Quantum-chromodynamic corrections to parity-violating asymmetries in *ep* reactions at high Q^2

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We examine quantum-chromodynamic (QCD) corrections to the Q^2 and y dependence of various asymmetries in *ep* deep-inelastic scattering produced by the parity-violating weak neutral currents of the Weinberg-Salam model. We find, for Q^2 in the range $10 \leq Q^2 \leq 10^4$ $(\mathrm{GeV}/c)^2$, that a small change in the value of $\sin^2\theta_W$ can mask the small ($\sim$ 5–10%) QCD corrections even if data are taken over the entire range of y. We conclude that QCD does not produce significantly large modifications to the weak asymmetries considered for the above range of Q^2.

INTRODUCTION

Since the recent observation at SLAC (Ref. 1) of apparent parity violation in inelastic scattering of polarized electrons off hydrogen and deuterium targets, there has been much theoretical interest in extracting the constraints these new data impose on models of the weak and electromagnetic interaction [specifically, models based on the $\mathrm{SU}(2)\otimes\mathrm{U}(1)$ gauge group[2]]. The calculation of the asymmetry measured by this experiment, i.e.,

$$A^- \equiv \left(\frac{\sigma(e_R^-)-\sigma(e_L^-)}{\sigma(e_R^-)+\sigma(e_L^-)}\right)_{\mathrm{wk+em}}, \tag{1}$$

was done by Cahn and Gilman[3] and by Marciano and Sanda[4] using the scaling parton model.[5] The results of the SLAC experiment were, however, obtained at low Q^2 and some doubt has been raised as to the validity of a direct, quantitative comparison of the experimental results and the predictions of the scaling parton model[5] due to quantum-chromodynamics[6] (QCD) and other corrections.[7]

In this paper we would like to examine various weak asymmetries of the kind (1) at higher values of Q^2 where the parton model with QCD corrections should give good quantitative results. In particular, we are interested in the effects QCD has on the values of the various weak asymmetries for various values of x and y (the well-known scaling variables) and Q^2. We will consider the scattering of polarized electrons and positrons from *unpolarized* protons only for $Q^2 \geq 10$ $(\mathrm{GeV}/c)^2$. We use the results of Ref. 3 together with the parametrizations of the Q^2-dependent structure functions given by Buras and Gaemers.[8] Below, we also consider the "asymmetry" parameters

$$R(e^{\pm}_{L,R}) \equiv \frac{\sigma^{\mathrm{wk+em}}(e^{\pm}_{L,R})}{\sigma^{\mathrm{em}}(e^{\pm}_{L,R})}, \tag{2}$$

$$\sigma^{\pm} \equiv \left(\frac{\sigma(e^{\pm}_L)}{\sigma(e^{\pm}_R)}\right)_{\mathrm{wk+em}}, \tag{3}$$

$$\chi \equiv \left(\frac{\sigma(e^-_R)+\sigma(e^-_L)}{\sigma(e^+_R)+\sigma(e^+_L)}\right)_{\mathrm{wk+em}}, \tag{4}$$

in addition to A^- defined above.

Our procedure is as follows. We follow the work of Cahn and Gilman.[3] (We consider first the case of n Z bosons.) We denote the coupling of a fermion (f) to the photon and $Z^{(\alpha)}$ bosons by ($\alpha = 1, \ldots, n$)

$$[\tfrac{1}{2}\gamma_\mu(1+\gamma_5)Q^{\gamma,Z^{(\alpha)}}_{Lf} + \tfrac{1}{2}\gamma_\mu(1-\gamma_5)Q^{\gamma,Z^{(\alpha)}}_{Rf}][A^\mu, Z^\mu_{(\alpha)}],$$

with the obvious requirement $Q^\gamma_{Lf} = Q^\gamma_{Rf} = Q^\gamma_f$. We then obtain the following cross sections:

$$d\sigma^i_{RR} \propto \left| \frac{Q^\gamma_{Re}Q^\gamma_{Ri}}{Q^2} + \sum_\alpha \frac{Q^{Z^{(\alpha)}}_{Re}Q^{Z^{(\alpha)}}_{Ri}}{Q^2+M_{Z_{(\alpha)}}{}^2} \right|^2,$$

