Quantum effects in the early universe. II. Effective action for scalar fields in homogeneous cosmologies with small anisotropy

J. B. Hartle and B. L. Hu
Department of Physics, University of California, Santa Barbara, California 93106
(Received 16 April 1979)

The one-loop contributions of conformally invariant scalar fields to the effective action are calculated for homogeneous cosmological models with small anisotropy. The dynamical equations which determine the classical geometry are displayed and the matrix elements of the stress-energy tensor between the initial and final vacuums are determined.

I. INTRODUCTION

The central problem of modern cosmology is to determine to what extent the currently observed features of the universe must be ascribed to initial conditions or to what extent they are determined by dynamical processes occurring over the course of its history. Perhaps the most striking structural feature of the universe today is its remarkable isotropy[1] and approximate homogeneity.[2] Beginning with the work of Misner[3,4] in 1968 there have been a number of investigations of physical processes which could drive the universe towards a state of homogeneity and isotropy. Much of this work has concentrated on the dissipation of anisotropy in homogeneous model cosmologies because of the conceptual and mathematical simplicity of these models. On the one hand, these investigations could contribute to an understanding of the present state of isotropy by showing how significant initial anisotropies could be damped. On the other hand, as has been stressed by Barrow and Matzner,[5] these investigations could show that there could be no significant amount of initial anisotropy dissipated without an unacceptable increase in the entropy of the universe. Either way these investigations are important for the fundamental question raised above.

One of the most effective processes for dissipating anisotropy is the production of particle pairs in the very early universe. Particles obeying conformally invariant wave equations, such as the photon and neutrino, will not be produced in conformally flat geometries such as the homogeneous, isotropic universes. This follows from a general argument of Parker.[6] Pair creation in anisotropic universes therefore tends to drive them towards an isotropic state. Calculations by Zel'dovich and Starobinsky[7] and by Hu and Parker[8] have shown that even quite large anisotropies can be dissipated in a few multiples of the Planck time. These calculations, however, are limited to times greater than of the order of one Planck time after the start of the universe in order to avoid the computational difficulties associated with the initial singularity, in particular, the back reaction of the produced particles. Initial conditions must therefore be imposed at some arbitrary early time not quite at the singularity.

In this series of papers[9,10] we shall carry out a model calculation of anisotropy damping which can be extended all the way back to the start of the universe and which takes into account the back reaction of the produced particles in a consistent quantum-mechanical way. The price for doing this is that our calculation is restricted to small anisotropies in a sense we shall describe below. Our calculation thus provides a useful complement to the earlier work which can deal with larger anisotropies but only in later time regimes.

At the outset we shall describe the model we consider, the reasons for its features and its limitations. We consider the dissipation of small amounts of anisotropy in homogeneous spatially flat universes containing classical radiation by the production of conformally invariant scalar particles. The classical geometry which describes such a Bianchi I universe can be put in the form[3]

$$ds^2 = a^2(\eta)[-d\eta^2 + (e^{2\beta(\eta)})_{ij}\,dx^i dx^j], \qquad (1.1)$$

where $\beta(\eta)$ is a symmetric, traceless, 3×3 matrix and both a and β depend only on the time. The assumption of spatially flat sections simplifies our calculations but not, we believe, in an essential way. The assumption of homogeneity is, of course, a central restriction of the model. The assumption of small anisotropy can be translated into a restriction on the size of β.

In these universes we shall consider the production of conformally invariant scalar particles obeying the wave equation

$$\Box^2\varphi - \tfrac{1}{6}R\varphi = 0\,. \tag{1.2}$$

The technical advantages of considering a single scalar degree of freedom rather than the greater number associated with the more realistic electromagnetic and neutrino cases are obvious, and not, we believe, an essential limitation of the model. A far more important omission is that we shall not consider the production of gravitons. There is no justification for this truncation of the theory other than the technical simplicity thereby achieved. Since gravity is not a conformally invariant theory it is as yet unclear whether there are significant physical issues which are neglected by restricting attention to fields obeying Eq. (1.2).

In addition to the scalar field the universe will be assumed to contain classical radiation whose energy-momentum tensor has vanishing trace. The presence of classical radiation allows the anisotropy of the universe to be maintained at a small value over its whole history. Without some supporting matter the anisotropy would inevitably grow large at some epoch because there are no homogeneous, isotropic vacuum solutions of Einstein's equations.

The dissipation of anisotropy by particle production in the above model will be studied using the effective-action method already outlined in the first paper of this series.[9] The central quantity is the vacuum persistence amplitude—the amplitude that an initial scalar particle vacuum $|0_-\rangle$ evolves into a final scalar particle vacuum $|0_+\rangle$. This is given in terms of the effective-action functional $\Gamma[\tilde{g}]$ by

$$\langle 0_+|0_-\rangle = \exp(i\Gamma[\bar{g}])\,, \tag{1.3}$$

where the classical geometry $\bar{g}$ is a solution of the variational problem

$$\delta\Gamma/\delta\tilde{g}_{\alpha\beta} = 0\,, \tag{1.4}$$

with appropriate boundary conditions to be described below. For the problem under consideration both initial and final particle vacuum states might be characterized by a three-geometry with a certain anisotropy. The probability that this anisotropy persists will be given by $|\langle 0_+|0_-\rangle|^2$. In the presence of particle production $\Gamma[\bar{g}]$ will be complex and this probability will be less than one. The deviation from unity is a measure of the dissipation of anisotropy.

Several approximations will be used in calculating the effective action. One has already been spelled out. We will consider only classical geometries of the homogeneous form in Eq. (1.1) with small anisotropies. The effective action can thus be developed in a perturbation series in β. This is restrictive but certainly controllable. The second approximation will be to evaluate the effective action in the one-loop approximation and then calculate only the contribution to it of a single quantized scalar field obeying Eq. (1.2). In this approximation

$$\Gamma[\tilde{g}] = S_E[\tilde{g}] - \tfrac{1}{2}i\{\mathrm{Tr}\,\ln[G(x,x')]\}_{\mathrm{reg}}\,, \tag{1.5}$$

where S_E is the classical gravitational action, G is the Green's function of the scalar field propagating in the background geometry $\tilde{g}$, and the subscript reg indicates that the trace must be suitably regularized.

