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Asymptotic freedom in the infinite-momentum frame

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We study the renormalization of SU(N) Yang-Mills theory through one loop in the null-plane gauge. We choose as a measure of the effective coupling the off-shell four-gluon correlation function with all legs having zero transverse momentum. We find that, in addition to self-energy and vertex corrections, two-gluon exchange contributes to coupling-constant renormalization in an essential way. In this gauge, asymptotic freedom is due to a residual attraction between two gluons in the ultraviolet domain.

I. INTRODUCTION

In this article we show how asymptotic freedom[1] arises in the null-plane gauge $[A^+\equiv(1/\sqrt{2})(A^0+A^3)=0]$. This gauge is the appropriate one for quantizing a gauge theory on a lightlike plane: $x^+\equiv(1/\sqrt{2})(x^0+x^3)=\text{constant}$.[2] Our main motivation for presenting this work is that null-plane quantization is a promising technique[3,4] for studying an SU(N) gauge theory in the limit $N\to\infty$.[3] Asymptotic freedom, the fact that the effective coupling constant gets large at low momentum, is commonly interpreted as a signal that the spectrum of non-Abelian gauge theories is dramatically different from that of Abelian gauge theories. If this is right, an understanding of the physics underlying asymptotic freedom should point the way to a better understanding of the instability which is at the heart of this difference. If we are going to study the $N\to\infty$ limit of quantum chromodynamics through the techniques of null-plane quantization, we should surely understand the dynamical manifestation of asymptotic freedom.

Physics looks different in different gauges. The physics of asymptotic freedom in the Coulomb gauge has been elucidated in the work of Appelquist, Dine, and Muzinich.[5] They study the effective coupling between two separated heavy-quark and antiquark color sources, and they find that asymptotic freedom manifests itself as an attraction between color electric field lines which implies a tendency for the field lines to be more collimated than in the Abelian case. Asymptotic freedom is the statement that this collimation increases with the separation of the sources. Our aim in this paper is to provide an analogous physical picture for the null-plane gauge.

There seems to be some controversy about the very existence of the null-plane gauge. For example, in an Appendix, Curtright and Ghandour[6] study a class of axial gauges ($n_\mu A^\mu=0$) with $n\cdot n\neq 0$ and try to reach the null-plane gauge by taking $n^2\to 0$ at the end of the calculation. For the gluon propagator, they find that this limit does not exist. Of course, since the gluon propagator is gauge variant, there is no reason the limit should exist.

Our approach will be to treat the null-plane gauge on its own terms: We set $A^+=0$ *ab initio* and pursue the consequences for one-loop calculations. The problem mentioned in the preceding paragraph stems from singularities in the gluon propagator as $P^+\equiv(1/\sqrt{2})(P^0+P^3)\to 0$: In the $A^+=0$ gauge the propagator is

$$D^{\mu\nu}(P)=\left(\frac{-i}{P^2}\right)\left(g^{\mu\nu}-\frac{P^\mu g^{\nu+}+P^\nu g^{\mu+}}{P^+}\right). \tag{1.1}$$

We regulate these singularities by cutting out the region of integration $-\epsilon\leq P^+\leq\epsilon$ and maintaining $\epsilon\neq 0$ throughout the calculation. We use the *same* ϵ *for all propagators*. This is essential for consistency. We then compute radiative corrections to the four-gluon correlation function. We find that each individual graph contributes an ϵ-dependent piece to coupling-constant renormalization. This ϵ dependence is expected to cancel in the sum of all contributions to a gauge-invariant quantity.

The ulatrviolet divergence of the four-gluon correlation function is one such gauge-invariant quantity. In the null-plane gauge, there are logarithmic divergences in two-gluon exchange graphs as well as in vertex and self-energy corrections. The ϵ dependence cancels when all these divergent parts are added together. In this gauge, wave-function renormalization works against asymptotic freedom: $Z<1$ as required by the null-plane commutation relations. The combination of wave-function renormalization and vertex renormalization leaves the ultraviolet part of the three-gluon correlation function ϵ dependent and with the wrong sign for asymptotic freedom. The ϵ dependence is finally canceled by the two-gluon exchange graphs yielding the correct coefficient of the logarithmic divergence. The two-gluon exchange contribution has the correct sign (corresponding to an attractive interaction) for asymptotic freedom.

