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Macroscopic parity-violating effects: Neutrino fluxes from rotating black holes and in rotating thermal radiation

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Two macroscopic effects of parity nonconservation are considered. (i) Particle emission by rotating black holes is shown to be asymmetric. In particular, neutrinos are emitted preferentially in the direction opposite to the hole's angular momentum. (ii) It is shown that in a rotating thermal radiation there exist equilibrium neutrino and antineutrino currents parallel to the angular velocity vector.

I. INTRODUCTION AND SUMMARY

Parity nonconservation was predicted theoretically by Lee and Yang[1] and discovered experimentally by Wu[2] in a famous experiment in which she observed electron emission by radioactive cobalt atoms. Wu found that the atoms emitted more electrons in the direction of their spins than in the opposite direction. Another manifestation of parity nonconservation is the left-handedness of neutrinos. The spin of neutrinos is always antiparallel to the direction of their motion. Antineutrinos are right-handed; their spin is parallel to the direction of their motion. In the present paper it will be shown that effects very similar to that observed by Wu occur on a macroscopic scale as well. (A brief account of a part of this work had been published earlier.[3,4])

In the next section of this paper particle emission by rotating black holes is shown to be asymmetric. In particular, more neutrinos are emitted in the direction antiparallel to the hole's angular momentum and more antineutrinos in the parallel direction. The characteristic parameter of asymmetry is[5] $-\Omega M$, where Ω is the angular velocity and M is the mass of the black hole. (The minus sign signifies that neutrinos are emitted preferentially in the direction opposite to that of the hole's angular momentum.) For a rapidly rotating black hole this parameter is of order -1. It is noted that the results obtained apply not only to black holes but to any rotating star emitting thermal neutrinos. If the chemical potential of neutrinos in the star is positive (negative), then the star accelerates like a rocket in the direction parallel (antiparallel) to its angular momentum.

In Sec. III it is shown that intrinsic parity nonconservation for neutrinos gives rise to an equilibrium neutrino current in a rotating thermal radiation. In a charge-symmetric radiation, when the chemical potential of neutrinos μ is zero, neutrino and antineutrino currents are exactly equal and opposite. A nonzero value of μ results in a net energy flux. The neutrino current density and energy flux are calculated as functions of angular velocity, temperature and chemical potential of neutrinos. It is argued that particles other than neutrinos can also develop currents as a result of parity-violating weak interactions. Possible astrophysical consequences of macroscopic parity-violating effects will be discussed elsewhere.

II. BACK-HOLE EVAPORATION

Hawking has demonstrated[6] that a black hole of mass M and angular velocity Ω emits particles at a constant rate given by

$$\frac{dN}{dt\,d\omega}=(2\pi)^{-1}\sum_{j,m,p}\Gamma_{\omega jmp}\left[\exp\left(\frac{\omega-m\Omega}{T}\right)\pm 1\right]^{-1}, \tag{1}$$

where

$$T=(4\pi M)^{-1}(M^2-a^2)^{1/2}[M+(M^2-a^2)^{1/2}]^{-1} \tag{2}$$

is the black-hole temperature,

$$a=J/M=2\Omega M[M+(M^2-a^2)^{1/2}], \tag{3}$$

J is the angular momentum of the black hole, and a can take all values from zero to M. (We assume that the black hole is uncharged.) The quantity $\Gamma_{\omega jmp}$ is the absorption coefficient for an incoming particle with energy ω, angular quantum numbers j and m, and polarization or helicity p. The upper and lower signs correspond to fermions and bosons, respectively.

Assuming for definiteness that the black hole's angular momentum is directed upwards, Eq. (1) implies that the hole emits more particles with the same (upward) direction of angular momentum than with the opposite one. One expects also that more particles are emitted with spin directed upwards than with spin directed downwards. Given the left-handedness of neutrinos and the right-handedness of antineutrinos, this means that neutrinos are emitted preferentially in the lower hemisphere and antineutrinos preferentially in the upper hemisphere.