$$d\sigma^i_{RL} \propto \left| \frac{Q^\gamma_{Re}Q^\gamma_{Li}}{Q^2} + \sum_\alpha \frac{Q^{Z^{(\alpha)}}_{Re}Q^{Z^{(\alpha)}}_{Li}}{Q^2+M_{Z_{(\alpha)}}{}^2} \right|^2 (1-y)^2,$$

$$d\sigma^i_{LL} \propto \left| \frac{Q^\gamma_{Le}Q^\gamma_{Li}}{Q^2} + \sum_\alpha \frac{Q^{Z^{(\alpha)}}_{Le}Q^{Z^{(\alpha)}}_{Li}}{Q^2+M_{Z_{(\alpha)}}{}^2} \right|^2,$$

$$d\sigma^i_{LR} \propto \left| \frac{Q^\gamma_{Le}Q^\gamma_{Ri}}{Q^2} + \sum_\alpha \frac{Q^{Z^{(\alpha)}}_{Le}Q^{Z^{(\alpha)}}_{Ri}}{Q^2+M_{Z_{(\alpha)}}{}^2} \right|^2 (1-y)^2,$$

where $d\sigma^i_{L(R)L(R)}$ represents the cross section for scattering left- (right-) handed electrons on left- (right-) handed quarks of type i. The total contribution is obtained by multiplying each $d\sigma^i$ by the corresponding "weight" factors given by the quark-antiquark distribution functions obtained from deep-inelastic scattering. Hence,

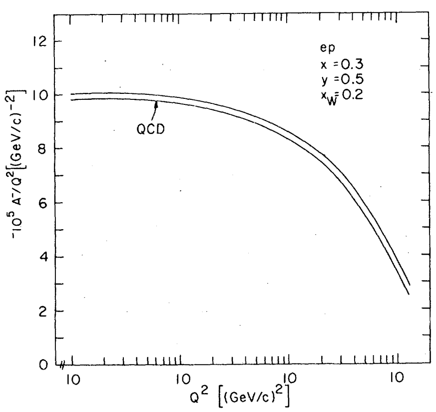


FIG. 1. Q^2 dependence of $-A^-/Q^2$ for deep-inelastic ep scattering.

$$d\sigma_R = \sum_i (d\sigma^i_{RL} + d\sigma^i_{RR}) q_i(x)\,,$$

$$d\sigma_L = \sum_i (d\sigma^i_{LL} + d\sigma^i_{LR}) q_i(x)\,,$$

where i runs over both quarks and antiquarks.

Within the Weinberg-Salam model, $n=1$ and the Q^Z_{L,R_i} are well-known functions of the weak isospins and the Weinberg angle given by[3] ($x \equiv \sin^2\theta_W$)

$$Q^Z_{Li} = \frac{e}{[x(1-x)]^{1/2}}\,(T_{3Li} - Q^\gamma_i x)\,,$$

$$Q^Z_{Ri} = \frac{e}{[x(1-x)]^{1/2}}\,(T_{3Ri} - Q^\gamma_i x)\,.$$

Once the $q_i(x)$ are known, we can immediately calculate the various asymmetries. To this end, and to include the QCD corrections, we choose the $q_i(x, Q^2)$ to be those of Buras and Gaemers[8] which should be valid for $2 \le Q^2 \le 15\,000$ (GeV/c)^2; it should be noted that these distributions contain only the leading-logarithmic QCD corrections. Instead of expanding the various cross sections in powers of $Q^2/M_Z{}^2$ we have kept the full expressions given above; our Q^2 dependence is thus a combination of the effects and of the Z-boson propagator and the Q^2-dependent distribution functions.

The relevant cross sections for positron scattering can easily be obtained from those of the electron.

