Measurement of the neutron magnetic moment

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The neutron magnetic moment μ_n has been measured with an improvement of 2 orders of magnitude in experimental accuracy. The separated-oscillatory-field magnetic resonance technique was employed using slow neutrons and protons (in flowing water) in the same magnetic field. We find the ratio of neutron to proton moment to be $\mu_n/\mu_p = -0.684\,979\,35(17)$ (0.25 ppm). Expressed in Bohr magnetons μ_B this gives $\mu_n/\mu_B = -1.041\,875\,64(26) \times 10^{-3}$. Expressed in nuclear magnetons μ_N, $\mu_n/\mu_N = -1.913\,041\,84(88)$.

I. INTRODUCTION

The observation that the neutron, a neutral particle, has a nonzero magnetic moment has historically been of considerable importance in the development of nuclear and particle physics. However, prior to any explicit measurement of the neutron magnetic moment, μ_n, there was an indication that $\mu_n \neq 0$ from measurements of the proton and deuteron magnetic moments, μ_p and μ_d, for, if the neutron and proton combine in a pure ${}^3S_1$ state to form the deuteron, one would expect $\mu_d = \mu_p + \mu_n$. From this relation, early measurements of μ_p and μ_d gave $\mu_n \simeq -1.8\mu_N$ where μ_N is the nuclear magneton.

In 1938, Frölich, Heitler, and Kemmer[1] used the "meson-exchange theory" to explain the anomalous moments of the proton and neutron. According to the simple Dirac theory, μ_p should equal μ_N and μ_n should be zero. The anomalous moments are defined as the difference between the actual magnetic moments and those predicted by the Dirac theory, which assumes the neutron and proton each to be structureless spin-$\frac{1}{2}$ particles. The principal conclusion of Frölich, Heitler, and Kemmer and those who later refined the theory[2-7] was that the anomalous moments of the neutron and proton should be approximately equal in magnitude and opposite in sign.[8] This prediction agrees with experimental results to within a few percent (the current value of the "anomalous" moments being $-1.91\mu_N$ for the neutron and $+1.89\mu_N$ for the proton). The actual absolute values of the anomalous moments are much more difficult to predict using this model.

The first actual measurement of μ_n was reported by Alverez and Bloch in 1940.[9] They produced neutrons by deuteron bombardment of Be and used the neutrons in a single-coil Rabi-type magnetic resonance apparatus.[10] The neutron polarization was accomplished by transmission through magnetized pieces of Swedish Iron. The magnetic-field measurement in the resonance region was performed by comparison with the proton cyclotron frequency. The result of Alverez and Bloch was $|\mu_n| = 1.93 \pm 0.02\,\mu_N$. In general, no information about the sign of μ_n is obtainable from an experiment (such as that of Alverez and Bloch) which employs an oscillating rather than a rotating field. With an assumed negative sign for μ_n, this result was in encouraging agreement with the exchange model.

The discovery by Kellog, Rabi, Ramsey, and Zacharias[11,12] that the deuteron has an electric quadrupole moment indicated that the deuteron could not be in a pure ${}^3S_1$ state but must have some D-state admixture.[13] Thus the additivity of neutron and proton moments in the deuteron could not be exact. It was therefore of great interest to obtain a more accurate measurement of μ_n in order to compare it more carefully with the values of μ_d and μ_p.

In 1947 Arnold and Roberts[14] measured μ_n using a single oscillatory field with neutrons from a reactor. They were able to obtain a great increase in accuracy by performing the field measurement with the then novel technique of NMR. Their result was $|\mu_n| = 1.9103 \pm 0.0012 \mu_N$. Bloch, Nicodemus, and Staub[15] were also able to obtain an accurate measurement of μ_n through the employment of NMR as a field-monitoring technique. They reported their result as a ratio of neutron to proton magnetic moments and found $|\mu_n/\mu_p| = 0.685\,001 \pm 0.000\,030$.

With these more accurate values of μ_n, in combination with the more accurate determinations of μ_p and μ_d from molecular-beam experiments, it was possible to compare critically the amount of $^3D_1$ admixture in the deuteron ground state predicted from the magnitude of the deuteron electric quadrupole moment as opposed to that predicted by the nonadditivity of the magnetic moments. The gratifying result was that both procedures yielded approximately the same admixture.

In 1949, Rogers and Staub,[16] using a rotating-field rather than oscillating-field, magnetic resonance technique, were able to determine directly the sign of the neutron moment. It was negative as had been expected.

The most accurate measurement of μ_n (at the inception of the work reported here) was that performed by Corngold, Cohen, and Ramsey.[17,18] That experiment benefited from several advances in experimental techniques. The neutron source was a graphite-moderated reactor which allowed a much higher flux of thermal neutrons than was previously available. Neutron polarization was accomplished by mirror reflection from magnetic materials which yielded higher polarizations with lower losses. The separated-oscillatory-field magnetic resonance technique,[19] which allows longer resonance regions and therefore narrower linewidths, was also employed. The result of Cohen, Corngold, and Ramsey, expressed as a ratio of μ_n to μ_p, was $\mu_n/\mu_p = -0.685\,039 \pm 0.000\,017$ (25 ppm).

The principal source of error in the Cohen, Corngold, and Ramsey measurement arose from field inhomogeneities and the method of field determination employed. A small proton NMR probe was used to map the field in the resonance region by placement in several discrete locations, before and after experimental runs. It was subsequently discovered that an unfortunate selection of shimming materials had given rise to field inhomogeneities on a scale too small to be resolved by the routine field measurement technique. The major portion of the error quoted above was an *a posteriori* estimate of the effect of these inhomogeneities.[20]

With the development of the quark models, a new and highly appealing result for the nucleon magnetic moments was possible. Bég, Lee, and Pais[21] and independently Sakita[22] proposed that if the quarks have spins and if the internal-symmetry group SU(3) of the baryons is broken *only* by electromagnetism, then there exist uniquely determined ratios among the magnetic moments of the members of the baryon octet. For the neutron-proton ratio, one obtained[23] the result $\mu_n/\mu_p = -\frac{2}{3}$. It should be noted that the uncertainty in this theoretical value exceeds the discrepancy between the experimental and theoretical values.

Until the present experiment, the magnetic moment of the neutron was much less accurately determined than the magnetic moments of the proton, electron, or muon. The magnetic moments of the electron and proton and the g factor of the positive muon were all known to a fractional error of less than 3×10^{-8}, whereas the fractional error in the neutron magnetic moment was 1000 times greater, approximately 3×10^{-5}. The possibility of a substantial improvement in the knowledge of a fundamental quantity provided a strong motivation for this work.

Much of the apparatus used in this experiment had been developed for use in a program to search for a neutron electric dipole moment (EDM).[24-29] In particular, the current work arose out of the most recent experiment of Dress *et al.*[24] performed at the Institut Max von Laue–Paul Langevin (ILL) in Grenoble. The neutron-beam apparatus was originally designed with the neutron electric dipole moment exclusively in mind. It was, however, highly suitable for measuring the magnetic moment of the neutron as well. The magnetic field was low, but this potential disadvantage was more than offset by the large pole piece gap. Such a gap implies low field inhomogeneities and therefore it permits a more accurate magnetic-field determination. In the previous most accurate experiment,[18] the accuracy of the result was primarily limited by the field inhomogeneity and the consequent difficulty in calibrating the magnetic field.

