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Dimensional regularization and supersymmetry at the two-loop level

P. K. Townsend and P. van Nieuwenhuizen
Institute for Theoretical Physics, State University of New York, Stony Brook, New York 11794
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The self-energy and vertex corrections of the massless Wess-Zumino model are evaluated at the two-loop level by means of ordinary dimensional regularization. Unexpectedly the results satisfy the Ward identities of global sypersymmetry.

I. INTRODUCTION

One of the important problems in supergravity is to find a regularization scheme which preserves local supersymmetry. For Yang-Mills gauge theories and Einstein gravity, dimensional regularization[1] is applicable as it preserves the local symmetries of these theories, but for supergravity and supersymmetric field theories, in general, it is not clear whether dimensional regularization is applicable. The reason is that Fierz rearrangements are required to prove the invariance of supersymmetric actions and these depend on the dimension of spacetime. That is, an action that is supersymmetric in four dimensions will not be in, say, two or six dimensions. This would appear to invalidate a naive extension of the dimension of spacetime away from $n=4$. If there does not exist a regularization scheme that preserves supersymmetry, then it is commonly believed that the supersymmetry Ward identities will contain anomalies. Since the original proofs of one-loop[2] and two-loop[3] finiteness of supergravity were based on such Ward identities, as well as the subsequent classification of higher-loop counterterms[4] (which assumes that counterterms are locally supersymmetric on-shell) it is clear that the issue of the existence of a valid regularization scheme has serious implications for the finiteness of supergravity. If there is no such scheme, supergravity may very well give infinite results at the two-loop level and be no better a quantum theory of gravity than ordinary Einstein gravity. Fortunately, there are indications that dimensional regularization does preserve supersymmetry and may provide an adequate regularization scheme for supergravity.

In this paper we consider a particular theory with global supersymmetry, the massless Wess-Zumino model,[5] and we consider certain two-point and three-point Green's functions.[6] By explicit calculation of all two-loop contributions to these functions (that is, not only the single and double poles in $n-4$, but also the finite parts) we find, somewhat surprisingly, that the supersymmetry Ward identities are exactly satisfied. Thus it would appear that alternative dimensional-regularization procedures (to be discussed in Sec. IV) requiring extensions of supersymmetric theories to n dimensions with the complications of extra terms in the action proportional to $n-4$ are not necessary. If one proceeds straightforwardly just as for any other, not necessarily supersymmetric, theory, one obtains the desired results.

Before proceeding with the calculations there is one difficulty to confront. The Langrangian is

$$\mathcal{L}=-\tfrac{1}{2}(\partial_\mu A)^2-\tfrac{1}{2}(\partial_\mu B)^2-\tfrac{1}{2}\bar{\chi}\not\partial\chi+\tfrac{1}{2}F^2+\tfrac{1}{2}G^2 +g[-F(A^2-B^2)+2GAB+\bar{\chi}(A+i\gamma_5 B)\chi]\,. \qquad (1.1)$$

Since B couples to $\bar{\chi}\chi$ with a γ_5 matrix, we must decide how to deal with several γ_5 matrices on a fermion line. There are two reasonable choices: Either one does the γ-matrix algebra in n dimensions or in four dimensions. It matters which we choose because the results differ according to the choice. In the former case, $\gamma_5\not p\gamma_5$ equals $+\not p$ if the first four components of p_μ vanish, while $\gamma_5\not p\gamma_5$ is always $-\not p$ in the latter case. From 't Hooft and Veltman's analysis of the axial anomaly using dimensional regularization[1] it is known that there γ_5 commutes with γ_n, $n>4$. In our case, however, one should do the γ-matrix algebra as if in four dimensions, that is, γ_5 anticommutes with all γ_n. Apparently, this rule was first discovered in a calculation of the quadrupole moment of the W boson.[7] Recently, the problem of multiple γ_5's in the one-loop axial anomaly with dimensional regularization has been thoroughly studied in Ref. 8, with the same conclusions. For the problem of higher-loop axial anomalies with dimensional regularization we refer the reader to Ref. 9. There are several arguments for this rule, one of which comes from considering diagrams which lead to a trace over four γ matrices and two γ_5's. One can pull the two γ_5's together in two different ways and these give different results, in general, if the algebra is done in n dimensions. Thus, to avoid ambiguities, one is forced to do the algebra in four dimensions. We will discuss this point further in a later section. For the calculations in the following sections we will do all γ_5-matrix algebra in four dimensions.

