

Implications of baryon magnetic moments for the quark model

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The implications of recent baryon-magnetic-moment measurements for various assumptions in the quark model are considered. The static quark model is seen to give a qualitative understanding of the baryon magnetic moments, but fails in a quantitative test ($\chi^2/DF = 21.5/5$), largely due to inconsistency of the now accurate determinations of the Λ^0 and Ξ^0 moments. Introducing nonstatic effects (orbital, relativistic, or exchange) still permits four independent sum rules to be written (for eight moments) if approximate SU(3) symmetry is assumed for quark-model wave functions. Two sum rules give good agreement with experiment, but those sum rules for any case involving the Ξ^- or Σ^- moments do not, suggesting that their experimental determination is inconsistent with such a quark model.

With the recent precise measurement of the Ξ^0 magnetic moment,¹ all possible measurements of octet baryon moments have been made. The only hoped-for improvement is an increase in the accuracy of the very difficult measurements of the Ξ^- (Ref. 2) and Σ^- (Ref. 3) moments. At this time it is appropriate to look at the implications of these octet magnetic-moment measurements for the quark model.

Recent papers by De Rújula, Georgi, and Glashow⁴ and Lipkin⁵ use spin-dependent interactions that are related to the strange-quark moment to determine the Λ^0 magnetic moment in good agreement with its recent experimental determination.⁶ The point of the present paper is to use all the measured moments to test, particularly, the SU(6) and SU(3) symmetry of the baryon wave functions.

The original static quark model with Dirac quark moments reproduced the famous result⁷ $\mu_p/\mu_n = -1.5$, so close to the experimental value⁸ -1.46 . The static quark model can be extended to the strange baryons by assuming that the only SU(3) breaking is in the strange-quark magnetic moment μ_s .⁹ We resolve the slight ambiguity caused by the fact that $1.5 \neq 1.46$ by assuming a small SU(2) breaking in the nucleon-quark magnetic moments μ_u and μ_d , as determined by the experimental proton and neutron moments. Then all the strange moments can be written in terms of the known nucleon moments and one parameter, the strange-quark (or Λ) moment μ_s .¹⁰ These magnetic moment predictions are given by Eqs. (5) of Ref. 10. The best overall fit of this model by adjusting μ_s to minimize χ^2 is compared to experiment in Table I. We see that the static quark model with arbitrary quark moments gives a qualitative understanding of the baryon moments, but fails in a quantitative test with a χ^2 of 21.5 for 5 degrees of freedom. We note from Table I that it is primarily the Λ^0 and Ξ^0 moments, because of

the accuracy of their determination, that preclude quantitative agreement.

If the assumption is made that the quark moments are Dirac moments so that $\mu = q/2m$, with q being the usual third integral quark charge, then we can infer the quark masses from Table I. These are $m_u = 338$ MeV, $m_d = 322$ MeV and $m_s = 509$ MeV. The strange-quark mass excess over the nucleon quarks is about what would be expected from baryon mass differences. However, the nucleon-quark mass difference (16 MeV) is considerable larger than would be expected from the neutron-proton mass difference and is too large to be considered as an electromagnetic breaking of SU(2) (charge independence).

The static assumption of the above quark model can be relaxed by considering nonstatic magnetic-moment contributions due to orbital, relativistic, and exchange effects. These contributions will only affect the magnetic-moment relations tested in Table I if there is also SU(6) breaking of the quark-model wave functions. Conversely, in the absence of the nonstatic effects, no amount of SU(6) or SU(3) breaking can change the quark-model predictions of Table I.¹⁰ So the qualitative agreement of Table I with experiment provides no indication of SU(6) or even SU(3) symmetry of the quark model provided the nonstatic effects are not too large.