Several developments have allowed the current work to obtain a substantial (2 orders of magnitude) improvement on the error in our knowledge of μ_n. Access to the cold source, a liquid deuterium moderator, at the ILL allowed the use of an intense beam of slow neutrons. This implied a narrower resonance linewidth (an improvement of approximately a factor of 5 over the previous best measurement[18]) and therefore a higher precision. The use of neutron guides along the entire length of the apparatus substantially reduced the divergence loss of neutrons resulting in higher counting rates. Most important as a source of improvement has been the method of field determination.

Following a suggestion of Purcell,[30] made some time ago, the field was monitored by obtaining a separated-oscillatory-field resonance signal from flowing water in the vicinity of the neutron beam. Indeed, with the use of neutron guides it was possible to contain the flowing liquid in exactly the same volume as that occupied by the neutron beam.

The use of a separated-oscillatory-field resonance technique with flowing water had been previously attempted by Benedek[31] and demonstrated by Sherman.[32] In contrast to these previous experiments,[31,32] the protons in the present experiment were polarized in an independent high-magnetic-field region. Furthermore, the change in proton polarization was detected by an NMR apparatus in yet another strong-field region. The use of strong polarization and detection fields, well away from the field to be measured, gave a large signal characteristic of a strong-field NMR experiment. This was true even when measuring a relatively weak unknown field.

The experiment reported here consisted, in principle, of a comparison between resonance frequencies for neutrons and protons (in water) obtained using the same separated-oscillatory-field geometry for both. A preliminary result of this work has been published[25] and a detailed description of the experiment is available in the form of an Institut Laue–Langevin internal report.[34]

We note that, as directly observed, our measurement provided the absolute value of the ratio between the resonance frequency for neutrons in vacuum to that of protons in a cylindrical sample of water. As will be shown, this experimentally determined quantity can be related, without loss of accuracy, to the ratio of the magnetic moments μ_n/μ_p.

II. APPARATUS

In this experiment, the Larmor-precession frequencies for neutrons and protons in the same magnetic field were compared. The apparatus consisted of a separated-oscillatory-field magnetic resonance spectrometer, capable of measuring almost simultaneously the neutron and proton (in water) Larmor frequencies.

For the purpose of describing the apparatus, it is convenient to consider separately the neutron source, the physical construction of the spectrometer, the neutron polarization and detection equipment, the proton polarization and detection equipment, and the electronics and computer control.

A. The neutron source

The source of neutrons for the present work was the cold source of the high-flux reactor located at the Institut Max von Laue–Paul Langevin (ILL) in Grenoble, France. The cold source is a 38-cm-diameter aluminum sphere containing liquid deuterium at a temperature of 25 °K placed in a neutron flux of approximately 3×10^{14} $n\,sec^{-1}\,cm^{-2}$. Neutrons emerging from the cold source are conducted through the biological shielding via curved neutron guides.

The present experiment used neutron guide H18 in the experimental hall at the ILL. This guide has a bend with a 25-m radius between the cold source and the guide tube exit. Such a bend acts as a filter, allowing only neutrons having a veloctiy less than some characteristic velocity to emerge from the beam port. The neutron beam from guide H18 is characterized by a typical velocity of ~ 180 $m\,sec^{-1}$.[36] The flux through the apparatus was approximately 1.5×10^{5} $n\,sec^{-1}$.

B. Physical construction of magnetic resonance region

The spectrometer is of the separated-oscillatory-field configuration, very similar in conception to the more familiar molecular-beam resonance spectrometers. Ramsey[10] has given a thorough description of the double-coil resonance technique.

The spectrometer utilizes a high stability, high homogeneity, large volume electromagnet. The magnet is a modification of the permanent magnet used in the recent neutron electric dipole moment searches.[24,25] Dress[25] *et al.* give a detailed description of the magnet's original construction. A cross section of the magnet can be seen in the inset of Fig. 1. The magnet had 244 turns of copper wire arranged in such a way as to give a minimal vertical gradient. The magnetic field was a nominal 18 G.

Measurements using a differential fluxgate, as well as those using the spectrometer itself as a diagnostic tool, indicated that the field gradients over the resonance region were less than 2 ppm cm^{-1}. The drift in the mean gradient over the resonance region was typically 1 ppm h^{-1} or less and was probably due to thermal effects.

A Plexiglass frame was inserted between the magnet pole pieces to support various components of the spectrometer. These included the separated-oscillatory-field coils, trim coils, and three glass tubes.

The oscillatory-field coils were 3 cm long and had 15 turns each. The coil separation was 61 cm. The coils were carefully wound on the Plexiglass frame so as to ensure that the axis of each coil

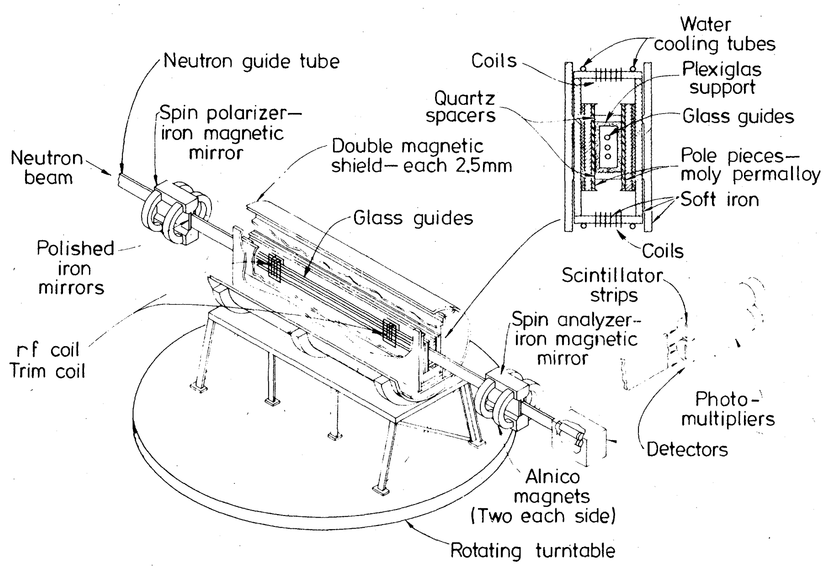

FIG. 1. Schematic view of spectrometer with detail of neutron polarization and detection equipment.

was parallel to the axis of the spectrometer.

The trim coils were small dc coils placed in the vicinity of the oscillatory-field coils to allow adjustment of the steady magnetic field near them. These trim coils were used to ensure that the field in the neighborhood of the oscillatory-field coils was equal to the mean field between the coils. A discussion of the motivation for such an adjustment will be given later.

The three glass tubes were of circular cross section with an 11 mm inside diameter. Their axes were separated by ~16 mm.

The magnet and spectrometer assembly (see Fig. 1) was mounted in a two-layer cylindrical Molypermalloy magnetic shield to reduce the effects of changing external magnetic fields. Rough measurements implied an overall transverse shielding factor of approximately 80.

The entire assembly was mounted on a large rotating platform. This allowed the orientation of the spectrometer with respect to the direction of the neutron velocity to be reversed.

C. Neutron polarization and detection

The apparatus for the polarization and detection of neutrons is identical to that employed in the most recent neutron EDM experiment. It is described in detail by Dress *et al.*[24] The neutrons leaving guide H18 were polarized by glancing reflection from a magnetized mirror. A polarization of approximately 75% was obtained. A similar magnetic mirror was employed as an analyzer. The neutrons were detected with a ^{6}Li loaded glass scintillator cemented to a photomultiplier. Individual neutron counts could be discriminated.

