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Chromoelectric-flux-tube model of particle production

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Quark confinement is assumed to be implemented by the generation of chromoelectric flux tubes with uniform energy density. Approximate formulas for the production of quark pairs in these tubes by tunneling are derived and various observable implications are studied. The non-Abelian nature of the SU(3) color group is taken partially into account thus yielding a quantitative estimate for the baryons-to-mesons ratio. Reasonable agreement with experiment is obtained.

I. INTRODUCTION

For a variety of reasons, related mainly to the short-distance behavior, quantum chromodynamics (QCD) appears to be the most promising theoretical framework for strong interactions.[1] To understand within this framework the long-distance behavior, is the outstanding challenge to QCD.

It is believed that confinement arises, in a $q\bar{q}$ system, say, because of a tubelike configuration of the color-electric field between the quarks. Such a configuration yields linear potentials and linear trajectories.[2]

Clearly, it is of great interest to find additional independent manifestations of the confining tube in hadronic processes. The present work is directed towards this goal. More definitely, we address the question of whether the basic features of high-energy multiple particle production can be related to the confinement tube. For simplicity we restrict ourselves to e^+e^- annihilation, although some of the qualitative features we discuss are expected to hold also for the more general case of hadron collisions.

Our basic picture for multiple quark-pair production is rather similar to what happens in QED in one space dimension.[3] The annihilation of the e^+e^- pair creates a $q\bar{q}$ pair, and this generates a cascade. The confining constant color-electric field between the q and $\bar{q}$ builds a tube within which new $q\bar{q}$ pairs may be created. Such a pair is pulled apart by the field to a point where it shields the field in between, and thus pairs off with the original $q\bar{q}$ to form smaller tubes of lower energy. This process repeats itself till all the available energy is used up.

The main observation we make is that within the above framework the elementary process of pair creation may be understood rather well. In fact, it is not very different from the idealized situation of noninteracting particles created by a uniform Abelian field, a problem which has been exactly solved by Schwinger.[4] As a result of this observation we get, with no adjustable parameters, predictions for the transverse-momentum spectrum, for the kaons-to-pions ratio, and for the baryons-to-mesons ratio. These predictions are consistent with experiment.[5] Our general picture of the process is seen to agree also with the width-to-mass ratio of meson resonances. We remark that a major difficulty in the way of a detailed phenomenological analysis is the need to identify the final quark configurations with clusters of a given energy.

The program of the paper is as follows. In Sec. II we introduce the flux-tube model, evaluate its parameters, and list the approximations which underly it. We then rederive Schwinger's pair-production formula by a semiclassical tunneling calculation. This method has an advantage over the exact calculation in that the limitations due to the finite size of the tube and the finite time available for the quark-pair materialization may be assessed. As a byproduct we derive the transverse-momentum distribution of the produced quarks. In Sec. VI we discuss the transverse-momentum spectrum of the produced mesons and in Sec. V the K/π ratio. In particular, a reasonable transverse-momentum cutoff is found and the production of strange quarks (and mesons) is damped due to their higher mass.

In Sec. VI, we take the non-Abelian nature of the color group into account. A remarkable simple mechanism for baryon production emerges and this yields a reasonable baryon-meson ratio. In Sec. VII the model is extended to the rather extreme case of resonance decay. Although the low-lying $q\bar{q}$ resonances can hardly be thought of as tubes, we still expect the flux-tube description to hold as a rough approximation. We are then able to calculate an average width/mass ratio. This ratio is sensitive to the thickness of the flux tube, which is the only unknown parameter in the model.

Section VIII is devoted to a critical discussion and comparison to other QCD-motivated approaches to multiple particle production.[6]

A detailed treatment of the cascade evolution and the related problem of particle multiplicities will be given elsewhere.

II. THE FLUX-TUBE MODEL

Hadron dynamics possesses two major characteristics which QCD is expected to explain. These are quark confinement and the generation of quark ("constituent") masses through the spontaneous breakdown of chiral SU(3) × SU(3) manifested by PCAC (partial conservation of axial-vector current). In what follows we shall have little to say about PCAC, but will assume certain reasonable properties of the mechanisms which govern the two effects. These assumptions will then be used in a rather extreme fashion to formulate an approximate semiquantitative theory of hadron production in high-energy e^+e^- annihilation .

The first assumption we make is that for distance scales of 1 GeV^{-1} (0.2 F) or more, quarks may be treated approximately as massive Dirac particles. The relevant masses will be the so-called "constituent" masses,[7] 350 MeV for the u, d quarks and 500 MeV for the s quark. The dynamical content of this assumption is that it assigns a relatively small scale to the mechanism which endows the (almost) massless "current" quarks with their "observed" masses. In line with own general approach, we shall not try to supply a theoretical justification to this assumption. It may, however, be noted, that phenomenologically the success of the constituent-quark model in explaining the hadronic spectroscopy supports our assumption.

Our second assumption is that in a $q\bar{q}$ system confinement is implemented through the generation of a chromoelectric flux tube of universal thickness for which the quark and antiquark act as source and sink. Again, no justification will be attempted beyond remarking that strong-coupling lattice gauge theories lead to such a picture.[8]

The flux-tube hypothesis will be used rather extremely. More specifically, we assume that the sole effect of the nonlinear quantum dynamics of the gauge fields is to compress the longitudinal chromoelectric field decreed by Gauss's law into a tube. The time scale on which this process occurs will be assumed to be rather short compared to hadronic scales (not more than ~1 GeV^{-1}). Moreover, long-range vacuum fluctuations will be assumed to be relatively weak and to occur on time scales which are long compared to those relevant for pair production. Thus, the chromoelectric field in the tube will be treated as a classical longitudinal Abelian field during the act of pair creation. In particular, the above assumptions require that the magnitude of $g^2/8\pi^2$, where g is the gauge coupling constant which governs the vacuum fluctuations of the relevant scale, be comparatively small; we shall indeed see below that with a reasonable choice for the radius of the tube this criterion is satisfied.

