Is there a strong van der Waals force between hadrons?

G. Feinberg
Department of Physics, Columbia University, New York, New York 10027

J. Sucher*
Laboratory for Solar Physics and Astronomy, Goddard Space Flight Center, National Aeronautics and Space Administration, Greenbelt, Maryland 20772

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We study the question of the existence of long-range forces that are stronger than electromagnetic forces between ordinary hadrons. A phenomenological analysis is carried out which puts limits on the magnitude of the coupling constant λ_N entering a hypothetical interhadronic potential $V_N(r) \sim (\lambda_N/r)(r_0/r)^{N-1}$, with a length scale $r_0 \sim 1$ F and $1 \leq N \leq 7$. Bounds on the value of λ_N are obtained from a variety of sources, including Eötvös- and Cavendish-type experiments, hyperfine structure of the hydrogen molecule, and the level structure of exotic atoms. The dispersion-theoretic approach to the asymptotic behavior of interparticle potentials is reviewed and used to analyze some of the theoretical implications of long-range forces. We stress the fact that long-range potentials require that the scattering amplitude $F(s,t)$ is not analytic at $t = 0$. Such a lack of analyticity is often connected with physical states whose mass spectrum extends down to zero. The implications of this for quantum chromodynamics (QCD) and the recent suggestions that QCD may imply the existence of a long-range force between hadrons are studied. A speculative scheme is considered which might yield such forces without requiring the existence of massless color gluons as observable particles.

I. INTRODUCTION

For many years one of the features which has distinguished the strong (and weak) interactions from the electromagnetic interactions is the range of the forces involved. In particular, it has long been accepted that any potential $V_{AB}^{\text{str}}(r)$ capable of describing, at least approximately, the low-energy elastic scattering of two hadrons A and B must fall off exponentially with the distance r between them. This is because in quantum theory the range of the force is inversely proportional to the mass m_0 of the lightest particle, or system of particles, which can be exchanged between A and B. Since the lightest known hadron, the neutral pion, has a mass $m_\pi > 0$, it has commonly been thought, since the time of Yukawa, that for large r,

$$V_{AB}^{\text{str}} \sim \frac{g_{\text{str}}^2}{4\pi} \frac{e^{-r m_\pi}}{r}, \tag{1.1}$$

where g_{str} is a dimensionless coupling constant typical of strong interactions (spin-dependent factors, if any, are suppressed).

In contrast to this, the corresponding potential V_{AB}^{em} arising from the electromagnetic interactions between the hadrons is, of course, just the Coulomb potential at large distances,

$$V_{AB}^{\text{em}} \sim Q_A Q_B \frac{e^2}{4\pi} \frac{1}{r}, \tag{1.2}$$

provided that the charge quantum numbers Q_A and Q_B are both nonvanishing. The Coulomb potential is the prototype example of a long-range potential, defined as one which falls off like an inverse power of r. In quantum theory, this long-range character is associated with the exchange of a single virtual photon, whose mass is zero.

If the particles are electrically neutral, but not self-conjugate, one-photon exchange is still possible. If both A and B have nonzero spin, this leads to the familiar inverse-cube dipole-dipole potential, proportional to the product of the magnetic moments of the particles. However, if both particles are spinless, one-photon exchange can give rise only to a short-range force. Nevertheless, even if $s_A = s_B = 0$, there is a long-range force of electromagnetic origin, the so-called van der Waals force, obtained by taking into account electromagnetic interactions to order e^4. In particular, consider atoms 1 and 2, both in S-wave ground states with separation r between the fixed nuclei. If one computes the second-order level shift arising from the electrostatic interaction between the electrons and nucleus of atom 1 and those of atom 2 one finds, for distances r which are large compared to atomic dimensions, the result