In Sec. II we present a detailed description of our computations. We close Sec. II with a comparison to charge renormalization in QED in the null-plane gauge. In Sec. III we discuss the physical picture emerging from our results and possible implications for our large-N program.

II. RADIATIVE CORRECTIONS TO GLUON-GLUON SCATTERING

A. Verbal description of the calculation

We shall restrict our study to SU(N) Yang-Mills theory. At certain points of the calculation, it will be illuminating to compare our calculation to scalar quantum electrodynamics. Because of infrared divergences we are compelled to keep the gluons off the mass shell. The number of kinematic variables to describe gluon-gluon scattering is clearly very large. We shall simplify our study by examining this process in a particular kinematic domain which we shall describe shortly. This will be sufficient to illustrate the way asymptotic freedom comes about in the null-plane gauge.

We shall compute the Fourier transform of the connected four-gluon correlation function

$$i\Gamma^4_{\mu_1\mu_2\mu_3\mu_4} \equiv \langle \mathrm{Tr}[A_{\mu_1}(x_1)A_{\mu_2}(x_2)A_{\mu_3}(x_3)A_{\mu_4}(x_4)]\rangle_c \tag{2.1}$$

through one loop in the null-plane gauge. We are representing the gluon field as an $N\times N$ anti-Hermitian matrix

$$A^{\mu}_{\alpha\beta}(x) = -A^{\mu *}_{\beta\alpha} .$$

In Feynman graphs the gluon line is represented as a double line with arrows indicating the direction of color flow. The correlation function (2.1), for example, has the graphical representation shown in Fig. 1.

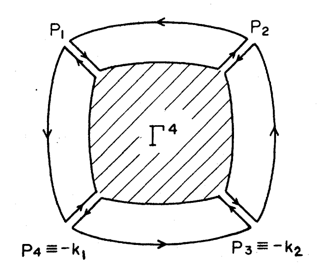

FIG. 1. Graphical representation of the correlation function (2.1). Each double line emerging from the central blob represents a gluon. The trace in (2.1) is represented by the lines connecting each gluon to its two neighbors.

In the $A^+=0$ gauge only the transverse components of the gauge field, A_i, $i=1,2$, are dynamically independent, and we shall only consider the case where all fields occurring in (2.1) are transverse. Thus the Fourier transform of Γ^4 will depend on four-momenta p_k and four polarizations i_k, $k=1,2,3,4$. The main kinematic restriction we make in this paper is to take the transverse momentum of each gluon to be zero:

$$\vec{p}_{\perp k}=0, \quad k=1,2,3,4 .$$

This restriction simplifies the momentum dependence of Γ_4 and also simplifies the discussion of wave-function renormalization since the longitudinal and transverse gluons do not mix when $p_\perp=0$.

In evaluating Feynman integrals we always do the l^- integration first. This corresponds to doing old-fashioned perturbation theory with energy denominators: The integration region in l^+ is broken into different domains corresponding to different x^+ orderings of the vertices. Next, the $\vec{l}_\perp$ integration is performed. These integrations are ultraviolet divergent, and we introduce a simple cutoff $\vec{l}_\perp^{\,2} \leq \Lambda^2$ in transverse-momentum space. Finally, the integration over l^+ is essentially the same as the integration over the usual Feynman parameter x and we will usually leave it unevaluated.

B. Dependence of Γ^4 on N

Because we are computing a color singlet, the internal-symmetry structure of the gauge group will only enter the calculation as an N-dependent factor multiplying the contribution of each diagram. One way of working out these factors using the double-line representation of the gluon line is to sum over all possible routings of color through the diagram. For a diagram of fixed momentum structure, there are many color routings corresponding to the twisting of various gluon lines. The N dependence for a particular color routing is simple $(-1)^T N^L$ where T is the number of twists and L is the number of closed index loops. The results of this procedure are summarized in Fig. 2.