D. Proton polarization and detection

The water (demineralized) flow originated from a 50-l reservoir placed approximately 10 m above the spectrometer (see Fig. 2). A constant gravity head to the spectrometer was ensured by placing an overflow outlet near the top of the reservoir. The rate of refill of the reservoir was greater than the flow rate through the spectrometer, the difference flowing out through the overflow.

The water was directed from the reservoir to the proton polarizer, which was a chamber placed in a permanent magnet with a field of ≈ 2 kG. The chamber had a volume of ≈ 500 cm^3 and contained a series of baffles to ensure that each small volume of water spent approximately the same amount of time in the magnetic field. At a nominal flow rate of 80 cm^3 sec^{-1}, each small volume of water spent several seconds in the polarizer volume. This time was somewhat more than the longitudinal relaxation time for protons in water.

The polarized protons then passed through a series of hand and electrovalves which could be arranged to direct the water through either the middle tube or one of the outer tubes. When water was sent through the outer tube, it was possible to evacuate the middle tube which could then be used as a neutron guide.

After leaving the spectrometer magnet the protons entered the proton-spin analyzer. This analyzer consisted of a high-homogeneity permanent magnet with a field of approximately 4 kG. A tube containing the flowing water passed between the pole pieces of the magnet (see insert Fig. 2) and through a sensing coil of 10 turns. This sensing coil was connected to a commercial

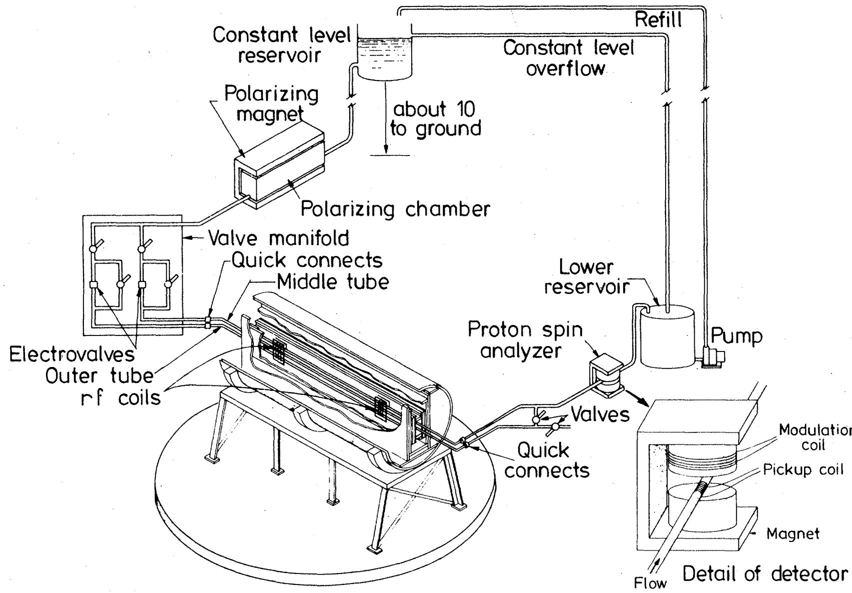

FIG. 2. Schematic view of spectrometer with detail of proton polarization and detection equipment.

"Q-meter NMR detector". An additional coil of 20 turns was wound around one of the pole pieces of the permanent magnet to allow modulation of the magnetic field.

The details of this magnetometer using the separated-oscillatory-field technique with flowing water are described by Pendlebury *et al.*[33] where particular reference is made to the apparatus described above.

It should be noted that Figs. 1 and 2 refer to the same machine with only different portions emphasized for the sake of clarity. They should be thought of as being two views of the same apparatus which must be "superimposed" to give a correct conception of the apparatus.

E. Electronics and computer control

The arrangement of the electronics equipment had two principal functions. The first was the measuring and recording of the signals which corresponded to changes in polarization of neutrons and protons in the spectrometer. The second was on-line control of the experiment. Both of these functions were carried out under the control of a PDP11/10 computer interfaced through CAMAC to appropriate electronic modules.

As was mentioned in Sec. II C., the neutron counting rate was such that individual neutrons arriving at the ^{6}Li scintillator could be distinguished. Therefore, pulses were counted which corresponded to individual neutrons. The output of the photomultiplier tube fed an Ortec 4S4 fast amplifier which in turn fed an EG G *T*2000/N fast discriminator. It was possible to adjust this electronic chain such that the background counting rate (beam off) was ≈ 1 sec^{-1} out of a total count rate $\approx 10^5$ sec^{-1}. The signal was then sent to a prescalar which adjusted the pulse shape to be compatible with a BORER 1004A scalar. This scalar was CAMAC compatible and thus the neutron count rate was available to the PDP11.

III. EXPERIMENTAL PROCEDURE

The aim of this experiment was to realize, as closely as possible, a situation in which it was possible to measure simultaneously beam type resonances for neutrons and protons in precisely the same measurement volume. If this situation were fully realized then the ratio of neutron to proton magnetic moments would be given by

$$\mu_n/\mu_p = \omega_n/\omega_p\,, \tag{1}$$

where ω_n and ω_p are, respectively, the neutron and proton resonance frequencies. From a direct measurement of μ_n/μ_p it would be possible to express the neutron moment in a variety of units through existing measurements of μ_p. It is clear, however, that the realization of a truly simultaneous measurement for neutrons and protons poses serious practical difficulties.

While it is possible to use a glass guide to direct free neutrons through the spectrometer, it is obviously not possible with free protons. One must instead use some proton rich molecule and direct it through the glass guide.

To do so with a molecular beam of proton rich molecules presents several problems as well. The space averaging by the narrow molecular beam would be different and the high velocity (typically an order of magnitude greater than the neutron velocity) which characterizes such beams would result in a broader linewidth for the proton signal, which would reduce the accuracy to which ω_p could be determined. Following the early suggestion of Purcell,[30] liquid water was selected as the medium with which to measure the proton resonance frequency.

Liquid water has several advantages which make it useful for this application. It contains a high density of protons, it can be easily contained in the glass guide tubes, it can be made to flow quite slowly which results in a narrow resonance line, and detection is quite simple using easily available commerical NMR apparatus. However, the use of liquid water as the medium in which to measure the proton frequency presents two problems.

The first problem results from the fact that the magnetic field in the local vicinity of the proton in a water sample differs from the field applied to the bulk sample. However, if the water is sufficiently pure and if the geometry is simple, corrections can be made to very high accuracy.

The second problem arises because it is clearly not possible to send flowing water and neutrons simultaneously through the same volume. This problem was overcome by making the proton and neutron resonance frequency determinations in two steps. Two different guide tubes were used in the spectrometer. One, which we call the middle tube, could be arranged to conduct either neutrons or water through the spectrometer. The second, or outer tube, conducted only water and served as a permanent field monitor.