It is well known that SU(6) is broken for the baryons, as evidenced by the 200–300 MeV mass gap between the spin $\frac{3}{2}$ and $\frac{1}{2}$ baryons, which seems large enough to show up in the baryon wave functions. More direct evidence for SU(6) breaking of the wave functions is seen from the large deviation of the ratio of the neutron to proton structure functions from the SU(6) value of $\frac{2}{3}$, especially as the scaling variable approaches 1.^{11,12}

A natural question is whether quantitative agreement can be achieved for the baryon moments in a broken-SU(6) quark model with nonstatic magnetic

TABLE I. Baryon magnetic moments (in nuclear magnetons) in the static quark model.

Baryon	Ref.	Experiment	Theory (Ref. 10)	χ^2
Λ^0	6	-0.6138 ± 0.0047	-0.612	0.1
Σ^+	8	2.83 ± 0.25	2.67	0.4
Σ^-	3	-1.48 ± 0.37	-1.09	1.1
Ξ^0	1	-1.20 ± 0.06	-1.44	15.5
Ξ^-	2, 8	-1.85 ± 0.75	-0.50	3.3
$(\Lambda\Sigma)$	14	$-1.82^{+0.18}_{-0.25}$	-1.63	1.1
Total χ^2				21.5

moment contributions. Although the predictions of Table I are lost, sum rules can still be derived for combinations of baryon moments that cancel out the nonstatic effects.¹³ This procedure is described in Ref. 13 and the sum rules [Eqs. (9)–(12)] can be written as

$$3\Lambda + \frac{1}{2}(\Sigma^+ + \Sigma^-) - (\Xi^0 + \Xi^-) = p + n = 0.88 (1.9 \pm 0.8), \quad (1)$$

$$\Xi^0 - \Xi^- - 2\sqrt{3}(\Lambda\Sigma) = p - n = 4.70 (7.0 \pm 1.1), \quad (2)$$

$$\Sigma^+ - \Sigma^- + \Xi^- - \Xi^0 = p - n = 4.70 (3.7 \pm 0.9), \quad (3)$$

where the particle symbols represent the magnetic moments and $(\Lambda\Sigma)$ represents the Λ^0 , Σ^0 transition moment measured by the Σ^0 lifetime.¹⁴ The experimental value of the left-hand side is written in parentheses following each equation. Each of these equations can be derived without the quark model by using SU(3) symmetry assumptions about the form of the magnetic moment operator. In the quark model, Eqs. (1) and (2) depend on SU(3) symmetry of the baryon wave functions, but no other assumption about quark moment contributions. Equation (3) depends on the slightly weaker assumption that the nonstatic corrections to a quark moment contribution depend only on the spin state of the quark relative to the other quarks. That is, that the odd u quark in the Ξ^0 behaves like the odd u quark in the neutron, with a similar connection between the identical u quarks in the Σ^+ and proton.

Equations (1)–(3) show poor agreement with the assumption of SU(3) symmetry of the wave functions but the experimental errors on μ_{Ξ^-} and μ_{Ξ^0} are too large to provide a real test.

We have not yet made any assumption relating quark moments to each other. The assumption is often made that the magnetic moments of the nucleon quarks are in the 2:1 ratio of their charges. This is a natural assumption in any quark model,

considering the nearness of μ_n/μ_p to the predicted $-\frac{2}{3}$, and is further suggested by the indications from deep-inelastic scattering that the quarks are point Dirac particles. If we take $\mu_u = -2\mu_d$ when they are in corresponding positions in baryon wave functions, then Eq. (3) can be broken down into two separate equations by canceling out any contribution of the unlike quark in each baryon¹³:

$$\Sigma^+ - \Sigma^- = 2p + n = 3.67 (4.31 \pm 0.45) \quad (4)$$

and

$$\Xi^0 - \Xi^- = p + 2n = -1.03 (0.65 \pm 0.75). \quad (5)$$

Even with the large errors, the Ξ sum rule Eq. (5) is over two standard deviations off. The Σ sum rule Eq. (4) is only a little better.