C. Gluon self-energy: Wave-function renormalization and vacuum polarization [see Figs. 2(b) and 2(c)]

We do not make any kinematic restrictions for this part of the calculation. Let us define the vacuum polarization tensor $\Pi_{\mu\nu}(k)$ as usual (see Fig. 3). Then because of our gauge choice $A^+=0$, only Π_{--}, Π_{-i}, and Π_{ij} enter any calculation. Treating these in turn, we have (assuming $k^+>0$)

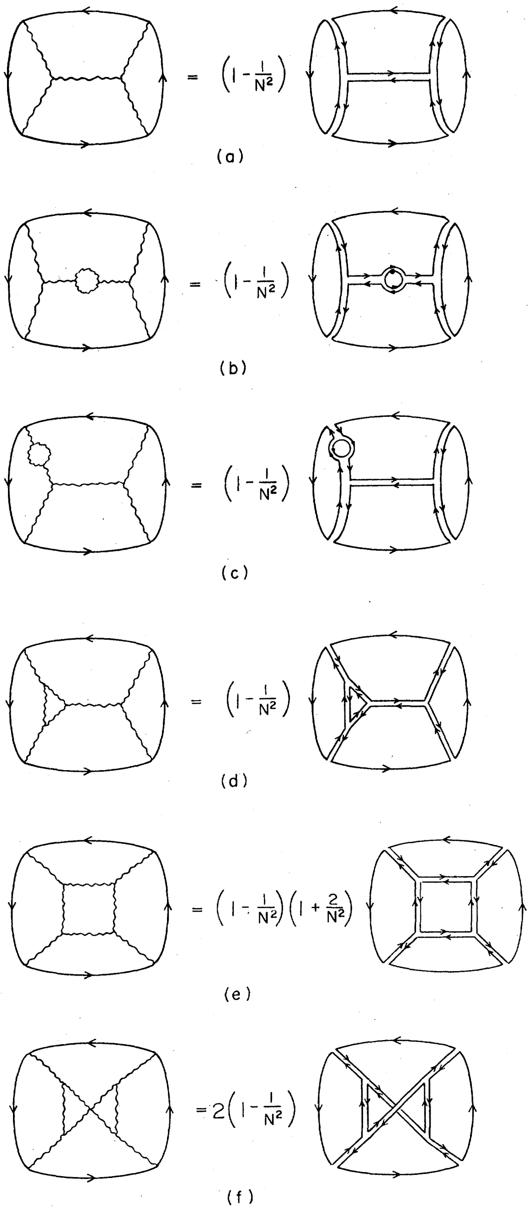

FIG. 2. Color dependence of some graphs contributing to (2.1). The graphs on the left-hand side represent the sum over all allowed color routings with the fixed space-time structure shown. Those on the right have the simple N dependence N^L, with L the number of closed index loops.

$$i\Pi_{--} = \frac{Ng^2}{(2\pi)^4}\int d^4l \frac{\delta_{ij}\delta_{ij}}{l^2(l-k)^2}(k^+ - 2l^+)^2$$
$$= 2k^{+2}\frac{Ng^2 i}{16\pi^2}\left[\int_0^{k^+}\frac{dl^+}{k^+}(k^+ - 2l^+)^2 \ln\frac{\Lambda^2}{\frac{l^+}{k^+}\left(1-\frac{l^+}{k^+}\right)k^2}\right]$$
$$= 2k^{+2}\frac{Ng^2 i}{16\pi^2}\left(\tfrac{1}{3}\ln\frac{\Lambda^2}{k^2}+4\right) \equiv ik^{+2}\Pi_1(k^2)\,. \qquad (2.2)$$

$$i\Pi_{-j} = \frac{Ng^2}{(2\pi)^4}\int d^4l(k_- - 2l_-)iD^{m\mu}(l)iD_m{}^{\nu}(l-k)$$
$$\times\left[(l_\nu + k_\nu)g_{\mu j} + (l_\mu - 2k_\mu)g_{\nu j} + (k_j - 2l_j)g_{\mu\nu}\right]$$
$$= 2k_j k_- \frac{Ng^2 i}{16\pi^2}\left(\tfrac{1}{3}\ln\frac{\Lambda^2}{k^2}+4\right) = ik_-k_j\Pi_1(k^2)\,. \qquad (2.3)$$

