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Fast-meson production and the recombination model

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Multiquark structure functions for the proton have been formulated assuming a statistical distribution of sea partons as in the Kuti-Weisskopf model. An alternative form for the recombination function has been obtained by imposing a cutoff on the rapidity gap of combining quarks. These multiquark structure functions and the recombination function have been used, within the framework of the recombination model of Das and Hwa, to discuss single-meson inclusive distributions, in the fragmentation region of the proton, in p-p collisions.

I. INTRODUCTION

The inclusive production, in hadron collisions, of fast mesons with small transverse momentum, p_T, has aroused considerable experimental[1-3] and theoretical[4-11] interest in recent years. It has been observed by Ochs[4] that the longitudinal momentum distribution of π^+, for example, produced in the fragmentation region by p-p collisions is quite similar to the valence u-quark distribution in a proton. The fragmentation of the leading quark gives a pion distribution that is steeper than that of the valence quark and contributes only a few percent to pion production.[5] This led Das and Hwa[5] to propose the recombination model for single-meson production within the framework of the quark-parton model. In this framework a fast meson with small p_T is formed in p-p collisions by a two-step process [see Fig. 1(a)]; a process by which a quark-antiquark pair is picked out from a proton followed by the recombination of this pair to form the produced meson. The inclusive distribution of π^+, for example, is therefore given by

$$\frac{x}{\sigma_T}\frac{d\sigma^{\pi^+}}{dx}=\int f_{u\bar{d}}(x_1,x_2)R(x_1,x_2;x)dx_1dx_2\,, \qquad (1)$$

where x_1, x_2, x are the fractions of the proton momentum carried by the u, $\bar{d}$, and π^+, respectively. $f_{u\bar{d}}(x_1,x_2)$ is the two-quark structure function, i.e., the probability density of finding simultaneously within the proton two partons $u,\bar{d}$ with momentum fractions x_1 and x_2, respectively. We incorporate momentum conservation for the process by writing the recombination function as

$$R(x_1,x_2;x)=\tilde{R}(x_1,x_2)\delta\left(\frac{x_1}{x}+\frac{x_2}{x}-1\right). \qquad (2)$$

Here, $\tilde{R}(x_1,x_2)$ is the recombination probability for the u and $\bar{d}$ quarks to form the π^+. In Eq. (2) we have ignored the possibility of many-body recombination. In order to exclude the contribution from wee partons,[12] which mainly produce pions in the central region, we impose the conditions $\tilde{R}(x_1,0)=\tilde{R}(0,x_2)=0$. Equation (1) involves two unknown functions, viz. $f_{u\bar{d}}(x_1,x_2)$ and $\tilde{R}(x_1,x_2)$.

In this paper we propose a form of the recombination function based on a point of view different from the one adopted by Das and Hwa.[5] Our choice is based on the assumption that the recombination probability falls off sharply when the rapidity gap between the combining quarks is greater than a preassigned value. This view is advantageous particularly for the consideration of recombination of more than two quarks, to form, for instance, a baryon. We use the Kuti-Weisskopf method[13,14] to formulate the multiquark structure functions that appear in Eq. (1). This formalism has distinct advantages when more complicated processes such as multimeson production, baryon production, etc., are considered.

We consider single-meson inclusive production as an input to determine the nature of the recombination function and of the multiquark structure functions. However, the fit is nontrivial in the sense that it is not *a priori* obvious that our prescription would reproduce the experimental

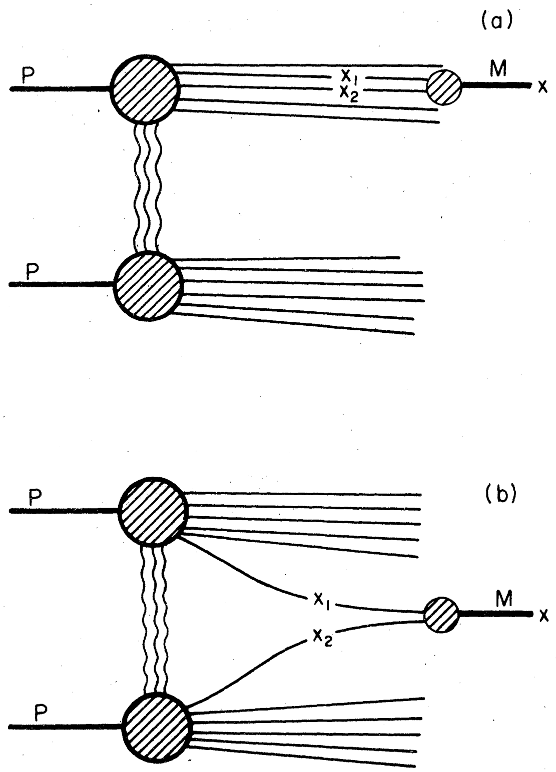

FIG. 1. The quark recombination mechanism when the recombining quarks are from (a) the same hadron, (b) different hadrons.

data. We use the results obtained here for the calculation of other processes in a forthcoming paper.

In Sec. II we suggest an alternative form for the recombination function. The modified Kuti-Weisskopf[13] model used for calculating the multiquark structure functions is discussed in detail in Sec. III. The details of the calculation are relegated to the Appendix. The parameters that appear in the structure functions have been chosen so as to reproduce the inclusive deep-inelastic *ep* and *en* scattering data. Single-meson inclusive production is discussed in Sec. IV. Concluding remarks are presented in Sec. V.

II. THE RECOMBINATION FUNCTION

In the absence of any viable theoretical model for the recombination function, it may be of interest to formulate the same from a point of view different from the one adopted by Das and Hwa.[5]

In general, the two-body recombination function should satisfy the conditions $\tilde{R}(x_1, 0) = \tilde{R}(0, x_2) = 0$, so as to exclude the contribution from wee partons. The combination of these with other quarks mostly gives rise to mesons in the pionization region and so does not contribute to the production of mesons with $x \geq 0.5$. Also, $\tilde{R}(x_1, x_2)$ being the probability for two quarks to form a meson satisfies $\tilde{R}(x_1, x_2) \leq 1$.

Das and Hwa (DH)[5] have proposed the following form for the recombination function:

$$\tilde{R}^{\mathrm{DH}}(x_1, x_2) = 4\alpha_M \frac{x_1 x_2}{x^2}, \quad \alpha_M \leq 1. \tag{3}$$

This satisfies all the conditions mentioned above. Equation (3) may be justified by using the quark-counting rule for the valence quarks in the produced meson.