$$V_{12}^{\text{inst}}(r) \sim -C_{12}/r^6 \quad (r \gg a_0) \tag{1.3}$$

as first shown by London.[1] Here C_{12} is a constant whose value depends on the structure of both atoms. However, the result (1.3) must be modified when r becomes very large, of the order of $\lambda_{\max}$, the largest wavelength associated with electric-dipole excitations of the atomic ground states. The retarded character of electromagnetic interactions then becomes important and, as first shown by Casimir and Polder,[2] has the effect of changing the r^{-6} behavior to an r^{-7} behavior. Using a nonrelativistic description for the atomic electrons and neglecting spin, these authors found that the inclusion of the effects of transverse-photon exchange between the electrons yields the result

$$V_{12}^{\text{ret}}(r) \sim -D_{12}/r^7 \quad (r \gtrsim \lambda_{\max}), \tag{1.4a}$$

where

$$D_{12} = \frac{-23}{4\pi}\, \alpha_{1;\text{nr}}^e \alpha_{2;\text{nr}}^e \tag{1.4b}$$

and the α's are electric polarizabilities of the atoms as they would be computed in nonrelativistic quantum theory, e.g.,

$$\alpha_{\text{nr}}^e = -2 \sum_{n\neq 0} \frac{|\langle \phi_n | e\vec{r}\cdot\hat{z} | \phi_0\rangle|^2}{E_0 - E_n} \tag{1.4c}$$

for a one-electron atom with ground-state wave function $\phi_0(r)$. The ratio $V_{12}^{\text{ret}}/V_{12}^{\text{inst}}$ is of order $(r\Delta E)^{-1}$ where ΔE is an average excitation energy.

From the point of view of a manifestly covariant formulation of quantum electrodynamics, the long-range potential of order e^4 between neutral spinless systems A and B can be regarded as arising from the exchange of two photons between A and B. Within such a framework one can show that[3,4]

$$V_{AB}^{\text{em}}(r) \sim -D_{AB}/r^7, \tag{1.5a}$$

where

$$D_{AB} = \frac{23}{4\pi}(\alpha_A^e \alpha_B^e + \alpha_A^m \alpha_B^m) - \frac{7}{4\pi}(\alpha_A^e \alpha_B^m + \alpha_A^m \alpha_B^e). \tag{1.5b}$$

Here, e.g., α_A^e and α_A^m are the static electric and magnetic polarizabilities of A, respectively. These polarizabilities are defined, in contrast to (1.4c), as the low-energy limits of appropriately chosen invariant amplitudes entering the description of the tensor amplitude $M^{\mu\nu} = M^{\mu\nu}(p_A', k'; p_A, k)$ associated with Compton scattering $\gamma + A \to \gamma + A$. If A and B have spin then (1.5a) still holds but D_{AB} has additional terms which depend on the spin orientation of A and B, that is, the long-range potential is spin-dependent.[5]

The result (1.5) will be referred to as the "D theorem." It is a substantial generalization of (1.4) because its derivation, which uses the techniques of dispersion theory, depends only on the validity of general principles such as Lorentz invariance, conservation of the electromagnetic current, analyticity, and unitarity.[4] The D theorem is therefore independent of any assumptions about the internal structure of the particles A and B, be they atoms, hadrons, or other kinds of elementary systems. However, it does assume that the exchanged photons propagate as free massless particles in going from A and B and neglects radiative corrections to the Compton amplitudes.[6] From a dispersion-theoretic point of view, the long-range character of the retarded van der Waals potential (1.5a) is a consequence of the fact that the smallest value of the invariant mass $Q^2 = (k_1 + k_2)^2$ of a system of two photons is zero.

With the advent of quantum chromodynamics (QCD) as a promising theory of strong interactions and of the structure of ordinary hadrons, the question of the interhadronic potential at large distances has received renewed attention. This is because in QCD hadrons are pictured as bound states of a few colored quarks and/or antiquarks interacting by the exchange of colored massless vector mesons, the so-called color gluons. Because the hadrons themselves are color-neutral, two hadrons cannot exchange a single color gluon, which belongs to an SU(3) octet. Therefore, no long-range force arising from single-gluon exchange occurs between any two hadrons, whether they have spin or not. However, the exchange of a pair of gluons is not forbidden on this basis because two gluons can be in a color-singlet state. This had led a number of authors to speculate that, in analogy with the van der Waals type of potentials arising from two-photon exchange, gluon exchange between hadrons can give rise to a long-range force between hadrons, i.e., a potential which falls off as an inverse power at large distances.[7-13] Some authors have considered the problem within the framework of "potential exchange," i.e., have described the interaction between quarks and antiquarks in terms of potentials (e.g., the linear potential popular from the charmonium model), and have computed a long-range potential between widely separated color-neutral quark aggregates.