RESULTS

(I) Let us first turn our attention to the asymmetry measured by parameter A^-. Figure 1 shows the Q^2 dependence of A^-/Q^2 for $Q^2 \ge 10$ (GeV/c)^2; the decrease at large Q^2 is due to the suppression of the Z-boson propagator. Note that the QCD result is ~5–10% smaller in magnitude than the prediction without QCD; this is a general result for all values of y and Q^2 in the range $10 \le Q^2 \le 10^4$ (GeV/c)^2 with $x \gtrsim 0.2$. We would expect, however, that these results would also apply for values of Q^2 outside the range examined so long as lowest-order QCD is valid.

The y dependence of A^-/Q^2 is shown in Fig. 2 for two very different values of Q^2. Note, however, that the QCD prediction is not significantly different from that of the non-QCD prediction. The effect of QCD is merely to shift the non-QCD prediction downwards without changing the y distribution significantly. Note that at both values of Q^2, the small changes produced by QCD can be

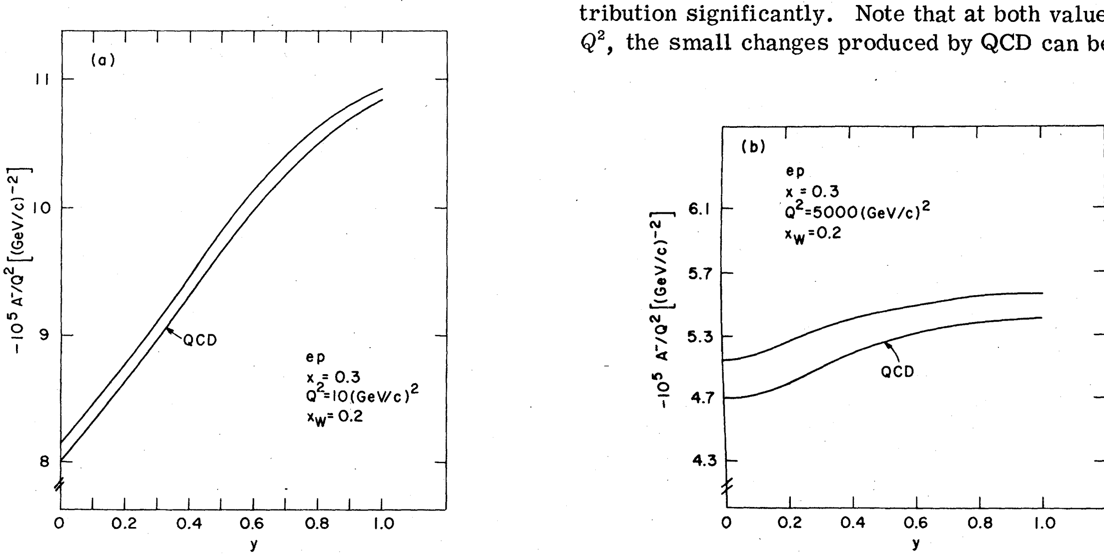
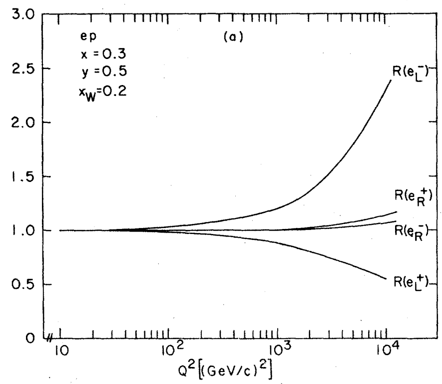


FIG. 2. (a) y dependence of $-A^-/Q^2$ for $Q^2=10$ (GeV/c)^2. (b) y dependence of $-A^-/Q^2$ for $Q^2=5000$ (GeV/c)^2.



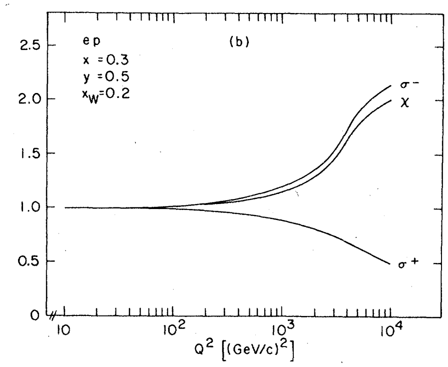


FIG. 3. (a) Q^2 dependence of $R(e^{\pm}_{L,R})$ for ep scattering. (b) Q^2 dependence of $\sigma^{\pm}$ and χ for ep scattering.

simulated by a small change in x_W, i.e., if one compares the scaling parton model with experiments, the values of x_W so obtained will be larger by a few percent. (Note that this argument only holds for $x_W \leq \frac{1}{4}$ for which $-A^-/Q^2$ has a positive slope.)