When the middle tube conducted neutrons, it was possible to make a nearly simultaneous determination of the neutron resonance frequency in the middle tube and the proton resonace frequency in the outer tube. When the middle tube was filled with flowing water, it was possible to make a nearly simultaneous measurement determination of the proton resonance frequency in both tubes. "Nearly simultaneous," in this case, means times on the order of tens of seconds, which is short compared with times corresponding to any drifts in the spectrometer characteristics. One can express these two different comparisons as ratios, R_1 and R_2, with

$$R_1 \equiv \frac{\omega_{nm}}{\omega_{p0}}, \quad R_2 \equiv \frac{\omega_{p0}}{\omega_{pm}}, \tag{2}$$

where ω_{nm} is the neutron resonance frequency in the middle tube and ω_{pm}, ω_{p0} are the proton resonance frequencies in the middle and outer tubes, respectively. It must be emphasized that ω_{pm}, ω_{p0} correspond to frequencies measured for protons in a cylindrical sample of water at a particular temperature.

If (1) the ratio of the average field in the outer tube to the average field in the middle tube remains constant in time, (2) the presence or absence of water in the middle tube does not affect the field average over the outer tube, and (3) the protons and neutrons take the field average over the middle tube in the same way, then one can write

$$\omega_n/\omega_p(\mathrm{cyl}, \mathrm{H_2O}, \theta) = R_1 R_2\,, \tag{3}$$

where $\omega_p(\mathrm{cyl}, \mathrm{H_2O}, \theta)$ refers to the resonance frequency for protons in a cylindrical sample of water at temperature θ. As will be shown, this ratio can be related, without loss of accuracy, to the ratio of the free precession frequencies and therefore to the ratio of magnetic moments.

The first assumption, that the ratio of the average fields stays constant, or at least varies only negligibly, is quite reasonable. The magnet had large pole pieces of high permeability separated by a distance considerably larger than the tube diameter or separation.

These characteristics tend to cause any field change to manifest itself as a change in the field as a whole rather than as a local change. Furthermore, the Molypermalloy magnetic shields reduced substantially the effect of any external field change as well as "smoothing" it. The absolute field variations over times comparable to a measurement (hours) were typically a few parts in 10^6. In any event it was a simple matter to measure the size of any variations in the fields at the two tubes by looking for variations in R_2 with time. As will be discussed, no such variations were observed within experimental error.

The second assumption, that the field average at the outer tube is unaffected by the presence or absence of water in the middle tube, is not strictly correct. In fact, there is a small effect due to the finite susceptibility of water. However, the effect is small and can be calculated quite accurately. It can also be measured as will be discussed later. The third assumption, the equivalence of the field averages taken by protons and neutrons, will be discussed in some detail later.

The actual experiment thus consisted of several steps. First, the spectrometer was adjusted to ensure certain symmetries in the spectrometer operating conditions. Then the ratio R_2 was measured, followed by a measurement of R_1. Then R_2 was again measured to ensure that no drifts had occurred. Finally the symmetry of the spectrometer operating conditions was rechecked to verify that no changes had occurred in the course of running.

This procedure was also followed with the spectrometer orientation reversed with respect to the neutron velocity. In addition several systematic checks were made, as is discussed in Sec. IV.

A. Preliminary adjustment of the spectrometer

Two important adjustments to the spectrometer were necessary to reduce possible sources of systematic error in the final result. The first of these was made to ensure that the average particle velocity through the spectrometer remained unchanged when the apparatus was rotated through 180°. The aim of this rotation was to account for a certain class of frequency shifts which included those due to coil phase errors, geometrical misalignments, and the Millman effect. These will be discussed in detail later. What is essential in this procedure is that the magnitude of velocity be the same for both orientations of the apparatus.

Two different diagnostics were employed to monitor the neutron velocity. These were (1) the magnitude of oscillating current which maximized the transition probability in the resonance region and (2) the linewidth of the resonance at optimal oscillating current. Both of these are approximately proportional to the mean neutron velocity. The neutron velocity distribution could be changed slightly by minor adjustment of the orientation of the polarizing mirrors. With careful attention, both optimal current and linewidth could be made equal, to within approximately two percent, for the two spectrometer orientations.

The second adjustment was made to ensure that the magnetic field at the separated coils was equal to the mean field between the coils. This was done by comparing the Rabi single-coil resonance frequency for each coil individually with the double-coil resonance for the complete spectrometer. The proton resonance was particularly useful for this procedure due to its narrow linewidth. The fields at the separated coils could then be adjusted to bring these three frequencies into equality with the use of the trim coils already described. This could be done to within approximately 0.01 mG (0.5 Hz for the proton resonance).

B. Line-center determination technique

In the present experiment, it was necessary to determine the resonance frequency with an error that was small compared with the linewidth of the resonance. In the case of the neutron signal, the ratio of the measurement accuracy to resonance linewidth was about $1\text{–}10^4$. It is therefore

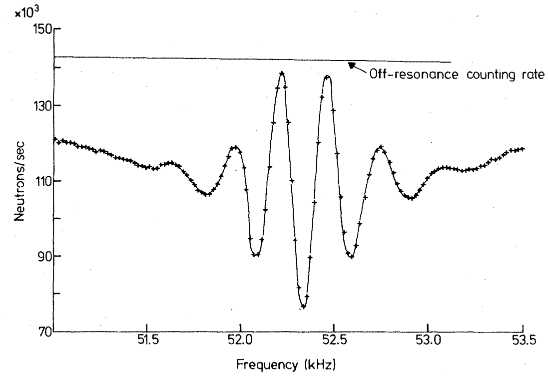

FIG. 3. Typical neutron resonance.

useful to discuss the technique by which the line centers were determined. In order to locate a resonance line to such a high accuracy it is necessary to have considerable knowledge of the resonance line shape. In the particular technique described below it is necessary to assume a symmetrical line shape. The validity of this assumption will be discussed in the section on systematic effects. Figures 3 and 4 show typical neutron and proton line shapes. It should be noted that any apparent asymmetries in Figs. 3 and 4 arise from noise or drifts over the long time period required for a complete scan.

In order to measure at the most sensitive frequency, the resonance was observed on either side of the central minima near the position of maximum (in absolute value) slope. The measurement procedure began with a careful plot of the resonance. From this plot, the full width at half maximum $\Delta\nu$, the absolute value of the slope at half maximum m, and an estimate of the central frequency ν_0 were determined. These values served as parameters in a computer operated routine which then made successive estimates of the actual central frequency.

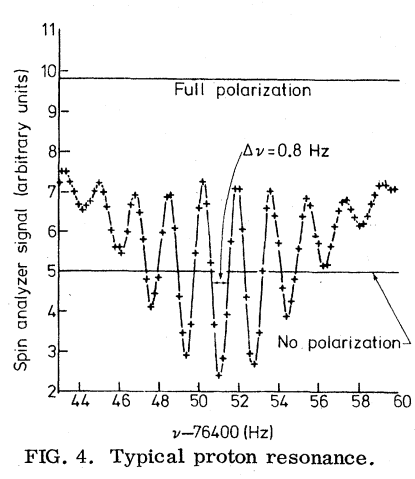

FIG. 4. Typical proton resonance.

This routine consisted of setting the frequency to $\nu_0 \pm \Delta\nu/2$, the position at which the absolute value of the slope is m. If I_+ and I_- represent the intensities of the resonance (counting rate for neutrons or NMR signal intensity for protons) then an improved approximation to the central frequency ν_0' is given by

$$\nu_0' = \nu_0 - \frac{I_+ - I_-}{2m}. \tag{4}$$

Equation (4) represents the best estimate of the central frequency to first order in $(I_+ - I_-)$. The actual line center determination consisted of a procedure in which each estimate was adjusted and a corrected value for the resonance frequency was computed from Eq. (4). In those cases where the correction in Eq. (4) was sufficiently small as to ensure that higher-order terms in $(I_+ - I_-)$ were negligible, the corrected value ν_0' was used as a measured value. In this way the data taking program was able to follow any drifts in the magnetic field. If the field drifts were too large (as evidenced by a large disparity between I_+ and I_-) particular data could be discarded.