A major purpose of the present paper is to analyze the question of a long-range interhadronic force from a phenomenological point of view. This is done in Sec. II, where we consider the bounds on such forces which can be obtained from the results of measurements involving a variety of techniques, including gravitational experiments both of the Eötvös and of the Cavendish type, observation of the hyperfine structure of molecular hydrogen, and study of the level structure of exotic systems, such as an antiproton bound in a low Bohr orbit about a sulfur nucleus.

In Sec. III we review briefly the dispersion-theoretic approach to the asymptotic behavior of interparticle forces and consider a number of theoretical issues relevant to the question of the existence of long-range hadronic forces. In Sec. IV we study multigluon exchange between hadrons. We find that the existence or nonexistence of long-range forces depends on how confinement is implemented. We also consider the approach based on potential exchange. We emphasize that if the confining q-$\bar{q}$ potential comes from multigluon exchange there is the danger that it has a color-independent part, which would lead to disastrous results for hadron-hadron forces. In Sec. V we summarize our results and comment on them as well as on related work of other authors.

II. EXPERIMENTAL EVIDENCE ABOUT LONG-RANGE FORCES BETWEEN HADRONS

Whatever the theoretical situation with regard to the predictions of QCD or other theories of hadrons about the occurrence of long-range forces between hadrons, it is of interest to investigate the experimental situation concerning these forces. Other analyses of this have appeared, but we do not entirely agree with the conclusions expressed in some of them.[8,10,14] We analyze only cases in which the hadrons are separated by many fermis, where there should be clean separation between long-range and Yukawa forces.

We consider the possibility of both a spin-independent (SI) interaction and, if both hadrons have spin, of a spin-dependent (SD) interaction between two hadrons. We parametrize these potentials as follows: The potential between individual hadrons, which may have spins S_1 and S_2, is taken to be either one of two types,

$$V_N^{\rm SI} = \frac{\lambda_N^{\rm SI}}{r}\left(\frac{r_0}{r}\right)^{N-1}\hbar c = \lambda_N^{\rm SI}\left(\frac{r_0}{r}\right)^{N} 200\ \text{MeV}, \tag{2.1}$$

$$V_N^{\rm SD} = \frac{\lambda_N^{\rm SD}}{r}\left(\frac{r_0}{r}\right)^{N-1}\hbar c\, \vec{S}_1\cdot\vec{S}_2 \simeq \lambda_N^{\rm SD}\left(\frac{r_0}{r}\right)^{N} 200\ \text{MeV}, \tag{2.2}$$

where $r \gg r_0$, with r_0 taken to be 1 F by convention. The λ_N are dimensionless constants that characterize the long-range potentials. We ask what information is available about $V_N^{\rm SI}$ or $V_N^{\rm SD}$ for various values of N.

It turns out that very different aspects of physics are sensitive to the existence of V_N for the various values of N. Since the underlying theory does not seem to be a good guide to what value of N, if any, is to be expected, we consider all integer values of N, from $N=1$ through $N=7$.

A. Gravitation experiments

We first consider what can be learned about long-range forces between hadrons from experiments involving gravitation. The first example of this was given long ago by Lee and Yang.[15] There are two types of experiments that are relevant, those in which the inverse-square character of gravity is tested and those in which the exact proportionality of gravitational and inertial mass is tested. The latter experiments are relevant because the hadronic long-range forces between two objects will be proportional to the number of hadrons in each object and this number is not strictly proportional to the inertial mass, if we compare objects of different atomic composition.