In Sec. III we consider the self-energy and vertex corrections to (1.1) at the one-loop level. The Ward identities are satisfied, but we believe that it is the two-loop calculation presented in Sec. IV which provides a really nontrivial test of dimensional regularization. We find that the Ward identities are satisfied if one uses minimal subtraction to renormalize the theory. All nonlocal $\ln p^2$ terms as well as extraneous ln2, $\ln\pi$, and Euler's constant γ terms cancel.[10,11] As a check on our results we calculate the two-loop β function and find that it is indeed finite as $\epsilon = 4-n$ goes to zero, owing to the expected conspiracy between $1/\epsilon^2$ poles at the two-loop level and $1/\epsilon$ poles at the one-loop level.[10] These positive results lead us to the conjecture that dimensional regularization preserves global supersymmetry to all orders in perturbation theory in any supersymmetric model.

II. WARD IDENTITIES AT THE ONE-LOOP LEVEL

The action of (1.1) is invariant under the global supersymmetry transformation

$$\begin{aligned} &\delta A = \bar{\epsilon}\chi, \quad \delta B = -i\bar{\epsilon}\gamma_5\chi, \\ &\delta\bar{\chi} = -\bar{\epsilon}\partial\!\!\!/(A + i\gamma_5 B) + \bar{\epsilon}(F + i\gamma_5 G), \\ &\delta F = \bar{\epsilon}\partial\!\!\!/\chi, \quad \delta G = i\bar{\epsilon}\gamma_5\partial\!\!\!/\chi. \end{aligned} \tag{2.1}$$

The Ward identities for the one-particle irreducible diagram can be obtained from the effective action Γ by functionally differentiating the equation[6]

$$\frac{\delta\Gamma}{\delta A}\chi_\gamma - i\frac{\delta\Gamma}{\delta B}(\gamma_5\chi)_\gamma + [-\partial\!\!\!/(A + i\gamma_5 B) + F + i\gamma_5 G]_{\gamma\beta}\frac{\delta\Gamma}{\delta\bar{\chi}_\beta} + \frac{\delta\Gamma}{\delta F}(\partial\!\!\!/\chi)_\gamma + i\frac{\delta\Gamma}{\delta G}(\gamma_5\partial\!\!\!/\chi)_\gamma = 0. \tag{2.2}$$

Differentiating by $A(y)$ and $\bar{\chi}_\alpha(x)$ and then setting all fields equal to zero yields

$$\frac{\delta^2\Gamma}{\delta A(y)\delta A(x)}\delta_{\gamma\alpha} - i\frac{\delta^2\Gamma}{\delta A(y)\delta B(x)}(\gamma_5)_{\gamma\alpha} + (\partial\!\!\!/_y)_{\gamma\beta}\frac{\delta^2\Gamma}{\delta\chi_\alpha(x)\delta\bar{\chi}_\beta(y)} - \frac{\delta^2\Gamma}{\delta A(y)\delta F(x)}(\overleftarrow{\partial\!\!\!/}_x)_{\gamma\alpha} - i\frac{\delta^2\Gamma}{\delta A(y\delta G(x)}(\gamma_5\overleftarrow{\partial\!\!\!/}_x)_{\gamma\alpha} = 0. \tag{2.3}$$

It is immediately apparent from the Lagrangian (1.1) that the two-point functions $\langle AF\rangle$, $\langle AG\rangle$, and $\langle AB\rangle$ are zero. We, therefore, obtain the Ward identity

$$\frac{\delta^2\Gamma}{\delta A(y)\delta A(x)}\delta_{\gamma\alpha} + (\gamma^\mu)_{\gamma\beta}\frac{\partial}{\partial y^\mu}\frac{\delta^2\Gamma}{\delta\chi_\alpha(x)\delta\bar{\chi}_\beta(y)} = 0. \tag{2.4}$$

After the following Fourier transform

$$\int e^{ipx}e^{iqy}\frac{\delta^2\Gamma}{\delta\chi_\alpha(x)\delta\bar{\chi}_\beta(y)}d^4x\,d^4y = (2\pi)^4\delta^4(p-q)\Gamma_{\chi_\alpha\bar{\chi}_\beta}(p) \tag{2.5}$$

and similarly for Γ_{AA} one finds the momentum-space identity

$$\Gamma_{AA}(p)\delta_{\gamma\alpha} - i(p\!\!\!/)_{\gamma\beta}\Gamma_{\chi_\alpha\bar{\chi}_\beta}(p) = 0. \tag{2.6}$$

which is represented graphically in Fig. 1. Notice that the classical action satisfies the identity (2.4) (with derivatives with respect to χ being left derivatives), or, equivalently, that the *inverse* propagators satisfy (2.6). This follows from the relation between the functional W for connected Green's functions and Γ:

$$\frac{\delta^2\Gamma}{\delta\psi(x)\delta\phi(y)} = \mp\left(\frac{\delta^2 W}{\delta j_\phi(y)\delta j_\psi(x)}\right)^{-1} \tag{2.7}$$