To describe this effect quantitatively we shall rewrite Eq. (1) in the form

$$\frac{dN}{dt\,d\omega\,do}=(2\pi)^{-1}\sum_{j,m}\Gamma_{\omega jm}f_{\omega jm}(\theta)\times\left[\exp\left(\frac{\omega-m\Omega}{T}\right)+1\right]^{-1}, \quad (4)$$

where $f_{\omega jm}(\theta)$ characterizes the angular distribution of particles in the mode (ω, j, m) and

$$2\pi\int_0^\pi f_{\omega jm}(\theta)\sin\theta\, d\theta=1\,. \quad (5)$$

From the neutrino field equations in the Kerr metric, it can be shown[3] that

$$f_{\omega jm}=|{}_{-L/2}Y_{j,-m}(\theta,\phi;a\omega)|^2\,, \quad (6)$$

where ${}_sY_{j,m}(\theta,\phi;a\omega)$ are the spin-weighted spheroidal harmonics introduced by Teukolsky[7,8] and L is the lepton number: $L=+1$ for neutrinos and $L=-1$ for antineutrinos.

The right-hand side of Eq. (4) can be calculated analytically in the case of low energies of the emitted particles ($M\omega\ll 1$) when the dominant contribution to the Hawking radiation is given by the mode with $j=\frac{1}{2}$ and $m=\pm\frac{1}{2}$. Neglecting higher powers of $M\omega$, the absorption coefficient for neutrinos in this mode is[9]

$$\Gamma_{\omega,1/2,m}=M^2\omega^2\,. \quad (7)$$

Note that $\Gamma_{\omega jm}$ is the same for neutrinos and antineutrinos. (This is a consequence of the CP invariance of the neutrino field equations.) Since $a\omega\ll 1$, the spin-weighted spheroidal harmonics reduce to the spin-weighted spherical harmonics

$${}_sY_{j,m}(\theta,\phi)={}_sY_{j,m}(\theta,\phi;0)\,, \quad (8)$$

for which closed analytical expressions are known.[10] In particular, for $L=\pm 1$ and $m=\pm\frac{1}{2}$,

$$|{}_{-L/2}Y_{1/2,-m}(\theta,\phi)|^2=(4\pi)^{-1}(1-2mL\cos\theta)\,. \quad (9)$$

From Eqs. (4) and (6)–(9) we get[11]

$$\frac{dN}{d\omega\,dt\,do}=(8\pi^2)^{-1}M^2\omega^2(\cosh\omega/T+\cosh\Omega/2T)^{-1}\times[e^{-\omega/T}+\cosh\Omega/2T-L\sinh(\Omega/2T)\cos\theta]\,. \quad (10)$$

As expected, the black hole emits more antineutrinos in the upper hemisphere ($0\le\theta<\pi/2$) and more neutrinos in the lower hemisphere ($\pi/2<\theta\le\pi$).

For a slowly rotating black hole ($M\Omega\ll 1$), Eq. (10) reduces to

$$\frac{dN}{dt\,d\omega\,do}=(8\pi^2)^{-1}M^2\omega^2(1-2\pi LM\Omega\cos\theta)\,. \quad (11)$$

The characteristic parameter of asymmetry in the angular distribution of neutrinos, defined as the difference of emission rates in the upper and lower hemispheres divided by the total emission rate, is of order $-M\Omega$. For a rapidly rotating black hole ($a\sim M$) this parameter becomes of order -1. In the limiting case $a=M$, $T=0$ [see Eq. (2)], and

$$\frac{dN}{dt\,d\omega\,do}=(8\pi^2)^{-1}M^2\omega^2(1-L\cos\theta)\,. \quad (12)$$

The asymmetry in the neutrino emission by rotating black holes has been investigated independently by Leahy and Unruh.[12] They performed numerical calculations to determine the angular dependence of the total neutrino flux on the polar angle and used analytic methods to study the case of low frequencies. In the latter case their results are in agreement with those of the present paper.

Particles other than neutrinos can be in both positive- and negative-helicity states, and in the free-field approximation the angular distribution of such particles is symmetric. However, if interactions are taken into account, an asymmetry can develop as a result of parity-nonconserving weak interactions.[13] This is most easily understood in the case of emission of particles which decay asymmetrically. For example, it is well known that in the muon decay $\mu^-\to e^-+\bar\nu_e+\nu_\mu$ ($\mu^+\to e^++\nu_e+\bar\nu_\mu$) more electrons (positrons) are emitted in the direction parallel (antiparallel) to the spin of the μ^- (μ^+). The muons produced by a rotating black hole are partially polarized, since the hole emits preferentially particles with the same (upper) direction of spin. This implies that after the muons decay, more electrons will be moving in the direction parallel to the hole's angular momentum and more positrons in the antiparallel direction.