C. Measurement of R_1 and R_2

The ratio R_1 has been defined as the ratio between the simultaneously determined neutron precession frequency in the middle tube and proton precession frequency in the outer tube. In preparation for a measurement of R_1, the middle tube was emptied of water and evacuated. A plot of the neutron resonance was then taken.

From the plot of the neutron resonance, the half width, the slope, and a nominal central frequency were taken. The half width was approximately 124 Hz. The slope m could be determined to an estimated accuracy of 10%. As will be discussed later, this error in the determination of the slope did not constitute a comparable error in the final frequency determination.

A plot of the proton resonance in the outer tube was also prepared and from it the slope, half width, and an estimate of the central frequency were made. These values, as well as those determined for the neutron case, were then used as initial values in an automatic routine which determined the resonance frequency for both the neutron and proton in accordance with the technique described in the previous section.

It was not possible to measure the proton and neutron resonances simultaneously, as the same oscillator and the same separated coils were to be used for each. As a result the measurements of the neutron and proton were alternated over periods of approximately 1 min and averaged in such a way as to minimize drifts in the magnetic field. It should be noted that suitable delays were incorporated in these procedures to allow any transients to become unimportant. A similar procedure was followed for the measurement of R_2, the ratio of the resonance frequency for water in the outer tube to that for water in the middle tube. It was not possible to make rapid alternations between measurements of R_1 and R_2 as this required a pumpdown for the middle tube in order to allow passage of the neutrons through the middle tube. A more detailed description of these procedures is given elsewhere.[34]

IV. EXPERIMENTAL RESULTS

The data used for the final determination of μ_n/μ_p were taken during a running period of about 200 h. The procedure used consisted of first measuring R_2 (the ratio of the proton frequency in the outer tube to the proton frequency in the middle tube) as described in the previous section. The middle tube was then evacuated and R_1 (the ratio of the neutron frequency in the middle tube to the proton frequency in the outer tube) was determined at each of several neutron rf powers. (The procedures for the actual measurement of R_1 and R_2 are described in the previous section.) Water was then reintroduced to the middle tube and the ratio R_2 was again measured. This procedure was followed in each of the two machine orientations. Table II summarizes the data obtained during the running period.

TABLE I. Tests for stability of R_2.

Run number	Orientation	$10^7(1-R_2)$
M1	2	3.0(1.1)
M2	2	6.0(1.7)
	Rotation	
M3	1	3.2(1.9)
M4	1	4.6(1.5)
	Rotation (twice)	
M5	1	4.8(1.3)
M6	1	4.0(0.9)
	Rotation	
M7	2	9.9(1.3)
M8	2	7.9(1.1)
	Rotation	
M9	1	2.6(1.7)
M10	1	2.0(1.4)
M11	1	4.0(1.4)
	Rotation	
M12	2	12.0(1.6)
M13	2	12.3(2.2)
M14	2	14.1(1.6)
M15	2	12.0(1.3)
M16	2	16.8(1.8)

Tests were also made to verify that the ratio R_2 did not vary with time. A summary of the results of these tests is shown in Table I.

A number of other systematic checks were also made. In one test, the cables to the rf coils were reversed and R_1 was measured. No variations in resonance frequencies were seen, as is expected by the short cable lengths. In another, the audio amplifier was capacitively coupled to the rf coils to see if any shift which could be accounted for by a dc offset in the amplifier output (which in principle did not exist) was noticeable. No shift was observed. As an additional control, the ratio of proton frequency at optimum rf power to proton frequency at $\frac{1}{2}$ rf power was measured. No variation was seen to within 2 parts in 10^7. This is to be expected as the low power delivered to the separated-oscillatory fields would not yield an observable Bloch-Siegert shift.

In an additional test, R_1 was measured with the neutron rf power at $9\times$ optimum. It was not possible to perform this measurement to high accuracy (presumably) due to effects of velocity spread. However, a large negative shift was seen, which is in accordance with the theory of the Bloch-Siegert shift as described in the following section.

It should be noted that the temperature of the water used in the proton resonances, previously defined as θ, was $(22\pm1)^\circ$C.

V. DISCUSSION

As discussed in Sec. III the directly measured quantity was the ratio of the neutron Larmor frequency to the Larmor frequency of protons in a cylindrical sample of water. From this ratio it would be possible to calculate the ratios of the magnetic moments of the neutron to proton. In this section we show that the measurement actually made corresponds to the appropriate ratio of Larmor frequencies. Section V F will be devoted to a calculation of μ_n/μ_p from the experimental ratio of the two Larmor frequencies.

The following topics will be discussed: validity of the two-tube procedures; field inhomogeneities, Bloch-Siegert effect, and coil phase errors; effects due to velocity distribution; line-center determination technique; miscellaneous small effects; and calculation of results.

A. Validity of the two-tube procedure

In Sec. III it was noted that if three conditions could be met, then the product $R_1R_2=\mu_n/\mu_p$ (cyl, H_2O, θ).

The first condition concerned the constancy of the ratio of the average field at the outer tube to that at the middle. As was mentioned, this is certainly not an unreasonable assumption and, furthermore, it is supported very strongly by the experimental results given in Table II.

The second condition dealt with the effect which filling the middle tube with water has on the field average at the outer tube. This could give rise to a slight error, for when R_2 is being measured the middle tube contains water whereas when R_1 is being measured the middle tube contains no water. Since water has a finite diamagnetic susceptibility, the presence or absence of water in the middle tube will slightly change the field at the outer tube.

The geometry is quite simple in this case, a cylinder (assumed to be infinite in length as it extends far beyond the region over which the field is averaged) in a field perpendicular to its axis. If the magnitude of the external field is H then the change in the mean field $\langle\Delta H\rangle=\beta H$, where $\beta=-2\pi^2r^2\kappa/R^2$ and r is the tube radius (assumed to be the same

TABLE II. Data used in extrapolation in Fig. 5.

Run number	Orientation	Neutron rf power	R_1	$10^7(1-R_2)$
M1	1	···	···	9.6(1.0)
M2	1	···	···	9.7(2.1)
M3	1	···	···	9.7(1.2)
PN1	1	0.80	0.684 999 56(15)	···
PN2	1	0.60	0.684 998 71(17)	···
PN3	1	0.40	0.684 997 80(19)	···
PN4	1	1.00	0.685 000 54(10)	···
MT4	1	···	···	9.4(1.7)
MT5	2	···	···	6.0(1.0)
PN5	2	1.00	0.685 003 08(16)	···
PN6	2	0.80	0.685 001 44(16)	···
PN7	2	0.60	0.685 000 37(17)	···
PN8	2	0.40	0.684 999 39(17)	···
MT6	2	···	···	6.0(13)

for both tubes), κ is the volumetric susceptibility of water, and R is the separation between the axes of the two tubes. In our case, $\beta = -4.8(0.3) \times 10^{-7}$ where the uncertainty is given by an estimate of the slight modification to the field due to the presence of the μ-metal pole pieces. This estimate was made on the basis of a simple argument involving magnetic "images." In order to account for this effect one must multiply the measured frequency ratio by the quantity $(1+\beta)$ in order to have a correct measurement of ω_n/ω_p (cyl, H_2O, θ).