The restriction to the one-loop terms in the effective action is an approximation which cannot be justified in its own context. This is because the one-loop terms result in significant corrections to the classical action in the early universe. Whether the corrections from higher loops result in small or large corrections to the results of the one-loop calculations is largely a matter of conjecture in the absence of any real ability to calculate these terms. On the positive side it can at least be said that the one-loop corrections make the physical amplitudes for particle production finite in contrast to those calculated in the test field approximation. One might, therefore, hope that the higher loops which are neglected will only result in further finite but qualitatively similar corrections to the one-loop results calculated here. Developing a more efficient approximation scheme for calculating quantum effects in the early universe remains an interesting but largely unattacked question.

In this paper we shall begin this program by calculating the effective action for small anisotropy and the equations of motion which follow from Eq. (1.4). Solution of these equations will be considered in a subsequent paper.[10] In Sec. II the effective action is calculated to second order in the anisotropy. In Sec. III the equations of motion are derived while in Sec. IV the matrix elements $\langle 0_+|T^{\alpha\beta}|0_-\rangle$ of the scalar field stress-energy tensor are calculated.

II. THE EFFECTIVE ACTION FOR SMALL ANISOTROPY

In this section we shall evaluate the one-loop effective action given by Eq. (1.5) to second order in the matrix β which controls the deviation of the assumed classical geometry from exact isotropy. We write the development $\Gamma[\tilde{g}]$ in powers of β as

$$\Gamma[a,\beta] = \Gamma_0[a] + \Gamma_1[a,\beta] + \Gamma_2[a,\beta] + \cdots\,, \tag{2.1}$$

where Γ_n is proportional to β^n. To evaluate the individual terms in this expansion both the classical gravitational action and the one-loop corrections must be expanded in powers of β.

The expansion of the classical action is straightforward. Its definition is[11]

$$S_E[\tilde{g}] = l^{-2}\int d^4x(-\tilde{g})^{1/2}R + (\text{surface terms}), \tag{2.2}$$

where $l = (16\pi G)^{1/2}$ is the Planck length and the surface terms are chosen to cancel the metric second derivatives in the action. Making use, for example, of the results of Misner[3] one finds for geometries of the form in Eq. (1.1)

$$S_E[a,\beta] = Vl^{-2}\int d\eta[-6(a')^2 + a^2\beta'_{ij}\beta'^{ij}] \tag{2.3}$$

to second order in β. Here V denotes the coordinate volume over which the spatial integration is carried out, a prime denotes a derivative with respect to η, and the Minkowski metric has been used to raise and lower indices on the β_{ij}, viz., $\beta^{ij} = \beta^i{}_j = \beta_{ij}$.

To evaluate the one-loop corrections we begin by writing down the action for the scalar field theory whose equation of motion is Eq. (1.2):

$$S_f[\varphi,\tilde{g}] = -\tfrac{1}{2}\int d^4x(-\tilde{g})^{1/2}(\tilde{g}^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi + \tfrac{1}{6}R\varphi^2). \tag{2.4}$$

Expanding the action in powers of β one finds

$$S_f[\varphi,a,\beta] = S_{f0}[\varphi,a] + S_{f1}[\varphi,a,\beta] + S_{f2}[\varphi,a,\beta], \tag{2.5}$$

where

$$S_{f0} = -\tfrac{1}{2}\int d^4x\, a^2[\eta^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi + (a''/a)\varphi^2], \tag{2.6}$$

$$S_{f1} = \int d^4x\, a^2\beta^{ij}\partial_i\varphi\partial_j\varphi, \tag{2.7}$$

$$S_{f2} = -\int d^4x\, a^2(\tfrac{1}{12}\beta'_{ij}\beta'^{ij}\varphi^2 + \beta^{ik}\beta_k{}^j\partial_i\varphi\partial_j\varphi). \tag{2.8}$$

Here $\eta^{\alpha\beta}$ denotes the Minkowski metric in rectangular coordinates.

The Green's function G may now be expanded in powers of β. Denote the Green's function in the limit of exact isotropy by G_0. This function was extensively discussed in paper I. If we denote symbolically the contribution to the wave equation of first-order action in Eq. (2.7) by V_1 and that of the second-order action in Eq. (2.8) by V_2 then we can write

$$G = G_0 + G_0(V_1 + V_2 + \cdots)G$$
$$= G_0 + G_0V_1G_0 + G_0V_2G_0 + G_0V_1G_0V_1G_0 + \cdots, \tag{2.9}$$

where the products are operator products. Inserting this into the expression for the effective action in Eq. (1.5) we find

$$\Gamma_0[a] = -6Vl^{-2}\int d\eta(a')^2 - \tfrac{1}{2}i(\mathrm{Tr}\ln G_0)_{\text{reg}}, \tag{2.10}$$

$$\Gamma_1[a,\beta] = -\tfrac{1}{2}i[\mathrm{Tr}(V_1G_0)]_{\text{reg}}, \tag{2.11}$$

$$\Gamma_2[a,\beta] = Vl^{-2}\int d\eta\, a^2\beta'_{ij}\beta'^{ij} - \tfrac{1}{2}i[\mathrm{Tr}(V_2G_0)]_{\text{reg}} - \tfrac{1}{4}i[\mathrm{Tr}(V_1G_0V_1G_0)]_{\text{reg}}. \tag{2.12}$$

The terms in the expansion of Γ of higher order than the lowest can be represented by a series of Feynman diagrams as shown in Fig. 1.

The effective action in the limit of exact isotropy for a conformally invariant scalar field was calculated in paper I. The one-loop contributions are the local action which gives rise to the trace anomalies in the equation of motion. Combining Eqs. (3.5) and (3.9) of paper I, one has

$$\Gamma_0[a] = -V\int_{-\infty}^{+\infty} d\eta\left[\frac{6(a')^2}{l^2} + 3\lambda\left(\frac{a''}{a}\right)^2 - \lambda\left(\frac{a'}{a}\right)^4\right], \tag{2.13}$$

where $\lambda = (2880\pi^2)^{-1}$.

To calculate the higher-order contributions Γ_1 and Γ_2, they must first be regularized, and we will use the method of dimensional regularization in the number of conformally related flat-space dimensions to do this. We continue the geometry and field theory to a number of dimensions n where the expressions in Eq. (2.11)

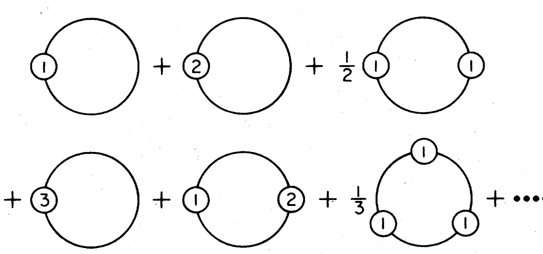

FIG. 1. Feynman diagram expansion of the effective action. The circles containing the number n represent the interaction V_n. The contributions of the last two diagrams displayed above are denoted by Γ_a and Γ_b, respectively, in Appendix A.

are finite. We then subtract a counterterm which is the integral of a local polynomial in the curvature. The coefficients in this polynomial have singularities at $n=4$ with residues such that the subtracted expression in Eq. (2.12) is finite.