Here, we propose an alternative form for $\tilde{R}(x_1, x_2)$. It is reasonable to suppose that the recombination probability for any pair of quarks falls off when they have very different longitudinal rapidities. (We recall that longitudinal rapidity is the variable that weighs the longitudinal phase space uniformly.) We realize this rapidity cutoff by choosing for the recombination function a Gaussian distribution in the rapidity gap, $y_1 - y_2$, of combining quarks. We expect this distribution to have a width of order unity. However, this criterion does not eliminate recombination between a wee and a nonwee parton since the use of the rapidity variable weighs the whole of the phase space uniformly. The elimination of wee quarks from the recombination process can be achieved most naturally if we recognize that although we have assumed a longitudinal approximation, there is, in practice, a small transverse momentum, p_T, carried by the partons. We therefore parametrize the recombination function as

$$\tilde{R}(x_1, x_2) = \alpha_M \left[\frac{|x_1|}{(x_1{}^2 + x_T{}^2)^{1/2}} \frac{|x_2|}{(x_2{}^2 + x_T{}^2)^{1/2}}\right] \times \exp\left(-\frac{1}{\Delta^2}(y_1 - y_2)^2\right). \tag{4}$$

Here, x_T is the transverse quark mass measured as a fraction of the proton momentum. The factor within the square brackets may be regarded as a correction to the longitudinal approximation for the phase space. Again, $\alpha_M \leq 1$.

Consideration of the quark rapidity gap as a measure of recombination probability leads to two distinct processes for meson formation, viz., (a) the recombination of two quarks from the same hadron [see Fig. 1(a)], and (b) the recombination of two quarks, one from each hadron [see Fig. 1(b)], to form the meson. Only the first of these processes has been considered by Das and Hwa.[5] For the process (a) the rapidity gap can be written in terms of the Feynman variable as

$$y_1 - y_2 = \ln\left[\frac{x_1 + (x_1^2 + x_T^2)^{1/2}}{x_2 + (x_2^2 + x_T^2)^{1/2}}\right]. \tag{5}$$

It is worth noting that the recombination function obtained using Eq. (5) depends only on the scaled value $\xi_i = x_i/x$ in the limit $x_T \to 0$. In this limit, we obtain

$$\tilde{R} = \alpha_M \exp\left[-\frac{1}{\Delta^2}\left(\ln\frac{x_1}{x_2}\right)^2\right]$$
$$= \alpha_M \exp\left[-\frac{1}{\Delta^2}\left(\ln\frac{\xi_1}{\xi_2}\right)^2\right]. \tag{6}$$

This form of the recombination function, with the momentum conservation condition $\xi_1 + \xi_2 = 1$, has been shown in Fig. 2 for several values of Δ. It is interesting to observe that the form in Eq. (6) with $\Delta = 2$ is very nearly the same as $\tilde{R}^{\rm DH}$ over the domain that excludes the boundary of ξ_1. The approach to zero of the recombination function at the boundary governs the details of the manner in which the wee partons are excluded from the recombination process.

For the process (b) the rapidity gap is written as

$$|y_1 - y_2| = \ln\{[|x_1| + (x_1^2 + x_T^2)^{1/2}]$$
$$\times [|x_2| + (x_2^2 + x_T^2)^{1/2}]/x_T^2\}. \tag{7}$$

The recombination probability for this process depends on both Δ and x_T besides the overall normalization α_M. From Eq. (4) and Eq. (7) we observe that the recombination probability for this process vanishes in the limit $x_T \to 0$. In general, the term $1/x_T^2$ in Eq. (7) causes the recombination of quarks from different protons to be considerably suppressed.

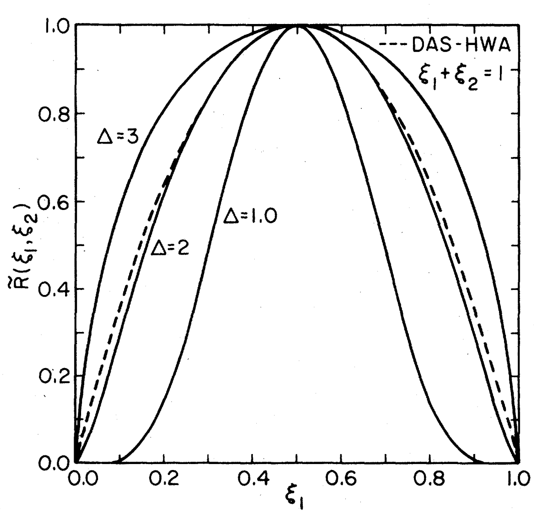

FIG. 2. The recombination function $\tilde{R}(\xi_1, \xi_2)$ with $\xi_1 + \xi_2 = 1$ for various values of Δ. The dashed curve is the recombination function used in Ref. 5 ($R^{\rm DH} = 4\xi_1\xi_2$).

III. MULTIQUARK STRUCTURE FUNCTIONS IN THE KUTI-WEISSKOPF MODEL

A. Formalism

For the sake of definiteness, we consider a proton with its three valence quarks u, u, d together with sea quarks and gluons. Following Kuti and Weisskopf[13] the sea quarks and gluons, which are collectively referred to as sea partons, are considered indistinguishable, and are assumed to be distributed statistically. The partition function for the parton distribution within a proton is, therefore, written as

$$\Phi = Z \int_0^1 dx_1 dx_2 dx_3 \Phi^V(x_1, x_2, x_3) \Phi^S(x_1 + x_2 + x_3). \tag{8}$$

We assume that

$$\Phi^V(x_1, x_2, x_3) = v_{u_1}(x_1) v_{u_2}(x_2) v_d(x_3). \tag{9}$$

Since the sea partons are statistically distributed, we have

$$\Phi^S(x) = \sum_{\{h_a = h_{\bar{a}}\}=0}^{\infty} \left(\prod_a \frac{1}{h_a!} \prod_{i=0}^{h_a} \int_0^1 dz_i^a S_a(z_i^a)\right)$$
$$\times \delta\left(1 - x - \sum_{i,a} z_i^a\right), \tag{10}$$

where a runs over the flavors of the sea partons, i.e., $a = u, d, s, \bar{u}, \bar{d}, \bar{s}, g$. $v_i(S_a)$ is the probability for finding a valence quark of type i (a sea quark labeled by index a) inside the proton *in the absence of any correlation*. These will be referred to as the primitive density functions. The explicit inclusion of the δ function in Eq. (10) takes into account the momentum conservation for all kinematic configurations and introduces a correlation among quarks and gluons. In writing Eq. (9) we have assumed that there are no other correlations amongst the valence quarks. We have also ignored the contributions from the charm and other quark seas. These can be incorporated without any difficulty.