As an experimental check on the magnitude of β, a test was made to measure the effect of a nearby tube filled with water on the ratio R_2. This "third" tube was located on the opposite side of the "middle" tube from the "outer" tube, and could be either empty or filled with water. The difference in R_2 with and without water in this third tube was found to be

$$R_{2,\,\mathrm{air}} - R_{2,\,\mathrm{H_2O}} = 3.8(1.1) \times 10^{-7} (\text{experimental}),$$

where

$$R_{2,\,\mathrm{H_2O}} = R_2 \text{ with water in third tube},$$

and

$$R_{2,\,\mathrm{air}} = R_2 \text{ with air in third tube}.$$

It should be noted that the effect of the finite susceptibility for air is negligible.

It would be expected that this difference would be due to differences in the value of the correction term β for different values of R, and, in fact, it agrees well with the theoretical value for that difference which is

$$R_{2,\,\mathrm{air}} - R_{2,\,\mathrm{H_2O}} = 3.6(0.2) \times 10^{-7} \text{ (theoretical)}.$$

The contribution of the thin glass tubes (which are stationary) on the differences in the values of R_2 is only of second order in the magnetic susceptibilities and can be neglected. The changes in the external field due to the proton polarization in the water are negligibly small.

The third condition required for the two tube procedures to be valid is that the field average taken by the protons and neutrons is the same. We first note that the average field gradient in the magnet (transverse to the flow) was less than 1 part in 10^6 per cm. Since the tube inner diameter was 1.1 cm, it follows that the degree to which the field averaging need be the same for protons and neutrons in order to have a negligible effect on the final error is not great (the final error quoted is 0.25 ppm). In fact the field averaging is equivalent for the neutrons and protons to a very high degree (see Pendlebury *et al.*[33] for a discussion of this point).

There are a number of conditions which, if satisfied, imply that the field average taken by the particles through a separated coil resonance spectrometer corresponds to a true volumetric field average over the volume occupied by the particles (see Pendelbury *et al.*[33] for a discussion of this problem with particular attention paid to the case for flowing liquid). We shall assume that the particles fill the volume uniformly. This condition is certainly satisfied by the flow of an incompressible liquid through a tube and should be satisfied by neutrons moving in a guide (if one observes far from any bends or discontinuities in the guide).

Given uniform filling, if either (1) the velocity distribution is independent of position or (2) the velocity distribution is characterized by a very narrow range of velocities,[33] then the field average will be a correct volumetric one.

Condition (1) is satisfied for neutron transmission through a long straight guide. For the water flow the velocity distribution is quite narrow (see the section on velocity distribution) approximately satisfying condition (2). It should be noted as well that even if condition (2) is not satisfied, an error will arise only if there is a fortuitous combination of spatial velocity distribution and field inhomogeneity. For example, in the case of a cylindrically symmetric flow pattern (observed to be valid for "steady turbulent flow") and a field characterized by only a first-order spatial variation (the major inhomogeneity over distances small compared with the pole piece separation) there will be no deviation from a correct volumetric average.

In the current experiment, since the conditions for accurate field averaging are satisfied and since the field gradients are very small to begin with, we believe that effects of field averaging are negligible (i.e., less than 1 part in 10^8).

B. Field inhomogeneities, Bloch-Siegert effect, and coil phase errors

If the Rabi resonance frequency at each of the two separated coils differs from the mean Larmor frequency in the region between the coils, there will be a shift in the value of the observed resonance frequency. This effect has been discussed by Ramsey,[10] Shirley,[37] and Code.[38]

Following the discussion of Code[38] we let $\bar{\omega}$ be the mean Larmor frequency in the region between the separated coils and ω_0 be the Rabi resonance frequency at each of the two separated coils. We assume them to be equal as is convenient for estimating the maximum error in the case of a static inhomogeneity, and describes as well the situation caused by the Bloch-Siegert effect. We let ω be the frequency applied to the separated-oscillatory fields. The resonance frequency ω_R can be determined by differentiating the transition probability $P(\omega)$ and setting the result equal to zero. That is,

$$\frac{\partial}{\partial\omega} P(\omega)\bigg|_{\omega=\omega_R} = 0 .$$

Using the expression for $P(\omega)$ given by Ramsey,[10] Code has shown that the resonance frequency can be given by

$$\omega_R = \bar{\omega} + K(\chi)(\omega_0 - \bar{\omega}) , \qquad (5)$$

with

$$K(\chi) = l \tan\chi/(L + 2l \tan\chi) .$$

In Eq. (5) χ is defined by $\chi = (\frac{1}{2} at)$ with $a = [(\omega_0 - \omega)^2 + (2\gamma H_1)^2]^{1/2}$, H_1 is the magnitude of the oscillating fields, γ is the gyromagnetic ratio of the particle of interest, and t is the time spent by the particle in each of the oscillating-field regions. It should be noted that near resonance χ is simply $\frac{1}{2}$ the expectation value of the angle rotated in the coil. l and L have been defined previously as the coil lengths and coil separation respectively.

As was mentioned previously, an essential part of the initial adjustment of the spectrometer was devoted to ensuring that $\omega_0 \approx \bar{\omega}$. This was done by adjusting the field in the vicinity of the separated coils by the use of two trim coils. This adjustment was performed by observing ω_0 and $\bar{\omega}$ with the proton resonance. (It should be noted that the Bloch-Siegert effect and the effect of coil phase errors were negligible for the protons.)

We note that even if $\omega_0 \neq \bar{\omega}$ there will be no shift, at optimal oscillating power, in the ratio between the neutron and proton resonance frequencies, for at optimal power $K(\chi_p) = K(\chi_n)$, where χ_p and χ_n are the values of χ for protons and neutrons respectively. However, in order to account for the Bloch-Siegert shift, it was necessary (as described below) to vary the oscillating-field intensity for the neutrons in a systematic way. Under these circumstances one would expect an error in the frequency ratio of approximately $|K(\chi_n) - K(\chi_p)|(\omega_0 - \bar{\omega})$. In the worst case, this term had a value less than 7 parts in 10^8. As is discussed in the section on experimental error, this quantity must be treated as a possible source of error in the measurement of μ_n/μ_p.

The question of accounting for the power-dependent Bloch-Siegert effect in the separated-oscillatory-field technique has been discussed by several authors.[10,37,38,39] With particular reference to the current experiment, Greene[39] has described an extrapolation technique based on the theory developed by Code and Ramsey.[38] This procedure involves measuring the resonance frequency at a number of oscillatory powers and extrapolating, using an expression equivalent to Eq. (5), to zero power. By incorporating reversals of the spectrometer it is also possible to account for coil phase errors or slight mechanical misalignment.