The scalar field theory is continued to n dimensions in such a way that it is always conformally invariant. This is largely a matter of convenience. The result is that the Green's function in the limit of exact isotropy G_0 can be conformally related to the usual flat-space Feynman Green's function through

$$G_0(x,x')=[a(\eta)]^{1-n/2}G_F(x,x')[a(\eta')]^{1-n/2}, \tag{2.14}$$

where

$$G_F(x,x')=\frac{-1}{(2\pi)^n}\int d^nk\,\frac{e^{ik\cdot(x-x')}}{k^2-i\epsilon}. \tag{2.15}$$

Inserting these expressions into Eqs. (2.11) and (2.12) we can find regularized expressions for these quantities by dimensionally regulating the resulting flat-space Feynman integrals in the usual way.[12] The terms $\mathrm{Tr}(V_1G_0)$ and $\mathrm{Tr}(V_2G_0)$, for example, are proportional, respectively, to $\int d^nk$ and $\int d^nk(k^2-i\epsilon)^{-1}$. In the dimensional procedure the regularized value of both of these quantities is zero. Thus we conclude, in particular, that

$$\Gamma_1[a,\beta]=0 \tag{2.16}$$

and that the second term in Eq. (2.12) vanishes. It is the third term in this expression which contains the interesting one-loop corrections. Writing $-(i/4)\mathrm{Tr}(V_1G_0V_1G_0)$ out and denoting it more compactly by I we have from Eq. (2.7)

$$I=-i\int d^nx\,a^n(\eta)\int d^nx'a^n(\eta')[a^{-2}(\eta)\beta^{ij}(\eta)][\partial_i\partial'_kG_0(x,x')][a^{-2}(\eta')\beta^{kl}(\eta')][\partial'_l\partial_kG_0(x',x)]. \tag{2.17}$$

Using Eqs. (2.14) and (2.15), this becomes

$$I=\int d^nx\int d^nx'\beta^{ij}(\eta)K_{ijkl}(x-x')\beta^{kl}(\eta'), \tag{2.18}$$

where

$$K_{ijkl}(x)=\int\frac{d^nk}{(2\pi)^n}e^{ik\cdot x}\hat{K}_{ijkl}(k) \tag{2.19}$$

and

$$\hat{K}_{ijkl}(k)=-i\int\frac{d^nk'}{(2\pi)^n}\frac{(k'+k)_i(k'+k)_kk'_jk'_l}{[(k+k')^2-i\epsilon](k'^2-i\epsilon)}. \tag{2.20}$$

This integral is most easily carried out by rotating both k^0 and $k^{0\prime}$ through an angle $+\pi/2$ in the complex plane so that the denominators become the norms of Euclidean four-vectors. The integral can then be carried out in a standard fashion[12] and the result is

$$\hat{K}_{ijkl}(k)=(\delta_{ij}\delta_{kl}+\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk})\times\frac{1}{16(4\pi)^{n/2}}\frac{\Gamma(2-n/2)[\Gamma(n/2-1)]^2}{(n^2-1)\Gamma(n-2)}(k^2)^{n/2}+\begin{pmatrix}\text{terms proportional to tensors}\\ \text{constructed from } k_i \text{ and } \delta_{ij}\end{pmatrix}. \tag{2.21}$$

We do not need to quote the last terms because they all give vanishing contributions to I since β is independent of the spatial variables. Near $n=4$

$$\hat{K}_{ijkl}(k)=-(\delta_{ij}\delta_{kl}+\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk})(k^2)^2\frac{1}{1920\pi^2}\times\left[\frac{1}{n-4}+\tfrac{1}{2}\ln(k^2)+\text{const}+O(n-4)\right]+\begin{pmatrix}\text{terms giving vanishing}\\ \text{contribution to } I\end{pmatrix}. \tag{2.22}$$

In these expressions k^2 is the Euclidean norm of the four-vector. The values to be used in Eq. (2.18) are obtained by rotating k^0 back through an angle $-\pi/2$ in the complex plane. The divergent part of I comes from the pole at $n=4$ in Eq. (2.22). Calculating the residue of the pole term in Eq. (2.22) to define the divergent part of I, one finds

$$I^{\text{div}}=-\frac{1}{960\pi^2(n-4)}\int d^4x\,\beta''_{ij}\beta''^{ij} \tag{2.23a}$$

$$=-\frac{1}{1920\pi^2(n-4)}\int d^4x(-\tilde{g})^{1/2}C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta}, \tag{2.23b}$$

where $C_{\alpha\beta\gamma\delta}$ is the Weyl tensor in four dimensions and the last equality is understood to hold to the quadratic order in β in which we are carrying out the calculation.

In the dimensional procedure the effective action is regularized by adding a counteraction which is the integral of a local polynomial in the n-dimensional curvature. For a conformally invariant field theory, such as the scalar field under consideration here, the counteraction must be constructed from curvature quantities which are conformally invariant or pure divergences in the limit of four dimensions. This requirement leads to a counteraction of the general form[13]

$$S_c = \frac{\mu_c^{\,n-4}}{n-4}\int d^n x(-\tilde{g})^{1/2}[A(R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} - 4R_{\alpha\beta}R^{\alpha\beta} + R^2) + B(R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} - 2R_{\alpha\beta}R^{\alpha\beta} + \tfrac{1}{3}R^2)]\,. \quad (2.24)$$

Here A and B are numerical constants and μ_c is an arbitrary parameter with the dimensions of an inverse length. The term multiplied by A is the argument of the Gauss-Bonnet identity in four dimensions and thus a pure divergence there. The term multiplied by B is the square of the Weyl tensor in four dimensions and thus conformally invariant. The addition of this counteraction will imply a trace anomaly of the form

$$T = \alpha\,\Box^2 R + \beta(R_{\alpha\beta}R^{\alpha\beta} - \tfrac{1}{3}R^2) + \gamma C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta}\,, \quad (2.25)$$

where α, β, and γ are related to A, B by[13]

$$\alpha = 2B/3\,, \quad \beta = -2A\,, \quad \gamma = A + B\,. \quad (2.26)$$

The values of A and B are fixed by the requirement that the counteraction in Eq. (2.24) cancel the divergences in the one-loop effective action. The works of a number of authors[14-16] using various methods of regularization have fixed these values for the scalar problem at hand at

$$A = -(5760\pi^2)^{-1}\,, \quad B = (1920\pi^2)^{-1}\,, \quad (2.27)$$

corresponding to trace anomaly parameters $\alpha = \beta = (2880\pi^2)^{-1}$. The present method of dimensional regularization reproduces these values. For the value of B we will demonstrate this immediately below by showing that with this value the counteraction cancels the divergence in the effective action calculated to second order in β which is displayed in Eq. (2.23). Some care is required in determining the value of A in perturbation theory because the argument of the Gauss-Bonnet identity is a pure divergence. To determine A the effective action must be computed for geometries in which the integrals of this divergence, which is to say the associated surface terms, do not vanish identically. We carry out such a computation explicitly in Appendix A.