III. NEUTRINO CURRENTS IN THERMAL RADIATION

Another macroscopic effect of parity nonconservation is the generation of equilibrium neutrino currents in a rotating thermal radiation. The physics of this effect is similar to that of the asymmetric radiation by black holes.

The Fermi distribution for neutrinos in a rotating system has the form[15]

$$f(\omega,m,L)=\left[\exp\left(\frac{\omega-m\Omega-\mu L}{T}\right)+1\right]^{-1}, \quad (13)$$

where Ω is the angular velocity, μ is the chemical potential of neutrinos, and m is the projection of the particle's total angular momentum on the direction of $\vec\Omega$. Intuitively, it is clear that for a given energy ω, particles with greater values of *orbital* angular momentum are distributed at larger distances from the rotation axis. Then from Eq. (13) it follows that in any finite region of space there are more particles with spin parallel to $\vec{\Omega}$ than with spin antiparallel to $\vec{\Omega}$. Since neutrinos are left-handed and antineutrinos are right-handed, we conclude that more neutrinos move in the direction antiparallel to $\vec{\Omega}$ and more antineutrinos move parallel to $\vec{\Omega}$.

Although Eq. (13) is a direct consequence of the argument in Landau and Lifshitz,[15] I failed to find its complete derivation in the literature. Since this equation is of major importance for what follows, its deviation is given in the Appendix.

To calculate the neutrino current density, we shall first find the appropriate spinor wave functions $\psi_{\omega p m L}$ (p is the momentum projection on the direction of $\vec{\Omega}$). It will be convenient to use cylindrical coordinates,

$$ds^2 = dt^2 - dr^2 - r^2 d\phi^2 - dz^2 . \tag{14}$$

The neutrino field equations in curvilinear coordinates can be written in the form[16]

$$\begin{aligned} &\gamma_\mu(\partial_\mu - \Gamma_\mu)\psi = 0 ,\\ &(1 - L\gamma^5)\psi = 0 . \end{aligned} \tag{15}$$

Here γ_μ are the Dirac matrices satisfying

$$\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2g^{\mu\nu} . \tag{16}$$

The matrices Γ_μ are determined by the equations

$$\begin{aligned} &\Gamma_\mu \gamma^\nu - \gamma^\nu \Gamma_\mu = \partial\gamma^\nu/\partial x_\mu + \Gamma^\nu_{\sigma\mu}\gamma^\sigma ,\\ &\mathrm{Tr}(\Gamma_\mu) = 0 , \end{aligned} \tag{17}$$

and $\Gamma^\nu_{\sigma\mu}$ are the usual Christoffel symbols. In the cylindrical coordinates (14) a suitable choice of the matrices γ_μ and Γ_μ is given by

$$\begin{aligned} &\gamma^t = \gamma^0, \quad \gamma^r = \gamma^1, \quad \gamma^\phi = \gamma^2/r, \quad \gamma^z = \gamma^3 ,\\ &\Gamma_t = \Gamma_r = \Gamma_z = 0, \quad \Gamma_\phi = \frac{i}{2}\begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}, \end{aligned} \tag{18}$$

where $\gamma^0, \ldots, \gamma^3$ are the Dirac matrices in the standard representation (see, e.g., Ref. 17) and σ_i are the Pauli matrices. The solutions of Eqs. (15) with appropriate boundary conditions[18] are now easily found in the form

$$\psi_{\omega p m L} = \frac{1}{4\pi}\begin{pmatrix} \eta \\ L\eta \end{pmatrix} \exp(ipz - i\omega t - im\phi) , \tag{19}$$

where the two-component spinor η is given by

$$\eta = \begin{pmatrix} i(\omega + pL)^{1/2} & J_{m+1/2}(\alpha r) \\ L(\omega - pL)^{1/2} & J_{m-1/2}(\alpha r) \end{pmatrix} , \tag{20}$$