The structure function for m valence ($m \leq 3$) and n sea partons can be written in terms of the functional derivative as

$$f_{q_1 \cdots q_m s_1 \cdots s_n}(x_1, \ldots, x_m, z_1, \ldots, z_n) = v_{q_1}(x_1) \cdots v_{q_m}(x_m) S_1(z_1) \cdots S_n(z_n) \frac{\delta^{m+n}\Phi}{\delta v_{q_1}(x_1) \cdots \delta v_{q_m}(x_m) \delta S_1(z_1) \cdots \delta S_n(z_n)}, \quad (11)$$

where the q_i's and the s_j's denote the valence quarks and sea partons. (The flavor index on the sea parton has been suppressed for simplicity.)

The actual computation of the multiquark structure functions is complicated by the presence of correlations between the partons introduced due to momentum conservation. The calculation is most conveniently carried out in terms of the Fourier representation since the δ function is factorizable therein. We then obtain

$$\Phi = \frac{Z}{2\pi} \int_{-\infty}^{\infty} d\xi e^{i\xi} \tilde{v}_{u_1}(\xi) \tilde{v}_{u_2}(\xi) \tilde{v}_d(\xi) e^{\tilde{S}(\xi)}, \quad (12)$$

with

$$\tilde{S} = \sum_a \tilde{S}_a$$

and

$$\tilde{v}_i(\xi) = \int_0^{\infty} e^{-i\xi x} v_i(x) dx, \quad (13a)$$

$$\tilde{S}_a(\xi) = \int_0^{\infty} e^{-i\xi x} S_a(x) dx. \quad (13b)$$

It is clear from the form of the partition function that the structure function consists of a product of primitive density functions (which would be the structure function in the absence of correlations) times a correlation function $G(z)$, where z is the fraction of the momentum left over after picking up the specified partons, i.e., $z = 1 - x$, where x is the total momentum carried by the partons that have been designated within the structure function. Since the specification of any number of sea partons does not alter their original Poisson distribution, the functional form of $G(z)$ depends only on the particular group of valence quarks that is picked out. Corresponding to the six groups that can be formed from the valence quarks in the proton, we have six correlation functions given by

$$G_0(1-y) \equiv \frac{\delta\Phi}{\delta S_a(y)} = \frac{Z}{2\pi} \int_{-\infty}^{\infty} d\xi e^{i\xi(1-y)} \tilde{v}_{u_1} \tilde{v}_{u_2} \tilde{v}_d e^{\tilde{S}}, \quad (14a)$$

$$G_u(1-y) \equiv \frac{\delta\Phi}{\delta v_{u_1}(y)} = \frac{Z}{2\pi} \int_{-\infty}^{\infty} d\xi e^{i\xi(1-y)} \tilde{v}_{u_2} \tilde{v}_d e^{\tilde{S}}, \quad (14b)$$

$$G_d(1-y) \equiv \frac{\delta\Phi}{\delta v_d(y)} = \frac{Z}{2\pi} \int_{-\infty}^{\infty} d\xi e^{i\xi(1-y)} \tilde{v}_{u_1} \tilde{v}_{u_2} e^{\tilde{S}}, \quad (14c)$$

$$G_{uu}(1-y=1-y_1-y_2) \equiv \frac{\delta^2\Phi}{\delta v_{u_1}(y_1)\delta v_{u_2}(y_2)} = \int_{-\infty}^{\infty} d\xi e^{i\xi(1-y)} \tilde{v}_d e^{\tilde{S}}, \quad (14d)$$

$$G_{ud}(1-y=1-y_1-y_2) \equiv \frac{\delta^2\Phi}{\delta v_{u_1}(y_1)\delta v_d(y_2)} = \int_{-\infty}^{\infty} d\xi e^{i\xi(1-y)} v_{u_2} e^{\tilde{S}}, \quad (14e)$$

$$G_{uud}(1-y=1-y_1-y_2-y_3) \equiv \frac{\delta^3\Phi}{\delta v_{u_1}(y_1)\delta v_{u_2}(y_2)\delta v_d(y_3)} = \int_{-\infty}^{\infty} d\xi e^{i\xi(1-y)} e^{\tilde{S}}. \quad (14f)$$

The structure functions are then obtained by multiplying the primitive structure functions by the appropriate correlation function.

The general form of any structure function is thus given by

$$f_{q_1 \cdots q_m s_1 \cdots s_n}(x_1, \ldots, x_m, z_1, \ldots, z_n) = v_{q_1}(x_1) \cdots v_{q_m}(x_m) S_1(z_1) \cdots S_n(z_n) G_{q_1 \cdots q_m}(Y),$$

with

$$Y = 1 - (x_1 + \cdots + x_m + z_1 + \cdots + z_n) \text{ and } m \leq 3. \quad (15)$$

The overall normalization is fixed by imposing momentum conservation, i.e.,

$$\int_0^1 dx\, x \left(f_{u_1}(x) + f_{u_2}(x) + f_d(x) + \sum_a f_{s_a}(x) \right) = 1. \quad (16)$$

In the original Kuti-Weisskopf model where the function Φ^V is assumed to be factorizable, Eq. (16) is equivalent to requiring that the total probability is normalized to unity. However, it is of interest to note that this equivalence is independent of the factorizable nature of the valence function.

B. Parametrization of structure functions

We recall that the correlation effect discussed earlier is due to the reduction of momentum available to the remaining partons when one or more quarks with a specified momentum are designated within a proton. This is a direct consequence of momentum conservation. If, however, the momentum fraction of the designated quarks is very small, the remaining partons can be distributed over almost the whole momentum space and, hence, are essentially uncorrelated with the small momentum quarks that have been picked out. Thus, the single-particle structure functions approach the primitive structure functions when the momentum fraction, x, of the designated parton approaches zero. The phenomenological choices of structure functions available in the literature[15] thus serve as a guide for our choice of primitive structure functions. Also, the observation, $\nu W_2^{en}(x=1)/\nu W_2^{ep}(x=1) \simeq 0.25$ suggests that $v_d(x)/v_u(x) \to 0$ as $x \to 1$. We parametrize our structure functions as

$$v_{u_1}(x) = v_{u_2}(x) = x^{-\alpha}(1+\beta x), \tag{17a}$$

$$v_d(x) = x^{-\alpha}(1-x)(1+\gamma x), \tag{17b}$$

$$S_a(x) = g_a x^{-1}(1-x)^{n_a}, \tag{17c}$$

We also choose

$$S_q = S_{\bar{q}}. \tag{17d}$$

The multiquark structure functions are calculated using Eqs. (14), (15), and (17). Details of the calculation are relegated to the Appendix. The single-quark structure functions are explicitly shown below.