Thus we see that at both low and high Q^2, QCD has very little effect on the y dependence of A^-/Q^2, i.e., most of the QCD correction is y independent over the Q^2 range studied here. (The small x_W shift mentioned above, however, is only significant if a highly accurate measurement of this variable is made *before* the asymmetry measurements we are discussing.)

We have also examined the x dependence of A^-/Q^2 at both low and high energies; we find that A^- is quite insensitive to x (at the 10% level) for large x (i.e., $x \geq 0.2$) at both high and low Q^2. For fixed large x, the Q^2 dependence of the x distributions is not significantly modified by more than ~10% by QCD. The x dependence will not be discussed further here and we will take $x = 0.3$ in what follows.

(II) We now turn to the other parameters defined in Eqs. (2)–(4). The deviation of any of these parameters from unity may be very difficult to observe for $Q^2 \lesssim 2$–300 (GeV/c)^2 but at much larger values of Q^2 [$\gtrsim 10^3$ (GeV/c)^2] the deviation is significant. Figures 3 and 4 show the Q^2 behavior of these quantities (without QCD corrections) and their y dependence at $Q^2 = 5000$ (GeV/c)^2 respectively. (We have not shown the QCD-corrected predictions for these quantities since they are very small compared with the scale of the figure.) As can be seen in Fig. 4, the QCD corrections are almost insignificant (<5%) for $R(e_L^-)$ and σ^-, small (~5%) for $R(e_L^+)$ and $R(e_R^{\pm})$, and somewhat larger (~5–10%) for χ and σ^+ independent of y.

Several authors[9] have considered combinations of the R's defined above which do measure true asymmetries, i.e., both parity and charge-conjugation violations by weak neutral currents. Consider the following parameters:

$$A^+ \equiv \left(\frac{\sigma(e_R^+) - \sigma(e_L^+)}{\sigma(e_R^+) + \sigma(e_L^+)} \right)_{\mathrm{wk+em}}, \tag{5}$$

$$B^{\pm} \equiv \left(\frac{\sigma(e_R^{\pm}) - \sigma(e_L^{\mp})}{\sigma(e_R^{\pm}) + \sigma(e_L^{\mp})} \right)_{\mathrm{wk+em}}, \tag{6}$$

$$C_{L,R} \equiv \left(\frac{\sigma(e_{L,R}^+) - \sigma(e_{L,R}^-)}{\sigma(e_{L,R}^+) + \sigma(e_{L,R}^-)} \right)_{\mathrm{wk+em}}. \tag{7}$$

Our results for A^+/Q^2 can be found in Figs. 5(a) and 5(b). Here we again see that at both low and high Q^2 the QCD corrections essentially scale the curve downwards (by ~10%) with little or no effect on the general shape of the y distribution. As in the case of A^- this would tend to give values of x_W which were slightly low by ~10% at both low and high Q^2. Note that the QCD correction to the shape of the y distribution is more pronounced at the higher Q^2 value.

Figures 6(a) and 6(b) show the y dependence of $B^{\pm}$ and $C_{L,R}$ at low and high Q^2 respectively. As expected, the QCD corrections are again quite small (~5–10%) for $B^{\pm}/Q^2$ and C_L/Q^2; somewhat larger corrections (~10–15%) are obtained for C_R/Q^2. These corrections can again be mimicked by a small change in the value of x_W; note that the corrections are such that the small shift in x_W is in the same direction for each of the parameters and in rough agreement with that obtained for $A^{\pm}/Q^2$.

We have not shown the Q^2 behavior of A^+, $B^{\pm}$, or $C_{L,R}$ since they are similar to that of A^-.