$J_k(x)$ are the familiar Bessel functions, m can take all positive and negative half-integer values, and $\alpha \equiv (\omega^2 - p^2)^{1/2}$. The wave functions $\psi_{\omega p m L}$ are normalized according to

$$\int \psi^\dagger_{\omega p m L}\psi_{\omega' p' m' L'}\, r\, dr\, d\phi\, dz = \delta_{mm'}\delta_{LL'}\delta(p - p')\delta(\omega - \omega'). \tag{21}$$

The lepton current density in z direction corresponding to the mode ($\omega p m L$) equals

$$\begin{aligned} j_{\omega p m L}(r) &= L\psi^\dagger_{\omega p m L}\gamma^t\gamma^z\psi_{\omega p m L} \\ &= (8\pi^2)^{-1}[(\omega + pL)J_{m+1/2}{}^2(\alpha r) \\ &\qquad - (\omega - pL)J_{m-1/2}{}^2(\alpha r)] . \end{aligned} \tag{22}$$

Choosing the z axis in the direction of the angular velocity vector $\vec{\Omega}$, we can now write the equilibrium neutrino current density in the form[19]

$$\langle \vec{J} \rangle = J\vec{\Omega}/\Omega , \tag{23}$$

where

$$J(r) = \int_0^\infty d\omega \int_{-\omega}^{\omega} dp \sum_{L=\pm1}\sum_m f(\omega, m, L) j_{\omega p m L}(r) . \tag{24}$$

The sum over m in Eq. (24) cannot be calculated analytically. However, noticing that of all functions $J_n(x)$ only $J_0(x)$ is nonzero at $x = 0$, we can easily calculate the current density J on the rotation axis ($r = 0$). In this case Eq. (24) reduces to

$$\begin{aligned} J(0) &= (4\pi^2)^{-1}\int_0^\infty d\omega\, \omega^2 \sum_{L=\pm1} [f(\omega, -\tfrac{1}{2}, L) - f(\omega, \tfrac{1}{2}, L)] \\ &= -(4\pi^2)^{-1}T^3\Big[\sinh\xi_1 \int_0^\infty (\cosh x + \cosh\xi_1)^{-1}x^2 dx \\ &\qquad + \sinh\xi_2 \int_0^\infty (\cosh x + \cosh\xi_2)^{-1}x^2 dx\Big], \end{aligned} \tag{25}$$

where $\xi_1 = (\Omega - 2\mu)/2T$, $\xi_2 = (\Omega + 2\mu)/2T$. The integrals in Eq. (25) can be evaluated,[20]

$$\int_0^\infty (\cosh x + \cosh\xi)^{-1}x^2 dx = \xi(\pi^2 + \xi^2)/3\sinh\xi , \tag{26}$$

and we find

$$J(0) = -\Omega T^2/12 - \Omega^3/48\pi^2 - \Omega\mu^2/4\pi^2 . \tag{27}$$

It is interesting that J does not vanish even at $T \to 0$. One has to remember, however, that Eq. (27) is valid only if the conditions of thermal equilibrium are satisfied. In particular, the size of the system must be much larger than the mean free path for neutrinos. This mean free path is a rapidly increasing function of inverse temperature T^{-1}, and thus the conditions of thermal equilibrium break at sufficiently low temperatures.

Another interesting case in which Eq. (24) can be treated analytically is the case of slow rotation, when $\Omega \ll T$ and $\Omega r \ll 1$, and the Fermi function in Eq. (24) can be expanded in powers of Ω. Assuming for simplicity that $\mu = 0$, we can write

$$f(\omega, m, L) = f(\omega) - m\Omega f'(\omega) + \tfrac{1}{2}m^2\Omega^2 f''(\omega) - \tfrac{1}{6}m^3\Omega^3 f'''(\omega) + \cdots , \quad (28)$$

where

$$f(\omega) = (e^{\omega/T} + 1)^{-1} . \quad (29)$$

Substituting this expansion in Eq. (24) and using the relations[21]

$$\sum_{n=-\infty}^{\infty} J_n^2(x) \begin{Bmatrix} 1 \\ n \\ n^2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0 \\ x^2/2 \end{Bmatrix} \quad (30)$$

we find, after a straightforward calculation

$$J = -\tfrac{1}{12}\Omega T^2 (1 + \Omega^2/4\pi^2 T^2 + 4\Omega^2 r^2 + \cdots) . \quad (31)$$