There are three parameters which determine the properties of the flux tube. These are the magnitude of the longitudinal field $\mathcal{E}$, the gauge coupling constant g, and the radius of the tube Λ. The energy per unit length stored in the tube is according to our assumptions:

$$k=\tfrac{1}{2}\mathcal{E}^2 A\,, \tag{1}$$

where $A=\pi\Lambda^2$ is the cross-sectional area. The parameter k represents the large distance attractive force which acts on the quarks, and can be shown[2] to be related to the Regge slope, α', through:

$$k=\frac{1}{2\pi\alpha'}=0.177\ (\text{GeV})^2\,. \tag{2}$$

A second relation among the parameters emerges on applying Gauss's law which equates the chromoelectric flux to the quark charge.

$$\mathcal{E}A=\tfrac{1}{2}g\,, \tag{3}$$

where the factor $\frac{1}{2}$ is due to the fact that the quarks couple to the gauge field through the SU(3) generators $\frac{1}{2}\lambda^a$. Equations (2), (3) upon eliminating A lead to

$$\tfrac{1}{2}g\mathcal{E}=2k=0.354\ (\text{GeV})^2\,. \tag{4}$$

Equation (4) is simply the force which acts on a free quark inside the tube, and as in an ordinary capacitor the magnitude of this force is twice that which operates on the end quarks. Note incidentally that the radius and coupling constant are related through

$$g^2=4\Lambda^2/\alpha'\,. \tag{5}$$

Most of our results are independent of Λ, and the only assumptions we need are that Λ is sufficiently large compared to the distance scale relevant for quark pair production and is small compared to the average distance travelled by the end quarks between two consecutive acts of pair production (the tube should be longer than its thickness). As will be seen $\Lambda\sim 2.5\ \text{GeV}^{-1}$ is a reasonable choice from this point of view. Using Eq. (5) we see that this value corresponds to $g^2/8\pi^2\sim 0.35$ which is still small compared to unity.

We may now give a qualitative description of the process of hadron production in e^+e^- annihilation. A pair of $q\bar{q}$ is produced at a point and start receding from each other at a velocity close to the velocity of light. As the distance between the quarks exceeds ~1 GeV^{-1} the tube is formed and lengthens with time. As in QED, a pair of $q\bar{q}$ can tunnel through the barrier formed by the constant field inside the tube, be pulled apart and materialize within a characteristic distance and at a rate determined by the transverse energy and the field strength. The new $\bar{q}$ (q) then forms a tube with the parent q ($\bar{q}$) through a process of screening which occurs also (by assumption) on a relatively short time scale. The tube thus splits and the process is repeated (N times) till all the kinetic energy has been converted into the chromostatic energy and quark transverse energies of the 2^N $q\bar{q}$ pairs which are identified with mesons. The produced quarks (and mesons) will be seen to have a calculable transverse-momentum spectrum which is cut off due to the barrier-penetration factor. This factor depends also on the quark mass so that strange-quark production is damped. When carried to extremes the above picture may also be used to yield an estimate for the lifetime of low-energy resonances. Finally, a simple and amusing extension of the model supplies a natural mechanism for baryon production. These subjects will be treated in some detail in what follows.

III. PAIR PRODUCTION THROUGH TUNNELING AND SCHWINGER'S FORMULA

The problem of a Dirac field which interacts with an external uniform electric field was solved by Schwinger almost thirty years ago.[4] Among other results Schwinger gives the following expression for the vacuum persistence probability:

$$|\langle 0_+|0_-\rangle|^2 = \exp\left\{\left[-\frac{g^2\mathcal{E}^2}{16\pi^3}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left(-\frac{2\pi m^2 n}{g\mathcal{E}}\right)\right]\int d^4x\right\}. \tag{6}$$

$g/2$ is the elementary charge, $\mathcal{E}$ is the external field, and m is the mass of the Dirac particle. This result clearly exhibits the instability of the vacuum due to the possibility of pair creation. In fact, the decay probability per unit time and per unit volume is immediately read off as being given by

$$P = -\frac{g^2\mathcal{E}^2}{16\pi^3}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left(-\frac{2\pi m^2 n}{g\mathcal{E}}\right). \tag{7}$$

The essential singularity in $g\mathcal{E}$ of Eq. (7) is a signal of the tunneling effect responsible for pair creation. This reflects itself also when one tries to think in the language of path integrals. In order to estimate the vacuum-one-pair transition, one has to find the dominant paths (histories) with the required boundary conditions. Whenever one deals with scalars these paths are build of sequences of states that are eigenstates of the field and satisfy some classical equation of development. Thus, *a priori*, at each intermediate state, the system may be in any one of a continuous infinity of states. It is this last statement that does not hold true for fermions, and the reason for this is that fermions obey the exclusion principle. Keeping this in mind, one may use path-integral reasoning also for fermions. Thus, we look for continuously varying sequences of states of the Fermi system that connect the vacuum to the one pair occupied state. Clearly these trajectories cannot break, by continuity, any of the conservation laws of discrete quantities. The final states are restricted by *all* the conservation laws, but the intermediate states are not. Whenever the final states are such that some of the intermediate states in the path will be forced to break energy conservation, the transition may occur only via tunneling. The description of the sequence of intermediate states for tunneling is facilitated by imagining the system to acquire purely imaginary linear momentum. Thus the equations of motion possess a formal classical meaning. Once the dominant trajectories are known, the amplitude for the transition is given by a properly weighted sum over the exponential of the classical action associated with each trajectory. Our problem is essentially one dimensional, and, therefore, we are in fact going to use explicitly only the most elementary version of WKB. This plus some probability arguments will turn out to be all we need in order to rederive Eq. (6).