At this point we should say something about the so-called hybrid model where the right-handed electron transforms as the lower member of a right-handed doublet, i.e., $t_3(e_R) = -\frac{1}{2}$. Although

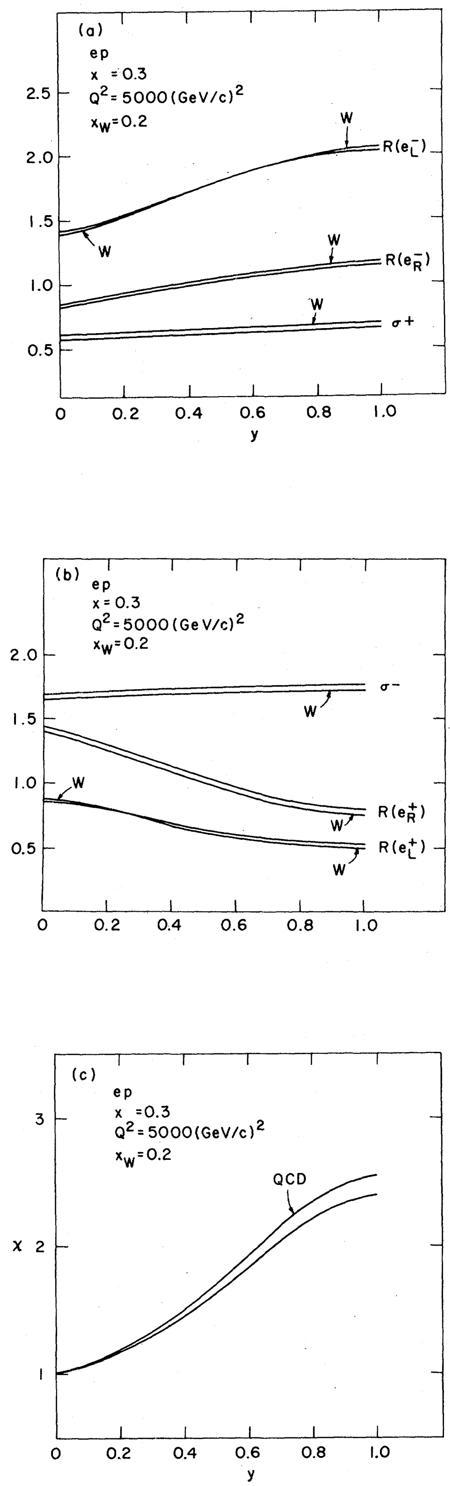


FIG. 4. (a) y dependence of $R(e^-_{L,R})$ and σ^+ at $Q^2 = 5000$ (GeV/c)^2; W=QCD prediction. (b) y dependence of $R(e^+_{L,R})$ and σ^- at $Q^2 = 5000$ (GeV/c)^2; W=QCD prediction. (c) y dependence of χ at $Q^2 = 5000$ (GeV/c)^2; W=QCD prediction.

this model seems to be ruled out by the SLAC[1] experiment, we consider it briefly for completeness. Because the electron neutral current is pure vector in this model, it is easy to see that

$$A^+ = A^- = B^+ = B^-, \quad C_L = C_R = 0\,. \tag{8}$$

Thus we need only examine one of these, A^- say, for various Q^2 and y values. In this case we find that the QCD corrections at low Q^2 are ~5% and somewhat larger (~10%) at high Q^2. We find that the QCD prediction is larger in magnitude than the non-QCD prediction for the hybrid model in contrast to the standard model. As in the standard model case, however, the shift in the distributions can be simulated by a small shift in x_W (but in the direction opposite to the standard model shift). Although QCD does shift the curve downward slightly, it is unlikely that the shift improves the poor agreement of the hybrid model with the SLAC experiment.