Note that this result is in agreement with Eq. (27). In the lowest approximation, the current density is given by

$$\vec{J} = -\tfrac{1}{12} T^2 \vec{\Omega} . \quad (32)$$

In a charge-symmetric rotating radiation when the chemical potential of neutrinos μ is zero, neutrino and antineutrino fluxes are exactly equal and opposite. A nonzero value of μ results in a net energy flux

$$\vec{S} = S\vec{\Omega}/\Omega , \quad (33)$$

where

$$S = \int_0^\infty d\omega \int_{-\omega}^{\omega} dp \sum_{L=\pm 1} \sum_m f(\omega, m, L)\, n_{\omega p m L}\, p \quad (34)$$

and

$$n_{\omega p m L}(\vec{x}) = \psi^\dagger_{\omega p m L}(\vec{x}) \psi_{\omega p m L}(\vec{x}) . \quad (35)$$

The energy flux S can be now easily calculated in the case of slow rotation. Expanding the Fermi function $f(\omega, m, L)$ in powers of Ω and using Eqs. (26) and (30) we get

$$\vec{S} = -\tfrac{1}{12}\vec{\Omega}(\mu T^2 + \mu^3/\pi^2) . \quad (36)$$

IV. DISCUSSION

(1) It is easily understood that the effect of asymmetric neutrino emission occurs not only in black holes but in any rotating star emitting thermal neutrinos. In the latter case, the chemical potential of neutrinos μ is not necessarily zero. If $\mu > 0$, then the star emits more neutrinos than antineutrinos, and therefore it accelerates like a rocket in the direction of its angular momentum.

(2) According to the Fermi distribution (13), the numbers of neutrinos in the states (ω, p, m) and $(\omega, -p, m)$ are equal. The neutrino current density arises because neutrinos with $p<0$ are distributed closer to the rotation axis than those with $p>0$. If we imagine a thermal radiation rotating in an infinite cylinder with perfectly reflecting walls, we expect a counterflow of neutrinos to develop near the walls of the cylinder. It is possible that the net neutrino current averages out to zero. (At least, the total momentum of neutrinos is equal to zero.) It would be interesting to check this explicitly and to find the current-density distribution for some reasonable boundary condition at the walls of the cylinder.[22]

(3) Particles other than neutrinos can be in both positive- and negative-helicity states, and application of the Fermi distribution (13), say, to electrons, gives a zero current density. However, electrons and other particles can develop currents in a nonequilibrium situation. One example is the black-hole evaporation, where, as was shown in Sec. II, electron currents can be generated by processes such as muon decay. Another interesting case is an expanding universe. Analysis based on grand unified theories suggests[14,23] that substantial deviation from local equilibrium in the early universe can occur for a short time at temperatures of order $10^{16}-10^{17}$ GeV. If the motion of matter in the universe has a rotational component, baryon and electric currents (as well as a net baryon number[14,23]) can be generated during this period. At $T \lesssim 10^{16}$ GeV the currents will be dissipated by collisions, and local equilibrium will be reestablished.

(4) Baryon and electric currents vanish in equilibrium in the ideal gas approximation (13). One expects, however, that nonzero equilibrium currents will result if parity-violating weak interactions are taken into account.[24]

(5) Baryon parity-violating currents can give rise to baryon density fluctuations in the early universe and electric currents can produce electromagnetic fields. Possible cosmological consequence of these effects are now being considered.

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APPENDIX: QUANTUM STATISTICAL DISTRIBUTION IN A ROTATING SYSTEM

Let us consider a closed system consisting of several kinds of particles (components). We shall assume that interactions between the particles are weak, so that the system can be treated as a mixture of ideal gases. The statistical properties of the system are determined by the values of its energy E, momentum $\vec{P}$, angular momentum $\vec{L}$, and the numbers of particles $N^{(i)}$ in all components.[25] Let us choose our z axis in the direction of $\vec{L}$. The total momentum of the system can be made zero by a suitable choice of inertial frame of reference. However, it will be convenient to assume that $P_x = P_y = 0$, while P_z is not necessarily zero.