In order to apply the strategy explained above, we need a convenient quantum-mechanical description of all the states that the vacuum may tunnel to. Since these states consist of pairs only, and the individual pair-creation events are uncorrelated as long as they do not interfere with the exclusion principle, we may be content with calculating the probabilities for the creation of all possible distinct one-pair states that have the same quantum numbers as the vacuum. We decide that any relevant one-pair state will be described by the following parameters: the longitudinal position of each component of the pair, the absolute value of the transverse momentum of the components of the pair, the longitudinal projection of the spatial angular momentum of each component, and the spin state of each component. The longitudinal direction is fixed by the external field, henceforth the z axis. The values of the spin and angular momentum in the z direction are restricted by the cylindrical symmetry of the system. For our WKB calculation we prefer to work with a different basis in which the two-dimensional transverse momentum is diagonalized. We are then free to forget about spin if we include a factor of 2 in the degeneracy of a one-pair state of given transverse momentum. We multiply by 2 and not by 4 because of J_z conservation. We will only consider pairs for which the components are pulled apart by the field, therefore, the pairs are longitudinally ordered. We further restrict the relevant final pairs by demanding that the distance between the particle and the antiparticle, d, satisfies

$$d=4E_T/g\mathcal{E}\,, \tag{8}$$

where $E_T=(p_T{}^2+m^2)^{1/2}$ is the transverse energy of each component. The classical meaning of this restriction is that the pair is created with zero longitudinal momentum. A state which does have some longitudinal momentum propagates in the semiclassical approximation with a pure phase and it does not tunnel. This kind of restriction is always used in semiclassical calculations of barrier penetration.

Having classified the possible final states we may go ahead and define the semiclassical trajectories. Any such trajectory is described by the following: At an arbitrary point along the z axis, a pair with transverse momentum p_T is formed. It must have the same energy as the vacuum and therefore each component has imaginary longitudinal momentum equal to iE_T. The electric field pulls the pair apart, the potential barrier which each component faces being $V(q) = E_T-\frac{1}{2}g\mathcal{E}q$, where q is the distance along the z axis from the point where the pair was initially formed. Thus the imaginary longitudinal momentum is given as a function of q by $p=i(E_T{}^2-\frac{1}{4}g^2\mathcal{E}^2q^2)^{1/2}$. Therefore, the action per component is

$$S=\int_0^{2E_T/g\mathcal{E}}|p|\,dq=\frac{\pi E_T{}^2}{2g\mathcal{E}}\,. \tag{9}$$

The total action is twice the above quantity and thus we get that the probability for a tunneling event to occur at momentum p_T is

$$P(\vec{p}_T)=\exp\left(-\frac{2\pi E_T{}^2}{g\mathcal{E}}\right). \tag{10}$$

Now it is clear that the vacuum-persistence probability is given by the probability that no such event happended at any instant since the existence of the field or for any location of the center of the pair or for any value of the transverse momentum or spin. The exclusion principle assures us that no more than one event with the above specification may occur. Thus the vacuum-persistence probability is given by

$$|\langle 0_+|0_-\rangle|^2=\prod_{\text{spin}}\prod_{z}\prod_{t}\prod_{\vec{p}_T}[1-P(\vec{p}_T)]\,. \tag{11}$$

In order to perform the infinite product we need some discretization scheme. To this end we introduce the spacial and temporal large dimensions of the problem: L_x, L_y, L_z, T. We now construct cells $(\Delta p_x, \Delta p_y, \Delta z, \Delta t)$ which have to be small enough for the exclusion principle to operate but still sufficiently large to allow events from different cells to be distinguishable. Since the transverse directions are bounded by L_x and L_y, we get $\Delta p_x=2\pi/L_x$, $\Delta p_y=2\pi/L_y$. The frequency $1/\Delta t$ counts the number of times per second the system gets ready to tunnel. The quantity which oscillates with this frequency is the number of virtual pairs at p_T. Therefore, $1/\Delta t=\omega/2\pi=E_T/\pi$.

The longitudinal extension of the cell must be clearly given by the total length of the barrier seen by the particle and the antiparticle. Thus, we get $\Delta z=4E_T/g\mathcal{E}$. Inserting this into (11) we get

$$\begin{aligned}|\langle 0_+|0_-\rangle|^2&=\exp\sum_{\substack{\text{spin}\\ \vec{p}_T,z,t}}\ln[1-P(\mathrm{p}_T)]\\ &=\exp(-L_xL_yL_zTP)\,, \qquad (12)\\ P&=-\frac{g\mathcal{E}}{8\pi^2}\int_{m^2}^{\infty}d(E_T{}^2)\ln\left[1-\exp\left(-\frac{2\pi E_T{}^2}{g\mathcal{E}}\right)\right],\end{aligned}$$

which fully reproduces Eq. (7). Needless to say, we make no claim for rigor in the derivation presented above, and especially not for the calculation of the numerical value of the factor in front of the last expression in Eq. (12). Nevertheless, we do learn two important lessons (for our purposes), which are not straightforward from Schwinger's treatment:

(a) The probability per unit time per unit volume to produce pairs with transverse momentum p_T is

$$-\frac{g\mathcal{E}}{8\pi^3}\ln[1-P(\vec{p}_T)]d^2p_T \underset{2\pi m^2\gg g\mathcal{E}}{\sim} \frac{g\mathcal{E}}{8\pi^3}\exp\left(-\frac{2\pi E_T{}^2}{g\mathcal{E}}\right)d^2\vec{p}_T\,. \tag{13}$$

(b) The above result stays approximately true as long as the semiclassical calculation we presented is applicable. Thus we know to what degree the uniformity of the field may be altered without seriously affecting the results presented above.

IV. JET STRUCTURE AND THE TRANSVERSE-MOMENTUM SPECTRUM

In Sec. II and Sec. III we formulated our model and evaluated the rate of the elementary act of quark pair production. The production of hadrons will be viewed as a cascade process: As the produced pair of quarks emerge from the barrier they in turn act as sources of color flux. Since the new $\bar{q}$ (q) moves toward the initial q ($\bar{q}$), it is energetically favorable to split the original tube into two somewhat shorter tubes—in other words, we assume that the nonlinear gauge field vacuum fluctuations will force complete screening within a characteristic short time interval. The process may now be repeated again and again, thus leading in the end to a distribution of $q\bar{q}$ pairs connected by short flux tubes which we identify with mesons.

The transverse-momentum distribution of the produced quarks is governed by Eq. (13). It will be noticed that for sufficiently large quark masses (the relevant range of values for the *constituent* quark masses is[7] 350–500 MeV), the approximate form holds and the distribution becomes mass independent.

$$dN(\vec{p}_T) \sim d^2p_T \exp\left(-\frac{2\pi p_T^{\,2}}{g\mathcal{E}}\right) \sim d^2p_T \exp(-8.9p_T^{\,2}), \qquad (14)$$

where p_T is measured in GeV/c. Equation (14) yields for the rms p_T the value 335 MeV/c. With this we find $E_T \sim 0.5$ GeV. We now remark that we are led to $\Delta z \sim 2.8$ GeV^{-1}, $\Delta t \sim 6.3$ GeV^{-1} and clearly self-consistency requires that these values be smaller than the average separation in space-time between consecutive pair-production events. The latter depends on the value assumed for the tube radius Λ, and as will be seen below there exists a reasonable range of values for Λ which satisfies our criterion.

In order to estimate the effect of the finite thickness of the tube, we follow the classical trajectory also in the transverse direction. The relevant equation is (q_T is the transverse co-ordinate):

$$\frac{dq_T}{dq} = \frac{p_T}{(E_T^{\,2} - \frac{1}{4}g^2\mathcal{E}^2q^2)^{1/2}} . \qquad (15)$$

This is easily solved to give

$$q(q_T) = \frac{2E_T}{g\mathcal{E}} \sin \frac{g\mathcal{E}q_T}{2p_T} . \qquad (16)$$

The pair materializes only when $q(q_T)$ reaches the value of $2E_T/g\mathcal{E}$. Therefore, if the radius of the tube is Λ we get

$$p_T < \frac{g\mathcal{E}\Lambda}{\pi} \equiv p_T^{\max} . \qquad (17)$$

A reasonable estimate for the radius Λ is 2.5 GeV^{-1}. Thus we get $p_T^{\max} = 0.55$ GeV^{-1} and we are safe. We would like to stress that the width of the tube is really an unknown parameter. Therefore, it might be useful to reverse the above calculation, use the experimentally observed transverse-momentum cutoff as input, and obtain a lower bound for Λ. This bound is typically of the order of 1.5 GeV^{-1}.

We now turn to the question of how Eq. (14) is reflected by experiment. First of all, we note the great similarity between Eq. (14) and the so-called "jet-model fit."[9] But we must keep in mind that we are dealing with quarks and in experiment one measures particles. As long as the total energy of any individual tube is large enough, the average transverse *velocity* $[= p_T/(E_T^{\,2} + p_L^{\,2})^{1/2}]$ of the newly produced quarks may be neglected, and we are free to assume that all pair-creation events occur in the laboratory frame. Thus, during the cascade, the transverse-momentum distributions of the produced quark pairs are uncorrelated Gaussians [Eq. (14)]. Therefore, the rms p_T of the whole configuration is larger than the rms p_T of each individual quark by $\sqrt{2}$. When the total energy of the pairs becomes low, in the final stages of the cascade, phase-space limitations make their influence felt and the transverse momenta of the decay products become highly correlated. Since the number of resultant particles increases without substantially increasing the total available transverse momentum, the influence of this last stage is to lower the rms p_T: Thus, eventually the effect of the factor $\sqrt{2}$ is expected to cancel and the average transverse momentum of the particles seen in the laboratory should be approximately 350 MeV. This estimate is clearly in agreement with the observed value.[5,9]

We end this section by observing that the Gaussian distribution [Eq. (14)] really holds only for momenta which are smaller than the cutoff. In fact, since the tube has a finite radius, there are large momentum fields at the edges which would generate power-law tails for the transverse-momentum distribution. This effect should be matched with the high-momentum behavior predicted from asymptotic freedom.