The combinations in Eqs. (4) and (5) can be considered as isolating different nucleon quark contributions. Equation (4) can be rewritten as

$$\mu_d(n) = -(2p + n)/4 = -0.918 \quad (4')$$

or

$$\mu_d(\Sigma^-) = -\frac{1}{4}(\Sigma^+ - \Sigma^-) = -1.08 \pm 0.11, \quad (4'')$$

and Eq. (5) can be written as

$$\mu_d(p) = p + 2n = -1.033 \quad (5')$$

or

$$\mu_d(\Xi^-) = \Xi^0 - \Xi^- = +0.65 \pm 0.75, \quad (5'')$$

where the notation $\mu_d(B)$ represents the d -quark contribution to the magnetic moment of baryon B .

If the baryon wave functions were symmetric under SU(6), then all of these measures of μ_d would be equal. The difference between $\mu_d(n)$ and $\mu_d(p)$ represents the spin dependence [or SU(6) breaking] of the nucleon wave functions. We note that these two contributions differ by 12% even though the SU(6) breaking of the nucleon moments is only 3% (the difference between 1.46 and $\frac{3}{2}$). So there is considerably more SU(6) breaking than indicated by looking only at the nucleon magnetic-moment ratio. SU(3) breaking is evidenced by the difference between $\mu_d(n)$ and $\mu_d(\Sigma^-)$ and also between $\mu_d(p)$ and $\mu_d(\Xi^-)$. The SU(3) breaking is seen to be particularly large in the measured Ξ moments, Eq. (5''). If we make the reasonable assumption that $\mu_d < 0$ for the negative d quark, then Eq. (5'') implies the inequality $\Xi^- > \Xi^0$ and even this is not well satisfied at present.

If the SU(6) breaking evidenced by $\mu_d(n)$ and $\mu_d(p)$ arises mainly due to relativistic effects, it can be characterized by equivalent energy denominators replacing the simple mass terms in the Dirac form $\mu = q/2m$. For $\mu_d(n)$ this energy would be 341 MeV and it would be 303 MeV for $\mu_d(p)$. This considerable energy difference (38 MeV) now

represents the magnitude of SU(6) breaking while preserving full SU(2) symmetry (charge independence), including equal u and d quark masses.

In order to get some idea which measurements prevent agreement with SU(3) wave functions, we can use Eqs. (4) and (5) to eliminate some of the moments in Eqs. (1) and (2). From Eq. (2) we get

$$(\Lambda\Sigma) = (\sqrt{3}/2)n = -1.65 \left(-1.82^{+0.18}_{-0.25} \right). \quad (6)$$

Equation (6), depending on SU(3) symmetry of the wave functions and $\mu_u = -2\mu_d$, is reasonably well satisfied.

We can write Eq. (1) in four different ways by using Eqs. (4) and (5) to eliminate various pairs of baryons:

$$3\Lambda + \Sigma^+ - 2\Sigma^0 = p - \frac{1}{2}n = 3.65 \left(3.41 \pm 0.28 \right), \quad (7a)$$

$$3\Lambda + \Sigma^- - 2\Sigma^0 = -p - \frac{3}{2}n = -0.08 \left(-0.90 \pm 0.42 \right), \quad (7b)$$

$$3\Lambda + \Sigma^+ - 2\Sigma^- = 3p + \frac{7}{2}n = 1.68 \left(4.71 \pm 1.52 \right), \quad (7c)$$

$$3\Lambda + \Sigma^- - 2\Sigma^- = p + \frac{5}{2}n = -1.99 \left(0.40 \pm 1.55 \right). \quad (7d)$$

Equations (7) also depend on SU(3) symmetry of the wave functions and $\mu_u = -2\mu_d$. We see that Eq. (7a) is satisfied, but any equation with either the Σ^- or Ξ^- is violated, even with the large errors.