with the minus sign for bosons and the plus sign for fermions. The second Ward identity we consider is obtained by differentiating (2.2) with respect to $\chi_\alpha(y)$, $A(x)$, $A(z)$:

$$(\partial\!\!\!/_x)_{\gamma\beta}\frac{\delta^3\Gamma}{\delta A(z)\delta\chi_\alpha(y)\delta\bar{\chi}_\beta(x)} + (\partial\!\!\!/_z)_{\gamma\beta}\frac{\delta^3\Gamma}{\delta A(x)\delta\chi_\alpha(y)\delta\bar{\chi}_\beta(z)} - (\partial\!\!\!/_y)_{\gamma\alpha}\frac{\delta^3\Gamma}{\delta A(z)\delta A(x)\delta F(y)} = 0, \tag{2.8}$$

where we have used that $\langle AAA\rangle$, $\langle AAB\rangle$, and $\langle AAG\rangle$ are zero, as follows again from the form of (1.1). We Fourier transform as follows:

$$\Gamma_{\phi\psi\chi}(p,q,r) = \int e^{ipx}e^{iqy}e^{irz}\frac{\delta^3\Gamma}{\delta\phi(x)\delta\psi(y)\delta\chi(z)} \times d^4x\,d^4y\,d^4z \tag{2.9}$$

to obtain the momentum-space version of (2.8)

$$(p\!\!\!/)_{\gamma\beta}\Gamma_{A\chi_\alpha\bar{\chi}_\beta}(r,q,p) + (r\!\!\!/)_{\gamma\beta}\Gamma_{A\chi_\alpha\bar{\chi}_\beta}(p,q,r) - (q\!\!\!/)_{\gamma\alpha}\Gamma_{FAA}(q,p,r) = 0. \tag{2.10}$$

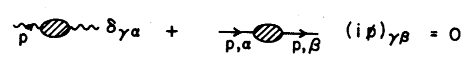

FIG. 1. A two-point function supersymmetry Ward identity.

This identity is represented graphically in Fig. 2. Again the classical action satisfies (2.8) and again the tree graphs in Fig. 2 satisfy (2.10) because of momentum conservation $p+q+r=0$ and because for fermions the relation between Γ and W graphs involves an extra minus sign as in (2.7).

We now turn to the one-loop evaluation of the two- and three-point functions Γ_{AA}, $\Gamma_{\chi\chi}$, $\Gamma_{A\chi\bar{\chi}}$, Γ_{FAA}. Once we have the complete result we will come back to the Ward identities and check whether they are satisfied.

The one-loop corrections to the A and χ propagators are easily evaluated. One finds (for Feynman rules and convention see Table I)

$$\begin{aligned} S(3\mathrm{a}) &= 4g^2\pi^{n/2}(p^2)^{n/2-2}(\not{p})_{\beta\alpha} \\ &\quad\times\Gamma(2-n/2)B(n/2-1,\ n/2-1), \\ S(3\mathrm{b}) &= -4ig^2\pi^{n/2}(p^2)^{n/2-1} \\ &\quad\times\Gamma(2-n/2)B(n/2-1, n/2-1), \end{aligned} \qquad (2.11)$$

where $S(3\mathrm{a})$ is the Green's function for an incoming spinor with spinor index α and momentum p and an outgoing spinor with index β and the same momentum. It is immediately clear that the first Ward identity is satisfied *for any n*. For our two-loop calculation of Sec. III we need also the pole parts of (2.11),

$$\begin{aligned} S(3\mathrm{c}) &= -S^{\mathrm{pole}}(3\mathrm{a}) = -\delta u^2\not{p}\pi^2\epsilon^{-1}, \\ S(3\mathrm{d}) &= -S^{\mathrm{pole}}(3\mathrm{b}) = \delta i u^2 p^2\pi^2\epsilon^{-1}. \end{aligned} \qquad (2.12)$$

Obviously, the poles separately satisfy the Ward identity because we have used minimal subtractions.

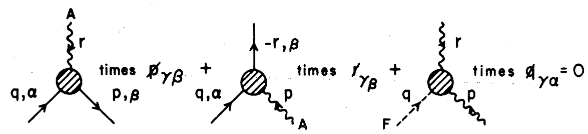

FIG. 2. A three-point function sypersymmetry Ward identity.