$$f_u(x) = 2x^{-\alpha}(1+\beta x)G_u(1-x), \tag{18a}$$

$$f_d(x) = x^{-\alpha}(1-x)(1+\gamma x)G_d(1-x), \tag{18b}$$

$$f_{s_a}(x) = g_a x^{-1}(1-x)^{n_a}G_0(1-x), \tag{18c}$$

The tail of these functions, when x approaches 1, is largely governed by the leading behaviors of the functions G_u, G_d, G_0 in the same limit. We find from Eqs. (A14)–(A16) that when $x \to 1$,

$$G_u(1-x) \propto (1-x)^{\delta_2 - 1}, \tag{19a}$$

$$G_d(1-x) \propto (1-x)^{\delta_2 - 1}, \tag{19b}$$

$$G_0(1-x) \propto (1-x)^{\delta_3 - 1}, \tag{19c}$$

which yields for the quark structure functions

$$f_u(x) \propto (1-x)^{\delta_2 - 1}, \tag{20a}$$

$$f_d(x) \propto (1-x)^{\delta_2}, \tag{20b}$$

$$f_{s_a}(x) \propto (1-x)^{\delta_3 + n_a - 1}, \tag{20c}$$

where $\delta_n = g + n(1-\alpha)$, $g = 2(g_u + g_d + g_s) + g_g$. The correlation functions do not, in general, obey the simple proportionality as indicated by Eq. (19). More generally, they are modified by a multiplicative power series $H(1-x, \delta)$ the form of which has been shown in Fig. 3. For further details, we refer the reader to the Appendix, Eqs. (A13)–(A20).

The parameters have been chosen to reproduce the low q^2, deep-inelastic inclusive e-p and e-n scattering data. We have chosen $\alpha = 0.5$, this being the intercept of the ρ trajectory. Also, for simplicity, we have chosen $n_d = n_s = n_g = 0$. This choice does not necessarily give the best fit to the data, even within our present parametrization. We find that the structure functions are rather

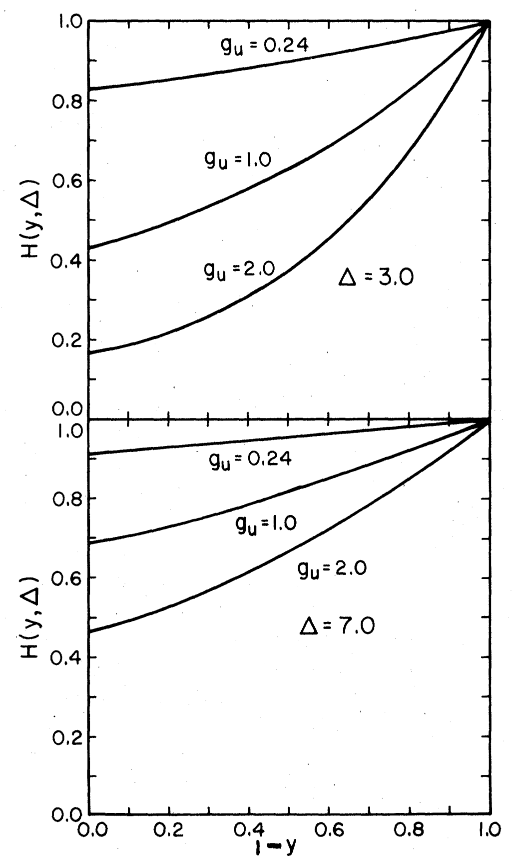

FIG. 3. The function $H(y, \Delta)$ defined in Eqs. (A11) and (A13). The parameters used are $n_u = 3$, $n_d = n_s = n_g = 0$ for various values of Δ and g_u. In this case, $H(y, \Delta)$ is independent of g_d, g_s, g_g. (The case when $n_u = n_d = 3$, $n_s = n_g = 0$ is effectively identical to the cases shown for appropriate values of g_u and g_d.)

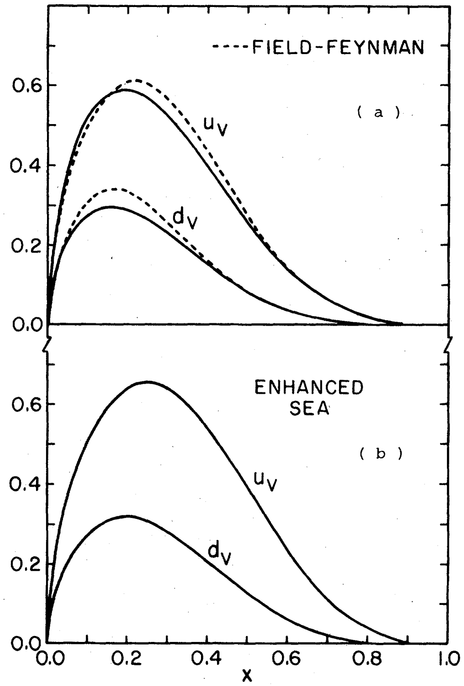

FIG. 4. (a) The valence structure functions as fixed from the deep-inelastic inclusive data. The dashed curves are the functions used by Field and Feynman (Ref. 15). (b) The valence functions after the enhancement of the quark sea. (See text.)

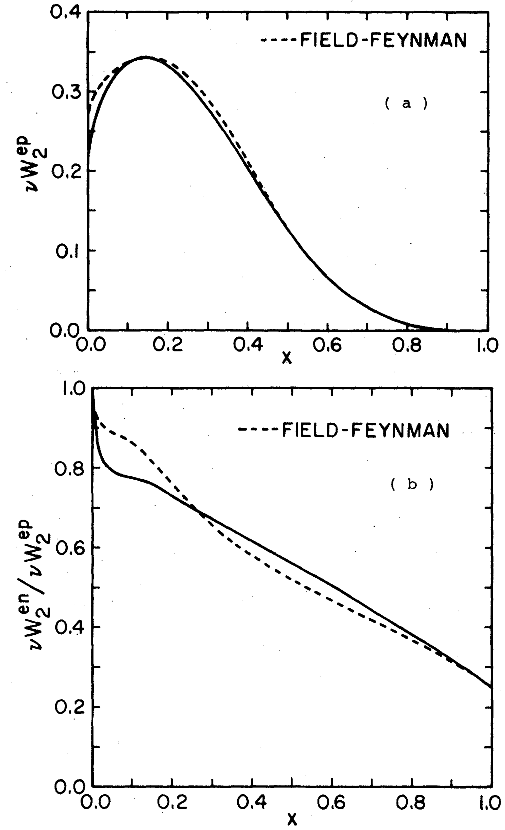

FIG. 5. (a) Our fit to νW_2^{ep} with the values of parameters given in the text. (b) The fit for $\nu W^{en}/\nu W_2^{ep}$. Shown as dashed curves are the fits obtained by Field and Feynman (Ref. 15).