The calculation of Π_{ij} is much more involved. The graph in Fig. 3(a) is quadratically divergent and is also divergent as our P^+ cutoff ϵ goes to zero. It is instructive to separate this quadratically divergent contribution into two pieces corresponding to the way the calculation is set up when the theory is quantized on the null plane. There is an "instantaneous" piece,

$$i\Pi_{ij}^{(a)\text{inst.}} \underset{\Lambda\to\infty}{\sim} -\frac{ig^2N}{16\pi^2}\left[\tfrac{1}{2}\int_{-\infty}^{\infty} dl^+\left(\frac{1}{|l^+ - k^+|}+\frac{1}{|l^+|}\right) + 8\left(\frac{k^+}{\epsilon}-1\right)\right]\Lambda^2\delta_{ij}\,, \qquad (2.4)$$

which corresponds to an induced nonlocal quartic interaction arising when the A_+ field is explicitly eliminated from the Hamiltonian. Thus this term is a part of the matrix element of the Hamiltonian in a one-gluon state. Then there is the "noninstantaneous" piece which comes from the iteration of trilinear vertices:

$$i\Pi_{ij}^{(a)\text{noninst.}} \underset{\Lambda\to\infty}{\sim} +\frac{ig^2N}{16\pi^2}\left[4+8\left(\frac{k^+}{\epsilon}-1\right)\right]\Lambda^2\delta_{ij}\,. \qquad (2.5)$$

The ϵ dependence cancels between these pieces yielding the result

$$i\Pi_{ij}^{(a)} \underset{\Lambda\to\infty}{\sim} \frac{ig^2N}{16\pi^2}\left[4-\tfrac{1}{2}\int_{-\infty}^{\infty} dl^+\left(\frac{1}{|l^+ - k^+|}+\frac{1}{|l^+|}\right)\right]\Lambda^2\delta_{ij}\,. \qquad (2.6)$$

The graph in 3(b) yields

$$i\Pi_{ij}^{(b)} = \frac{ig^2N}{16\pi^2}\Lambda^2\delta_{ij}\int_{-\infty}^{\infty}\frac{dl^+}{|l^+|}\,, \qquad (2.7)$$

which formally cancels the second term in (2.6), leaving

$$i\Pi_{ij} \underset{\Lambda\to\infty}{\sim} \frac{ig^2N}{16\pi^2}\delta_{ij}(4\Lambda^2)\,. \qquad (2.8)$$

This result would imply a downward shift in the gluon (mass)^2

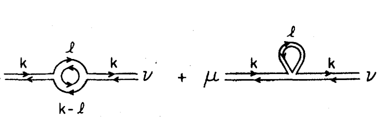

FIG. 3. Graphs contributing to the vacuum polarization tensor. It is understood that there are no propagators for the external legs in the definition of $\Pi_{\mu\nu}$.

$$\delta\mu_G{}^2 = -\frac{g^2N}{16\pi^2}(4\Lambda^2)\,, \tag{2.9}$$

which violates Lorentz invariance. Thus an explicit gluon-mass counterterm must be added to the Hamiltonian to cancel this effect.

In light-cone quantization, it is very natural to normal-order the interaction terms in the Hamiltonain. If this is done, the graph in Fig. 3(b) will be zero and the instantaneous piece (2.4) of the graph in Fig. 3(a) will be zero. Thus the second-order self-energy will yield a mass shift [cf. (2.5)]

$$\delta\mu^2 = -\frac{g^2N}{16\pi^2}\left[4+8\left(\frac{k^+}{\epsilon}-1\right)\right]\Lambda^2 . \tag{2.10}$$

We must accordingly start with a gluon bare mass

$$\mu_0{}^2 = \frac{g^2N}{16\pi^2}\Lambda^2\left[4+8\left(\frac{k^+}{\epsilon}-1\right)+O(g^2)\right] \tag{2.11}$$

in order for one-loop corrections to be consistent with Lorentz invariance.

