Constituent description of *NN* elastic scattering observables at large angles

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We suggest that the constituent picture of nucleon-nucleon elastic scattering can be tested by the spin-correlation measurements A_{ll}, A_{ss}, A_{nn}, and A_{sl}. These measurements provide a means for isolating various reaction mechanisms, including possible quantum-chromodynamic instanton effects. We give specific model calculations to illustrate these ideas.

I. INTRODUCTION

Nucleon-nucleon elastic scattering at high energy and fixed c.m. angle (i.e., $s,t\to\infty$; s/t fixed) is a particularly interesting process from the standpoint of understanding quark dynamics. If quantum chromodynamics (QCD) is correct and no infrared or confinement phenomena introduce mass scales, perturbation theory is thought to be a trustworthy guide to the scattering of composite systems. From it one obtains the results known as dimensional counting[1,2]:

$$\frac{d\sigma}{dt}(A+B\to C+D) \underset{\substack{s\to\infty \\ t/s\,\text{fixed}}}{\sim} \frac{F(\theta)}{s^{N-2}} \text{ (modulo logarithms)}, \tag{1.1}$$

where $N=n_A+n_B+n_C+n_D$ is the minimum number of fundamental constituents of the composite particles A, B, C, and D. For instance, for nucleon-nucleon scattering $n_A=n_B=n_C=n_D=3$, the number of quarks in a nucleon, so that Eq. (1.1) gives $d\sigma/dt\sim s^{-10}F(\theta)$. This behavior is in good agreement with experiment, as are the predictions for the proton and pion form factors, and for photoproduction and meson-nucleon and Compton scattering.[2,3]

The counting rules may be applied to nonhadronic systems such as $ee\to ee$ or $e\gamma\to e\gamma$ where their validity is well understood and where there are calculable subasymptotic corrections. The application of the rules to hadronic composite systems is more problematical in that we do not know the regime of validity of an asymptotic expression such as Eq. (1.1). Indeed, even at the Born approximation, certain diagrams [e.g., Fig. 1(a)] may be present in QCD which give results which do not agree with Eq. (1.1), e.g., giving for pp, $d\sigma/dt\sim s^{-8}F(\theta)$.[4] No evidence for these contributions has been found in the data up to $s\sim 60$ GeV^2,[2,5] even though naively they might dominate. Thus the constituent picture of elastic nucleon-nucleon scattering needs to be better understood. The reward is potentially great: insight into confinement and other nonperturbative effects.

A means of obtaining more detailed information on QCD dynamics is to study the spin dependence of fixed-angle pp and pn elastic scattering at high energy: Since QCD dictates how gluons (vectors) interact with quarks (spinors), we anticipate calculable predictions for the nucleon spin amplitudes.

The available data[6-8] [Figs. 2(a)–2(c)], albeit at relatively low values of s and t, do show interesting spin structure. It is not evident that constituent hard-scattering models can be applied to these data. For example, dimensional counting in QCD for the nucleon form factor leads to t^{-2}, which shows up only for $t\gtrsim 5$ GeV/c^2. Nevertheless, the nucleon-nucleon spin data may be giving an early

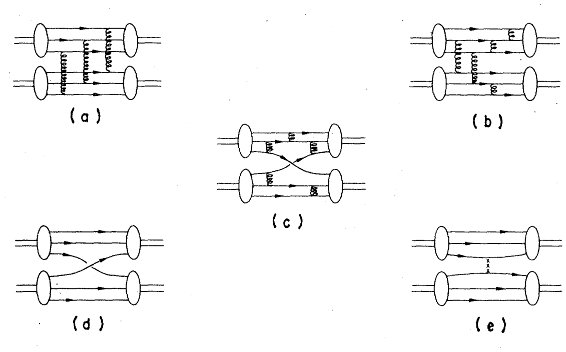

FIG. 1. (a)–(e) Typical diagrams for fixed-angle, large-s NN scattering. The wave functions are generally taken to be those of free quarks in the appropriate SU(6) configuration (this is to be regarded as merely an approximation, which gives the dominant helicity and s dependences correctly). (a) Landshoff diagrams, (b) and (c) QCD diagrams giving s dependence of Eq. (1.1) (dimensional counting), and (d) QIM diagram and (e) instanton diagram.

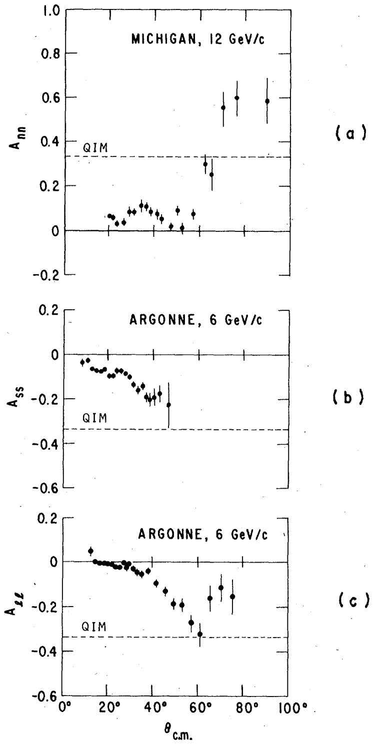

FIG. 2. Data on A_{nn} for $pp \to pp$ at 12 GeV/c (Ref. 8) compared with the asymptotic QIM prediction (4.10). (b) Data on A_{ss} for $pp \to pp$ at 6 GeV/c (Ref. 7) compared with the asymptotic QIM prediction (4.10). (c) Data on A_{ll} for $pp \to pp$ at 6 GeV/c (Ref. 6) compared with the asymptotic QIM prediction (4.10).

signal for behavior which one could hope to understand in the QCD framework.