insensitive to the specific values of β and γ. Our choice of parameters is shown below.

$$\alpha = 0.5, \quad \beta = 3.5, \quad \gamma = 5.0,$$

$$g_u = g_d = 0.12, \quad g_s = 0.09, \quad g_g = 2.84,$$

$$n_u = 3, \quad n_d = n_s = n_g = 0.$$

We thus have $\delta_2 = 4.5$, $\delta_3 = 5.0$, $g = 3.5$.

In Fig. 4 we show our valence quark structure functions. In Fig. 5, we display the results of our calculation of the deep-inelastic processes. These have been compared with those of Field and Feynman.[15] We obtain a good fit to νW_2^{ep}. Although our fit to the ratio, $\nu W_2^{en}/\nu W_2^{ep}$, for $x \leq 0.2$ is rather unsatisfactory, we are able to fit the data very well for intermediate and large values of x.

IV. FAST-MESON PRODUCTION IN HADRON COLLISIONS

The interaction between hadrons, in the plateau region of the rapidity axis, within the framework of the parton model, is due to exchange of wee partons.[16] The momentum distribution of partons is, in general, not expected to remain unaltered during the course of hadron collisions. In particular, the quark sea is expected to be enhanced due to the conversion of gluons to q-$\bar{q}$ pairs. Ochs's observation,[4] however, suggests that the distribution of the fast valence quarks is relatively unaltered by the hadronic interaction. We thus assume that the structure functions for the valence quarks are the same as the ones determined by the low-q^2, deep-inelastic e-p and e-n scattering data. (We recall that the photon probes the instantaneous parton distribution within the proton.)

It is of interest to see how Ochs's phenomenological observation can be realized in the Kuti-Weisskopf model. For definiteness, we consider π^+ production. The relevant two-quark structure function is given by

$$f_{u_v s_d}(x_1, x_2) = 2v_u(x_1)S_d(x_2)G_u(1-x_1-x_2). \qquad (21a)$$

This may be compared and contrasted with the form used by Das and Hwa,[5] which is

$$F^{\rm DH}_{u_v s_d}(x_1, x_2) = f_u(x_1)f_{s_d}(x_2)(1-x_1-x_2). \qquad (21b)$$

Using Eq. (1) we obtain

$$\frac{x}{\sigma_T}\frac{d\sigma^{\pi^+}}{dx} = 2G_u(1-x)\int v_u(x_1)S_d(x_2)R(x_1,x_2;x)dx_1dx_2\,, \quad (22)$$

The integral on the right-hand side of Eq. (22) is a rather slow varying function of x, so that the shape of the π^+ spectrum is essentially governed by the function $G_u(1-x)$. We thus see that the π^+ spectrum has approximately the same shape as the correlation function $G_u(1-x)$ and, hence, to leading order in $(1-x)$, the same shape as the valence u-quark distribution in the proton. We thus see that the correlation between the partons introduced by momentum conservation leads to a natural explanation of Ochs's observation, which is not as obviously understood using the form Eq. (21b) for the two-quark structure function.

It is clear from Eq. (1) that the data that should be compared with the predictions of the recombination model is the data that has been integrated over the transverse momentum, i.e., with

$$x\frac{d\sigma}{dx} = \int E\frac{d^3\sigma}{dp^3}d^2p_T\,. \quad (23)$$

It is rather misleading to compare the single-particle $\pi^\pm, K^\pm$ data at some fixed value of p_T since the p_T dependence is different for each inclusive reaction. In fact, the Fermilab-Illinois Collaboration[1] has obtained the following fits for the p_T distributions:

$$g_h(p_T) = A_h(1+p_T^2/a_h^2)^{-4}\,, \quad (24)$$

with

$$a_{\pi^+}^2 = 0.66\pm 0.1\,, \quad a_{\pi^-}^2 = 0.74\pm 0.01$$

$$a_{K^+}^2 = 0.64\pm 0.03\,, \quad a_{K^-}^2 = 0.9\pm 0.1\,,$$

$$a_p^2 = 0.41\pm 0.02\,, \quad a_{\bar p}^2 = 1.2\pm 0.3\,.$$

The above data yield[17] $\langle p_T\rangle_{\text{valence}} \simeq 0.2$ GeV and $\langle p_T\rangle_{\text{sea}} \simeq 0.35$ GeV. It is interesting to note that the sea quarks carry, on an average, a larger transverse momentum than the valence quark. This is consistent with the enhancement of the sea, possibly by conversion of gluons.

The Fermilab-Illinois[1] and the CERN ISR data[2] have been analyzed and the p_T-integrated, single-meson inclusive distributions have been extracted therefrom by Roberts, Hwa, and Matsuda.[18] We use the results of their analysis to compare our calculation with the data. We make the following choice of parameters: (a) The same values of the valence parameters, α, β, γ as obtained from the e-p, e-n scattering data are used (see Sec. III). (b) $n_u = n_d = 3$, $n_s = 0$, $g_u = 0.486$, $g_d = 0.608$, $g_s = 0.048$, $g_g = 1.216$, (c) $\Delta = 1.7$. The overall magnitude has been adjusted to fit the π^+ data. We find that $\alpha_M = \frac{1}{3}$ for the Fermilab-Illinois data and $\alpha_M = 1/4.2$ for the ISR data. The difference reflects the fact that the meson production magnitude is not pinned down accurately by present experiments.

For the formation of $\pi^\pm$ and K^+, the contributions from both the valence-sea and the sea-sea recombination have been considered, whereas only the latter contributes to K^- production. We note that the correlation function $G(1-x)$ has different asymptotic forms, $(1-x)^{3.5}$ and $(1-x)^4$, for the valence-sea and the sea-sea structure functions, respectively. This is different from the approach used by Duke and Taylor[6] who assumed the factorizable form with the same phase-space factor $(1-x)$ for both the valence-sea and the sea-sea structure functions as in Eq. (21b).

Our predictions are compared with the above-mentioned data in Fig. 6 and Fig. 7.