Figure 5 (taken from Greene[39]) shows the extrapolation procedure. The intercept corresponds to a frequency ratio of 0.684 996 31. We note that this value does not include the effects of finite

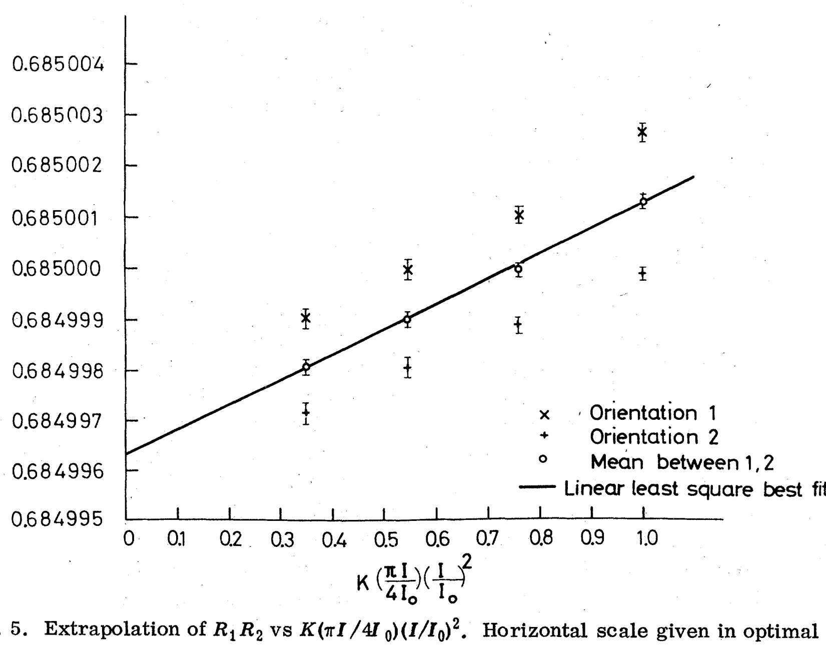

FIG. 5. Extrapolation of R_1R_2 vs $K(\pi I/4I_0)(I/I_0)^2$. Horizontal scale given in optimal units.

velocity distribution which will be considered in the next section.

C. Effects due to velocity distribution

One of the virtues of the Ramsey double-coil resonance technique is that the resonance frequency is relatively independent of the particle velocity. There are, however, some possible sources of error due to finite velocity distribution.

It is useful, first of all, to make some estimate of the velocity distributions for the particles used in the present experiment. Unlike those in molecular beam experiments using effusive sources,[10] the velocity distributions of the neutrons and protons used in the present experiment cannot be deduced theoretically with any confidence. Fortunately, accurate determinations of the distributions are not necessary, and an approximate estimate is available from the qualitative features of the Ramsey double-coil pattern.

We note that the number of subsidiary minima which appear on the Ramsey pattern is dependent on the velocity spread of the particles which yielded the pattern. In the limit of a single velocity for all particles, one would expect many such minima, the only limitation on the number of subsidiary resonances then being the width of the single-coil resonance envelope. This limitation would correspond to roughly L/l side lobes. Since we in general have far fewer than L/l side lobes, we conclude that the primary mechanism for the degradation of the Ramsey signal is finite velocity spread.

Let n be the number of observed subsidiary minima. Then at the frequency which would correspond to the $(n+1)$th minimum, coherence has been lost between particles whose velocities differ by Δv, the velocity spread of the spectrum. Let λ_1 be twice the Ramsey linewidth associated with particles of velocity $\overline{v}+\Delta v/2$. We have let $\overline{v}$ be the mean velocity of the particles. The condition for lost coherence is that the phases of the Ramsey pattern for the fast and slow particles be opposite.[40] Very roughly, this becomes

$$(n+1)\lambda_2 \approx (n+\tfrac{1}{2})\lambda_1 . \tag{6}$$

Noting that λ is proportional to v, we can conclude that

$$\frac{\Delta v}{\overline{v}} \approx \frac{1}{2n+1} . \tag{7}$$

Equation (7) is merely a rough estimate. It should, however, be approximately correct. For the neutron and proton velocity distributions, Eq. (7) gives $\frac{1}{7}$ and $\frac{1}{11}$ respectively for $(\Delta v/\overline{v})$.

A possible shift in the observed resonance frequency will arise from the finite velocity spread due to the nonlinearity in the factor $K(\chi)$ in Eq. (5). In order to estimate the size of this effect we follow a procedure similar to that described by Code and Ramsey,[38] except that we now include a finite velocity distribution. It is convenient to define the distribution $h(t)$, the probability that a particle will spend time t in each of the separated oscillatory regions. The function $h(t)$ clearly contains the same information as the velocity distribution. We find ω_R by solving the following equation:

$$0 = \frac{\partial}{\partial\omega}\int_{\text{all } t} P(\omega, t')h(t')dt' \bigg|_{\omega=\omega_R} \tag{8}$$

If one assumes a narrow velocity distribution and evaluates Eq. (8) near resonance, one finds that the resonance frequency ω_R can be given by

$$\begin{aligned}\omega_R - \overline{\omega} = (\omega_0 - \overline{\omega}) \\ \times\left[K(\overline{x}) + \frac{l}{L}\left(\frac{\Delta t}{L}\right)^2\left(1 - \frac{\tan\overline{\chi}}{\overline{\chi}} + 2\tan^2\overline{\chi}\right)\right. \\ \left. + O\left(\frac{l}{L}\right)^2\left(\frac{\Delta t}{t}\right)^2\right] ,\end{aligned} \tag{9}$$

where $\overline{t}$ is the mean of $h(t)$, $\overline{\chi}=\frac{1}{2}a\overline{t}$, and Δt is the second central moment of $h(t)$.

The first two terms in Eq. (9) correspond to the result of Code and Ramsey.[38] The third term gives rise to an additional shift which is not completely accounted for in the extrapolation technique used in the previous section. The higher-order terms do not contribute noticeably in this experiment.

The effect of this shift due to a finite velocity distribution, when included in the extrapolation technique of the previous section, gives a proportional shift in the frequency ratio $|\mu_n/\mu_p(\text{cyl}, H_2O, \theta)|$ of -0.14 ppm.

We note that this shift may be slightly in error due to an imprecise determination of the second central moment of the velocity distribution. Furthermore, this shift is too small to be explicitly observed with our apparatus. We therefore assign a generous error, equal in magnitude to the shift of 0.14 ppm to be included in our final experimental error.

When this shift is combined with the extrapolated result of Sec. V B, the result of Sec. V B is adjusted to a ratio of 0.684 996 21.

D. Discussion of line-center determination technique

In considering the implications of the line-center determination technique, a few remarks are in order pertaining to the question of line symmetry.

We have discussed the effect of a field difference between the rf and interference regions of the spectrometer. This difference gives a shift to the center of the double-coil resonance minimum within the single-coil envelope. There will be a slight effect on the line shape which can be viewed as arising from the now noncentral placement of the double-coil resonance within the single-coil resonance. One can imagine that the two sides of the double-coil resonance are therefore unequally "weighted" by the single-coil envelope.

The shift has been calculated on the basis of the known effects, equivalent to field inhomogeneities. We estimate an asymmetry on the order of $(l/L)^4$ which implies a few parts in 10^6. Since the final error we quote corresponds to "splitting" the neutron line to 1 part in 10^4, one would not expect any degradation in accuracy due to this effect. The proton line being approximately 100 times narrower presented no problem from the point of view of line symmetry.

Field inhomogeneities within the rf coils, however, could distort the single-coil envelope and thus alter the symmetry of the double-coil pattern. No such asymmetries of the single coil were seen. Furthermore, one would not expect to see such distortions unless the field variations (expressed as frequencies) were large compared with the single-coil linewidth.[10] The proportional field variations within the coil were of order 10^{-6}, which corresponds to less than 10^{-2} Hz. Data were rejected if the initial guess for the central frequencies ν_0 used in the determination of R_1 and R_2 differed from the derived result by more than 0.2 Hz.