If one approximates the scattering by sums of Feynman diagrams involving quarks, then in the limit that the Mandelstam invariants s, t, and u are large compared to any quark mass, the quarks may be treated as massless. This has observable consequences for np and pp elastic scattering, since for massless quarks the QCD vector interaction preserves the quark helicity. We shall see in Sec. IV that this helicity conservation at the quark level implies $A_{ss} = -A_{nn}$ and $A_{sl} = P_0 = 0$ at all angles. In addition, at 90° in the c.m. pp has the general constraint $A_{nn} - A_{ss} - A_{ll} = 1$, so in that case $A_{ll}(\pi/2) = 2A_{nn}(\pi/2) - 1$. Our conventions for spin asymmetries correspond to those in Refs. 6–8.

Violations of these relations could have a very exciting explanation since in QCD there are instanton and possibly other nonperturbative effects which are not helicity conserving. Owing to the particular isospin and double-helicity-flip structure of instanton effects, pp and pn polarization experiments may provide a means for isolating them. We discuss this possibility briefly in Sec. III. It may be noted in passing that one needs some such mechanism to explain the nonconservation of the axial-vector baryon current.[9]

We may also hope to learn more about the dynamics following from perturbative approximations to QCD. The lack of a unique, completely convincing explanation for the absence of the so-called Landshoff contribution which gave $d\sigma/dt(pp \to pp) \sim s^{-8}F(\theta)$ reflects itself in the fact that at present we cannot give a "first-principle" derivation of the nucleon-nucleon helicity amplitudes even in perturbation theory.[2,3] However, that is precisely the value of studying this phenomenon. We can identify several different models for NN scattering, each of which is a conceivable consequence of QCD, obtain their predictions for the helicity amplitudes, and by comparing these to experiments, learn something concrete about the dynamics at work (or conceivably obtain evidence against QCD in general). Three possible QCD-inspired models naturally present themselves.

The first is a model we might call "perturbative QCD," in which the helicity, energy, and angular dependence of the NN scattering amplitude is given to first approximation by the total Born amplitude for the bare quark systems to scatter [see Figs. 1(b) and 1(c)], projected onto the appropriate SU(6) wave function for the nucleon. In view of the fact that present data indicate the absence of Landshoff contributions [Fig. 1(a)],[5] we should probably include in this model only diagrams leading to s^{-10} behavior [e.g., Figs. 1(b) and 1(c)].

This model could be further modified by only including diagrams with quark interchange [e.g., Fig. 1(c)]. Computing the consequences of these models requires considerable effort, involving the summation of many (interfering) amplitudes and lengthy traces. Thus one is led to consider a still simpler model [Fig. 1(d)] which we call the "quark-interchange model" (QIM) because it is reminiscent of the spirit of the original constituent-interchange model[10]: Quarks are interchanged between the nucleons by an interaction which conserves helicity and is independent of the helicity of exchanged and spectator quarks. Then [neglecting SU(6) breaking in the nucleon wave functions] the $pp \to pp$ and $pn \to pn$ helicity amplitudes are obtained by projecting the possible quark exchange amplitudes on the SU(6) wave functions for the proton and neutron. In Sec. II we present the results of this QIM model for the quark-helicity-conserving pp and np amplitudes.

Section IV is devoted to the experimental consequences of the various possibilities for the helic-

TABLE I. *NN* observables using polarized beams and targets.

$\phi_1 = \langle ++\|\phi\|++\rangle$	$\sigma = \frac{1}{2}(\|\phi_1\|^2+\|\phi_2\|^2+\|\phi_3\|^2+\|\phi_4\|^2+4\|\phi_5\|^2)$
$\phi_2 = \langle ++\|\phi\|--\rangle$	$\sigma P_0 = -\mathrm{Im}(\phi_1+\phi_2+\phi_3-\phi_4)\phi_5^*$
$\phi_3 = \langle +-\|\phi\|+-\rangle$	$\sigma A_{sl} = \mathrm{Re}(\phi_1+\phi_2-\phi_3+\phi_4)\phi_5^*$
$\phi_4 = \langle +-\|\phi\|-+\rangle$	$\sigma A_{nn} = \mathrm{Re}(\phi_1\phi_2^*-\phi_3\phi_4^*+2\|\phi_5\|^2)$
$\phi_5 = \langle ++\|\phi\|+-\rangle$	$\sigma A_{ss} = \mathrm{Re}(\phi_1\phi_2^*+\phi_3\phi_4^*)$
	$\sigma A_{ll} = \frac{1}{2}(-\|\phi_1\|^2-\|\phi_2\|^2+\|\phi_3\|^2+\|\phi_4\|^2)$

ity amplitudes, and the sensitivity of spin-spin observables to the presence of instantonlike interactions is illustrated.