Retaining just the pole term in Eq. (2.22), combining it with the counteraction in Eq. (2.25) at the value of B in Eq. (2.27), and expanding the result about $n=4$ and to quadratic order in β one finds

$$I^{\text{pole}} + (S_c)_{\text{order } \beta^2} = \lambda V \int_{-\infty}^{+\infty} d\eta \left\{\left[-\left(\frac{a'}{a}\right)^2 - \left(\frac{a''}{a}\right)\right]\beta'_{ij}\beta'^{ij} + 3[\ln(\mu_c a) + \tfrac{1}{2}]\beta''_{ij}\beta''^{ij}\right\}, \quad (2.28)$$

where as before $\lambda = (2880\pi^2)^{-1}$. The pole at $n=4$ in Eq. (2.23) has been canceled. Some relations for curvature quantities in n dimensions useful in deriving this result are recorded in Appendix B.

The remainder of Γ_2 comes from the logarithmic and finite terms in Eq. (2.22) inserted in Eq. (2.19) and then in Eq. (2.18). The spatial integrations are easily carried out because of the homogeneity of the geometry. There remains only the frequency integral in Eq. (2.19) with $\tilde{K}(k)$ evaluated at $\vec{k}=0$. The $\ln\omega^2$ term in Eq. (2.22) acquires a negative imaginary part when rotated back through an angle of $-\pi/2$ in the complex plane to the values used in Eq. (2.19). Combining the resulting expression with Eq. (2.28) and the quadratic part of the Einstein action from Eq. (2.3) one finds the following expression for Γ_2:

$$\Gamma_2[a,\beta] = V\left(\int_{-\infty}^{+\infty} d\eta \left\{\left[\left(\frac{a}{l}\right)^2 - \lambda\left(\frac{a''}{a}\right) - \lambda\left(\frac{a'}{a}\right)^2\right]\beta'_{ij}\beta'^{ij} + 3\lambda\,[\tfrac{1}{2}i\pi + \ln(\mu a)]\beta''_{ij}\beta''^{ij}\right\} - 3\lambda\int_{-\infty}^{+\infty} d\eta \int_{-\infty}^{+\infty} d\eta'\,\beta''_{ij}(\eta)\tilde{K}(\eta-\eta')\beta''^{ij}(\eta')\right)\,. \quad (2.29)$$

Here we have defined

$$\tilde{K}(\eta) = \frac{1}{\pi}\int_0^{\infty} d\omega\cos(\omega\eta)\ln\omega \quad (2.30)$$

and combined all scales which enter in the same way into a single regularization scale μ. The answer is complex and nonlocal.

It is not difficult to verify explicitly that this expression gives the correct trace anomaly in Eq. (2.25) to quadratic order in β and with $\alpha=\beta=\lambda$ through the relation

$$T = \frac{1}{a^3}\frac{\delta}{\delta a}[\Gamma - S_E]\,. \quad (2.31)$$

III. EQUATIONS WHICH DETERMINE THE CLASSICAL GEOMETRY

To second order in the anisotropy the contribution of a conformally invariant scalar field to the one-loop effective action at homogeneous spatially flat geometries is given by the sum of Eqs. (2.13) and (2.29) derived in Sec. II. The equations of motion which determine the classical geometry are found by varying this effective action with respect to a and to β_{ij}. In this section we shall derive these equations.

We consider first the equation

$$\delta\Gamma_2[a,\beta]/\delta\beta^{ij}=0. \tag{3.1}$$

The action Γ_2 depends only on the first and second derivatives of β_{ij}. A first integral of Eq. (3.1) is therefore easily seen to be given by re-expressing Γ_2 in terms of

$$\kappa_{ij}=\beta'_{ij} \tag{3.2}$$

and calculating

$$\delta\Gamma_2[a,\kappa]/\delta\kappa^{ij}=-2C_{ij}, \tag{3.3}$$

where c_{ij} is a 3×3 matrix of constants. The quantity κ_{ij} differs by only a scale factor from the shear σ_{ij}:

$$\sigma_{ij}=\kappa_{ij}/a. \tag{3.4}$$

Explicitly one has from Eq. (2.29) for Eq. (3.3)

$$3\lambda\frac{d}{d\eta}\left\{-\left[\tfrac{1}{2}i\pi+\ln(\mu a)\right]\frac{d\kappa_{ij}}{d\eta}+\int_{-\infty}^{+\infty}d\eta'\tilde{K}(\eta-\eta')\frac{d\kappa_{ij}}{d\eta'}\right\}+\left[\left(\frac{a}{l}\right)^2-\lambda\left(\frac{a'}{a}\right)^2-\lambda\left(\frac{a''}{a}\right)\right]\kappa_{ij}=c_{ij}. \tag{3.5}$$

This is a linear integro-differential equation for κ_{ij} given the scale factor $a(\eta)$.