The one-loop corrections to the FAA and $A\chi\chi$ vertices in Fig. 2 vanish individually. The corrections to the $A\chi\chi$ vertex cancel because we do the γ_5-matrix algebra in four dimensions. This allows us to pull together the two γ_5's due to a virtual B line, giving an overall minus sign. One can see here that this cancellation would not take place if γ_5 would not anticommute with all γ matrices. For the particular case of $\Gamma_{\chi\bar{\chi}}$, the difference would be proportional to $\not{p}_{n-4}$, which vanishes if the external momentum is purely four dimensional. To see this, recall that $\gamma_5(\not{k}-\not{p})\gamma_5 = -\not{k}_4 + \not{p} + \not{k}_{n-4}$ in n dimensions, instead of $-\not{k}_4 + \not{p} - \not{k}_{n-4}$. The extra $\not{k}_{n-4}$ piece vanishes after combining denominators, shifting the integration variable, and using symmetrical integration. This means that the identity (2.6) would be violated at the two-loop level. At the one-loop level the Ward identity analogous to (2.6), but with $\Gamma_{BB}(p)$ instead of $\Gamma_{AA}(p)$, would be violated. This is becuase the B self-energy corresponding to Fig. 3(b) would have the trace of $\gamma_5\not{k}\gamma_5(\not{k}-\not{p}) = -k_4\cdot(k-p)_4 + k_{n-4}\cdot(k-p)_{n-4}$. Now the $(k_{n-4})^2$ term does contribute to $\Gamma_{BB}(p)$ while there is no analogous contributions to $\Gamma_{\chi\bar{\chi}}$.

TABLE I. Feynman rules and conventions.

$A^{\bullet}A^{\bullet} = -i/p^2$, $B^{\bullet}B^{\bullet} = -i/p^2$, (wavy line)
$F^{\bullet}F^{\bullet} = +i$, $G^{\bullet}G^{\bullet} = +i$, (dashed line)
$\chi^{\bullet}\bar{\chi}^{\bullet} = -\not{p}/p^2$, (solid line)
$\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$, $\delta_{\mu\nu} = (++++)$, $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$, $\gamma_5{}^2 = 1$
$\int d^nk(k^2+M^2)^{-\alpha} = i\pi^{n/2}\Gamma(\alpha-n/2)\Gamma(\alpha)^{-1}(M^2)^{n/2-\alpha}$
$\int d^nk(k^2)^{-\alpha}(k-p)^{-2} = i\pi^{n/2}\Gamma(\alpha+1-n/2)\Gamma(\alpha)^{-1}B(n/2-\alpha, n/2-1)(p^2)^{n/2-\alpha-1}$
$\int d^nk(k^2)^{-\alpha}(k-p)^{-2}k_\mu = i\pi^{n/2}\Gamma(\alpha+1-n/2)\Gamma(\alpha)^{-1} \times B(n/2-\alpha+1, n/2-1)(p^2)^{n/2-\alpha-1}p_\mu$
$B(a,b) = \Gamma(a)\Gamma(b)\Gamma(a+b)^{-1}$
$g = uM^{\epsilon/2}$ with u dimensionless and $\epsilon = 4-n$
$p = p_4 + p_{n-4}$ with p_{n-4} the projection of p in $n-4$ dimensions

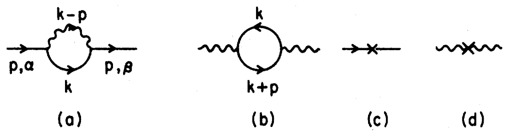

FIG. 3. One-loop corrections to the two-point functions.

III. WARD IDENTITIES AT THE TWO-LOOP LEVEL

We now repeat the analysis of Sec. II but with two-loop diagrams. Since the action and one-loop counterterms are given, the self-energies and vertices are calculated as in any renormalizable theory without reference to supersymmetry. Only after these results are obtained will we check whether the Ward identities are satisfied. For details of two-loop calculations one might consult Ref. 11.

The fermion self-energy corrections of Fig. 4(a) are obtained by inserting into the integrand of Fig. 3(a) the factor

$$1+\left(\frac{-i}{k^2}\right)\frac{1}{(2\pi)^n}S(3\mathrm{b})\,. \tag{3.1}$$

Similarly, Fig. 4(b) is obtained by inserting the factor (3.1), but with S(3b) replaced by S(3d). The complete results are (note that both A and B contribute equally)

$$\begin{aligned}S(4\mathrm{a})=&-32g^4 2^{-n}\Gamma(2-n/2)B(n/2-1,n/2-1)\\&\times(p^2)^{n-4}\not{p}\,\Gamma(4-n)[\Gamma(3-n/2)]^{-1}\\&\times[B(n-3,n/2-1)-B(n-2,n/2-1)]\,,\end{aligned} \tag{3.2a}$$

$$\begin{aligned}S(4\mathrm{b})=&\,2g^2u^2\pi^{-2+n/2}\epsilon^{-1}\Gamma(2-n/2)\\&\times B(n/2-1,n/2-1)(p^2)^{n/2-2}\not{p}\,.\end{aligned} \tag{3.2b}$$

