unitarity sum.[10] One has

$$\frac{f_\rho}{f_\phi}\frac{\mathrm{Im}T_{\phi\rho}(m_\phi^2)}{m_\phi\Gamma_\phi} = \sum_i \frac{f_\rho}{f_\phi}\frac{g_{\rho i}}{g_{\phi i}} B(\phi\to i)$$
$$\equiv \sum_i y(i) , \qquad (3)$$

where $g_{\rho i}$, $g_{\phi i}$, and $B(\phi\to i)$ are the coupling constants and branching ratios for a given intermediate state i. Due to isospin symmetry, the first two purely hadronic contributions just differ in a sign and tend to cancel. The cancellation, however, is far from being exact and the whole contribution $y(K\bar{K})$ becomes proportional to $B(\phi\to K^+K^-) - B(\phi\to K^0\bar{K}^0)$. One can calculate $y(K\bar{K})$ making use of different sets of the available experimental data,[13] combined with different theoretical relations such as ρ universality or exact SU(3) symmetry for the coupling constants. In all cases, the calculation leads essentially to the same result:

$$y(K\bar{K}) = 0.050 \pm 0.006 . \qquad (4)$$

We remark that the nonvanishing of these contributions is originated by the small electromagnetic (EM) mass differences of kaons. However, their effects have been greatly amplified by working at the ϕ peak, near the different $K\bar{K}$ thresholds, where the available phase spaces for the charged and neutral decay modes are substantially different. As a consequence, Eq. (4) represents a reasonably well understood contribution and the dominant one to the unitarity sum (3).

Since the η particle is not ideally mixed, the remaining contribution to Eq. (3) is essentially given by the EM- and OZI-allowed intermediate state $\eta\gamma$.[10] In contrast with the previous case, our theoretical understanding of the radiative decays of vector mesons[14] and, particularly, those involving the η-η' mixing angle θ_P (Ref. 15) is confused, thus making any SU(3) evaluation of their contribution rather unreliable. In spite of this, an upper limit for $|y(\eta\gamma)|$ can be derived, for any mixing angle, from SU(3) and the experimental[13] rate $\Gamma(\omega\to\pi\gamma)=890$ keV. One gets $|y(\eta\gamma)|<0.018$, showing that $y(K\bar{K})$ is still the dominant term in the unitarity sum (3). Experimentally, the situation has recently been improved with the measurements[16] $\Gamma(\phi\to\eta\gamma)=55\pm12$ keV and $\Gamma(\rho\to\eta\gamma)=50\pm13$ keV. With these results and $f_\rho/f_\phi \simeq -\sqrt{2}/3$, one can easily obtain

$$-y(\eta\gamma) = 0.014 \pm 0.005 . \qquad (5)$$

This is in good agreement with the previous upper bound, but cannot be contrasted with other reliable estimates. Its sign has been fixed by SU(3) and will be discussed later on.

The global unitarity correction of the ϕ resonance to the pion form factor at $s=m_\phi^2$ can be predicted combining the contributions (4) and (5). One obtains

$$|F_\pi^{\rho+\phi}(m_\phi^2)|^2 = (0.88\pm0.03)\,|F_\pi^\rho(m_\phi^2)|^2 , \qquad (6)$$

i.e., the dominant ρ-meson tail is expected to present a depletion of events on the ϕ peak. One predicts a 12% effect which is of the same order (but has a different shape) as the ϕ vacuum polarization detected in $e^+e^-\to\phi\to\mu^+\mu^-$ and, therefore, could be reasonably measured.[17] Such a measurement would be of interest in order to test our ideas on unitarity corrections and, in addition, it would provide valuable information on the η couplings. For instance, the relative sign of the strange- and nonstrange-quark pieces in the η wave function could be established. Indeed, the results quoted in Eqs. (5) and (6) have been deduced assuming the relative minus sign, which follows from SU(3) and the generally accepted values of θ_P;[15] in the opposite case, one should change sign in Eq. (5), thus obtaining a 23% reduction of events on the ϕ peak. The available data[7,8] do not seem to present such a drastic reduction and, in this sense, they favor the first possibility. We notice that the determination of this relative sign cannot be achieved from the analysis of mass formulas

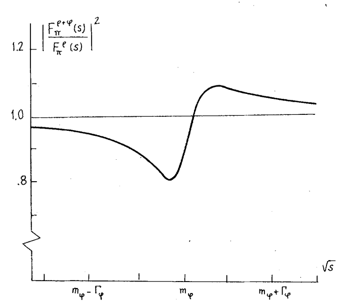

FIG. 1. Modulation on the ρ tail of the pion form factor around the ϕ-resonance peak. Unitarity effects and one-photon intermediate state have been included.

or decay rates involving the η-η' system. It requires the interference of several amplitudes, and the present analysis indicates that a moderately improved measurement of $|F_\pi(m_\phi^2)|$ may be sufficient for such a purpose.