Having removed the quadratic divergence by an explicit counterterm, it is now straightforward to compute Π_{ij}, with the result

$$i\Pi_{ij} = k_ik_j\Pi_1(k^2) - k^2\Pi_2\left(k^2,\frac{k^+}{\epsilon}\right),$$

where

$$\begin{aligned}\Pi_1(k^2) &= \frac{g^2N}{16\pi^2}\left(\tfrac{2}{3}\ln\frac{\Lambda^2}{k^2}+8\right),\\ \Pi_2(k^2) &= \frac{g^2N}{16\pi^2}\left\{\left(8\ln\frac{k^+}{\epsilon}-\frac{22}{3}\right)\ln\frac{\Lambda^2}{k^2}+\int_{\epsilon/k^+}^{1-\epsilon/k^+}dx\left[\frac{4}{x}+\frac{4}{1-x}-8+4x(1-x)\right]\ln\frac{1}{x(1-x)}\right\}.\end{aligned} \tag{2.12}$$

We finally quote these results as a renormalization of the different components of the gluon propagator

$$\begin{aligned}D^{ij}(k) &= \frac{-i\delta_{ij}}{k^2}[1-\Pi_2(k^2,k^+/\epsilon)]\,,\\ D^{i-}(k) &= \frac{-ik_i}{k^2}[1-\Pi_2(k^2,k^+/\epsilon)]\,,\\ D^{--}(k) &= \frac{i}{k^{+2}}[1-\Pi_1(k^2)] - \frac{i\vec{k}^2}{k^{+2}k^2}[1-\Pi_2(k^2,k^+/\epsilon)]\,.\end{aligned} \tag{2.13}$$

D. The proper three-point function [see Fig. 2(d)]

As we consider higher-point functions, the restriction to gluons with zero transverse momentum simplifies the problem considerably. For example, in this case the only nonzero three-point function is the one involving two transverse gluons and one longitudinal gluon. With the kinematics shown in Fig. 4, we have for the ultraviolet-divergent part (we assume $k^+>p^+>0$)

$$i\Gamma^3_{ij-} \underset{\Lambda\to\infty}{\sim} \delta_{ij}\frac{ig^2N}{16\pi^2}\ln\frac{\Lambda^2}{\mu^2}\left[\tfrac{10}{3}(k^++p^+)-2(k^++p^+)\ln\frac{k^+p^+}{\epsilon^2}+2(k^{+2}+p^{+2})\left(\frac{1}{p^+}\ln\frac{k^+-p^+}{k^+}+\frac{1}{k^+}\ln\frac{k^+-p^+}{p^+}\right)\right], \tag{2.14}$$

where μ is some renormalization point.

E. Gluon-gluon scattering at zero transverse momentum [see Figs. 2(a)–2(f)]

That we must consider the four-point function to understand asymptotic freedom in the null-plane gauge is apparent from the foregoing results. To see this, combine the ultraviolet-divergent parts of all the graphs that renormalize the three-point function for all $p_\perp=0$ (see Fig. 5):

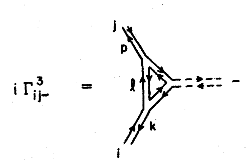

FIG. 4. Radiative corrections to the three-point function with $\vec{k}_\perp=\vec{p}_\perp=0$. The dotted line represents the longitudinal gluon.

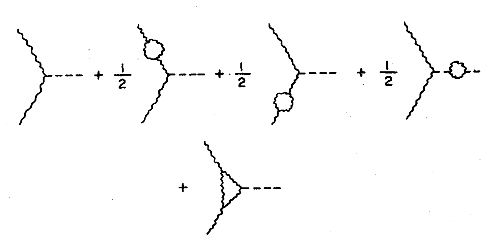

FIG. 5. The graphs contributing to the renormalized three-point function through one loop.

$$\Gamma^{3}_{ij-}+(k_-+p_-)\delta_{ij}[1-\tfrac{1}{2}\Pi_1(k-p)-\tfrac{1}{2}\Pi_2(p)-\tfrac{1}{2}\Pi_2(k)]$$
$$\underset{\Lambda\to\infty}{\sim}\delta_{ij}(k_-+p_-)\left\{1+\frac{g^2N}{16\pi^2}\ln\frac{\Lambda^2}{\mu^2}\left[2\ln\frac{k^+p^+}{\epsilon^2}-\tfrac{10}{3}-2\frac{k^{+2}+p^{+2}}{k^++p^+}\left(\frac{1}{p^+}\ln\frac{k^+-p^+}{k^+}+\frac{1}{k^+}\ln\frac{k^+-p^+}{p^+}\right)\right]-\tfrac{1}{3}+\tfrac{22}{3}-4\ln\frac{k^+p^+}{\epsilon^2}\right\}$$
$$\sim\delta_{ij}(k_-+p_-)\left\{1+\frac{g^2N}{16\pi^2}\ln\frac{\Lambda^2}{\mu^2}\left[\tfrac{11}{3}-2\ln\frac{k^+p^+}{\epsilon^2}-2\frac{k^{+2}+p^{+2}}{k^++p^+}\left(\frac{1}{p^+}\ln\frac{k^+-p^+}{k^+}+\frac{1}{k^+}\ln\frac{k^+-p^+}{p^+}\right)\right]\right\},\qquad(2.15)$$