V. THE SUPPRESSION OF STRANGENESS AND THE K/π RATIO

In the exact SU(3)-flavor limit we expect roughly equal numbers of K's and π's to be produced in high-energy collisions. This is true, in particular, in the present model. $s\bar{s}$, $u\bar{u}$, and $d\bar{d}$ pairs are equally produced and the $(q_i\bar{q}_j)$ pairs transform into mesons: $s\bar{d}$, $s\bar{u}$, $\bar{s}u$, and $\bar{s}d$ into strange and $u\bar{d}$, $d\bar{u}$, $u\bar{u}$, $d\bar{d}$, and $s\bar{s}$ into nonstrange mesons.

In reality the observed ratio $r=\langle N_K\rangle/\langle N_\pi\rangle$ is rather small, $r=0.1-0.05$,[5] which in the framework of QCD can only be attributed to SU(3)-symmetry breaking in the quark masses.

We would like to suggest that the strong suppression of K versus π production arises from two fairly distinct sources, the first being essentially kinematical, relatively well understood, and related directly to the physical K-π mass difference. The second reduction factor is of a dynamical origin and arises specifically within the present model. It is related to the quark mass difference and reflects the larger barrier for tunneling of heavier quarks.

(a) The basic assumption that $q_i\bar{q}_j$ pairs do turn directly into pseudoscalar mesons is definitely wrong. One would expect within the framework of the model that $q_i\bar{q}_j$ could pair, and with a larger statistical weight, into ^3S (i.e., vector mesons) and conceivably other $0^+1^+2^+$ multiplets. Even when the coupling for the decay of these heavier mesons respects SU(3), many kaonic decay channels are closed (e.g., $\rho, \omega \not\to \bar{K}K$, $Q \not\to KK\bar{K}$, while the corresponding pionic decays can clearly occur ($\rho\to 2\pi$, $\omega\to 3\pi$, $A_1\to 3\pi$). Thus, even when we begin with SU(3) equipartition at the level of the original "clusters," the preferential cascading into pions can enhance r by a factor of about 3–4.[10] Most recent analyses[11] of multiple particle production data directly revealed the resonance production. Indeed, it is being estimated that 70%–80% of the observed pions are not "first generation" or "directly produced," but rather emerge from the decay of resonances. The effect of this on the observed pion multiplicity is then indeed a factor of approximately 4.

(b) With our present picture for multiple particle production there is also a genuine dynamical suppression of $s\bar{s}$ pair production, as compared to $u\bar{u}$ and $d\bar{d}$ quark production. Thus, already at the level of resonances or clusters there will be a preponderance of nonstrange versus strange objects (ρ and ω rather than K^*, A_1 rather than Q, etc). This will then reduce the expected final ratio, r, by a further multiplicative factor of $\langle N_{\bar{s}s}\rangle/\langle N_{\bar{u}u}\rangle$ or $\langle N_{\bar{s}s}\rangle/\langle N_{\bar{d}d}\rangle$. From Eq. (2) we see that this reduction comes explicitly in the exponential tunneling factor and, in the same way as the transverse-momentum distribution, is independent of g or the size of the tube. The only relevant parameters once $g\mathcal{E}$ is known [see Eq. (4)] are the quark masses m_s and $m_u\sim m_d$. Since the momentum scale $\langle p_T\rangle\sim 0.35$ and the inverse tunneling distance $1/d\sim 0.36$ are quite small, the relevant values are the constituent quark masses, $m_s=0.5$, $m_u\sim m_d=0.35$. Nonetheless, we may take into consideration that the tunneling process starts at short distances and therefore we should consider varying our estimates down to say: $m_s=0.45$, $m_u\sim m_d=0.3$. The resultant ratios are

$$\begin{aligned} \frac{\langle N_{\bar{s}s}\rangle}{\langle N_{\bar{d}d}\rangle} &= \frac{\langle N_{\bar{s}s}\rangle}{\langle N_{\bar{u}u}\rangle} = 0.3 \\ &\text{for } m_s=0.5,\ m_u=m_d=0.35\,, \\ \frac{\langle N_{\bar{s}s}\rangle}{\langle N_{\bar{d}d}\rangle} &= \frac{\langle N_{\bar{s}s}\rangle}{\langle N_{\bar{u}u}\rangle} = 0.34 \\ &\text{for } m_s=0.45,\ m_u=m_d=0.30\,. \end{aligned} \tag{18}$$

This is the range of values required to achieve the observed K/π ratio.

VI. THE EFFECTS OF COLOR AND THE SUPPRESSION OF BARYON PRODUCTION

In the literature we are aware of, it has always been taken that the pairs created by the field within the tube are of such a nature that they cause complete screening and thus this mechanism produces only mesons. To explain the observed baryons in e^+e^- annihilation one would seemingly need a different mechanism. Although the number of baryons in the final states is strongly suppressed, the need of a new type of mechanism to account for a qualitative effect is a little bit disquieting. Moreover, a rather large part of the observed suppression has its origin in nondynamical effects, thus the dynamical effect though still small may not be really negligible. The nondynamical effects are similar to those encountered in the treatment of the K/π ratio. Any final baryon has appeared as a result of the decay of a highly excited baryonic state and therefore it appears in the laboratory accompanied by many mesons. On top of this there is a trivial factor of $\frac{2}{3}$ related to the fact that baryons are built of three quarks, whereas mesons contain only two. Therefore, the meson-to-baryon ratio is nondynamically enhanced.

The main purpose of this section is to show that a more careful analysis of the process of pair creation supplies a mechanism which may also be responsible for the creation of baryons. There exists a dynamical suppression of baryons and the relevant factor may easily be calculated.