We assume that the gravitational potential between two objects is given exactly by the Newtonian expression

$$V_{\rm grav}^{12} = -\frac{Gm_1m_2}{r},$$

where m_1 and m_2 are the masses and $G \simeq 6.7 \times 10^{-8}$ cgs units. Corrections due to general relativity are unimportant in the present context.

To compare with V_N, we can rewrite $V_{\rm grav}^{12}$ approximately as

$$V_{\rm grav}^{12} \simeq -\frac{Gm_p^2}{r}N_1N_2 \sim -\frac{2.4\times10^{-55}}{r}N_1N_2 \ \ \text{(cgs units)} \tag{2.3}$$

with N_i the number of nucleons in object "i." While this expression masks the lack of proportionality of nucleon number and mass, it is convenient for the purpose of rough comparisons.

$V_{\rm grav}^{12}$ can be compared to the potential V_N^{12} between the two objects arising from the potential $V_N^{\rm SI}$ acting between their constituents. We assume that the objects are far enough apart in comparison to their size so that r can be replaced by the separation R of their centroids and neglect screening of one nucleon by another. We also assume that $V_N^{\rm SI}$ is the same for any two nucleons. This appears likely if the potential arises from gluon exchange. Under these assumptions we obtain from $V_N^{\rm SI}$ an interaction energy

$$V_N^{12} \simeq \frac{\lambda_N^{\rm SI}}{R}\left(\frac{r_0}{R}\right)^{N-1}(3\times10^{-17})N_1N_2, \tag{2.4}$$

so that

$$\frac{V_N^{12}}{V_{\rm grav}^{12}} \simeq \lambda_N^{\rm SI}\left(\frac{r_0}{R}\right)^{N-1}(1.2\times10^{38})(1+\delta). \tag{2.5}$$

Here the term $1+\delta$ corrects for the fact that the gravitational potential involves the masses rather than the number of nucleons.

1. Eötvös-type experiments

Now consider the extent to which V_N^{12} might be present. Experiments done by Eötvös and others[16] compare the acceleration imparted by the Earth to various substances containing different numbers and types of hadrons, but of the same mass. A typical result is that the difference in acceleration is less than one part in 10^8. Later experiments by Renner improve this to one part in 10^9. Let us compare the force on hydrogen and on copper, two of the substances involved:

$$F(\mathrm{H}) \simeq \frac{GM_\oplus M_\mathrm{H}}{R_\oplus{}^2} - \frac{\lambda_N^{\mathrm{SI}} N}{r_0^2}\left(\frac{r_0}{R_\oplus}\right)^{N+1} N_\oplus N_\mathrm{H} \hbar c\,,$$

$$F(\mathrm{Cu}) \simeq \frac{GM_\oplus M_\mathrm{Cu}}{R_\oplus{}^2} - \frac{\lambda_N^{\mathrm{SI}} N}{r_0^2}\left(\frac{r_0}{R_\oplus}\right)^{N+1} N_\oplus N_\mathrm{Cu} \hbar c\,.$$

Then with $M_\mathrm{H} = M_\mathrm{Cu} \approx N_\mathrm{Cu} m_p$ we get

$$\begin{aligned} F(\mathrm{H})/F(\mathrm{Cu}) &\simeq 1 - \frac{N\lambda_N^{\mathrm{SI}}}{r_0^2}\left(\frac{r_0}{R_\oplus}\right)^{N+1} \frac{N_\oplus (N_\mathrm{H} - N_\mathrm{Cu})}{GM_\oplus M_\mathrm{Cu}/R_\oplus{}^2}\hbar c \\ &\simeq 1 - N\lambda_N^{\mathrm{SI}}\left(\frac{r_0}{R_\oplus}\right)^{N-1} \\ &\quad \times (\hbar c/Gm_p^2)(N_\mathrm{H}/N_\mathrm{Cu} - 1) \\ &\simeq 1 - N\lambda_N^{\mathrm{SI}} \times 1.2 \times 10^{38} \\ &\quad \times (N_\mathrm{H}/N_\mathrm{Cu} - 1)(r_0/R_\oplus)^{N-1}\,. \end{aligned}$$