III. REAL PART OF THE AMPLITUDE

We now proceed to the discussion of $\mathrm{Re}\,T_{\phi\rho}(s)$ which, according to Eq. (1), will be relevant only at the vicinity of the ϕ peak. The K^+K^- and $K^0\bar{K}^0$ intermediate states give a globally vanishing contribution since the strong asymmetry due to the different thresholds plays no role. Similarly, the contribution of the $\eta\gamma$ intermediate state cancels very approximately with that coming from $\eta'\gamma$ and contributing also to $\mathrm{Re}\,T_{\phi\rho}(s)$. This is a consequence of the relation $g_{\phi\eta\gamma}g_{\rho\eta\gamma} = -g_{\phi\eta'\gamma}g_{\rho\eta'\gamma}$, whose validity follows from the OZI rule and the ideal mixing of the ω-ϕ system, and does not depend on the η-η' mixing angle. The same is true for other meson-photon intermediate states, which, as discussed by Lipkin[4] and Renard,[6] give globally vanishing contributions to this doubly forbidden $\mathrm{Re}\,T_{\phi\rho}(s)$. Note that these cancellations occur inside the same SU(2) or SU(3) multiplets and, consequently, are expected to be accurate and cleaner than those required when dealing with ω-ϕ systems.[6]

The conclusion of the preceding paragraph is that the total contribution to $\mathrm{Re}\,T_{\phi\rho}$ of the physical intermediate states may safely be neglected. However, $\mathrm{Re}\,T_{\phi\rho}$ receives an important contribution through the virtual, one-photon conversion $\phi\to\gamma\to\rho$. This was first considered by Gatto[18] in the related framework of ρ-ω mixing, and can be fully evaluated in the VMD formalism. For values of $\sqrt{s}$ near the ϕ peak, one immediately gets[13]

$$\frac{f_\rho}{f_\phi}\frac{m_\phi^2}{m_\rho^2}\,\mathrm{Re}\,T_{\phi\rho}(s) \simeq -\frac{e^2}{f_\rho^2}\frac{m_\rho^4}{s} \simeq -5.1\times10^{-4}m_\phi^2\,. \quad (7)$$

With this result and those quoted in Eqs. (4) and (5), one can calculate the ratio $F_\pi^{\rho+\phi}(s)/F_\pi^\rho(s)$ making use of Eq. (1). The square of this ratio has been plotted in Fig. 1 and shows a non-negligible modulation on the ρ tail. The effect is due to the presence of the ϕ meson and contains the corrections arising from unitarity and one-photon intermediate states contributing to $T_{\phi\rho}(s)$.

IV. GLUONIC CONTRIBUTIONS

The traditional[18] one-photon diagram may be considered (apart from the fact that it involves a virtual $q^2\sim m_\phi^2$ photon) as a zero-order QCD diagram, i.e., the one-photon, zero-gluon contribution. The next-order diagram, containing one photon and two gluons, represents a genuine and on-mass-shell gluonic correction to the previous contribution. It is not necessarily a negligible correction, since the strong coupling constant at these energies, $\alpha_s(m_\phi^2)$, is certainly larger than at the region of the ψ-particle mass.

In order to estimate the contribution of the one-photon-and-two-gluons intermediate state in the $\phi\to\rho$ conversion, we compare this process with the closely related $\omega\to\rho$ conversion. The corresponding amplitudes satisfy the equation[19]

$$T(\phi\to 2g+\gamma\to\rho) = \frac{\sqrt{2}\,Q_s}{Q_u+Q_d}\,\frac{\alpha_s^2(m_\phi^2)}{\alpha_s^2(m_\omega^2)}\,T(\omega\to 2g+\gamma\to\rho)\,,$$

where $Q_{u,d,s}$ are the relevant quark charges. From this equation and taking into account that the vector-meson propagators can be reasonably approximated using the experimental relations

$$m_\omega^2 - m_\rho^2 \ll m_\rho\Gamma_\rho \ll m_\phi^2 - m_\rho^2\,,$$

one obtains

$$\Gamma(\phi\to 2g+\gamma\to\rho\to\pi^+\pi^-) \simeq 2\,\frac{\alpha_s^4(m_\phi^2)}{\alpha_s^4(m_\omega^2)}\left(\frac{m_\rho\Gamma_\rho}{m_\rho^2-m_\phi^2}\right)^2\frac{m_\omega^2 p_\phi^3}{m_\phi^2 p_\omega^3} \times\Gamma(\omega\to 2g+\gamma\to\rho\to\pi^+\pi^-)\,, \quad (8)$$

where p_ϕ and p_ω are the pion momenta in the final ϕ and $\omega\to\pi\pi$ decays appearing in the phase-space factors.