In order to see how baryons get produced we must remember that our underlying theory is a theory of three types of charges which may interact in the static limit by eight kinds of electric fields. The quarks at the end of the tube act, due to confinement, as colored capacitor plates. We consider them as classical sources (constrained by the confinement hypothesis) of the electric field within the tube. Inside the tube the field acts as an external field in the Yang-Mills equations for the color Fermi triplets. Pairs may thus be produced, and after some time they are far enough to be subjected in turn to the confinement hypothesis. Then they may screen the existing field and two hyperexcited mesons are created. We are going to show that this will not always be the case. The result of the new pair creation may also be a configuration of four quarks with no screening occurring. Such a configuration will decay also by pair production, most probably into a pair of very excited baryons.

The confinement hypothesis means that a system of quarks must be neutral if it is to separate. The quarks do that by acting as sources or sinks of color tubes of electric field. This means that the lines of force which terminate or start at a quark do not spread in a spherically symmetric way, but are squeezed into a finite number of tubes. Therefore, the forces between any two constituents of the system are linear.

In our process a photon comes in and creates a color singlet $q\bar{q}$ state. Let the three colors be denoted by 1, 2, 3, and let the SU(3) color matrices be given in the standard representation. Thus, the state the photon has created is

$$|s\rangle=\frac{1}{\sqrt{3}}[\,|1\bar{1}\rangle+|2\bar{2}\rangle+|3\bar{3}\rangle]\,. \tag{19}$$

After some time elapses the state of the system will be given by

$$|s;t\rangle=\frac{1}{\sqrt{3}}[e^{-iHt}|1\bar{1}\rangle+e^{-iHt}|2\bar{2}\rangle+e^{-iHt}|3\bar{3}\rangle]\,. \tag{20}$$

Our basic approximation is to treat the chromoelectric field which develops between the quarks as a classical field. In particular, the 3-vector potential is assumed to vanish so that only longitudinal electric fields are present. We further assume that the time scale of pair creation is sufficiently short to allow us to neglect the precession of the color frames of the quarks. Thus, the state $e^{-iHt}|x\bar{x}\rangle$ $(x=1,2,3)$ contains exactly one x quark and one $\bar{x}$ antiquark. Hence, we may focus attention on any of the three terms in Eq. (20)—no mixing occurs.

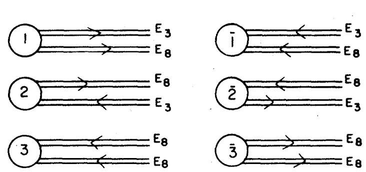

FIG. 1. Quarks as sinks and sources of the diagonal chromostatic field lines (E_3 and E_8).

Consider now a $x\bar{x}$ system subjected to the confinement hypothesis. For $x=1$, we may apply Gauss's law and use the experimental value of the energy per unit length within the tube:

$$\begin{aligned}&\int \vec{\nabla}\cdot\vec{\mathrm{E}}^a dV=E_z^a A=\tfrac{1}{2}g\lambda_{11}^a\,,\\ &\tfrac{1}{2}A\sum_{a=1}^{8}(E_z^a)^2=\frac{1}{2\pi\alpha'}\,.\end{aligned} \tag{21}$$

In Eq. (21) A denotes the area of the cross section of the tube $(A=\pi\Lambda^2)$, g is the coupling constant in the Yang-Mills Lagrangian, while λ_{ij}^a are the Gell-Mann SU(3) matrices. From Eq. (21) we find the electric field which binds the 1 and $\bar{1}$.

$$\begin{aligned}&E_z^a(1-\bar{1})=\delta^{a3}E^3+\delta^{a8}E^8\,,\\ &\tfrac{1}{2}gE^3=\frac{3}{4\pi\alpha'}=0.265\ (\mathrm{GeV})^2\,,\\ &\frac{1}{2\sqrt{3}}gE^8=\frac{1}{4\pi\alpha'}=0.088\ (\mathrm{GeV})^2\,.\end{aligned} \tag{22}$$

Equation (22) and its two companions for the quarks $(2\bar{2})$, $(3\bar{3})$ may be represented graphically by Fig. 1.

If the end quarks are, say, of the $x=1$ type, the dynamics within the tube may be approximately described by a triplet of Fermi fields subjected to an external field given by a diagonal matrix:

$$\left[i\partial\!\!\!/-m_{\mathrm{eff}}-iz\frac{1}{\pi\alpha'}\begin{pmatrix}1&0&0\\0&-\frac{1}{2}&0\\0&0&-\frac{1}{2}\end{pmatrix}\right]\psi=0\,. \tag{23}$$

The system decouples and we see that there are three competing channels by which the tube may decay. The field which tends to create quarks of the same type as the end quarks (1) is twice as strong as the field which tends to create quarks of the other two colors (2 and 3). If the pair created is $(1\bar{1})$, the 1 will be attracted towards the $\bar{1}$ at one of the ends of the tube. A glance at Fig. 1 shows that the confinement hypothesis will cause screening and therefore the dissociation of the tube into two mesonic configurations. Suppose now a pair $(2\bar{2})$ has been created. Now the sign of the electric field is reversed, and the 2 will move towards the end which is occupied by a 1. Screening is impossible and the application of our confinement hypothesis leads to the system described in Fig. 2, which cannot dissociate.

As time elapses the distance between the 2 and $\bar{2}$ grows and the field in between favors the creation of a new pair, this time of type 3 (see Fig. 1).