The difference $(N_\mathrm{H}/N_\mathrm{Cu}) - 1$ is approximately the same as the binding energy per nucleon in Cu divided by the proton rest energy, since the two samples are constrained to have the same total mass. Therefore,

$$\left|\frac{N_\mathrm{H}}{N_\mathrm{Cu}} - 1\right| \sim 10^{-2}\,.$$

We conclude that

$$N|\lambda_N^{\mathrm{SI}}| \left(\frac{r_0}{R_\oplus}\right)^{N-1} \times 10^{36} \lesssim 10^{-9}\,,$$

or, suppressing the absolute value sign on λ here and henceforth,

$$\lambda_N^{\mathrm{SI}} < \frac{10^{-45}}{N}\left(\frac{R_\oplus}{r_0}\right)^{N-1}. \tag{2.6}$$

In Table I we show the limit for λ_N^{SI} implied by this relation, using the value $R_\oplus/r_0 \simeq 6\times 10^8/10^{-13} = 0.6\times 10^{22}$.

We see that an inverse first power potential is ruled out by many orders of magnitude, as originally pointed out by Lee and Yang,[15] and that an inverse-square potential is also ruled out by an immense factor. Because of the large ratio $R_\oplus/r_0$, higher-power potentials would not contribute significantly to these experiments and so are not ruled out.

TABLE I. Limits on the strength λ_N^{SI} of an inverse-power spin-independent potential $V_N^{\mathrm{SI}}(r)$, defined by Eq. (2.1) of text, inferred from Eötvös-type experiments and Eq. (2.6).

N	Limit on $\lvert\lambda_N^{\mathrm{SI}}\rvert$ [a]	Limit on $\lvert\lambda_N^{\mathrm{SI}}\rvert$ [b]
1	10^{-45}	10^{-47}
2	10^{-23}	10^{-20}
3	10^{-2}	10^{7}

[a] Based on the results of Eötvös *et al.*, Ref. 16.
[b] Based on the results of Braginsky and Panov, Ref. 18.

Modified versions of Eötvös's experiment have been performed by Roll, Krotkov, and Dicke[17] and by Braginsky and Panov.[18] These experiments measure the acceleration imparted to various substances of the same mass by the sun. For these experiments, the difference in acceleration is smaller, the limit being 10^{-12}. However, the factor corresponding to $|N_1/N_2 - 1|$ is also smaller, being perhaps 10^{-3}. Furthermore, the ratio $R_\oplus/r_0$ is now much larger $\sim 10^{27}$, so this experiment, while more sensitive to an inverse-first-power potential, is less sensitive to higher-inverse-power potentials, as noted in Table I.

2. Cavendish-type experiments

We turn next to experiments measuring the gravitational force between laboratory sized objects of a type originally done by Cavendish. Although these experiments are not nearly so precise as the Eötvös type, they have two advantages for our present purpose. One is that the distances are much smaller, so they are much more sensitive to potentials that fall off faster than r^{-2}. Also, the Cavendish-type experiments in some cases actually measure the power dependence of the force and so are directly sensitive to extra terms coming from V_N, rather than only to a variation of the force from one substance to another.

A survey of the results of Cavendish-type experiments has been given by Long[19] who also, in a later paper, reports on an experiment of his own which appears to show an actual deviation from a pure Newtonian potential.[20] His results are consistent with the conclusion that

$$r\frac{d}{dr}(r^2F)\Big/(r^2F)\Big|_{r=10\,\mathrm{cm}} \lesssim 10^{-2}\,, \tag{2.7}$$

where F is the total force between two objects of laboratory size which are presumably shielded from obvious external electromagnetic forces.

We shall take this value as a limit rather than as a real effect.