II. SPIN-SPIN ASYMMETRIES IN THE QIM

The conventions we use for the spin observables for *NN* elastic scattering, defined in terms of Jacob-Wick[11] helicity amplitudes, are shown in Table I. There are 5 independent helicity amplitudes for $NN \to NN$. Other amplitudes can be related to these by parity, time-reversal invariance, or identical-particle symmetry (Table II).

In order to relate nucleon-nucleon amplitudes to amplitudes involving quarks, we make the natural assumption that the spin of the nucleon is carried by quarks. In the limit of light quarks we neglect any helicity-flip contributions from gluon exchanges so that the helicity of a given quark line in Fig. 1(d) is preserved. We can suppress color indices if we keep track of the ordering of the quarks in a nucleon. To illustrate, we will for simplicity suppress the flavor degree of freedom and write the quark configuration labels

$$|N(+)\rangle = |q(+)q(+)q(-)\rangle + |q(+)q(-)q(+)\rangle + |q(-)q(+)q(+)\rangle = W_1 + W_2 + W_3\,, \tag{2.1}$$

$$|N(-)\rangle = |q(-)q(-)q(+)\rangle + |q(-)q(+)q(-)\rangle + |q(+)q(-)q(-)\rangle = W_4 + W_5 + W_6\,. \tag{2.2}$$

The helicity content of an interchange diagram such as Fig. 1(d) can be specified by requiring that the first quark in each configuration W_i and W_j be interchanged to produce $W_{\bar{i}}$ and $W_{\bar{j}}$. If we denote by W_x any set of helicities such as $|q(+)q(+)q(+)\rangle$ which does not overlap with $|N(\pm)\rangle$, we have the following possibilities for exchange amplitudes:

$|N(+)\rangle|N(+)\rangle \to |N(+)\rangle|N(+)\rangle$, $\langle ++|\phi|++\rangle = \phi_1$:

$$\begin{array}{l} \underline{W_iW_j \to W_{\bar{i}}W_{\bar{j}}} \\ W_1W_1 \to W_1W_1 \\ W_1W_2 \to W_1W_2 \\ W_2W_1 \to W_2W_1 \\ W_2W_2 \to W_2W_2 \\ W_3W_3 \to W_3W_3 \\ W_1W_3 \to W_5W_x \\ W_2W_3 \to W_4W_x \\ W_3W_1 \to W_xW_5 \\ W_3W_2 \to W_xW_4 \end{array} \tag{2.3}$$

$|N(+)\rangle|N(-)\rangle \to |N(+)\rangle|N(-)\rangle$, $\phi_3 = \langle +-|\phi|+-\rangle$:

$$\begin{array}{l} \underline{W_iW_j \to W_{\bar{i}}W_{\bar{j}}} \\ W_1W_6 \to W_1W_6 \\ W_2W_6 \to W_2W_6 \\ W_3W_4 \to W_3W_4 \\ W_3W_5 \to W_3W_5 \end{array} \tag{2.4}$$

$|N(+)\rangle|N(-)\rangle \to |N(-)\rangle|N(+)\rangle$, $-\phi_4 = -\langle +-|\phi|-+\rangle$ (-1 = Jacob-Wick phase factor):

$$\begin{array}{l} \underline{W_iW_j \to W_{\bar{i}}W_{\bar{j}}} \\ W_1W_4 \to W_5W_2 \\ W_1W_5 \to W_5W_1 \\ W_2W_4 \to W_4W_2 \\ W_2W_5 \to W_4W_1 \\ W_3W_6 \to W_xW_x \end{array} \tag{2.5}$$

No other possibilities are possible in the QIM

TABLE II. Particle symmetry.

$$\phi_1^I(\theta) = (-)^{I+1}\phi_1(\pi-\theta)$$
$$\phi_2^I(\theta) = (-)^{I+1}\phi_2(\pi-\theta)$$
$$\phi_3^I(\theta) = (-)^{I}\phi_4(\pi-\theta)$$
$$\phi_5^I(\theta) = (-)^{I}\phi_5(\pi-\theta)$$
$$\langle pp|\phi|pp\rangle = \phi^{I=1}$$
$$\langle np|\phi|np\rangle = \tfrac{1}{2}\phi^{I=1} + \tfrac{1}{2}\phi^{I=0}$$
$$\langle np|\phi|pn\rangle = \tfrac{1}{2}\phi^{I=1} - \tfrac{1}{2}\phi^{I=0}$$