The equation for $a(\eta)$,

$$\delta\Gamma[a,\beta]/\delta a=0, \tag{3.6}$$

is to all orders in β the equation

$$R=-(l^2/2)T, \tag{3.7}$$

where T is the trace anomaly in Eq. (2.25). This follows immediately from the conformal invariance of the scalar field or directly from Eqs. (2.13) and (2.29). To second order in β Eq. (3.7) is

$$-\frac{a^2}{\lambda l^2}\left[6\frac{a''}{a}+\text{tr}(\kappa^2)\right]=\frac{-3a''''}{a}+\frac{12a'a'''}{a^2}+9\left(\frac{a''}{a}\right)^2-24\left(\frac{a'}{a}\right)^2\left(\frac{a''}{a}\right)+6\left(\frac{a'}{a}\right)^4$$
$$+\text{Tr}\left[\tfrac{3}{2}(\kappa')^2+2\left(\frac{a'}{a}\right)(\kappa^2)'-\tfrac{1}{2}(\kappa^2)''+2\left(\frac{a'}{a}\right)'\kappa^2\right]. \tag{3.8}$$

Here κ stands for the matrix κ_{ij}, κ' for the matrix κ'_{ij}, and Tr denotes a trace over the spatial indices. Equation (3.8) is a fourth-order, nonlinear differential equation for a given κ. While it is complicated, it at least has the virtue of being local. Equations (3.5) and (3.8) are two coupled equations for a and β_{ij} and thus the entire classical geometry. They could be recast in other forms. For example, Eq. (3.8) could be replaced with a nonlocal *third*-order integro-differential equation which is the energy integral following from the lack of explicit dependence of the effective action on η (or alternatively from the equation $T^{\alpha}_{0;\alpha}=0$, see Sec. IV). With suitable boundary conditions these two equations determine the classical geometry, the vacuum persistence amplitude, the particle production probabilities, and the back reaction of the produced particle. We expect to return to a discussion of the solutions for these quantities in a subsequent paper.[10]

IV. VACUUM MATRIX ELEMENTS OF THE STRESS-ENERGY TENSOR

The equations of motion for the classical geometry $\delta\Gamma[\tilde{g}]/\delta\tilde{g}_{\alpha\beta}=0$ can be put in the form

$$G_{\alpha\beta}=\tfrac{1}{2}l^2[T^{\text{rad}}_{\alpha\beta}+\langle 0_+|T_{\alpha\beta}|0_-\rangle], \tag{4.1}$$

where $G_{\alpha\beta}\equiv R_{\alpha\beta}-\frac{1}{2}\tilde{g}_{\alpha\beta}R$ is the Einstein tensor of the background with metric $\tilde{g}_{\alpha\beta}$, $T^{\text{rad}}_{\alpha\beta}$ is the stress-energy tensor for the classical radiation, and $T_{\alpha\beta}$ is that for the quantized scalar field. By writing out these equations we can identify the matrix elements. While we have already displayed the dynamical equations for the classical geometry in Sec. III, we shall here calculate these matrix elements to quadratic order in the anisotropy as they may be useful in comparing our results with other calculations. To shorten our expressions we shall write simply $T_{\alpha\beta}$ for $\langle 0_+|T_{\alpha\beta}|0_-\rangle$. All components will be quoted in the orthonormal frame defined in Eq. (B2) of Appendix B unless otherwise noted.

From Eq. (4.1) and Eq. (1.5) it follows that if we define

$$\Lambda = \Gamma[\tilde{g}] - S_E[\tilde{g}] \tag{4.2}$$

then in a coordinate basis

$$T^{\alpha\beta} = \frac{2}{(-\tilde{g})^{1/2}} \frac{\delta\Lambda}{\delta\tilde{g}_{\alpha\beta}}. \tag{4.3}$$

With the metric parametrized as in Eq. (1.1) variation with respect to a gives the trace of $T_{\alpha\beta}$ according to

$$T = \frac{1}{a^3} \frac{\delta\Lambda}{\delta a}. \tag{4.4}$$

Variation with respect to β_{ij} gives *to linear order in* β_{ij} the following components written in the orthonormal frame:

$$T^{ij} - \tfrac{1}{3}\delta^{ij}T^k{}_k = \frac{1}{a^4}\frac{\delta\Lambda}{\delta\beta_{ij}}. \tag{4.5}$$

In these relations if

$$\Lambda = \int d\eta\, \mathcal{L}(a'', a', a, \beta'', \beta) \tag{4.6}$$

and y represents one of the variables (a, β_{ij}), then

$$\frac{\delta\Lambda}{\delta y} = \left(\frac{\partial\mathcal{L}}{\partial y''}\right)'' - \left(\frac{\partial\mathcal{L}}{\partial y'}\right)' + \frac{\partial\mathcal{L}}{\partial y}. \tag{4.7}$$

The remaining component of $T_{\alpha\beta}$ can be determined to quadratic order in β as follows: Since a spatial vector cannot be constructed from β_{ij} we have

$$T^{0i} = 0 \tag{4.8}$$

identically. Equation (4.5) is already accurate to quadratic order since there are no trace-free tensors of quadratic order which can be constructed from the β_{ij}. The component T^0_0 can be found by solving the conservation law $T^\beta_{0;\beta} = 0$ which is a consequence of the fact that $T_{\alpha\beta}$ is constructed from an action, [Eq. (4.3)]. Noting that the connection coefficients in the orthonormal basis are (see Appendix B)

$$\Gamma_{0ij} = -\frac{1}{a}\left(\frac{a'}{a} + s\right)_{ij}, \quad \Gamma_{ij0} = -\frac{1}{a}t_{ij}, \tag{4.9}$$

with the others vanishing or connected to these by symmetries, we find the conservation law implies

$$\frac{1}{a^4}(a^4T^0_0)' = \frac{a'}{a}T + s_{ij}(T^{ij} - \tfrac{1}{3}\delta^{ij}T^k{}_k). \tag{4.10}$$

Inserting Eqs. (4.4) and (4.5) this can be written to quadratic order in β_{ij} as

$$(a^4T^0_0)' = a'\frac{\delta\Lambda}{\delta a} + \beta'_{ij}\frac{\delta\Lambda}{\delta\beta_{ij}}. \tag{4.11}$$

This can be integrated because $\mathcal{L}$ has no explicit dependence on η. The result is

$$T^0_0 = \frac{1}{a^4}\left[\text{const} + a'\left(\frac{\partial\mathcal{L}}{\partial a''}\right)' + \beta'_{ij}\left(\frac{\partial\mathcal{L}}{\partial\beta''_{ij}}\right)' - a''\left(\frac{\partial\mathcal{L}}{\partial a''}\right) - \beta''_{ij}\frac{\partial\mathcal{L}}{\partial\beta''_{ij}} - a'\frac{\partial\mathcal{L}}{\partial a'} - \beta'_{ij}\frac{\partial\mathcal{L}}{\partial\beta'_{ij}} + \mathcal{L}\right]. \tag{4.12}$$

The constant can be found by appropriate boundary conditions derived from the definitions of the initial and final states. In our example it will vanish because there are no scalar quanta in the initial and final states.

Since

$$T^{ij} = (T^{ij} - \tfrac{1}{3}\delta^{ij}T^k{}_k) + \tfrac{1}{3}\delta^{ij}(T - T^0_0) \tag{4.13}$$

all the components of the stress-energy vacuum matrix elements are determined from Eqs. (4.4), (4.5), (4.8), (4.12), and (4.13).