The numerator in the logarithmic derivative on the left-hand side of (2.7) gets a contribution from V_N and not from $F_{\rm grav}$. The denominator can accurately be taken to be $r^2 F_{\rm grav}$. Thus (2.3), (2.4), and (2.7) imply that

$$\frac{\lambda_N^{\rm SI} N(N-1)(r_0/R)^{N-1}(3\times 10^{-17})N_1N_2}{2.4\times 10^{-55}N_1N_2} < 10^{-2},$$

and with $R=10$ cm we get

$$\lambda_N^{\rm SI} < \frac{(10^{14})^{N-1}}{1.2\times 10^{40}}\frac{1}{N(N-1)}. \tag{2.8}$$

This limit is useless for $N=1$, but becomes stronger than the Eötvös limit for $N=2$, and much stronger, indeed useful, for an inverse-cube potential. Thus for $N=3$ we get

$$\lambda_3^{\rm SI} < 10^{-12}. \tag{2.9}$$

No useful limit is obtained for higher inverse-power potentials. A more qualitative version of this argument has been given by Fujii and Mima.[10]

3. *Sensitivity to spin-dependent interactions*

Before leaving the realm of Cavendish experiments, a comment is in order about the sensitivity of such experiments to spin-dependent long-range interactions. An experiment can test such interactions only if the objects involved have some average nuclear spin alignment. There is reason to think that this is the case in many Cavendish experiments. Consider a metal object not shielded from the Earth's magnetic field of about 1 G. Suppose that the nuclear spin is not zero. Then the interactions of the Earth's field with the nuclear magnetic moment will split the nuclear I_z levels. If the atom has no net electronic angular momentum, this splitting is given directly by

$$\Delta E \simeq \frac{g_I e\hbar H}{m_p c} \sim 10^{-11}\ \text{eV} \tag{2.10}$$

and the fractional difference in nuclear spin occupation is

$$\frac{\Delta N_n}{N_n} \simeq \frac{\Delta E}{k_B T} \simeq 10^{-9}. \tag{2.11}$$

On the other hand, if there is a net electron spin, the situation is more complicated. However, under normal conditions in a nonferromagnetic solid, the estimate given in Eq. (2.11) still appears to be correct.[21]

The results imply that for the two metal objects used in a typical Cavendish experiment there may be an average excess nuclear spin occupation along the direction of the Earth's field of about one part in 10^9. Suppose now that a potential such as $V_N^{\rm SD}$ exists between the nucleons of the two objects. This will lead to an additional force between the objects, given by

$$F_N^{\rm SD} \sim (\Delta N_n/N_n)^2(\partial V_N^{\rm SD}/\partial R)N_1N_2$$
$$= 10^{-18}N\lambda_N^{\rm SD}(r_0/R)^{N+1}\times\frac{3\times 10^{-17}}{r_0^2}N_1N_2. \tag{2.12}$$

The contribution of this to the logarithmic derivative in Eq. (2.7) is

$$N(N-1)\lambda_N^{\rm SD}(r_0/R)^{N-1}\times\frac{3\times 10^{-35}}{2.4\times 10^{-55}},$$

and, using (2.7), we infer that

$$\lambda_N^{\rm SD} < \frac{1}{N(N-1)}\frac{(10^{14})^{N-1}}{10^{22}}. \tag{2.13}$$

This leads to a limit

$$\lambda_2^{\rm SD} < 10^{-8} \tag{2.14}$$

and no useful result for higher-inverse-power potentials.[21a]

In summary, the Cavendish-type experiments constrain inverse-square and inverse-cube spin-independent potentials and inverse-square spin-dependent potentials to be very small (if the length parameter is chosen as 1 F), but do not significantly constrain other inverse-power potentials. The results are summarized in Table II.

B. Experiments on hydrogen molecules

Because of the rapid decrease with distance of the potentials with $N>3$, one may hope to do better in detecting them with measurements on hadrons that are separated by microscopic distances. In ordinary matter this means isolated molecules or solid bodies. The problem that arises in each of these cases is that it is not easy to calculate the effects of ordinary electro-

TABLE II. Limits on the strength $\lambda_N^{\rm SI}$ of $V_N^{\rm SI}(r)$ and on the strength $\lambda_N^{\rm SD}$ of an inverse-power spin-spin potential $V_N^{\rm SD}(r)$, defined by Eq. (2.2) of text, inferred from Cavendish-type experiments of Long (Ref. 20) and Eqs. (2.8) and (2.12).