The total entropy of the system is given by

$$S = \sum_i S^{(i)}, \tag{A1}$$

where $S^{(i)}$ is the entropy of the ith component:

$$S^{(i)} = \sum_n [(1 + f_n^{(i)}) \ln(1 + f_n^{(i)}) - f_n^{(i)} \ln f_n^{(i)}] \tag{A2}$$

for bosons and

$$S^{(i)} = -\sum_n [f_n^{(i)} \ln f_n^{(i)} + (1 - f_n^{(i)}) \ln(1 - f_n^{(i)})] \tag{A3}$$

for fermions.[15] Here the summation is taken over all one-particle quantum states, and $f_n^{(i)}$ is the average occupation number of the nth state in the ith component. The quantum state of a particle can be characterized by its energy ϵ, z component of momentum p, z component of total angular momentum m, and polarization or helicity h. In this case, n in $f_n^{(i)}$ stands for a set of four numbers (ϵ, p, m, h). The occupation numbers $f_n^{(i)}$ can be found from the requirement that the entropy S is a maximum under the subsidiary conditions

$$\sum_n f_n^{(i)} = N^{(i)}, \tag{A4}$$

$$\sum_{i,n} f_n^{(i)} \begin{pmatrix} \epsilon_n^{(i)} \\ m_n^{(i)} \\ p_n^{(i)} \end{pmatrix} = \begin{pmatrix} E \\ L \\ P \end{pmatrix}, \tag{A5}$$

where $L = L_z$ and $P = P_z$. One could add the conditions $P_x = P_y = L_x = L_y = 0$, but from the symmetry of the problem it is clear that these conditions will be satisfied automatically.

Using the method of Lagrange multipliers, we can write

$$\frac{\partial}{\partial f_n^{(i)}} \sum_j \left\{ S^{(j)} - \sum_{n'} f_{n'}^{(j)} [\alpha_j + \beta \epsilon_{n'}^{(j)} + \gamma m_{n'}^{(j)} + \delta p_{n'}^{(j)}] \right\} = 0, \tag{A6}$$

where α_i, β, γ, and δ are constants. From Eqs. (A2), (A3), and (A6) we find

$$f_n^{(i)} = [\exp(\alpha_i + \beta\epsilon + \gamma m + \delta p) \pm 1]^{-1}, \tag{A7}$$

where the upper and lower signs are for fermions and for bosons, respectively, and I have dropped the indices of ϵ, m, and p. To determine the constants in Eq. (A7), we note that Eq. (A6) implies

$$dE = \frac{1}{\beta} dS - \sum_i \frac{\alpha_i}{\beta} dN^{(i)} - \frac{\gamma}{\beta} dL - \frac{\delta}{\beta} dP. \tag{A8}$$

Comparing this with[15]

$$dE = T dS + \sum_i \mu_i dN^{(i)} + \Omega dL + V dP, \tag{A9}$$

where μ_i is the chemical potential of the ith component, Ω is the angular velocity, and V is the center-of-mass velocity of the system, we find

$$\beta = T^{-1}, \quad \alpha_i = -\mu_i / T, \quad \gamma = -\Omega / T, \quad \delta = -V/T \tag{A10}$$

and

$$f_n^{(i)} = \{\exp[T^{-1}(\epsilon - \mu_i - \Omega m - Vp)] \pm 1\}^{-1}. \tag{A11}$$

In the rest frame $V = 0$, and (A11) reduces to Eq. (13). Note that the values of Ω and V are the same for all components. In the frame where $V = 0$, the momentum density of particles other than neutrinos is equal to zero. (This is a direct consequence of the reflectional symmetry.) The momentum density of neutrinos in this system does not vanish, although the total neutrino momentum is zero. [See Sec. IV, point (2).]

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[18]It should be noted that a uniformly rotating system cannot be infinite, its maximum size in the plane of rotation being $r \sim \Omega^{-1}$. We shall assume, however, that the infinite-space neutrino wave functions give an adequate description of the system far from the boundaries.

[19]A somewhat more rigorous derivation can be found in Ref. 4, where the neutrino current density is calculated directly from the definition $\vec{J} = \mathrm{Tr}(\rho \vec{J})/\mathrm{Tr}\rho$.

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