CONCLUSION

In this paper we have analyzed the QCD corrections to various asymmetry parameters which measure P and/or C violation in ep deep-inelastic scattering. We found that at both low and high Q^2, independent of the value of y, QCD corrections to the standard model amount to roughly 10%. These corrections can be approximately simulated by a small shift in the value of x_W for any of the mentioned asymmetries; the shifts for the various asymmetries have roughly the same magnitude and all have the same sign. At present, only the low-Q^2 region can be examined by existing experimental facilities; the high-Q^2 region must await CHEEP[10] or ISABELLE with the electron-ring option.[11]

APPENDIX

In this appendix, we give the low-Q^2 theoretical results for $A^\pm$, $B^\pm$, and $C_{L,R}$; we follow the notation used by Cahn and Gilman.[3] For low Q^2 ($\ll M_Z^2$) the denominators of $A^\pm$, $B^\pm$, and $C_{L,R}$ can be approximated by twice the electromagnetic cross section. We find

$$\frac{d\sigma(e_R^-)}{2d\sigma_{\rm em}} \simeq -\frac{G_F Q^2}{2\sqrt{2}\pi\alpha}\,\frac{\sum_i Q_i f_i(x)}{\sum_i Q_i^2 f_i(x)} \times [g_V^i(g_V^e + g_A^e) + g_A^i(g_V^e + g_A^e)F(y)]\,, \tag{A1}$$

$$\frac{d\sigma(e_L^-)}{2d\sigma_{\rm em}} \simeq -\frac{G_F Q^2}{2\sqrt{2}\pi\alpha}\,\frac{\sum_i Q_i f_i(x)}{\sum_i Q_i^2 f_i(x)} \times [g_V^i(g_V^e - g_A^e) - g_A^i(g_V^e - g_A^e)F(y)]\,, \tag{A2}$$

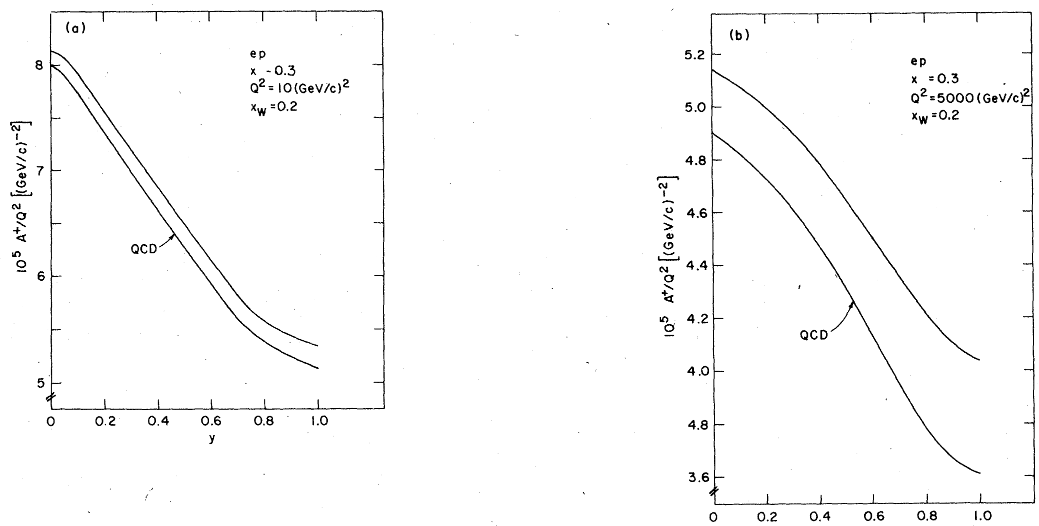


FIG. 5. (a) y dependence of A^+/Q^2 for $Q^2 = 10$ (Gev/c)^2. (b) y dependence of A^+/Q^2 for $Q^2 = 5000$ (GeV/c)^2.