N	Limit on $\lvert\lambda_N^{\rm SI}\rvert$	Limit on $\lvert\lambda_N^{\rm SD}\rvert$
1	...	...
2	10^{-26}	10^{-8}
3	10^{-12}	10^{6}

magnetic interactions with sufficient precision to enable one to separate out the effect of any new interaction.

One exception to this seems to be a series of experiments by Ramsey and collaborators[22] on spin-dependent interactions in the hydrogen molecule. The result of their analysis is that any anomalous spin-dependent interaction between two protons in H_2 cannot be greater than 4×10^{-4} of the interaction between the proton magnetic moments provided that the anomalous interaction leads to a splitting of the F levels of orthohydrogen. This is not the case for V_N^{SD} of Eq. (2.2). However, we can consider a tensor interaction

$$V_N^T(r)=\frac{\lambda_N^T}{r}\left(\frac{r_0}{r}\right)^{N-1}\mathcal{S}\hbar c\,, \tag{2.15}$$

where $\mathcal{S}=\vec{s}_1\cdot\vec{s}_2-3\,\vec{s}_1\cdot\hat{r}\,\vec{s}_2\cdot\hat{r}$ with $\vec{s}_i$ the proton spin. This does split the F levels. These experiments can then be used to give information about λ_N^T for $N\leq 4$.[22a]

For $N=3$ the comparison can be made directly because V_3^T is proportional to the magnetic-dipole interaction of the protons, $H_{\rm dip}=(2.8e\hbar/m_pc)^2\mathcal{S}r^{-3}$. It follows that the ratio of the corresponding level shifts $\delta E_3=\langle V_3^T\rangle$ and $\delta E_{\rm dip}=\langle H_{\rm dip}\rangle$ is given by

$$\begin{aligned}(\delta E_3)/(\delta E_{\rm dip})&=(r_0/\lambda\!\!\!^{-}_p)^2\alpha^{-1}(2.8)^{-2}\lambda_3^T\\ &\sim 400\lambda_3^T\,.\end{aligned} \tag{2.16}$$

According to Ref. 22, the measurement of the hyperfine structure agrees with a calculation in which only $H_{\rm dip}$ is included to within the experimental error in measurement, which is four parts in ten thousand. It follows that

$$\lambda_3^T<10^{-6}\,, \tag{2.17}$$

which is a significant restriction on an anomalous, spin-dependent, inverse-cube potential.

We can also extract a better limit for other λ_N^T from these data. We obtain in general

$$\lambda_N^T<10^{-6}(r_0/R)^{3-N}\,, \tag{2.18}$$

where R is the separation of the protons in orthohydrogen, which is 0.74×10^{-8} cm. For λ_2^T this gives

$$\lambda_2^T<10^{-11}\,, \tag{2.19}$$

a better result than the Cavendish experiment. For λ_4^T we get

$$\lambda_4^T<0.1\,. \tag{2.20}$$

No significant results are obtained for $N>4$. The results are summarized in Table III.

TABLE III. Limits on the strength λ_N^T of an inverse-power tensor potential $V_N^T(r)$ defined by Eq. (2.15) of text, inferred from measurement of hyperfine structure of molecular hydrogen by Ramsey and coworkers (Ref. 22) and Eq. (2.18).

N	Limit on $\lvert\lambda_N^T\rvert$
1	10^{-16}
2	10^{-11}
3	10^{-6}
4	10^{-1}
5	10^{4}

C. Hadronic atoms

A very convenient system for studying long-range forces between hadrons is a hadronic atom such as (π^--nucleus), (K-nucleus), or ($\bar{p}$-nucleus). In such atoms the orbiting hadron is sufficiently loosely bound that it can be accurately treated as a hydrogenic Coulomb bound state, whose properties are well understood.[23] Nevertheless, it is close enough to a number of other hadrons so that the long-range potentials we are considering could contribute significantly to the atomic energy levels.