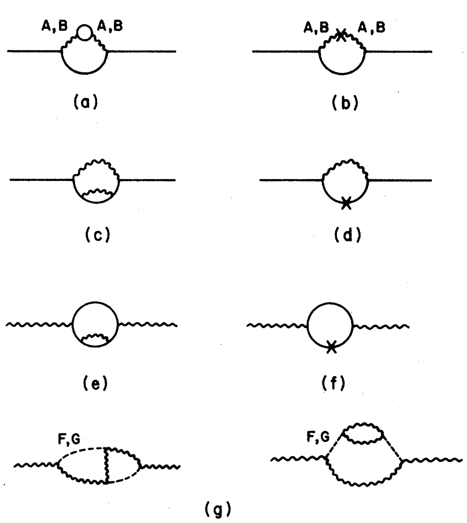

FIG. 4. Two-loop corrections to the two-point functions.

Note that S(4a) + S(4b) still contain a $1/\epsilon^2$ pole even though the insertion S(3b) + S(3d) is itself finite. This is a general feature. In fact, the $1/\epsilon^2$ term in S(4a) differs by a factor of (−2) from that of S(4b), so that ln$M$ contributions to the $1/\epsilon$ pole terms, due to the expansion of $g=uM^{\epsilon/2}$, cancel. For the diagrams of 4(c) and 4(d) one finds in a similar way

$$\begin{aligned}S(4\mathrm{c})=&-32g^4 2^{-n}(p^2)^{n-4}\not{p}\,\Gamma(4-n)[\Gamma(3-n/2)]^{-1}\\&\times\Gamma(2-n/2)B(n/2-1,n/2-1)\\&\times B(n-2,n/2-1)\,,\end{aligned} \tag{3.3a}$$

$$\begin{aligned}S(4\mathrm{d})=&\,4g^2u^2\pi^{-2+n/2}\epsilon^{-1}\Gamma(2-n/2)\\&\times B(n/2,n/2-1)(p^2)^{n/2-2}\not{p}\,.\end{aligned} \tag{3.3b}$$

Again, the fermion self-energy inserted into the fermion self-energy is free from ln$M$ contributions to pole terms. One might wonder whether expanding 2^{-n} and $\pi^{n/2}$ about $n=4$ would lead to $(\ln 2)/\epsilon$ and $(\ln\pi)/\epsilon$ terms. It does in the Green's functions but not in the renormalization constants Z, since to obtain the latter one must divide by one extra factor of $(2\pi)^{-n}$. The $(\ln p^2)/\epsilon$ terms already cancel in the Green's functions and hence in the Z factors. Also, γ/ϵ terms obtained by expanding $\Gamma(\epsilon)=\epsilon^{-1}-\gamma$ cancel.[11] These cancellations happen here separately for each self-energy insertion, but in massive ϕ^4 theory one needs to sum all graphs before $(\ln\mu)/\epsilon$ terms cancel at the two-loop level.[12]

We now turn to the two-loop corrections to the A propagator. Inserting the finite one-loop corrections to the fermion propagator into the one-loop boson propagator, Figs. 4(e) and 4(f), one finds

$$\begin{aligned}S(4\mathrm{e})=&-64ig^4 2^{-n}\Gamma(2-n/2)\\&\times B(n/2-1,n/2-1)B(n-2,n/2-1)\\&\times(p^2)^{n-3}\left[\frac{\Gamma(3-n)}{\Gamma(2-n/2)}-\frac{\Gamma(4-n)}{\Gamma(3-n/2)}\right],\end{aligned} \tag{3.4a}$$

$$\begin{aligned}S(4\mathrm{f})=&-4iu^2g^2\pi^{-2+n/2}\epsilon^{-1}\Gamma(2-n/2)\\&\times B(n/2-1,n/2-1)(p^2)^{n/2-1}\,.\end{aligned} \tag{3.4b}$$

The cancellation of $1/\epsilon$ poles proportional to ln$M$, ln2, lnπ, lnp^2, and γ is again a useful check on this result. Finally, we turn to the purely bosonic graphs of Fig. 4(g). The F and G propagators are simply $+i$ so that these graphs are equivalent to the one shown in Fig. 5, which occurs in the

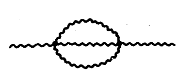

FIG. 5. Equivalent graph to those of Fig. 4(g) in $\mathcal{L}(A,B,\chi)$ after F and G have been eliminated from $\mathcal{L}(A,B,\chi,F,G)$.