For the explicit effective action given in Eq. (2.29) we find the following results for $T^{ij} - \frac{1}{3}\delta^{ij}T^k_k$, T, and T^0_0 accurate to quadratic order in β. The remaining components are determined by Eqs. (4.8) and (4.13):

$$T^{ij} - \tfrac{1}{3}\delta^{ij}T^k_k = \frac{2\lambda}{a^4}\frac{d}{d\eta}\left(3\frac{d}{d\eta}\left\{[\tfrac{1}{2}i\pi + \ln(\mu a)]\frac{d\kappa^{ij}}{d\eta} - \int_{-\infty}^{+\infty} d\eta'\bar{K}(\eta - \eta')\frac{d\kappa^{ij}}{d\eta'}\right\} + \left[\left(\frac{a''}{a}\right) + \left(\frac{a'}{a}\right)^2\right]\kappa^{ij}\right), \tag{4.14}$$

$$T = \frac{\lambda}{a^4}\left\{6\left[-\frac{a''''}{a} + 4\frac{a'''a'}{a^2} + 3\left(\frac{a''}{a}\right)^2 - 8\frac{a''}{a}\left(\frac{a'}{a}\right)^2 + 2\left(\frac{a'}{a}\right)^4\right] + \text{Tr}\left[4\left(\frac{a'}{a}\right)'(\kappa^2) + 4\left(\frac{a'}{a}\right)(\kappa^2)' + 3(\kappa')^2 - (\kappa^2)''\right]\right\}, \tag{4.15}$$

$$T^0_0 = -\frac{\lambda}{a^4}\left\{6\left[\left(\frac{a'}{a}\right)\left(\frac{a'''}{a}\right) - 2\left(\frac{a'}{a}\right)^2\left(\frac{a''}{a}\right) - \frac{1}{2}\left(\frac{a''}{a}\right)^2 + \frac{1}{2}\left(\frac{a'}{a}\right)^4\right] - 2\left[\left(\frac{a''}{a}\right) + 2\left(\frac{a'}{a}\right)^2\right]\kappa_{ij}\kappa^{ij} + 2\left(\frac{a'}{a}\right)\kappa'_{ij}\kappa^{ij} + 3\kappa'_{ij}D^{ij} - 6\kappa_{ij}D'^{ij}\right\}. \tag{4.16}$$

Here

$$D_{ij} = [\tfrac{1}{2}i\pi + \ln(\mu a)]\beta''_{ij} - \int_{-\infty}^{+\infty} d\eta' \bar{K}(\eta - \eta')\beta''_{ij}\,. \tag{4.17}$$

ACKNOWLEDGMENTS

The authors are grateful for helpful conversations with D. Boulware, M. Duff, and A. Lapedes. This work was supported in part by the National Science Foundation.

APPENDIX A: THIRD-ORDER DIVERGENCES

In this appendix we discuss the determination of the coefficient of the argument of the Gauss-Bonnet identity in the counteraction Eq. (2.24).

The argument of the Gauss-Bonnet identity

$$\mathcal{G} = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} - 4R_{\alpha\beta}R^{\alpha\beta} + R^2 \tag{A1}$$

is a pure divergence which for the metric of Eq. (1.1) has the following expansion in powers of β_{ij}:

$$\mathcal{G} = \frac{8}{a^4}\frac{d}{dn}\left[\mathrm{Tr}\left(-\tfrac{1}{2}\kappa^2\frac{a'}{a} + \tfrac{1}{3}\kappa^3 + \cdots\right)\right]. \tag{A2}$$

In this relation, which follows easily from the expansion of the curvature squares obtained in Appendix B [Eqs. (B10)–(B12)], $\kappa_{ij} = \beta'_{ij}$ and the trace is over the spatial indices.

To evaluate the coefficient of $\mathcal{G}$ in the counteraction one needs to calculate the divergent parts of the unregulated one-loop effective action for geometries for which the volume integral of $\mathcal{G}$ over the whole spacetime does not vanish *and* for which the integral expressing the regulated one-loop effective action is also finite. We have found it easier to find geometries which satisfy these conditions in third order in β than in the second order. For example, one could consider a geometry in which β has an asymptotic behavior such that κ vanishes as $\eta \to -\infty$ and approaches a constant as $\eta \to +\infty$, but for which κ', a'/a, and a''/a fall off sufficiently rapidly to make the regulated one-loop effective action [Eq. (2.29)] finite. In the integral of $\mathcal{G}$ over all spacetime there will then be no second-order contribution but there will be a contribution from the integral of the third-order term in Eq. (A2). Since this simple example will also illustrate how to deal with the divergences in third order, we shall now pursue it to calculate the coefficient of $\mathcal{G}$ in the counteraction, called A in Eq. (2.24).

Continuing the expansion begun in Eqs. (2.10)–(2.12) one finds the following expression for the third-order effective action:

$$\Gamma_3[a,\beta] = -\tfrac{1}{2}i\,\mathrm{Tr}(V_3G_0 + V_2G_0V_1G_0 + \tfrac{1}{3}V_1G_0V_1G_0V_1G_0)\,. \tag{A3}$$

Diagrammatically, this is the sum of the last three diagrams shown in Fig. 1. The first term of these three is proportional to $\int d^nk$ and $\int d^nk(k^2 - i\epsilon)^{-1}$ which vanish identically in the dimensional procedure. The next two terms we denote by Γ_a and Γ_b, respectively.

Of the two terms in V_2 given Eq. (2.8) only the second term will give a nonvanishing contribution to Γ_a. The first term will be proportional to $\mathrm{Tr}(\beta)$ and its derivatives all of which vanish identically. For the remaining contribution to Γ_a we find

$$\Gamma_a = -2\int d^nx \int d^ny\, \beta^{ij}(x) K_{ijkl}(x,y) \times \beta^{km}(y)\beta_m{}^l(y)\,, \tag{A4}$$

where K_{ijkl} is given by Eqs. (2.19) and (2.20). Near $n=4$ the parts of K_{ijkl} which give a nonvanishing contribution to Γ_a are given in Eq. (2.22). The divergent part is the residue of the pole term. We shall now evaluate this for the geometry described above. The divergent behavior for β resulting from the constant asymptotic behavior of κ means that some attention is required in evaluating Eq. (A4). The function $K^{\mathrm{pole}}_{ijkl}(x,y)$ obtained from Eqs. (2.19) and the pole term in Eq. (2.22) will be a fourth-order differential operator acting on δ-function distributions. Integration by parts may be used to evaluate expression (A4), but in view of the divergent asymptotic behavior for β care must be taken to retain the nonvanishing surface terms. The result is

$$\Gamma_a^{\mathrm{div}} = \frac{1}{240\pi^2(n-4)}\int d^4x\,\mathrm{Tr}\left[(\kappa')^2\kappa - \tfrac{2}{3}(\kappa^3)'\right], \tag{A5}$$

where we have written all surface terms as the integral of the total derivative $(\kappa^3)'$.