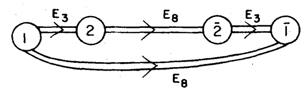

FIG. 2. A $q\bar{q}q\bar{q}$ configuration in which screening is impossible and thus it cannot split.

Now one may easily convince himself that fission will occur as in Fig. 3, and two baryonic configurations will be created.

We learned that the field within the tube may create quarks of a different type than the end quarks. If this happens, after one additional generation the four-quark configuration will prefer to split into a pair of clusters of the baryon antibaryon type. Baryons are dynamically suppressed because the nonscreening pair is produced by a field which is only half as strong as the field which creates screening pairs. Strange baryons are further suppressed due to the higher s-quark mass.

The relative probability to create nonscreening pairs is calculated with the help of Eqs. (7) and (23):

$$\frac{\langle N_{2\bar{2}}\rangle}{\langle N_{1\bar{1}}\rangle}=\frac{\langle N_{3\bar{3}}\rangle}{\langle N_{1\bar{1}}\rangle}=0.09$$

$$\text{for } m_s=450\text{ MeV and } m_u=m_d=300\text{ MeV}, \tag{24}$$

$$\frac{\langle N_{2\bar{2}}\rangle}{\langle N_{1\bar{1}}\rangle}=\frac{\langle N_{3\bar{3}}\rangle}{\langle N_{1\bar{1}}\rangle}=0.06$$

$$\text{for } m_s=500\text{ MeV and } m_u=m_d=350\text{ MeV}.$$

Therefore, our dynamical suppression factor is

$$\left(\frac{N_B}{N_M}\right)_{\text{dynamical}}=0.12\text{–}0.18. \tag{25}$$

Assuming that the nondynamical effect contributes another factor of the order of 5, we get for the baryons-to-pions ratio a value between 0.02–0.03. To the best of our knowledge this does not contradict experiment.[5]

VII. THE WIDTH OF THE TUBE AND Γ/M FOR MESONS

To a certain extent we possess a semiclassical picture for the meson resonances. A quark and an antiquark are bound by a tube of electric field. Energy is exchanged between the quarks and the field. The system rotates and oscillates, and one may imagine that the energy levels are given by a Bohr-Sommerfeld quantization rule. These levels are not real stationary states of the system. Pairs may be produced within the tube and the system may decay. Therefore, the levels acquire a semiclassically calculable width.

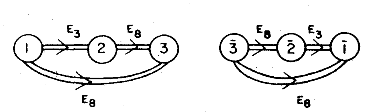

FIG. 3. Baryon configurations.

Suppose that the four-volume occupied by the tube is $V_4(t)$, where t is the meson lifetime as measured in its rest frame. Then we may use Eq. (6) to infer that the probability for the system not to decay is given by

$$\exp[-V_4(t)p], \tag{26}$$

where

$$p=\sum_{\text{flavor}}\frac{(g\mathcal{E})^2}{16\pi^3}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left(-\frac{2\pi m_F{}^2 n}{n^2}\right)$$

$$=\begin{cases}0.86\times10^{-3}\text{ GeV}^{-4}\text{ for } m_s=0.50\\ \qquad\qquad\text{and } m_u=m_d=0.35\,,\\ 1.18\times10^{-3}\text{ GeV}^{-4}\text{ for } m_s=0.45\\ \qquad\qquad\text{and } m_u=m_d=0.30\,.\end{cases}$$

$V_4(t)$ is a quantity which may be calculated by solving the classical equations of motion. In principle, one could include also the shape of the tube as a dynamical degree of freedom as it is done in various string models. We will take the much simpler case in which the tube is rigid and its width, 2Λ, is a constant parameter. We will assume that Λ is some number between 2 and 3 GeV^{-1}. This is the first time we encounter the need to have a quantitative estimate for Λ. In the equations which follow we will take Λ as 2.5 GeV^{-1}.[12] The sensitivity of the numbers to the value of Λ will be exhibited afterwards.

As our picture is very crude, we shall be content with estimating $V_4(t)$ in two extreme situations which may serve as reasonable bounds. The first extreme is to assume that the resonance is a one-dimensional oscillating system. In the second picture, very much similar to the one used in the derivation of the linearity of the Regge trajectories, the meson is assumed to be a rotating tube of a constant length. To simplify calculations even more we take the end quarks of the tubes to be massless in both cases. Therefore, the ends of the tube always move with the velocity of light.

If the length of the rotating tube is L then the mass M of the system will be given by

$$M=2k\int_0^{L/2}\frac{de}{(1-v_T^{\,2})^{1/2}}$$
$$=2k\int_0^{L/2}\frac{de}{[1-(2e/L)^2]^{1/2}}$$
$$=\frac{k\pi L}{2}\,, \tag{27}$$

where k is given by Eq. (2). The four-volume covered by this configuration is

$$V_4(t)=\pi\Lambda^2 Lt\,. \tag{28}$$

By Eq. (26) we see that the width is

$$\Gamma=\pi\Lambda^2 Lp\,. \tag{29}$$

Thus we get that the width-to-mass ratio is given by

$$\left(\frac{\Gamma}{M}\right)_{\rm rot}=\frac{2\Lambda^2}{k}p=(6.0\text{–}8.5)\times 10^{-2}\,. \tag{30}$$