Lagrangian of (1.1) after F and G have been eliminated. The first graph of Fig. 4(g) is actually zero because of the cancellations between the F and G internal lines. The calculation of Fig. 4(g) or Fig. 5 gives the same result. [In general, this is not true because graphs which are one-particle irreducible (1PI) for $\mathcal{L}(A,B,\chi)$ are not necessarily 1PI for $\mathcal{L}(A,B,\chi,F,G)$. The reader can easily construct examples.] One finds[12]

$$S(4g)=32ig^4 2^{-n}\Gamma(4-n)(3-n)^{-1}(p^2)^{n-3} \times B(n/2-1,n/2-1)B(n/2-1,n-2)\,. \tag{3.5}$$

One can understand why this graph has only a first-order pole in ϵ. Generally, a two-loop graph constructed by inserting a *finite* one-loop correction is free from $(\ln p^2)/\epsilon$ singularities, as we saw for the self-energy corrections. Since the previous graphs were all of this type, Fig. 4(g) cannot have a $(\ln p^2)/\epsilon$ term because nonlocal counterterms are forbidden on general grounds and therefore can have only a $1/\epsilon$ pole.

Now we can check the two-point Ward identity. Defining

$$\begin{aligned} A&=32g^2 iB(n/2-1,n/2-1)2^{-n}\,,\\ B&=g^2\Gamma(4-n)\epsilon^{-1}B(n/2-1,n-2)(p^2)^{n-3}\,,\\ C&=u^2\epsilon^{-1}(p^2)^{n/2-1}\pi^{n/2}\Gamma(2-n/2)2^{n-4}\,, \end{aligned} \tag{3.6}$$

we have the following summary of the above results:

$$\begin{aligned} &S(4a)=AB\left(\frac{2-\epsilon}{1-\epsilon}\right)i\not p^{-1}\,, && S(4b)=-ACi\not p^{-1}\,,\\ &S(4c)=AB2i\not p^{-1}\,, && S(4d)=-ACi\not p^{-1}\,,\\ &S(4e)=AB\left(4-\frac{2\epsilon}{\epsilon-1}\right)\,, && S(4f)=-2AC\,,\\ &S(4g)=AB\left(\frac{\epsilon}{\epsilon-1}\right)\,. \end{aligned} \tag{3.7}$$

Upon multiplying $S(4a+4b+4c+4d)$ by $i\not p$ one sees that the g^4 and g^2u^2 terms separately satisfy the Ward identity (as they should, since u and g are independent constants). Note that this result holds true not just for the poles in $1/\epsilon^2$ and $1/\epsilon$ but also the finite parts *even away from* $n=4$.

We have still to consider the two-loop corrections to the $A\chi\chi$ and FAA proper vertices. One need consider only the *nonplanar* two-loop graphs because all one-loop corrections vanished. Indeed, one may check explicitly that all planar two-loop graphs cancel. The contributions for the $A\chi\chi$ vertex are shown in Figs. 6(a) and 6(b) while the contributions to the AAF vertex are shown in Figs. 7(a) and 7(b). We choose to evaluate these

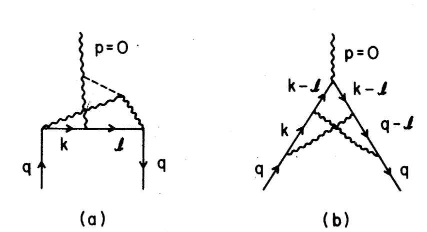

FIG. 6. Two-loop corrections to the $A\chi\chi$ vertex.

graphs with zero momentum carried away by the A particle in Fig. 6 and zero momentum carried away by one of the A particles in Fig. 7. This does not introduce infrared divergences, as is clear from the results

$$\begin{aligned} S(6a)&=-\frac{128ig^5}{(2\pi)^n}\int\frac{\not l\not k\, d^nk\, d^nl}{k^2l^2(q-k)^2(p-l)^2(k-l)^2}\,,\\ S(6b)&=\frac{128ig^5}{(2\pi)^n}\int\frac{\not l\not k\, d^nk\, d^nl}{k^2l^2(q-k)^2(q-l)^2(k-l)^2}\,,\\ &\quad-\frac{128ig^5}{(2\pi)^n}\not q\int\frac{\not k\, d^nk\, d^nl}{k^2l^2(q-k)^2(q-l)^2(k-l)^2}\,,\\ S(7a)&=-\frac{128ig^5}{(2\pi)^n}\int\frac{d^nk\, d^nl}{l^2(l-q)^2(k-l)^2(k-q)^2}\,,\\ S(7b)&=\frac{128ig^5}{(2\pi)^n}\int\frac{d^nk\, d^nl}{(k-l)^2l^2(l-q)^2(k-q)^2}\,,\\ &\quad-\frac{128ig^5}{(2\pi)^n}\int d^nk\, d^nl[(k-l)^2k^2(k-q)^2l^2(l-q)^2]^{-1}\\ &\qquad\times(k\cdot q)\,. \end{aligned} \tag{3.8}$$