The contribution Γ_b of the last diagram in Fig. 1 may be written

$$\Gamma_b = \tfrac{4}{3}\int d^nx\int d^ny\int d^nz\,\beta^{ij}(x)\beta^{kl}(y)\beta^{mn}(z)G_{ijklmn}(x,y,z)\,, \tag{A6}$$

where

$$G_{ijklmn} = \int\frac{d^np}{(2\pi)^n}\int\frac{d^nq}{(2\pi)^n}\int\frac{d^nr}{(2\pi)^n}(2\pi)^n\delta^{(n)}(p+q+r)\,e^{i(p\cdot x+q\cdot y+r\cdot z)}\hat{G}_{ijklmn}(p,q,r) \tag{A7}$$

with

$$\hat{G}_{ijklmn}=-i\int\frac{d^nk}{(2\pi)^n}\frac{(k-p)_ik_jk_k(k+q)_l(k+q)_m(k-p)_n}{(k^2-i\epsilon)[(k+q)^2-i\epsilon][(k-p)^2-i\epsilon]}. \tag{A8}$$

In Eq. (A6), β^{ij} is a function of η alone. As a consequence, if G_{ijklmn} is expanded in invariants multiplied by tensors constructed from δ_{ij} and the momenta p, q, and r, only the tensor constructed purely from the δ_{ij} will give a nonvanishing contribution to Eq. (A6). That term may be identified by Feynman parametrizing Eq. (A8), translating the integration variable to eliminate terms in the denominator odd in k, and retaining only the loop momenta in the numerator. Denoting this term by $G^{(6)}_{ijklmn}$ we have

$$G^{(6)}_{ijklmn}=2\int_0^1 d\alpha_1\int_0^1 d\alpha_2\int_0^1 d\alpha_3\delta\left(1-\sum\alpha_i\right)(-i)\int\frac{d^nk}{(2\pi)^n}\frac{k_ik_jk_kk_lk_mk_n}{(k^2+H-i\epsilon)^3}, \tag{A9}$$

where

$$H=\alpha_1\alpha_2p^2+\alpha_2\alpha_3q^2+\alpha_1\alpha_3r^2. \tag{A10}$$

The result for this integral is proportional to

$$Q_{ijklmn}=\frac{1}{n(n+2)(n+4)}\delta_{(ij}\delta_{kl}\delta_{mn)}, \tag{A11}$$

where the parentheses denote complete symmetrization and the numerical factor normalizes Q_{ijklmn} to have total trace (over spacetime indices) unity. In particular,

$$G^{(6)}_{ijklmn}=2Q_{ijklmn}\int_0^1 d\alpha_1\int_0^1 d\alpha_2\times\int_0^1 d\alpha_3\delta\left(1-\sum\alpha_i\right)J, \tag{A12}$$

where

$$J=-i\int\frac{d^nk}{(2\pi)^n}\frac{k^6}{(k^2+H-i\epsilon)^3}. \tag{A13}$$

After rotating the k^0 contour by $e^{i\pi/2}$ this integral may be evaluated by standard dimensional regularization techniques.[12] The result has a pole at $n=4$. Near $n=4$

$$J^{\text{div}}=-\frac{12}{(4\pi)^2}\frac{1}{n-4}H^2. \tag{A14}$$

Inserting this in Eq. (A12) and carrying out the integrals over the Feynman parameters one finds

$$G^{(6)\,\text{div}}_{ijklmn}=-\frac{1}{120\pi^2(n-4)}Q_{ijklmn}\times(p^4+q^4+r^4+p^2q^2+p^2r^2+q^2r^2). \tag{A15}$$

Finally, inserting this in Eq. (A7) the result in Eq. (A6) and evaluating the resulting expression with the attention to surface terms already discussed for Γ_a we find for the divergent part of the last diagram

$$\Gamma_b^{\text{div}}=-\frac{1}{720\pi^2}\frac{1}{(n-4)}\int d^4x\,\text{Tr}[3(\kappa')^2\kappa-\tfrac{4}{3}(\kappa^3)']. \tag{A16}$$

For the total divergent part of the third-order one-loop effective action we therefore find from the sum of Eq. (A16) and Eq. (A5)

$$\Gamma_3^{\text{div}}=-\frac{1}{360\pi^2(n-4)}\int d^4x\,\text{Tr}[\tfrac{1}{3}(\kappa^3)']. \tag{A17}$$

This divergence is to be canceled by the counteraction in Eq. (2.24). Using the expansion in Eq. (A2) and Eq. (B10) to evaluate the expansion of the Weyl tensor, we find the following for the third-order counteraction's divergent part:

$$\frac{8(A+B)}{n-4}\int d^4x\,\text{Tr}[\tfrac{1}{3}(\kappa^3)']. \tag{A18}$$

Comparing this with Eq. (A17) we see that the counteraction has the correct form to cancel the divergence in this order and that $A+B=(2880\pi^2)^{-1}$. The value of B has already been determined from the cancellation of the divergence in second-order perturbation theory as $B=(1920\pi^2)^{-1}$ [cf. Eq. (2.23b)]. The result of the dimensional regularization scheme for A is therefore

$$A=-(5760\pi^2)^{-1}, \tag{A19}$$

in agreement with that found by other methods.