The contribution given by Eq. (8) to the $\Gamma(\phi\to\pi^+\pi^-)$ decay rate has to be compared with the one-photon (VMD) contribution $\Gamma(\phi\to\gamma\to\rho\to\pi^+\pi^-)$. This turns out to be approximately given by[18,20]

$$\Gamma(\phi\to\gamma\to\rho\to\pi^+\pi^-) \simeq \left(\frac{e^2}{f_\rho f_\phi}\,\frac{m_\rho^2}{m_\rho^2-m_\phi^2}\right)^2 \times\left(\frac{p_\phi}{p_\rho}\right)^3\Gamma(\rho\to\pi^+\pi^-)\,.$$

From this result, Eq. (8) and $m_\rho \simeq m_\omega$, $p_\rho \simeq p_\omega$ one deduces the following expression for the relative importance of the two contributions to $\phi \to \pi^+\pi^-$:

$$\Gamma(\phi \to 2g+\gamma \to \rho \to \pi^+\pi^-)/\Gamma(\phi \to \gamma \to \rho \to \pi^+\pi^-)$$

$$= 2\frac{\alpha_s^4(m_\phi^2)}{\alpha_s^4(m_\omega^2)} \frac{f_\rho^2 f_\phi^2}{e^4} \frac{\Gamma_\rho^2}{m_\phi^2} \frac{\Gamma(\omega \to 2g+\gamma \to \rho \to \pi^+\pi^-)}{\Gamma(\rho \to \pi^+\pi^-)}$$

$$\simeq 16\frac{\alpha_s^4(m_\phi^2)}{\alpha_s^4(m_\omega^2)} . \qquad (9)$$

The last numerical step in Eq. (9) follows from the use of the experimental results[13] $f_\rho f_\phi/e^2 \simeq -3 f_\rho^2/\sqrt{2}e^2 \simeq -660$, $\Gamma_\rho \simeq \Gamma(\rho \to \pi\pi) = 155$ MeV, $\Gamma(\omega \to \pi\pi) = 130$ keV and from the assumption that the $\omega \to 2g+\gamma \to \rho \to \pi\pi$ decay chain accounts for all the $\omega \to \pi\pi$ decay width. This assumption can be partially justified by observing that the one-photon transition $\omega \to \gamma \to \rho \to \pi\pi$ represents only a negligible[18,20] fraction of the measured $\Gamma(\omega \to \pi\pi)$. However, QCD terms involving one photon and more than two gluons, as well as other type of mechanisms (quark-mass differences,[20] unconventional mixing models,[21] etc.) can also contribute to the EM $\omega \to \pi\pi$ decay rate. For these reasons, the right-hand side of Eq. (9) has to be considered as a rough estimate. In spite of this and the fact that $\alpha_s(m_\phi^2) < \alpha_s(m_\omega^2)$, it seems realistic to conclude that the one-photon-plus-two-gluons intermediate state gives a contribution to the $\phi \to \pi^+\pi^-$ decay rate which cannot be neglected in front of that of the one-photon conversion.

The main point of the preceding discussion is that these two contributions are expected to interfere (with unknown phase) giving rise to a detectable effect. The presence and amount of gluonic corrections can, therefore, be established by simply comparing the experimental data on $F_\pi(s)$ around the ϕ peak with the curve plotted in Fig. 1, where only nongluonic effects have been included. This seems to be compatible with the available experimental results,[7,8] but their error bars are so large that no reliable conclusion can be deduced from this comparison.

V. CONCLUSIONS

In conclusion, a measurement of the pion form factor in the ϕ-resonance region with a reasonable precision seems to be highly desirable. Its value on the peak $F_\pi(m_\phi^2)$ would test our conventional ideas on unitarity corrections in this particularly well understood case of doubly forbidden transitions. Similarly, it would provide valuable (and otherwise inaccessible) information concerning the quark content of the η particle and the puzzling η-η' complex.

Independently, experimental data on $F_\pi(s)$ immediately below and above the ϕ peak seem to be appropriate in order to test some basic ideas of QCD and, in particular, those related to the presence of gluonic corrections.

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