Addings $S(6a)+S(6b)$ we find the following *finite* contribution to the $A\chi\chi$ S matrix:

$$S(6)=-\frac{128ig^5}{(2\pi)^n}\not q\int\frac{\not k\, d^nk\, d^nl}{k^2l^2(q-k)^2(k-l)^2(q-l)^2}\,. \tag{3.9}$$

Similarly, adding $S(7a)+S(7b)$ we find

$$S(7)=-\frac{128ig^5}{(2\pi)^n}\quad\frac{d^nk\, d^nl\, k\circ q}{(k-l)^2k^2(k-q)^2l^2(l-q)^2} \tag{3.10}$$

which is also finite. Since the integral in (3.9) must be proportional to $\not q$, (3.9) is only a function of q^2 and can be evaluated by first taking the trace. In this way it follows that $S(6)$ equals $S(7)$. Setting

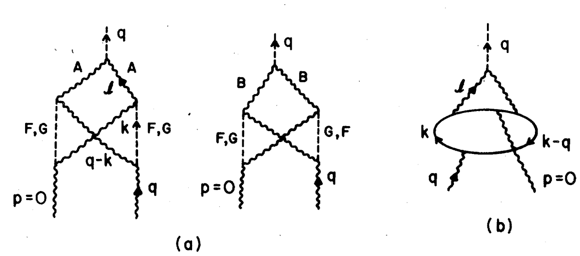

FIG. 7. Two-loop corrections to the FAA vertex.

$p=0$ in Fig. 2, Fig. 2 reduces to the requirement $S(6)-S(7)=0$. Hence we see that this Ward identity is also satisfied not only for the $1/\epsilon^2$, $1/\epsilon$ poles and finite parts at $n=4$, but even for any n away from $n=4$.

As a check on our result we evaluate the two-loop β function. For the renormalization constants of Ref. 6 we have found

$$Z_3(A)=Z_3(\chi)=1-u^2(2\pi^2\epsilon)^{-1} - u^4(4\pi^4)^{-1}(\epsilon^{-2}-\tfrac{1}{2}\epsilon^{-1})+O(u^6),$$
$$Z_1=1+O(u^6). \qquad (3.11)$$

For the two-loop β function we find

$$\beta(u)=\frac{-\frac{1}{2}\epsilon}{d/dn\ln(uZ_1/Z_3{}^{3/2})}=\frac{3}{4\pi^2}u^3-\frac{3}{8\pi^4}u^5. \qquad (3.12)$$

The check on our results is that $\beta(u,\epsilon)$ is finite as $\epsilon\to 0$; this requires that the square of the coefficient of the u^2/ϵ term in $Z_1{}^2/Z_3{}^3$ is equal to the coefficient of the u^4/ϵ^2 term, as indeed it is.

Finally, we note that we have taken everywhere $\mathrm{tr}(\gamma_\mu\gamma_\nu)=4\delta_{\mu\nu}$, with $\delta_{\mu\nu}$ the n-dimensional δ function. If we had taken $\mathrm{tr}(\gamma_\mu\gamma_\nu)=\delta_{\mu\nu}2^{n/2}$ and at the same time stayed within the minimal subtraction scheme, then there would have been $(\ln 2)/\epsilon$ poles which would have violated the Ward identities. Of course, one can always make finite recalibrations of the coupling constants, etc., to eliminate these terms and restore the Ward identities, but we have preferred to work consistently using minimal subtractions. Taking the trace to be $4\delta_{\mu\nu}$ means that there are also four components for χ in $4+\epsilon$ dimensions. Thus we have equal numbers of bosons and fermions even in $4+\epsilon$ dimensions, and this may explain our positive results. Since this argument breaks down for vector fields, the spin-$(1,\frac{1}{2})$ system should be analyzed. These results are reminiscent of the axial anomaly as obtained in Ref. 1. There one defines $\mathrm{Tr}(\gamma_5\gamma_{\alpha_1}\cdots\gamma_{\alpha_6})=4\delta_{\alpha_1\alpha_2}\epsilon(\alpha_3\alpha_4\alpha_5\alpha_6)$ + five more terms, but considers the Kronecker δ's an n-dimensional objects. This choice is equivalent to a γ_5 which anticommutes with only the first four γ matrices; in other words, $\gamma_5=\gamma_1\gamma_2\gamma_3\gamma_4$. At the one-loop level the normalization factor 4 could have been replaced by $2^{n/2}$, but for the higher-loop axial anomaly it could make a difference and the choice 4 rather than $2^{n/2}$ seems to be the correct one.[9]

IV. DISCUSSION

Straightforward evaluation of some two- and three-point functions at the two-loop level, using dimensional regularization, has shown that the results obey the supersymmetry Ward identities, even away from $n=4$. This is a hopeful sign for supergravity, where the stakes are higher. The proofs of two-loop finiteness are based either on helicity conservation or on the assumption that the on-shell divergences are locally supersymmetric. Both these statements are equivalent to Ward identities. Supersymmetry anomalies would thus invalidate the assumptions of these proofs and if, subsequently, supergravity would turn out to be infinite at the two-loop level, it would lose its most spectacular success. We intend to investigate this problem.