APPENDIX B: CURVATURE IDENTITIES

In this appendix we shall list some useful identities concerning the curvature tensor for the metric

$$ds^2=a^2[-d\eta^2+(e^{2\beta})_{ij}dx^idx^j] \tag{B1}$$

in $n-1$ spatial dimensions. β is an $(n-1)\times(n-1)$ trace-free symmetric matrix and both it and a are functions of the time coordinate η alone. Our results for tensor components in this appendix will always be quoted in the orthonormal frame

$$\omega^0=ad\eta,\quad \omega_i=(ae^{\beta})_{ij}dx^j. \tag{B2}$$

It is not difficult to convince oneself that the Riemann tensor components do not involve the dimension n explicitly so that the calculations of Misner[3] may be taken over directly. The nonvanishing ones are

$$R^0{}_{i0j}=\frac{1}{a^2}\left[\frac{a''}{a}-\left(\frac{a'}{a}\right)^2+s'+s\left(\frac{a'}{a}\right)+s^2+st-ts\right]_{ij}, \tag{B3}$$

$$R_{ijkl}=\frac{1}{a^2}\left[\left(\frac{a'}{a}+s\right)_{ik}\left(\frac{a'}{a}+s\right)_{jl}-\left(\frac{a'}{a}+s\right)_{il}\left(\frac{a'}{a}+s\right)_{kj}\right]. \tag{B4}$$

Here s and t are the matrices defined by

$$s=\tfrac{1}{2}[(e^{\beta})'(e^{-\beta})+(e^{-\beta})(e^{\beta})'], \tag{B5}$$

$$t=\tfrac{1}{2}[(e^{\beta})'(e^{-\beta})-(e^{-\beta})(e^{\beta})'], \tag{B6}$$

where a prime denotes a derivative with respect to η and in matrix expressions quantities such as a'/a are understood to be proportional to the unit matrix. The resulting nonvanishing Ricci tensor components and scalar curvature are

$$R^0{}_0=\frac{1}{a^2}\left\{(n-1)\left[\frac{a''}{a}-\left(\frac{a'}{a}\right)^2\right]+\mathrm{Tr}(s^2)\right\}, \tag{B7}$$

$$R_{ij}=\frac{1}{a^2}\left\{\left[\frac{a''}{a}+(n-3)\left(\frac{a'}{a}\right)^2\right]+s'+(n-2)s\left(\frac{a'}{a}\right)+st-ts\right\}_{ij}, \tag{B8}$$

$$R=\frac{1}{a^2}\left\{(n-1)\left[2\frac{a''}{a}+(n-4)\left(\frac{a'}{a}\right)^2\right]+\mathrm{Tr}(s^2)\right\}, \tag{B9}$$

where Tr indicates a trace over the spatial indices.

To the lowest few orders in β these results imply the following expressions for the squares of the Weyl tensor, Ricci tensor, and scalar curvature. They are expressed in terms of $\kappa_{ij}=\beta'_{ij}$:

$$C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta}=\frac{2}{a^4}\mathrm{Tr}(\kappa'^2+4\kappa'\kappa^2)+O(\beta^4), \tag{B10}$$

$$R_{\alpha\beta}R^{\alpha\beta}=\frac{1}{a^4}\left\{(n-1)\left[n\left(\frac{a''}{a}\right)^2-4\left(\frac{a''}{a}\right)\left(\frac{a'}{a}\right)^2+(n^2-5n+8)\left(\frac{a'}{a}\right)^4\right]+\mathrm{Tr}\left[2(n-1)\left(\left(\frac{a''}{a}\right)-\left(\frac{a'}{a}\right)^2\right)\kappa^2+\left(\kappa'+(n-2)\left(\frac{a'}{a}\right)\kappa\right)^2\right]\right\}+O(\beta^4), \tag{B11}$$

$$R^2=\frac{n-1}{a^4}\left\{(n-1)\left[2\left(\frac{a''}{a}\right)+(n-4)\left(\frac{a'}{a}\right)^2\right]^2+2\left[2\left(\frac{a''}{a}\right)+(n-4)\left(\frac{a'}{a}\right)^2\right]\mathrm{Tr}(\kappa^2)\right\}+O(\beta^4). \tag{B12}$$

The square of the Riemann tensor can then be computed from

$$R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}=C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta}+\frac{4}{(n-2)}R_{\alpha\beta}R^{\alpha\beta}-\frac{2}{(n-2)(n-1)}R^2. \tag{B13}$$

Finally, we note that if the argument of the Gauss-Bonnet identity in four dimensions is written

$$\mathcal{G}=R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}-4R_{\alpha\beta}R^{\alpha\beta}+R^2, \tag{B14}$$

then in terms of the n-dimensional Weyl tensor this can be written

$$\mathcal{G}=C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta}+4\left(\frac{1}{n-2}-1\right)R_{\alpha\beta}R^{\alpha\beta}+\left(1-\frac{2}{(n-2)(n-1)}\right)R^2. \tag{B15}$$

If $\mathcal{F}$ is the combination of the squares of curvatures which give the square of the Weyl tensor in four dimensions

$$\mathcal{F}=R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}-2R_{\alpha\beta}R^{\alpha\beta}+\tfrac{1}{3}R^2, \tag{B16}$$

then in terms of the n-dimensional Weyl tensor

$$\mathcal{F}=C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta}+(n-4)\left[-\frac{2}{(n-2)}R_{\alpha\beta}R^{\alpha\beta}+\frac{(n+1)}{3(n-2)(n-1)}R^2\right]. \tag{B17}$$

[1]G. F. Smoot, M. V. Gorenstein, and R. A. Muller, Phys. Rev. Lett. 39, 898 (1977).
[2]P. J. E. Peebles, *Physical Cosmology* (Princeton University Press, Princeton, N. J., 1971); R. Kron (unpublished).
[3]C. W. Misner, Astrophys. J. 151, 431 (1968).
[4]R. A. Matzner and C. W. Misner, Astrophys. J. 171, 415 (1972).
[5]J. D. Barrow and R. A. Matzner, Mon. Not. R. Astron. Soc. 181, 719 (1977); see, however, M. S. Turner and D.N. Schramn, Nature (to be published).
[6]L. Parker, Phys. Rev. 169, 1057 (1969).
[7]Ya. B. Zel'dovich and A. A. Starobinsky, Zh. Eksp. Teor. Fiz. 61, 2161 (1971) [Sov. Phys.—JETP 34, 1159 (1971)].
[8]B. L. Hu and L. Parker, Phys. Rev. D 17, 933 (1978).
[9]M. V. Fischetti, J. B. Hartle, and B. L. Hu, preceding paper, Phys. Rev. D 20, 1757 (1979).
[10]J. B. Hartle and B. L. Hu (unpublished).
[11]Our conventions are the same as C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973).
[12]See, e.g., G. Leibbrandt, Rev. Mod. Phys. 47, 849 (1975).
[13]M. Duff, Nucl. Phys. B125, 334 (1977).
[14]J. S. Dowker and R. Critchley, Phys. Rev. D 16, 3390 (1977).
[15]S. M. Christensen, Phys. Rev. D 17, 946 (1978).
[16]L. S. Brown, Phys. Rev. D 15, 1469 (1977).