so we can see immediately that helicity conservation at the quark level implies

$$\phi_2 = \langle ++ |\phi| -- \rangle = 0 , \tag{2.6}$$

$$\phi_5 = \langle ++ |\phi| +- \rangle = 0 . \tag{2.7}$$

The factor (−1) in Eq. (2.5) is worth comment. In the c.m. system we have in the initial state three quarks directed by convention along $\theta = 0$, and three along $\theta = \pi$. In the final state we have three quarks directed along $\theta = \theta_{\text{scatt}}$ and three along $\theta = \pi - \theta_{\text{scatt}}$. Four of the quarks have been rotated through θ_{scatt} and two through $\pi - \theta_{\text{scatt}}$. In the Jacob-Wick helicity basis (defining scattering in the x-z plane),[11] this rotation introduces a phase (−1) if the two quarks rotated through $\pi - \theta_{\text{scatt}}$ have opposite helicity. This phase convention is included in our definition of the observables in Table I. The crossing properties of our calculation are completely determined. If $f(\theta)$ gives the amplitude for scattering four quarks through an angle θ and two through $\pi - \theta$, then $f(\pi - \theta)$ is the amplitude for scattering four quarks through $\pi - \theta$ and two through θ. By the QIM assumptions this amplitude is independent of helicity and flavor.

Including flavor in the diagram with quark interchange introduces only small complications. The SU(6) wave functions[12] are

$$\sqrt{18}\,|p(+)\rangle = 2\,|u(+)u(+)d(-)\rangle - |u(+)d(+)u(-)\rangle - |d(+)u(+)u(-)\rangle + 2\,|u(+)d(-)u(+)\rangle - |u(+)u(-)d(+)\rangle$$
$$- |d(+)u(-)u(+)\rangle + 2\,|d(-)u(+)u(+)\rangle - |u(-)u(+)d(+)\rangle - |u(-)d(+)u(+)\rangle , \tag{2.8}$$

$$-\sqrt{18}\,|p(-)\rangle = 2\,|u(-)u(-)d(+)\rangle - |u(-)d(-)u(+)\rangle - |d(-)u(-)u(+)\rangle + 2\,|u(-)d(+)u(-)\rangle - |u(-)u(+)d(-)\rangle$$
$$- |d(-)u(+)u(-)\rangle + 2\,|d(+)u(-)u(-)\rangle - |u(+)u(-)d(-)\rangle - |u(+)d(-)u(-)\rangle , \tag{2.9}$$

$$-\sqrt{18}\,|n(+)\rangle = 2\,|d(+)d(+)u(-)\rangle - |d(+)u(+)d(-)\rangle - |u(+)d(+)d(-)\rangle + 2\,|d(+)u(-)d(+)\rangle - |d(+)d(-)u(+)\rangle$$
$$- |u(+)d(-)d(+)\rangle + 2\,|u(-)d(+)d(+)\rangle - |d(-)d(+)u(+)\rangle - |d(-)u(+)d(+)\rangle , \tag{2.10}$$

$$\sqrt{18}\,|n(-)\rangle = 2\,|d(-)d(-)u(+)\rangle - |d(-)u(-)d(+)\rangle - |u(-)d(-)d(+)\rangle + 2\,|d(-)u(+)d(-)\rangle - |d(-)d(+)u(-)\rangle$$
$$- |u(-)d(+)d(-)\rangle + 2\,|u(+)d(-)d(-)\rangle - |d(+)d(-)u(-)\rangle - |d(+)u(-)d(-)\rangle . \tag{2.11}$$

Note that the coefficients in (2.8)–(2.11) are not all unity.

To keep track of all the ways of exchanging quarks with various helicities and flavors, let $C^i_{\alpha\beta\gamma}$ be the coefficient of the SU(6) wave function for particle i where α, β, γ represent both the flavor and helicity indices. Thus in the process $ab \to cd$ (where a, b, c, d label the isospin and helicity of the nucleon) the helicity and flavor of quarks 2 and 3 for particles a and b are unchanged. Under the QIM assumptions this exchange is independent of the spectator quarks' flavor and helicity, and contributes

$$T_{11} = A_{\alpha_1'\alpha_2'\alpha_1\alpha_2} C^a_{\alpha_1\beta_1\gamma_1} C^c_{\alpha_1'\beta_1\gamma_1} C^b_{\alpha_2\beta_2\gamma_2} C^d_{\alpha_2'\beta_2\gamma_2} . \tag{2.12}$$

Here repeated indices are summed, and $A_{\alpha_1'\alpha_2'\alpha_1\alpha_2}$ is the amplitude for the chosen quarks to interact. The QIM further assumes that

$$A_{\alpha_1'\alpha_2'\alpha_1\alpha_2} \propto \delta_{\alpha_1'\alpha_2}\,\delta_{\alpha_2'\alpha_1}\frac{f(\theta)}{s^2} .$$

There are eight other contributions to the total scattering, as a result of choosing different initial quarks to interact. The sum of all diagrams is

$$\langle cd|\mathcal{T}|ab\rangle = A_{\alpha_1'\alpha_2'\alpha_1\alpha_2} M^{ac}_{\alpha_1\alpha_1'} M^{bd}_{\alpha_2\alpha_2'} . \tag{2.13}$$

For the QIM, the result in matrix form is

$$\langle cd|\mathcal{T}|ab\rangle \propto \mathrm{Tr}(M^{ac}M^{bd}) . \tag{2.14}$$

The calculation thus reduces to determining for each helicity and isospin configuration the matrix $M^{ij}_{\alpha\alpha'}$, which has a simple expression in terms of the SU(6) wave functions

$$M^{ij}_{\alpha\alpha'} = C^i_{\alpha\beta\gamma}C^j_{\alpha'\beta\gamma} + C^i_{\beta\alpha\gamma}C^j_{\beta\alpha'\gamma} + C^i_{\beta\gamma\alpha}C^j_{\beta\gamma\alpha'} . \tag{2.15}$$

These numbers can be computed directly from the wave function.