We can provide one more test of the quark model using the assumption that $\mu_u = -2\mu_d$, but no other assumption. We can solve for the magnetic moment contribution μ_s of the strange quark as it appears in different baryons by using $\mu_u = -2\mu_d$ to cancel out any possible nucleon quark contribution. This leads to

$$\mu_s(\Sigma) = -\Sigma^+ - 2\Sigma^- = 0.13 \pm 0.83 \quad (8)$$

and

$$\mu_s(\Xi) = \frac{1}{4}\Xi^0 + \frac{1}{2}\Xi^- = -1.23 \pm 0.19. \quad (9)$$

With SU(6) symmetry these moments would be equal and would also equal $\mu_s(\Lambda)$, given [assuming SU(6) symmetry or no nonstatic effects] by

$$\mu_s(\Lambda) = \Lambda = -0.614 \pm 0.005. \quad (10)$$

There is no evidence of any SU(6) symmetry in Eqs. (8)–(10), but beyond that, there is little evidence of *any* connection between the strange-quark contribution in the three different strange baryons. It is difficult to see how any quark model could give such different strange-quark contributions, unless the experimental errors are stretched, fortuitously, in the right directions.

We can eliminate the Σ^- and Ξ^- moments from Eqs. (8) and (9) by using Eqs. (4) and (5) to yield

$$\mu_s(\Sigma) = 7.34 - 3\Sigma^+ = -1.15 \pm 0.75 \quad (8')$$

and

$$\mu_s(\Xi) = 0.52 + \frac{3}{4}\Xi^0 = -0.38 \pm 0.05. \quad (9')$$

The error in $\mu_s(\Sigma)$ given by Eq. (8') is so large because the strange quark makes very little contribution to the Σ^+ moment in any quark model. Equations (8') and (9') still show considerable variation in μ_s due to SU(6) breaking [assuming approximate SU(3)]. The variation is much larger than the corresponding SU(6) breaking indicated for μ_d by Eqs. (4') and (5'). This probably indicates that both SU(3) and SU(6) broken wave functions are required to reproduce the baryon moments.

Our conclusions for the implications of the baryon magnetic moments for the quark model are as follows:

(1) Nonstatic (orbital, relativistic, exchange current) effects are small enough for the static quark model to give a qualitative understanding of baryon moments.

(2) Nonstatic effects, coupled with SU(6)-broken wave functions are large enough to prevent quantitative agreement with present experiments (Table I). The key experiments preventing quantitative agreement are the now accurate measurements of the Λ^0 and Ξ^0 moments.

(3) SU(3) symmetry of the baryon wave functions with [Eqs. (4)–(7)] or without [Eqs. (1)–(3)] the assumption that $\mu_u = -2\mu_d$ is not quantitatively compatible with present experiments even considering large errors.

(4) It is difficult to see how any quark model could be compatible with present experiments [Eqs. (5'), (8)–(10)].

(5) The two SU(3) equations [Eqs. (6) and (7a)] that do not involve the Σ^- and Ξ^- moments are satisfied. This means that the disagreement for SU(3) wave functions is primarily due to these two measurements, which are the most difficult moments to measure.

It is particularly important to improve the measurements of the Σ^- and Ξ^- moments to see whether these moments are, in fact, incompatible with a reasonable quark model.

Note added in proof. A recent measurement¹⁵ of $\mu_{\Sigma^+} = 2.30 \pm 0.14$ has changed the world average to $\mu_{\Sigma^+} = 2.33 \pm 0.13$. This increases the χ^2 of Table I to 28 and extends the disagreement with SU(3) in the quark model to the Σ^+ in Eqs. (7a) and (8'). In addition, Eq. (8) now implies that $\mu_s(\Sigma) = +0.63 \pm 0.75$ which is difficult to reconcile with any quark model. This means that now virtually all hyperon moment measurements contribute to disagreement with a reasonable quark model [and especially one with SU(3)-symmetric wave functions].

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