Various authors have proposed modified versions of dimensional regularization for supersymmetric theories, apparently never entertaining the possibility that the dimensional regularization might work by itself. We have to admit that they might be right, since we have no general proof for our *conjecture*: *Naive dimensional regularization works for all loops in all supersymmetric models*. For example, Delbourgo and Ramon-Medrano[13] consider $n>4$ in the complex dimensional plane and take $2^{(n/2)}$ supersymmetry generators. Siegel, in a very recent paper,[14] considers $n<4$ and regularizes amplitudes by demanding that the dimensionally reduced theories at $n<4$ coincide with the analytically continued amplitudes.

Another paper on the relation between supersymmetry and dimensional regularization is the calculation of the two-loop β function in a supersymmetric Yang-Mills model.[15] Also there exists a calculation[16] of the two-loop β function for the Wess-Zumino model and for a generalized, nonsupersymmetric Wess-Zumino model with two coupling constants, using dimensional regularization. These authors use the same rule $\{\gamma_5,\gamma_\mu\}=0$ for $\mu=1,N$ as in Ref. 7, and use also the rule $\mathrm{tr}I=4$ in order to maintain equal numbers of Fermi and Bose degrees of freedom away from $N=4$. They find that there is only one renormalization constant in the supersymmetric limit. This is a nontrivial check on the supersymmetric structure of the theory using dimensional regularization. These authors did not compute finite parts, since these were irrelevant for their stability analysis. In addition, T. Curtright and D. Z. Freedman (private communication) considered a year ago the use of dimensional regularization in the spin

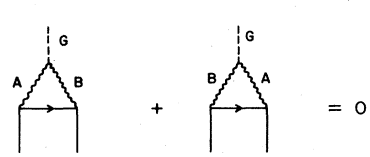

FIG. 8. Graph illustrating γ_5 ambiguity.

$(1, \frac{1}{2})$ non-Abelian gauge model. On-shell Ward identities could be enforced up to the two-loop level, using the trace rule $\mathrm{tr}I = 2(N-2)$, but off-shell there were difficulties.

Crucial for our results was the rule that in a spinor line with several γ_5 matrices one contracts first pairs of γ_5 matrices in four-dimensions; that is, these γ_5 matrices anticommute with all γ matrices. That eliminates all γ_5 for our cases, but in axial anomalies the remaining γ_5 is treated, according to, for example Ref. 1, in n dimensions. As stressed by these authors, the position of the last γ_5 leads to the same ambiguities as the routing of momenta in cutoff schemes. In our model with single axial vertices, such ambiguities arise if one contracts two γ_5 matrices, using four-dimensional γ algebra. Since one encounters diagrams with one A and one B line departing from a spinor line, such as in Fig. 8, it matters whether the remaining γ_5 is moved from left to right in four or in n dimensions. In four dimensions one obtains zero, in n dimensions one does not. This kind of ambiguity is, of course, well known from the axial anomaly. If some other symmetry (vector conservation in the axial anomaly, for example) is required to be satisfied, then the position of γ_5 and its n-dimensional character may be fixed. Similarly here, since no other symmetry is present, we have an ambiguity as where to move γ_5, and whether in four or in n dimensions. However, as is well known, in the axial-vector case, leaving γ_5 in its natural position and setting it equal to $\gamma_1\gamma_2\gamma_3\gamma_4$ leads to vector conservation (and the correct axial anomaly).

Similarly, in our case, contracting pairs of γ_5 matrices away in four dimensions preserves supersymmetry. The important point is that one can satisfy supersymmetry at all; the convenient point is that it can be done by doing γ_5-matrix algebra in four dimensions.

Our results are an encouraging indication that dimensional regularization in its original form as introduced by 't Hooft and Veltman[1] is compatible with global supersymmetry. For the on-shell counterterms of supergravity, this is all one needs, since it has been shown that for the S matrix global supersymmetry and Lorentz invariance imply local supersymmetry.[17]

Clearly, it is desirable to confirm our results in other models, in particular, in models with vector fields A_μ, since in n dimensions A_μ contains n components rather than four, which would violate the equality of boson and fermion states.

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