which is ϵ dependent, and the coefficient of $\ln(\Lambda^2/\mu^2)$ has the wrong sign for asymptotic freedom.

To understand what is going on, consider gluon-gluon scattering with all $p_\perp$ set equal to zero. The lowest-order contribution to the four-point correlation function (2.1) is given by a single longitudinal gluon exchange [see Fig. 6(a)]. Let us focus on the graph with singularities in the t (i.e., p_1,p_4) channel:

$$i\Gamma^{4(t)}_{i_1i_2j_2j_1}=ig^2N^4\left(1-\frac{1}{N^2}\right)\delta_{i_1j_1}\delta_{i_2j_2}\frac{(k_1^++p_1^+)(k_2^++p_2^+)}{(k_1^+-p_1^+)^2}+O(g^4).\qquad(2.16)$$

In addition to vertex and self-energy corrections, there are also many graphs involving double-gluon exchange which correct (2.16). To calculate all of them would be very laborious, but we can save ourselves a lot of work if we keep only the contributions with a quadratic singularity as $k_1^+\to p_1^+$. It is not hard to verify that this quadratic singularity is only present in the double-gluon exchange graphs of Fig. 6(b), where the dotted lines correspond to longitudinal gluons, i.e., their propagator is

$$D^{\mu\nu}_{\text{longitudinal}}\equiv D^{--}g^{\mu+}g^{\nu+}.\qquad(2.17)$$

These graphs are relatively simple to evaluate. We quote here only their ultraviolet-divergent part, keeping only terms with the quadratic singularity as $k_1^+\to p_1^+$:

$$i\Gamma^{4(t)[1\,\mathrm{PIR}]}_{i_1i_2j_2j_1}\underset{\substack{\Lambda\to\infty\\ k_1^+\to p_1^+}}{\sim}i\delta_{i_1j_1}\delta_{i_2j_2}N^4\left(1-\frac{1}{N^2}\right)g^2\frac{4p_1^+p_2^+}{(k_1^+-p_1^+)^2}\left[8\frac{g^2N}{16\pi^2}\ln\frac{\Lambda^2}{\mu^2}\ln\frac{k_1^+-p_1^+}{\epsilon}+O(g^4,(k_1^+-p_1^+))\right].\qquad(2.18)$$

F. Combination of terms

We now assemble all contributions to the ultraviolet divergence of the full connected four-point function for $\vec{p}_i=0$, $k_1^+\to p_1^+$:

$$i\Gamma^4\underset{\substack{\Lambda\to\infty\\ k_1^+\to p_1^+}}{\sim}ig^2N^4\left(1-\frac{1}{N^2}\right)\frac{4p_1^+p_2^+}{(k_1^+-p_1^+)^2}\delta_{i_1j_1}\delta_{i_2j_2}A^4,\qquad(2.19\text{a})$$

where

$$A^4=1+\frac{g^2N}{16\pi^2}\ln\frac{\Lambda^2}{\mu^2}\left\{8\ln\frac{k_1^+-p_1^+}{\epsilon}+\left[2(\tfrac{11}{3})-4\ln\frac{p_1^+p_2^+}{\epsilon^2}-4\ln\frac{(k_1^+-p_1^+)^2}{p_1^+p_2^+}\right]\right\}=1+\frac{22}{3}\left(\frac{g^2N}{16\pi^2}\right)\ln\frac{\Lambda^2}{\mu^2},\qquad(2.19\text{b})$$

where the quantity in square brackets is the contribution from the full three-point function [see Eq. (2.15)]. To compare with known results, we note that our Feynman rules were obtained from the Lagrangian

$$\mathcal{L}=\tfrac{1}{4}\operatorname{Tr}[F_{\mu\nu}F^{\mu\nu}],$$

with

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu+g[A_\mu,A_\nu].$$

With this definition of g, the conventional α_s is $g^2/2\pi$. Thus our result corresponds to running coupling constant

$$N\alpha_s(q^2)=\frac{12\pi}{11\ln(q^2/\mu^2)},$$

which is the well-known result.[1]