where

$$g_V^{e,i} \equiv (T_{3L}^{e,i} + T_{3R}^{e,i} - 2Q^{e,i}\sin^2\theta_W)\,, \tag{A3}$$

$$g_A^{e,i} \equiv (T_{3R}^{e,i} - T_{3L}^{e,i})\,, \tag{A4}$$

$$F(y) \equiv \frac{1-(1-y)^2}{1+(1-y)^2}\,, \tag{A5}$$

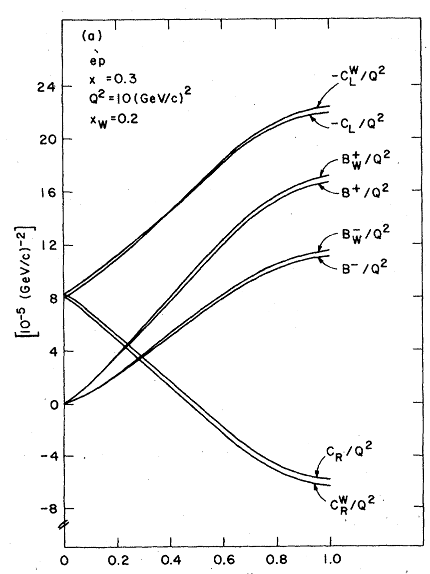
$Q_i \equiv$ charge of the ith quark with distribution function $f_i(x)$. For $d\sigma(e^+_{L,R})$ we simply let $g_A^e \to -g_A^e$ in Eqs. (A1) and (A2). Thus, we find

$$A^{\pm} \simeq \frac{G_F Q^2}{\sqrt{2}\pi\alpha}\,\frac{\sum_i Q_i f_i(x)}{(\sum_i Q_i^2 f_i(x))}\,[\pm g_A^e g_V^i - g_V^e g_A^i F(y)]\,, \tag{A6}$$

$$B^{\pm} \simeq \frac{G_F Q^2}{\sqrt{2}\pi\alpha}\,\frac{\sum_i Q_i f_i(x)}{\sum_i Q_i^2 f_i(x)}\,[g_A^i(g_V^e \mp g_A^e)F(y)]\,, \tag{A7}$$

$$C_{L,R} \simeq \frac{G_F Q^2}{\sqrt{2}\pi\alpha}\,\frac{\sum_i Q_i f_i(x)}{\sum_i Q_i^2 f_i(x)}\,[\pm g_A^e g_V^i - g_A^e g_A^i F(y)]. \tag{A8}$$

In the limit where we can neglect sea quarks and QCD corrections we find for ep deep-inelastic scattering with $f_u(x) = 2f_d(x)$:



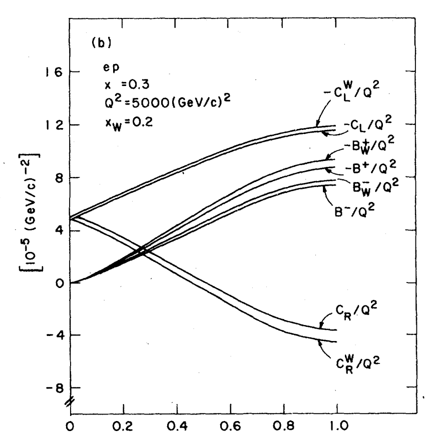


FIG. 6. (a) y dependence of $B^{\pm}/Q^2$ and $C_{L,R}/Q^2$ for $Q^2 = 10$ (GeV/c)^2; W=QCD prediction. (b) y dependence of $B^{\pm}/Q^2$ and $C_{L,R}/Q^2$ for $Q^2 = 5000$ (GeV/c)^2; W=QCD prediction.

$$A^{\pm}=\frac{G_F Q^2}{2\sqrt{2}\pi\alpha}\left[\pm(1+2T_{3R}^e)(\tfrac{5}{6}-2x_W)+\tfrac{5}{6}(1-2T_{3R}^e-4x_W)F(y)\right], \qquad (A9)$$

$$B^{\pm}=\frac{+5G_F^2Q^2}{6\sqrt{2}\pi\alpha}F(y)\times\begin{cases}-1+2x_W & (+),\\ 2T_{3R}^e+2x_W & (-),\end{cases} \qquad (A10)$$

$$C_{L,R}=\frac{G_F Q^2}{2\sqrt{2}\pi\alpha}(1+2T_{3R}^e)\left[\pm(\tfrac{5}{6}-2x_W)+\tfrac{5}{6}F(y)\right]. \qquad (A11)$$

We do not give expressions for $R(e^{\pm}_{L,R})$, $\sigma^{\pm}$, or χ here since they are quite complex.

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