The error in the determination of the resonance slope m does not give a corresponding error in frequency determination since the correction terms $(I_+ - I_-)/2m$ is in general on order of only 0.1 Hz. Thus an error in m of 10% would give rise to an error of order 0.01 Hz (a few parts in 10^7). Furthermore, since the field was always drifting (drift rates were approximately 0.2–0.3 Hz/h), a given data set consists of many determinations in which the correction term was positive or negative. This tends to reduce further any error due to a poor initial estimate of the slope.

E. Miscellaneous small effects

We mention, briefly, some small effects relevant to the separated-oscillatory-field technique which do not contribute shifts or errors of significant size.

The separated-oscillatory-field method, in the limit of infinitesimally short oscillatory-field regions eliminates first-order Doppler shifts and broadenings. However, in the case of oscillatory-field regions of finite length, a slight fractional broadening of the resonance line will occur $[\approx l\,\Delta v/(LC)]$. For the current experiment this will be largest for the neutron case but even there it is negligible, being approximately 10^{-9}. The second-order Doppler effect will shift the resonance frequency but the fractional shift is of order $(v/c)^2$ and therefore is negligible.

A frequency shift can arise due to spectral impurities in the signal applied to the separated oscillatory coils. The oscillator and amplifier used were rated as having less than 70 dB for harmonic distortion. The maximum error which might be expected due to distortion would be substantially less than one part in 10^8. The flat response of the oscillator and amplifier also ensured that no detectable shifts would occur due to changes in signal amplitude on opposite sides of the resonance.

It is conceivable that a shift would occur if the flowing water were slightly charged. This would give rise to a small magnetic field. However, even if such an effect were present it would change sign on rotation of the spectrometer and therefore be eliminated by the data analysis procedure.

Since the neutrons may be viewed as spending a small amount of time actually in the glass of the guide tube during a given bounce, a slight wall shift might be expected due to the diamagnetism of the glass. This effect is substantially diminished by the short duration of such bounces and the long time between bounces and it is entirely negligible in this measurement.

F. Calculation of experimental results

The extrapolated value from Sec. V B., when corrected for by the small shifts described in Secs. V A and V C, is

$$|\omega_n/\omega_p(\mathrm{cyl}, H_2O, 22^\circ)| = 0.684\,995\,88 \pm 0.000\,000\,16 \ (0.24\ \mathrm{ppm}). \quad (10)$$

The error on the result in Eq. (10) corresponds to the total error due to statistical uncertainty (0.17 ppm), errors arising from the correction for finite velocity distribution (0.14 ppm), uncertainties due to possible field inhomogeneities (0.07 ppm) for each of two independent orientations (therefore net error of 0.05 ppm), and an uncertainty in the correction due to the presence or absence of water in the middle tube (0.03 ppm).

VI. CALCULATION OF μ_n

It is possible, using previously determined physical quantities, to use the result of Eq. (10) to express μ_n in a variety of units. Most of these quantities involve the magnetic characteristics of water. Therefore it is necessary to demonstrate that the demineralized water used in the actual measurements in fact had the properties of distilled water. That is, we need to be sure that no impurities were introduced into the water during the course of the experiment.

A comparison with distilled water was carried out using a high-precision Perkin-Elmer R32 NMR spectrometer at the University of Sussex. A typical spectrum, obtained using two samples of water actually used in the final data, as well as a sample of distilled water, is shown in Fig. 6. Resonances from all samples observed showed no differences in resonance frequencies to better than 1 part in 10^8.

From the result of Eq. (10) the magnetic moment of the neutron in Bohr magnetons can be calculated using the relation

$$\frac{\mu_n}{\mu_B}=\frac{\omega_n}{\omega_p(\mathrm{cyl},\mathrm{H_2O},22^\circ)}\,\frac{\omega_p(\mathrm{cyl},\mathrm{H_2O},22^\circ)}{\omega_p(\mathrm{sph},\mathrm{H_2O},22^\circ)}\times\frac{\omega_p(\mathrm{sph},\mathrm{H_2O},22^\circ)}{\omega_p(\mathrm{sph},\mathrm{H_2O},35^\circ)}\,\frac{\mu_p'}{\mu_B}, \tag{11}$$

where the notation "sph" denotes a spherical sample, μ_p' is the effective proton moment in a spherical sample of H_2O at 35°C as measured by Philips, Cook, and Kleppner,[41] and the sign is that determined by Rogers and Staub.[16]

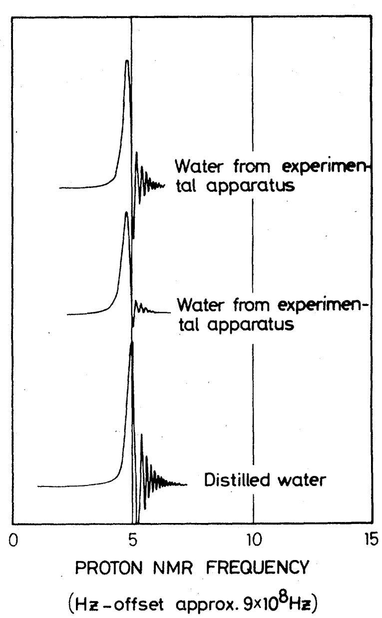

FIG. 6. Comparison between NMR signal from experimental water and distilled water. Vertical scale in arbitrary units.

The first ratio in Eq. (11) is our experimental result. The second ratio is given by $1-(2\pi K/3) = 1+1.505(2)\times10^{-6}$ from the data summarized by Pople *et al.*[42] where K is the volumetric susceptibility of water. The third ratio is taken to be $1+1.4(1)\times10^{-7}$ from the results of Hindman[43] as interpreted in a footnote in Philips *et al.*[41] With these results, the ratio of the neutron magnetic moment to the Bohr magneton is given by

$$\mu_n/\mu_B=-1.041\,875\,64(26)\times10^{-3}\ (0.25\ \mathrm{ppm})\,. \tag{12}$$

Using the previously determined values of μ_e/μ_B given by Winkler, Kleppner, Myint, and Walther,[44] the neutron moment can be expressed in terms of the free electron moment μ_e and free proton moment μ_p as

$$\mu_n/\mu_e=1.040\,668\,84(26)\times10^{-3}\ (0.25\ \mathrm{ppm}) \tag{13}$$

and

$$\mu_n/\mu_p=-0.684\,979\,35(17)\ (0.25\ \mathrm{ppm})\,. \tag{14}$$

We note that the result of Eq. (14) lies outside the limit of error quoted by Cohen, Corngold, and Ramsey[18] for the previous measurement of μ_n/μ_p. This is probably due to a failure to account adequately for certain unexpected field inhomogeneities in the previous work.[20]

Our result can also be expressed in terms of the nuclear magneton μ_N. However, in this case the accuracy is slightly degraded due to an uncertainty in the electron to proton mass ratio. Using the value of m/M obtained by combining results from Philips *et al.*[41] and Cohen and Taylor,[45] we find

$$\mu_n/\mu_N=-1.913\,041\,84\,(88)\ (0.45\ \mathrm{ppm})\,. \tag{15}$$

We note that the results reported above here differ by −0.14 ppm from our previous reported result appearing in Greene *et al.*[35] This is due to a more careful treatment of the velocity-distribution effect discussed in Sec. V E. We also note that this shift lies well within the error quoted previously.

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