In the limit where all kinematic variables are large we write the scattering amplitude for $NN \to NN$,

$$\phi_i(s, \theta) = \frac{c}{s^4}\,\phi_i(\theta) . \tag{2.16}$$

Keeping track of crossing we have the angular factors

$pp \to pp$:

$$\begin{aligned}
&\phi_1(\theta)=124F(\theta)+124F(\pi-\theta)\,,\\
&\phi_2(\theta)=0\,,\\
&\phi_3(\theta)=56F(\theta)+68F(\pi-\theta)\,,\\
&\phi_4(\theta)=-68F(\theta)-56F(\pi-\theta)\,,\\
&\phi_5(\theta)=0\,;
\end{aligned} \tag{2.17}$$

$np \to np$:

$$\begin{aligned}
&\phi_1(\theta)=56F(\theta)+68F(\pi-\theta)\,,\\
&\phi_2(\theta)=0\,,\\
&\phi_3(\theta)=88F(\theta)+100F(\pi-\theta)\,,\\
&\phi_4(\theta)=32F(\theta)+32F(\pi-\theta)\,,\\
&\phi_5(\theta)=0\,.
\end{aligned} \tag{2.18}$$

These amplitudes have also been obtained independently by Brodsky, Carlson, and Lipkin.[13] They have an elegant method based on group theory for obtaining these results. Their method amounts to arguing that the matrix M^{ij} has the general form

$$M^{ij}=\sum_{\lambda} C_\lambda \Theta_\lambda\,, \tag{2.19}$$

where the Θ_λ are the 16 matrices formed from combining the SU(2) of helicity with the SU(2) of isospin. For no spin or isospin flip, the relevant terms for QIM are

$$M^{ii}=\frac{n_u+n_d}{4}+\frac{n_u-n_d}{4}\tau_3+\frac{n_+-n_-}{4}\sigma_3 + \frac{n_{u^+}+n_{d^-}-n_{u^-}-n_{d^+}}{4}\sigma_3\tau_3\,, \tag{2.20}$$

where the numbers n_α are the average number of quarks of type α. The Wigner-Eckart theorem can be used to obtain the matrix M^{ij} for helicity and/or isospin flip. The resulting matrices are

$$\begin{aligned}
&M^{p\pm,p\pm}=\tfrac{3}{4}+\tfrac{1}{4}\tau_3\pm\tfrac{1}{4}\sigma_3\pm\tfrac{5}{12}\sigma_3\tau_3\,,\\
&M^{p\pm,p\mp}=-\tfrac{1}{2}\sigma_- -\tfrac{5}{6}\sigma_-\tau_3\,,\\
&M^{n\pm,n\pm}=\tfrac{3}{4}-\tfrac{1}{4}\tau_3\pm\tfrac{1}{4}\sigma_3\mp\tfrac{5}{12}\sigma_3\tau_3\,,\\
&M^{n\pm,n\mp}=-\tfrac{1}{2}\sigma_+ +\tfrac{5}{6}\sigma_+\tau_3\,.
\end{aligned} \tag{2.21}$$

The results in (2.17) and (2.18) follow from multiplying two of these matrices and taking the trace, that is, one evaluates the sum of the product of the coefficients for the different possibilities. It should be noted that with their method, σ represents not ordinary spin, but what they term H spin, which is related to the helicity.

III. THE CONTRIBUTION OF INSTANTONS

In perturbative QCD models, assuming that the u and d quarks in the nucleon are very light, we neglect the amplitude for gluon exchange to flip the helicity of a quark, as this amplitude is proportional to the quark mass. However, we may be leaving out something very important if we consider only these perturbative effects.

The existence of instanton solutions to QCD gives the theory a multiple vacuum structure which is central to our present understanding of the $U_A(1)$ problem. The QCD Lagrangian with two massless flavors of quarks is symmetric under $SU(2)\otimes SU(2)\otimes U_B(1)\otimes U_A(1)$. However, the consequences of a conserved axial-vector baryon current (either a fourth Goldstone boson with the quantum numbers of the η, or parity doubling of all observed hadrons) are not observed in nature. Instantons induce an anomaly in axial-vector baryon current conservation and thus break the undesired symmetry.[9]

Instantons also result in an effective interaction between the very light u and d quarks. The chiral nature of the interaction is such that both flavors must have the same initial helicity, and both are flipped in the final state. For NN scattering it is no longer necessary that ϕ_2 vanish, though ϕ_5 is unaffected by the instanton induced interaction.