The following comments are in order:

(i) The contribution of the process shown in Fig. 1(b) turns out to be negligible ($\leq 0.2\%$) for $x_T = 0.01$ for $x \geq 0.4$. For larger values of x_T ($x_T = 0.05$) the maximum contribution of this process increases to about 2% and for $x_T = 0.1$

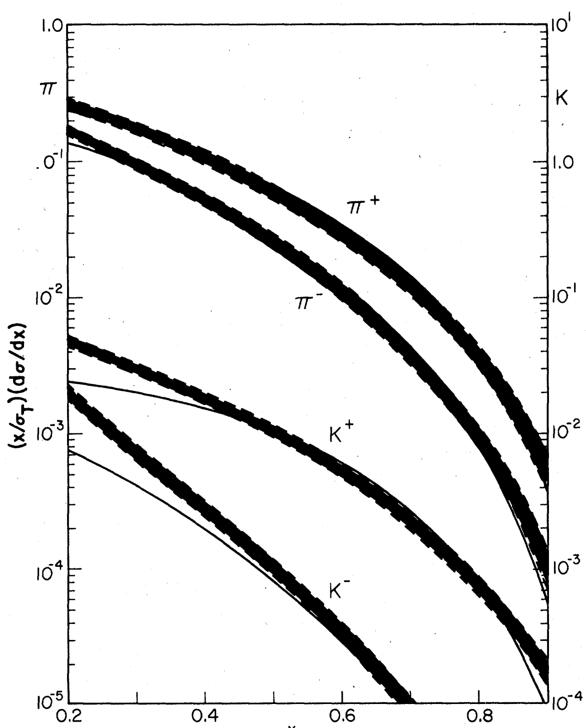

FIG. 6. The x distribution of $(x/\sigma_T)(d\sigma/dx)$ for the process $pp \to Mx$ with $M = \pi^\pm$, $K^\pm$. The solid curve is our calculation. The shaded area represents the experimentally allowed domain as analyzed from the Fermilab-Illinois collaboration (Ref. 1) data (see Ref. 17). The magnitude has been fitted by choosing $\alpha_M = \frac{1}{3}$.

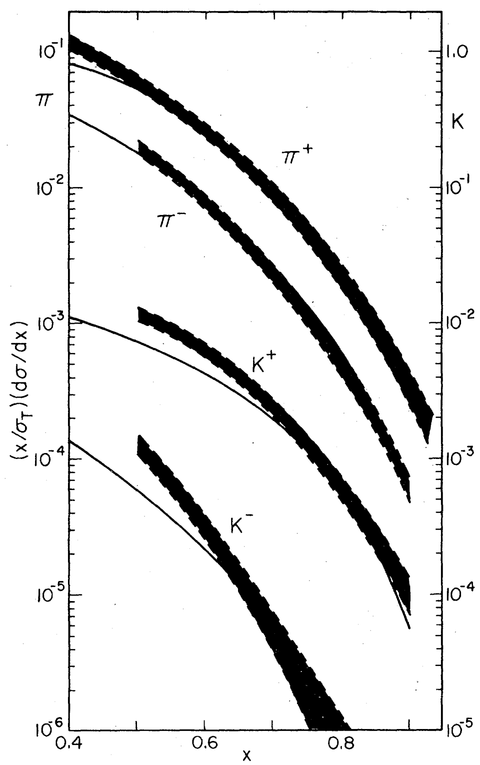

FIG. 7. The same as Fig. 6, except the comparison is with the CHLM Collaboration data (Ref. 2). α_M has been chosen to be 1/4.2.

to about 6% for $x \geq 0.4$.

(ii) The shape of the meson spectrum is relatively insensitive to the specific choice of the cutoff, Δ, used in the recombination function for $1 \leq \Delta \leq 2.5$. A choice of Δ in this domain reproduces the data reasonably well. Variation of Δ, of course, gives a sizeable effect on the magnitude of the single-meson spectrum. The condition $\alpha_M \leq 1$ implies a lower limit ($\Delta_L \simeq 0.5$) on Δ. The actual value of Δ is, however, expected to be larger than the naive lower limit since the probability to form a pseudoscalar meson by recombination is considerably less than one due to both the color suppression effect[8] and the formation of other mesons.

(iii) The contribution to pseudoscalar-meson production from the cascade decays of vector mesons[8,11] is no more than 25% for $x \geq 0.5$. The process of Fig. 1(a) is thus the dominant contribution and is plotted in Figs. 6 and 7.

(iv) For $x \gtrsim 0.6$, the Fermilab-Illinois data show that the π^+ and π^- spectra have almost the same shape. The K^+ spectrum is significantly flatter than the $\pi^\pm$ spectra. This suggests $n_s < n_u \simeq n_d$. We found that the choice $n_u \simeq n_d \simeq 3$ and $n_s \simeq 0$ reproduces the data adequately. A little larger value of n_s would yield an improved fit for the K^+ spectrum.

(v) The amount of K^- production relative to the K^+ is a crucial factor in the determination of the magnitude of the u-quark sea. We found a rather modest enhancement of the quark sea (the enhanced quark sea and the gluons carry 29% and 26% of the proton momentum respectively in our model) yields the best fit to the data. On the contrary, Duke and Taylor,[6] using a factorizable form of two-quark structure functions as in Eq. (21b), require a greater enhancement of the quark sea (corresponding to almost complete conversion of gluons to quark pairs) to fit the K^+ and K^- data. In the Kuti-Weisskopf model, full saturation yields flatter distributions for the meson spectra which are in disagreement with the $\pi^\pm$ and $K^\pm$ data.

(vi) In the extended Kuti-Weisskopf model, the enhancement of the quark sea causes an alteration in the distribution of the valence quarks. The origin of this is obviously the correlation that has been introduced by momentum conservation. The valence structure functions prior to, and after the enhancement of the sea have been shown in Fig. 4 for the same values of α, β, and γ. It is clear that the change in the valence structure functions is small ($\leq 10\%$). This is completely consistent with our intuitive expectations.

V. CONCLUDING REMARKS

In this paper, we have analyzed the recombination model for fast-meson production with a view to study the multiquark structure functions and the recombination function. We have given a general prescription for obtaining multiquark structure functions using an extension of the Kuti-Weisskopf procedure. In contrast to the original Kuti-Weisskopf model, we chose an asymmetric sea with $(1-x)$ powers that are arbitrarily different from those of the valence quarks. The valence-quark distributions are fixed so as to reproduce the deep-inelastic, low-q^2 ep and en scattering data. We have proposed an alternative form for the recombination function, essentially by imposing a cutoff, Δ, on the rapidity gap between the quarks that combine to form the meson. Our form is very similar to the one used by Das and Hwa for $\Delta \simeq 2$. Also, it can be readily extended to include multiquark recombination processes such as baryon production.