G. Charge renormalization in scalar QED

In the preceding subsection we saw that double-gluon exchange was essential to obtain the correct renormalization of the Yang-Mills coupling constant. In quantum electrodynamics we know that double-photon exchange does *not* contribute to charge renormalization. It is instructive to compare and contrast these situations.

In Fig. 7 we draw the scalar QED graphs analogous to the ones we focused on for the Yang-Mills theory. We again choose the null-plane gauge. The self-energy of the scalar field [Fig. 7(a)] yields (after dropping the quadratic divergence which is pure mass renormalization)

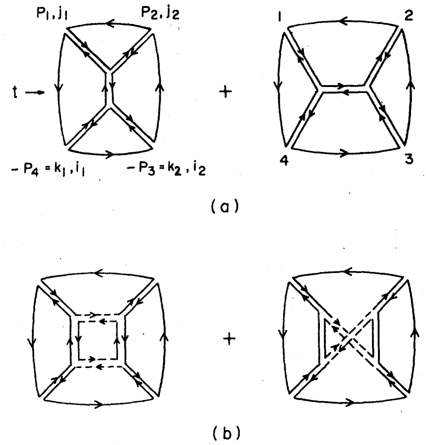

FIG. 6. (a) Lowest-order graphs contributing to the correlation function (2.1). (b) One-particle irreducible radiative corrections to (a) which contain the singularity $1/(k_1^+ - p_1^+)^2$.

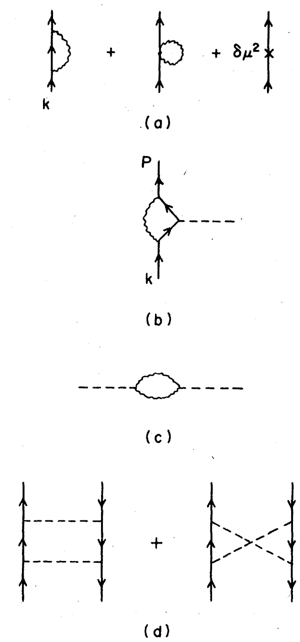

FIG. 7. Graphs contributing to charge renormalization in scalar electrodynamics.

$$-4k^2\frac{e^2 i}{16\pi^2}\left[\left(\ln\frac{k^+}{\epsilon}-1\right)\ln\frac{\Lambda^2}{\mu^2} - \int_\epsilon^{k^+-\epsilon}\frac{dl^+}{k^+}\left(\frac{k^+}{l^+}-1\right)\ln\frac{\mu^2}{l^+/k^+(1-l^+/k^+)k^2}\right],$$

which corresponds to a wave-function renormalization constant

$$Z_2(k^+)\underset{\Lambda\to\infty}{\sim} 1-\frac{e^2}{4\pi^2}\left(\ln\frac{k^+}{\epsilon}-1\right)\ln\frac{\Lambda^2}{\mu^2}.$$

The ultraviolet divergence of the vertex correction [Fig. 7(b)] is

$$-ie(k^+ + p^+)\left[1+\frac{e^2}{16\pi^2}\ln\frac{\Lambda^2}{\mu^2}\left(2\ln\frac{k^+p^+}{\epsilon}-4\right)\right]$$
$$\equiv -ie(k^+ + p^+)\frac{1}{Z_1}$$

and we have

$$\frac{Z_2(k^+)^{1/2}Z_2(p^+)^{1/2}}{Z_1(k^+p^+/\epsilon^2)}=1,$$

which we know is a consequence of the Ward identity.