Briefly, we review how the effective four-fermion interaction comes about. The generating functional for fermion Green's functions in Euclidean space may be written[14]

$$Z(\eta,\bar\eta)=\int \mathfrak{D}A\,\mathfrak{D}\Psi\,\mathfrak{D}\bar\Psi \times\exp\left[-\int d^4x(\tilde{\mathcal{L}}(x)+\bar\Psi_i \not{D}\Psi_i+\bar\Psi_i\eta_i+\bar\eta_i\Psi_i)\right], \tag{3.1}$$

where η and $\bar\eta$ are c-number fermion sources and $\tilde{\mathcal{L}}$ is the usual QCD Lagrangian containing gauge-fixing and ghost terms. We have suppressed the integration over the ghost fields, and over the flavors of heavy quarks.

Integrating over the fermion fields one obtains

$$Z(\eta,\bar\eta)=\sum_{Q=-\infty}^{\infty}\int[\mathfrak{D}A]_Q(\mathrm{Det}\not{D}_\perp)^{N_f} \times\prod_{\lambda=1}^{|Q|}\prod_{i=1}^{N_f}\int d^4x_1\bar\eta_i(x_1)\chi_\lambda(x_1)\int d^4x_2\bar\chi_\lambda(x_2)\eta_i(x_2)\exp\left[-\int d^4x_3\tilde{\mathcal{L}}(x_3)+\int d^4x\,d^4y\,\bar\eta(x)S_\perp(x,y)\eta(y)\right], \tag{3.2}$$

where N_f is the number of massless flavors, Q is the topological charge of the gauge fields, and $[\mathfrak{D}A]_Q$ indicates that the integration is restricted to fields of topological charge Q. $S_\perp$ and $\not{D}_\perp$ are the propagator and the gauge-covariant derivative operator restricted to functions orthogonal to the zero eigenvectors of $\not{D}$. The χ_λ are the zero eigenvectors, i.e., $\not{D}\chi_\lambda = 0$. There are $|Q|$ independent eigenfunctions. Fermion Green's functions are obtained by taking functional derivatives of (4.2) evaluated at $\eta_i = \bar{\eta}_i = 0$. Contributions from gauge field configurations with $|Q| \neq 0$ will vanish unless there are $|Q|$ incoming and $|Q|$ outgoing fermions of *each* light flavor.

Working in the approximation that there are two light flavors, u and d, instantons and anti-instantons generate a four-fermion interaction. The S-matrix element for scattering may be written[14]

$$\mathfrak{M} \sim \bar{u}_{\alpha_4}(p_4)\bar{u}_{\alpha_3}(p_3)u_{\alpha_2}(p_2)u_{\alpha_1}(p_1) \times [(1\mp\gamma_5)_{\alpha_4\alpha_1}(1\mp\gamma_5)_{\alpha_3\alpha_2} + (1\mp\gamma_5)_{\alpha_4\alpha_2}(1\mp\gamma_5)_{\alpha_3\alpha_1}] \times \int_0^\infty d\rho\, \mathfrak{D}(\rho)e^{-4m\rho}(4\pi\rho)^4 . \quad (3.3)$$

where $\mathfrak{D}(\rho)$ is the density of instantons or anti-instantons of size ρ and m is the u,d quark mass. It is best to calculate with a small quark mass to begin with and let the mass go to zero at the end of the calculation. We have suppressed the flavor indices in (3.3).

Inserting this between explicit spinors and keeping track of the flavor dependence of the interaction gives

$$\langle p_4 -; p_3 - |\mathfrak{M}(s,\theta)| p_1 +; p_2 +\rangle \propto \epsilon_{f_1 f_2}\, s \sin^2(\theta/2) \int_0^\infty d\rho\, \mathfrak{D}(\rho)e^{-4m\rho}(4\pi\rho)^4 , \quad (3.4)$$

where $\epsilon_{f_1 f_2}$ implies that the two initial quarks must have distinct flavors.

We assume that Eq. (3.4) gives the necessary helicity and isospin structure of instanton-induced qq scattering. However, the s dependence is clearly not to be taken seriously. Instantons provide an effective four-fermion interaction. Just as the Fermi four-fermion weak interaction violates unitarity at large s, so does the single-instanton approximation. A more reliable approximation in the case of instantons would be the dilute-gas approximation.[14,15] Furthermore, the instanton-induced effects in NN scattering involve interactions between off-mass-shell quarks, whereas Eq. (3.4) assumes the quarks are all on the mass shell. Even if we knew the energy dependence of the effect with some certainty, it would still not be clear how to determine the absolute normalization since there is a great deal of controversy about the normalization of effects which depend on integrals over instanton sizes.[15,16] Phenomenologically, it may be possible to summarize the instanton effects by parameters describing the SU(6) breaking of nucleon wave functions.