With our choice of multiquark structure functions and the recombination function, we are able to reproduce the single-meson inclusive spectra in p-p scattering reasonably well. Analysis of other processes such as baryon production, multimeson production, single-meson production with a Drell-Yan trigger, etc. would serve as further tests for the model and also as ways of pinning down the parameters more closely. The formalism can also be used to study meson production in lepton-hadron collisions with a view to obtaining more information about the quark fragmentation function.

It is extremely instructive to use the model to calculate some of the processes mentioned above for π-p and K-p scattering at small p_T. This is of interest since it would provide an explicit way of determining the valence-quark structure functions in the π and K mesons. These can then be compared with those obtained from high-p_T processes.

We address these questions in forthcoming papers.

Note added. It has been pointed out by L. Van Hove [CERN report No. TH-2580 (unpublished)] that the σ_T that appears in Eq. (1) should be the inelastic cross section with diffraction dissociation excluded. The relevant cross section, $\sigma \approx 28$ mb, for p-p collisions requires that α_M be increased by a factor of about 1.4.

ACKNOWLEDGMENTS

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APPENDIX

Here we present the details of the calculation of the multiquark structure functions based on the partition function defined in Eqs. (8)–(10). In the calculation, frequent use has been made of the formulas (A1)–(A3).

$$\int_0^\infty dx e^{-i\xi x} x^\beta = \Gamma(\beta+1) e^{-(i\pi/2)\xi(\beta+1)}, \qquad \text{(A1)}$$

$$\int_0^\infty dx \frac{e^{-i\xi x}}{(x^2+x_T{}^2)^{1/2}} \sim \ln(2/\xi x_T) + \text{const}, \quad \text{as } x_T \to 0, \qquad \text{(A2)}$$

$$\int_{-\infty}^\infty d\xi e^{i\xi y} \xi^{-\gamma} = \frac{2\pi e^{i\pi/2}(e^{i\pi/2}y)^{\gamma-1}}{\Gamma(\gamma)}. \qquad \text{(A3)}$$

The $\tilde{v}_i$ and $\tilde{S}_a$ defined in Eq. (13) are calculated using Eqs. (17), (A1), and (A2). We obtain

$$\tilde{v}_{u_1} = \tilde{v}_{u_2} = \Gamma(1-\alpha)\xi'^{-(1-\alpha)}[1+\beta(1-\alpha)\xi'^{-1}], \qquad \text{(A4)}$$

$$\tilde{v}_d = \Gamma(1-\alpha)\xi'^{-(1-\alpha)}[1+(\gamma-1)(1-\alpha)\xi'^{-1} - \gamma(1-\alpha)(2-\alpha)\xi'^{-2}], \qquad \text{(A5)}$$

$$\tilde{S}_a = g_a\left[\ln\frac{C}{\xi'} + \sum_{k=1}^{n_a}(-1)^k \binom{n_a}{k}\Gamma(k)\xi'^{-k}\right], \qquad \text{(A6)}$$

$$\tilde{S}_{\bar{q}} = \tilde{S}_q, \qquad \text{(A7)}$$

where $\xi' = \xi e^{i\pi/2}$, C is a constant and $a = u, d, s, \bar{u}, \bar{d}, \bar{s}, g$.

The multiquark structure functions are calculated using the six correlation functions $G_0(y)$, $G_u(y)$, $G_d(y)$, $G_{uu}(y)$, $G_{ud}(y)$, $G_{uud}(y)$ defined by Eqs. (14a)–(14f). These, in turn, are calculated from Eqs. (17), (A4)–(A7). $\exp(\tilde{S})$ has been written as

$$\exp(\tilde{S}) = (C\xi'^{-1})^g \exp\left(\sum_{k=1}^{N}(-1)^k a_k \Gamma(k)\xi'^{-k}\right), \qquad \text{(A8)}$$

where

$$N = \max(n_a), \qquad \text{(A9)}$$

$$g = \sum_a g_a, \qquad \text{(A10)}$$

and

$$a_k = \sum_a g_a \binom{n_a}{k}, \qquad \text{(A11)}$$

with the definition, $\binom{n}{l} = 0$ if $l > n$.

The integrations involved in Eq. (15) are easily performed by expanding the exponential in (A8) and using (A2). As an illustration, we explicitly write out $G_{uud}(y)$

$$G_{uud}(y) = ZC^{\delta_0} y^{\delta_0 - 1} H(y, \delta_0)/\Gamma(\delta_0), \qquad \text{(A12)}$$

where $\delta_n = g + n(1-\alpha)$, as defined in Sec. III B, and

$$H(y,\Delta) = 1 + \sum_{p=1}^\infty \frac{1}{p!} \sum_{k_1 \cdots k_p = 1}^{N} \frac{\Gamma(\Delta)\prod_{i=1}^p a_{k_i}\Gamma(k_i)}{\Gamma(\Delta + \sum_{i=1}^p k_i)} (-y)^\kappa, \qquad \text{(A13)}$$

where $\kappa = \sum_{i=1}^p k_i$. The function $H(y, \Delta)$, as defined by (A13), looks rather complicated. It is analytic for y in $(-\infty, \infty)$. Moreover, the series (A13) is absolutely convergent. Numerical methods have been used for its evaluation.

The other correlation functions are calculated in an identical manner. The overall normalization is fixed by the condition [Eq. (16)] of momentum conservation. We obtain

$$G_0(y) = \frac{[\Gamma(1-\alpha)]^3}{N} y^{\delta_3 - 1} \sum_{l=0}^4 C_l \frac{H(y, \delta_3 + l)}{\Gamma(\delta_3 + l)} y^l, \qquad \text{(A14)}$$

$$G_u(y) = \frac{[\Gamma(1-\alpha)]^2}{N} y^{\delta_2 - 1} \sum_{l=0}^3 C_l^u \frac{H(y, \delta_2 + l)}{\Gamma(\delta_2 + l)} y^l, \qquad \text{(A15)}$$

$$G_d(y) = \frac{[\Gamma(1-\alpha)]^2}{N} y^{\delta_2 - 1} \sum_{l=0}^2 C_l^d \frac{H(y, \delta_2 + l)}{\Gamma(\delta_2 + l)} y^l, \qquad \text{(A16)}$$