The total charge renormalization is given by vacuum polarization [Fig. 7(c)] which yields

$$Z_3 = 1-\frac{e^2}{48\pi^2}\ln\frac{\Lambda^2}{\mu^2}.$$

But what about the double longitudinal photon exchange [Fig. 7(d)]? The calculation of these graphs involves precisely the same Feynman integrals as our Yang-Mills calculation. The ultraviolet parts of the graph with crossed and uncrossed photon lines exactly cancel for QED.[7] In the Yang-Mills theory the graph with uncrossed lines has more factors of N and is not quite canceled by the graph with crossed gluon lines. It is this graph which is responsible for asymptotic freedom.

III. DISCUSSION

We close this article with a few remarks about the results of this paper.

We first comment on our regulation of the singularities at $P^+ = 0$. Recall that it was essential to use the same ϵ in all propagators. Otherwise, the cancellation of the ϵ dependence would leave incorrect finite pieces. Our ϵ-regulation procedure is certainly adequate order by order in perturbation theory. However, since our aim is to go beyond perturbation theory, a more systematic prescription is desirable. The most natural choice is to discretize P^+, i.e., to write $P^+ = lb$, $l = 1, 2, \ldots$ with b fixed and nonzero. Integrals over P^+ are replaced by sums over l, $\int_0^\infty dP^+ \to b\sum_{l=1}^\infty$, and P^+ conserving δ functions are replaced by Kronecker δ's: $\delta(lb - mb) \to (1/b)\delta_{l,m}$. To our knowledge this prescription was proposed by Casher[2] and then independently by the author[4] in the context of setting up a nonperturbative formalism for graph summation.

Our second remark has to do with the peculiar nature of null-plane calculations. It is well known that the null-plane vacuum plays only a passive role in the dynamics. For this reason, renormalization effects cannot be readily interpreted as endowing the vacuum with the properties of e.g., a dielectric medium. Null-plane dynamics is essentially the dynamics of field quanta interpreted as mutually interacting nonrelativistic particles moving in transverse space with $P^+=lb$ playing the role of the "mechanical mass." In quantum chromodynamics, these field quanta are the quarks and gluons. Quark confinement corresponds to the possibility that the only finite-energy single-particle states are color-singlet bound states of quarks and gluons. This point of view has been pursued by the author in an earlier publication.[8]

Now let us explain why we believe the picture of asymptotic freedom emerging in this paper is suggestive of quark confinement. We have seen that the first graph in Fig. 6(b) carries the sign of asymptotic freedom. This sign corresponds to the added rung, being an attractive interaction between the gluons. What is important here is that this effective interaction is of dimension 4, i.e., would correspond to an effective term in the Hamiltonian $-g^2A^4$. Such an interaction is always strong enough to bind massless particles, producing a tachyon which should imply that the perturbative vacuum is unstable in a conventional description.[9] In the language of null-plane dynamics we would put it differently: This attraction could make states with, say, a quark and antiquark; and many gluons have a (perhaps infinitely) lower energy than the state with a quark, antiquark, and Coulomb field. Whether this mechanism is sufficient to imply quark confinement is not yet answered but it is certainly suggestive.

These ideas take a much more concrete form in the large-N limit. In Ref. 8 we have shown that the large-N limit formulated on the null plane leads to a many-body chain problem. The ends of the chain are a quark and an antiquark and the constituents of the chain are gluons. A particle in the chain only interacts with its nearest neighbors. If we take longitudinal gluon exchange as a model for the nearest-neighbor interaction, the effective potential will be

$$V_{\text{eff}}(\vec{x}_\perp, k_1^+, k_2^+) \propto -Ng^2 f(k_1^+, k_2^+)\delta(\vec{x}_\perp)$$

with $f(k_1^+, k_2^+)>0$. We solved a model system in which the P^+ of each particle was fixed and the number of particles was fixed and large. The system became the dual string in the limit that the number of particles became infinite. The rest tension T_0 of the string was related to the two-body bound state of the nearest-neighbor potential, which we took to be a transverse δ function. We believe that the Landau tachyon in the running coupling constant is essentially this two-body bound state and claim that the results of this paper support in some measure this belief. The location of this tachyon μ_0 is used as a way of parametrizing scaling violations to deep-inelastic scattering with the result $\mu_0 \simeq 500$ MeV. Interpreting the Landau tachyon as the two-body bound state of the chain problem leads to the approximate relation between T_0 and μ_0 derived in Ref. 8.

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