Nonetheless, we can isolate instanton-induced effects by comparing np and pp scattering. Using the flavor and helicity dependence of Eq. (3.4), we can calculate the ratio of ϕ_2 for pp and np scattering using the counting techniques of Sec. II. We find

$$\phi_2(pp)^{\text{inst}}/\phi_2(np)^{\text{inst}} = -\tfrac{8}{17} . \quad (3.5)$$

IV. THE PREDICTIONS FOR NN OBSERVABLES

One of the most direct consequences of perturbative QCD models discussed above is that, of the ten real numbers representing five complex amplitudes at a given angle and energy, only three are expected to be nonzero. The fact that all of the amplitudes are expected to be real immediately implies

$$P_0 = 0 . \quad (4.1)$$

This requirement of real amplitudes at large angles must be relaxed in the forward, Regge, region where amplitudes pick up a Regge phase. The vanishing of the polarization may therefore be used as a preliminary test to ensure that the model is being compared with data in a regime in which it might be expected to hold.

The second consequence which follows only from the assumption of helicity conservation among the quarks (neglecting components of the wave function in which the valence quarks alone do not carry the helicity of the nucleon) is that the two amplitudes, $\phi_2(\theta)$ and $\phi_5(\theta)$, vanish. The absence of these amplitudes has the result Eq. (4.1), as well as

$$A_{nn} = -A_{ss} , \quad (4.2)$$

$$A_{sl} = 0 \quad (\text{independent of } \theta) . \quad (4.3)$$

It is evident that these results are also true of the Landshoff contribution, if present at higher energy. Furthermore, the predictions (4.1), (4.2), and (4.3) are not changed by modifying the SU(6) wave functions of the nucleon, as long as components having total quark helicity unequal to the nucleon helicity are not introduced, nor are they changed by allowing nonzero amplitudes to have a relative phase. These predictions represent a fairly direct test of the underlying helicity conservation. Since this may be broken by very interesting nonperturbative effects generated by instantons, it is important to test for (4.2) and (4.3), as well as (4.1).

At 90° in the c.m. system for pp scattering we have some additional simplification in the problem since from Table I

$$\phi_3(\pi/2) = -\phi_4(\pi/2). \tag{4.4}$$

Note also that, independent of the dynamics of quark helicity conservation, symmetry arguments give

$$\phi_5(\pi/2) = 0. \tag{4.5}$$

We can use (4.4) to write

$$1 - A_{nn}(\pi/2) + A_{ss}(\pi/2) + A_{ll}(\pi/2) = 0 \tag{4.6}$$

for pp scattering. A measurement of A_{ll} at 90° combined with either a measurement of A_{ss} or A_{nn} can be used in conjunction with (4.2) and (4.6) to test for quark helicity conservation.

Now let us concentrate on those results which rely specifically on the QIM amplitudes given in (2.17) and (2.18). In the QIM at 90° everything can be described in terms of one parameter

$$x = \frac{\phi_3(\pi/2)}{\phi_1(\pi/2)}. \tag{4.7}$$

Using the SU(6) wave functions and the amplitudes (2.17) we get

$$x_{\rm QIM} = \tfrac{1}{2}. \tag{4.8}$$

This implies

$$A_{ll}(\pi/2) = A_{ss}(\pi/2), \tag{4.9}$$

which can be combined with (4.6) and (4.2) to give immediately

$$A_{nn}(\pi/2) = \tfrac{1}{3}, \quad A_{ll}(\pi/2) = A_{ss}(\pi/2) = -\tfrac{1}{3}, \tag{4.10}$$

for pp scattering.

The pp spin-spin asymmetries have very little angular dependence because $\phi_3(\theta)$ and $\phi_4(\theta)$ are almost symmetric around $\pi/2$. For instance, with our SU(6) assumptions,

$$\frac{\phi_{\rm anti}}{\phi_{\rm sym}} = \frac{68-56}{68+56} = \frac{3}{31} \tag{4.11}$$

is quite small. The analytic expressions for the asymmetries as a function of angle are given in Table III.

The results for $np \to np$ are similar to those of pp. The magnitudes of the three np asymmetries A_{nn}, A_{ss}, and A_{ll} are equal. Their signs in each case are opposite to the corresponding ones in pp, and the common value is approximately 0.44 (see Table III).

It is instructive to compare the QIM results Eq. (4.10) to those of elastic electron-electron scatter-

TABLE III. QIM observables.

	$F(\theta) = \frac{1}{31}[s(\theta) + a(\theta)]$
	$F(\pi-\theta) = \frac{1}{31}[s(\theta) - a(\theta)]$
$pp \to pp$	$\frac{d\sigma}{d\Omega}(\theta) = 3s(\theta)^2\left(1 + \frac{3}{31^2}\frac{a(\theta)^2}{s(\theta)^2}\right)$
	$A_{nn}(\theta) = -A_{ss}(\theta) = -A_{ll}(\theta)$
	$A_{nn}(\theta) = \frac{1}{3}\,\dfrac{1 - \frac{3^2}{31^2}\frac{a(\theta)^2}{s(\theta)^2}}{1 + \frac{3}{31^2}\frac{a(\theta)^2}{s(\theta)^2}} \approx \frac{1}{3}$
	$P_0(\theta) = A_{sl}(\theta) = 0$
$np \to np$	$\frac{d\sigma}{d\Omega}(\theta) = \frac{3\times 571}{31\times 31}s(\theta)^2\left[1 - \frac{6\times 13}{571}\frac{a(\theta)}{s(\theta)} + \frac{3}{571}\frac{a(\theta)^2}{s(\theta)^2}\right]$
	$A_{nn}(\theta) = -A_{ss}(\theta) = -A_{ll}(\theta)$
	$P_0(\theta) = A_{sl}(\theta) = 0$
	$A_{nn}(\theta) = -\frac{47\times 16}{3\times 571}\,\dfrac{1 - \frac{3}{47}\frac{a(\theta)}{s(\theta)}}{1 - \frac{6\times 13}{571}\frac{a(\theta)}{s(\theta)} + \frac{3}{571}\frac{a(\theta)^2}{s(\theta)^2}} \approx -0.44$
	$\frac{d\sigma}{d\Omega}(np) \Big/ \frac{d\sigma}{d\Omega}(pp) \approx \frac{571}{961} \approx 0.6$