$$G_{uu}(y)=\frac{\Gamma(1-\alpha)}{N}\,y^{\delta_1-1}\sum_{l=0}^{2}C_l^{uu}\frac{H(y,\delta_1+l)}{\Gamma(\delta_1+l)}\,y^l\,, \tag{A17}$$

$$G_{ud}(y)=\frac{\Gamma(1-\alpha)}{N}\,y^{\delta_1-1}\sum_{l=0}^{1}C_l^{ud}\frac{H(y,\delta_1+l)}{\Gamma(\delta_1+l)}\,y^l\,, \tag{A18}$$

$$G_{uud}(y)=\frac{1}{N}\,y^{\delta_0-1}\frac{H(y,\delta_0)}{\Gamma(\delta_0)}\,, \tag{A19}$$

where

$$N=[\Gamma(1-\alpha)]^3\sum_{l=0}^{4}C_l\frac{H(1,\delta_3+l)}{\Gamma(\delta_3+l)}\,. \tag{A20}$$

This leads to $G_0(1)=1$ which is equivalent to the condition of momentum conservation.

The coefficients $C_l, C_l^u, C_l^d, C_l^{uu}, C_l^{ud}$ are just the coefficients of the lth power of ξ'^{-1} in the expansion of the polynomials, $\tilde{v}_{u_1}\tilde{v}_{u_2}\tilde{v}_dA^{-3}$, $\tilde{v}_{u_1}\tilde{v}_dA^{-2}$, $\tilde{v}_{u_1}\tilde{v}_{u_2}A^{-2}$, $\tilde{v}_dA^{-1}$, $\tilde{v}_{u_1}A^{-1}$, respectively, with $A=\Gamma(1-\alpha)\xi'^{-(1-\alpha)}$. They have been tabulated below.

$$\begin{aligned}
&C_0=1\,,\\
&C_1=(2\beta+\gamma-1)(1-\alpha)\,,\\
&C_2=[\beta^2+2\beta(\gamma-1)-\gamma(2-\alpha)/(1-\alpha)](1-\alpha)^2\,,\\
&C_3=[\beta^2(\gamma-1)-2\beta\gamma(2-\alpha)/(1-\alpha)](1-\alpha)^3\,,\\
&C_4=-\beta^2\gamma(2-\alpha)(1-\alpha)^3\,,
\end{aligned} \tag{A21}$$

$$\begin{aligned}
&C_0^u=1\,,\\
&C_1^u=(\beta+\gamma-1)(1-\alpha)\,,\\
&C_2^u=[\beta(\gamma-1)-\gamma(2-\alpha)/(1-\alpha)](1-\alpha)^2\,,\\
&C_3^u=-\beta\gamma(2-\alpha)(1-\alpha)^2\,,
\end{aligned} \tag{A22}$$

$$\begin{aligned}
&C_0^d=1\,,\\
&C_1^d=2\beta(1-\alpha)\,,\\
&C_2^d=\beta^2(1-\alpha)^2\,,
\end{aligned} \tag{A23}$$

$$\begin{aligned}
&C_0^{uu}=1\,,\\
&C_1^{uu}=(\gamma-1)(1-\alpha)\,,\\
&C_2^{uu}=-\gamma(2-\alpha)(1-\alpha)\,,
\end{aligned} \tag{A24}$$

$$\begin{aligned}
&C_0^{ud}=1\,,\\
&C_1^{ud}=\beta(1-\alpha)\,.
\end{aligned} \tag{A25}$$

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[1]J. R. Johnson *et al.*, Phys. Rev. Lett. 39, 1173 (1977); Phys. Rev. D 17, 1292 (1977).

[2]J. Singh *et al.*, Nucl. Phys. B140, 189 (1978).

[3]D. Cutts *et al.*, Phys. Rev. Lett. 40, 141 (1978), and paper submitted to the XIX International Conference on High Energy Physics, Tokyo, 1978 (unpublished).

[4]W. Ochs, Nucl. Phys. B118, 397 (1977).

[5]K. P. Das and R. C. Hwa, Phys. Lett. 68B, 459 (1977).

[6]D. W. Duke and F. E. Taylor, Phys. Rev. D 17, 1788 (1977).

[7]J. Ranft, Phys. Rev. D 18, 1491 (1978).

[8]M. Teper, Rutherford Laboratory Report No. RL-78-022, 1978 (unpublished).

[9]R. C. Hwa and R. G. Roberts, Rutherford Laboratory Report No. RL-78-078, 1978 (unpublished).

[10]R. C. Hwa, invited talk at the IX International Symposium on High Energy Multiparticle Dynamics, Tabor, Czechoslovakia, 1978 (unpublished).

[11]J. F. Gunion and S. J. Brodsky, in *Proceedings of the Seventh International Colloquium on Multiparticle Productions, Tutzing, Germany, 1976,* edited by J. Benecke *et al.* (Max-Planck-Institut fur Physik und Astrophysik, Munich, 1976), p. 289; Phys. Rev. Lett. 37, 402 (1976); R. Blankenbecler and S. J. Brodsky, Phys. Rev. D 10, 2973 (1974).

[12]R. P. Feynman, *Photon-Hadron Interactions* (Benjamin, Reading, Massachusetts, 1972).

[13]J. Kuti and V. F. Weisskopf, Phys. Rev. D 4, 3418 (1971).

[14]T. DeGrand and H. Miettinen, Phys. Rev. Lett. 40, 612 (1978).

[15]R. D. Field and R. P. Feynman, Phys. Rev. D 15, 2590 (1977).

[16]R. P. Feynman, in *Proceedings of the Third International Conference on High Energy Collisions, Stony Brook,* 1969, edited by C. N. Yang *et al.* (Gordon and Breach, New York, 1969), p. 237.

[17]Approximation of the p_T distribution, $g_h(p_T)$, by a Gaussian of the form, $g_h(p_T)\simeq A_h\exp(-4p_T^2/a_h^2)$ yields, using $\langle p_T\rangle_{\rm valence}=0.2$ GeV and $\langle p_T\rangle_{\rm sea}=0.35$ GeV, the transverse masses, $a_p^2=0.48$, $a_{\pi^+}^2=a_{K^+}^2=0.65$, $a_{K^-}^2=0.98$, $a_{\bar{p}}^2=1.47$. This is consistent with the observations made by the Fermilab-Illinois collaboration.

[18]R. G. Roberts, R. C. Hwa, and S. Matsuda, Rutherford Laboratory Report No. RL78-040, 1978 (unpublished).