If the maximal length of the resonance in the one-dimensional picture is L, the mass of the configuration is

$$M=kL\,. \tag{31}$$

Since the four-volume covered in one oscillation is $(\pi\Lambda^2L^2)/2$, the four-volume covered during a time t is, on the average, given by

$$V_4(t)=\frac{\pi\Lambda^2 L}{2}t\,. \tag{32}$$

Therefore, the width of the oscillating tube is

$$\Gamma=\tfrac{1}{2}\pi\Lambda^2 Lp\,, \tag{33}$$

leading to

$$\left(\frac{\Gamma}{M}\right)_{\rm osc}=\frac{\pi}{4}\left(\frac{\Gamma}{M}\right)_{\rm rot}=0.78\left(\frac{\Gamma}{M}\right)_{\rm rot}=(4.7\text{–}6.6)\times 10^{-2}\,. \tag{34}$$

Both the above results will change by approximately 40% when Λ is varied by 0.5 GeV^{-1}. The ranges of values already present in Eqs. (30) and (34) result from varying the masses of the constituent quarks.

From the experimental point of view the numbers are not bad at all, e.g.,

$$\left(\frac{\Gamma}{M}\right)_\rho\sim 0.19,\ \left(\frac{\Gamma}{M}\right)_f\sim 0.13,\ \left(\frac{\Gamma}{M}\right)_B\sim 0.10\,,$$
$$\left(\frac{\Gamma}{M}\right)_{A_2}\sim 0.08,\ \left(\frac{\Gamma}{M}\right)_{K^*}\sim 0.06,\ \left(\frac{\Gamma}{M}\right)_{\omega(1675)}\sim 0.08,$$
$$\left(\frac{\Gamma}{M}\right)_{g(1680)}\sim 0.10,\ \left(\frac{\Gamma}{M}\right)_Q\sim 0.10\,,$$
$$\left(\frac{\Gamma}{M}\right)_{K_N(1420)}\sim 0.07,\ \left(\frac{\Gamma}{M}\right)_{L(1770)}\sim 0.08\,.$$

It is amusing to note that in the string limit $(\Lambda\to 0)$ the resonance width vanishes, as may be appropriate in a dual model.

VIII. DISCUSSION

In the present work we have identified a component of multiparitcle production—namely quark pair production via tunneling in the confining tube of chromoelectric field. It is important to note the following: (a) The mechanism considered makes a sizable contribution. Specifically, it can account for most of the width of resonances. It is therefore gratifying that we also find that (b) the model accounts well for the qualitative features of multiple particle production in e^+e^- annihilation.

We assume that in hadron collisions an exchanged quark turns the hadrons into fast moving 3 and $\bar{3}$ color objects. The same tube model would then directly apply, and we would expect similar experimental features and, in particular, similar multiplicities. As noted by Brodsky and Gunion,[6] this is consistent with the available data.

However, the detailed gluon bremsstrahlung model of these authors and others, though also QCD (or rather QED) motivated is quite different from ours. It is assumed there that the gluons which are emitted in a typical dx/x (i.e., uniform in rapidity) spectrum separate into $q\bar{q}$ pairs which recombine into an equal number of color singlet clusters, $n_g=n_c$,[13] and each of these clusters eventually decays into a fixed number of final stable mesons.

Gluon pair creation does not lead to screening and splitting of the electric flux tube, as opposed to quark pair creation. Nevertheless, *a priori*, it may happen. The reason that we have neglected this possibility is that we believe that gluon production is damped by the penetration factor. Indeed, the indication, from the low-lying spectrum of hadrons (and the apparent absence of low-lying "glue balls") is that the effective constituent gluon mass is large ($\gtrsim$1 GeV). This strongly damps the production of gluons (the relative damping is of the order of 10^{-3}).

It is very likely that even if our pair-tunneling model does supply the explanation for the bulk of multiparticle production, other mechanisms operate alongside it. In particular, the incompleteness of the present picture is evident when we try to explain some rare processes. As an example we could consider an Okubo-Zweig-Iizuka (OZI) forbidden process such as $e^+e^-\to\phi+n\pi$. In our model a newly produced quark pair ($s\,\bar{s}$ say) separates under the pull of the color-electric field—the $\bar{s}$ getting closer to a parent q

and the s moving towards a parent $\bar{q}$. The configuration tends to separate into two $q\bar{s}$ and $s\bar{q}$ tubes. Therefore, ϕ or f' will be produced only in association with K's or K^*'s. Thus, in the first approximation our model exactly satisfies the OZI rule. It is an intriguing conjecture that the present picture of pair creation within the tube does provide a physical framework for the whole standard planar duality concept, a framework which is completely distinct from the $1/N$ expansion.

The discussion of this conjecture and the possibility of estimating nonplanar (e.g., OZI-rule violation) corrections is, however, beyond the scope of the present work.

We wish to end with a brief review of the various assumptions and approximations used in our schemes. The basic theoretical assumptions involved the total neglect of quantum and nonlinear effects beyond their role in fixing the effective mass and coupling constant, and the generation of confinement. Without a detailed understanding of the mechanisms involved, it is hard to assess the validity of these assumptions. We may, however, draw some encouragement from the self-consistency of the various estimates derived above. In particular, the value of the $g^2/8\pi^2$—the parameter which governs loop corrections seems to be relatively small ($\sim\frac{1}{3}$ for $\Lambda = 2.5$ GeV^{-1}). This may mean that the flux tube represents some average of a structure which contains smaller classical objects—thus fixing the scale which determined g at a value small compared to hadronic sizes.[14]

If the above discussion holds, then the other approximations, and, in particular, the neglect of the non-Abelian effect for finite time, may not be inconsistent.

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