ing. In the latter case, the interaction due to photon exchange is purely a vector-vector coupling, properly antisymmetrized. At 90° c.m. angle, the results are

$$A_{nn}(\pi/2) = -A_{ss}(\pi/2) = \tfrac{1}{9}, \quad A_{ll}(\pi/2) = -\tfrac{7}{9}, \tag{4.12}$$

for the spin-spin asymmetries, and

$$\frac{d\sigma}{dt} \propto s^{-2} \tag{4.13}$$

for the s dependence of the fixed-angle cross section. Even though one may imagine that the underlying interaction between quarks is also a vector interaction,[17] the bound-state nature of the objects being scattered has a profound influence on both the s dependence of the cross section and its spin dependence.

To illustrate the sensitivity of the data on spin-spin observables to the presence of helicity-nonconserving interactions, we can construct a simple amalgamation of the QIM amplitudes given in (2.17) and (2.22) together with the amplitude

$$\phi_2(\theta) = c[8g(\theta) + 8g(\pi - \theta)], \tag{4.14}$$

with $g(\theta) = 1/(1-\cos\theta)^3$ as suggested by (3.4). The factor "8" is obtained from counting arguments for pp scattering using SU(6) wave functions and the flavor and helicity dependence of (3.4). Corresponding arguments for np give a factor of 17. The constant c is unknown and relates the magnitude of a quark-helicity-nonconserving amplitude to a QIM amplitude at a given energy. Our convention on the QIM normalization is such that

$$\phi_2(\pi/2)/\phi_1(\pi/2) = 2c/31 .$$

Choosing the normalization for $\phi_1(\pi/2)$ corresponds to fixing that of $F(\theta)$ in (2.17) and (2.18) in the QIM. Since $F(\theta)$ appears in symmetric form, $F(\pi/2) = \frac{1}{2}$; the angular dependence for $\cos\theta \neq -1,1$ can be estimated using t^{-2} for the nucleon form factor[18]:

$$F(\theta) = \frac{0.5}{(1-\cos\theta)^4} . \tag{4.15}$$

This shape is in rough agreement with the shape of the large-angle elastic cross section.

The detailed form of $F(\theta)$ is not important for the QIM predictions of spin-spin asymmetries since the different helicity amplitudes have approximately the same angular dependence. However, the instanton-induced effect appears in A_{nn} as an interference with QIM so for the amalgamated model the predictions have some sensitivity to the choice of $F(\theta)$.

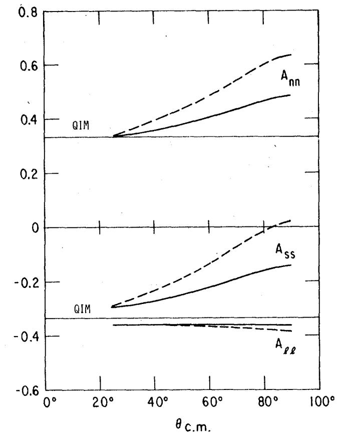

FIG. 3. Asymptotic behavior of $pp \to pp$ asymmetries A_{nn}, A_{ss}, and A_{ll} obtained by combining the QIM amplitudes with an instanton generated $\phi_2(\theta)$ as described in the text. The solid line $c = 2$, dashed line $c = 4$.

The spin-spin observables for $pp \to pp$ are plotted as a function of angle in this amalgamated model in Fig. 3 for two values of the constant c, $c = 2$ and $c = 4$. We see that the dominant effect is to increase A_{nn} and A_{ss} while having comparatively little effect on A_{ll}. At 90° we see that it is possible to reproduce the magnitude of A_{nn} with $c = 4$. The corresponding modifications of QIM in np are similar. At 90° with $c = 4$, for example, $A_{nn} = -0.57$ and $A_{ss} \approx 0.31$. Assuming the QIM contributions are reliable, a comparison of np and pp spin-spin data provides a test of the helicity and flavor dependence of instantonlike effects.

The necessity for some mechanism in addition to QIM is evident from the structure observed[8] in A_{nn} near 90° at $p_{lab} = 12$ GeV/c. However, these deviations from QIM might be understandable without instanton effects or helicity nonconservation. A measurement of either A_{ll} or A_{ss} near 90° could be decisive.

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