An important figure of merit for a hadronic atom is the average distance of the orbiting hadron from the nucleus. This is given by the formula

$$\bar{r}\simeq\frac{200}{Z}\frac{m_\pi}{m_r}\frac{3n^2-l(l+1)}{2}\,\mathrm{F}\,,$$

where n and l are the usual quantum numbers of the state and m_r is the reduced mass of the bound hadron, which is approximately its physical mass except for light atoms. This formula indicates the advantage of using the heaviest orbiting hadrons, the heaviest atoms, and of observing the deeper-lying states, since these will have the smaller $\bar{r}$ and so the greatest $V_N(r)$. Of course, if the hadrons get too close to the nucleus, the effects of pion exchange and other hadron exchange become large and complicate the analysis. Furthermore, in hadronic atoms the most tightly bound states of heavy atoms have high capture probabilities and it is quite difficult to observe transition x rays. As a result, no transitions have been observed in which $\bar{r}$ is less than about 25 F for initial or final states. Whether it is possible to improve on this value of $\bar{r}$ in future experiments is not known to us.

The existing experiments generally involve the observation of electric-dipole transitions between two states, both of which have $l=n-1$, so that $\Delta l=1$ and $\Delta n=1$.[24] We must therefore estimate the contribution to the energy shift of such states coming from V_N. To do this we need the expectation value of r^{-m} in such atomic states. A straightforward calculation gives

$$\langle r^{-m}\rangle_{n,n-1} = a^{-m}F_m(n)\,, \qquad (2.21a)$$

where $\langle\ \rangle_{n,n-1}$ denotes the expectation value in a state with quantum numbers $n, l=n-1$, $a=\hbar/Z\alpha m_r c$, and

$$F_m(n) = \frac{(2n-m)!}{(2n)!}\left(\frac{n}{2}\right)^{-m}, \qquad (2.21b)$$

provided that $2n \geq m$. (If $2n<m$, $\langle r^{-m}\rangle$ diverges and the nucleus must be treated as an extended source.) Equation (2.21) disagrees with a formula for a similar quantity given by Fishbane and Grisaru,[8] who have also studied the effect of V_6 and V_7 on energy levels of hadronic atoms. Their formula does not reduce to known cases correctly. For r^{-6}, the case they consider, their result is too small by a large factor, of order 30 to 100.[23a]

From Eq. (2.4) we find the shift due to $V_N^{\rm SI}$ in a hadronic atom to be

$$\Delta E_N(n) = \lambda_N^{\rm SI}\,(r_0/a)^N(\hbar c/r_0)F_N(n)A$$
$$= \lambda_N^{\rm SI}[Z\alpha r_0/(\hbar/m_p c)]^N F_N(n)A \times 200\ {\rm MeV}\,. \qquad (2.22)$$

To get the energy shift of a transition x ray, we use (2.22) to compute $\Delta E_N(n) - \Delta E_N(n-1)$.

The factor of A in (2.22) arises because the total potential acting at the position of the orbiting hadron gets an approximately equal contribution from all of the hadrons in the nucleus. This is under the assumption that the hadron-hadron potential is isospin independent, which would be a good approximation if this potential arises from gluon exchange. An isovector hadron-hadron potential would give a similar result for ΔE, but with A replaced by $N-Z$.

In Table IV we list the experimental values for the transition energies of various kaonic and antiprotonic atom transitions. The experimental values given are differences between observed values and the results of a calculation of the QED prediction of the transition energy. The difference can in many cases be explained as the result of a residual known strong-interaction contribution.[23] However, to get conservative limits on $\lambda_N^{\rm SI}$, we have taken the *total* difference between experiment and theory as an upper limit to the contribution of long-range forces.

In the case of pionic atoms, the energies of individual levels rather than transition energies have been measured. These are also given in Table IV. In the final column of that table, upper limits for $\lambda_N^{\rm SI}$ are given for various N and for the interaction of the bound particle and the isoscalar combination of nucleons. It can be seen that spin-independent potentials down to r^{-5} and a length scale of 1 F are pretty much ruled out by the data, while potentials such as r^{-6} and r^{-7} are not, unless the length parameter r_0 is several fermi.

TABLE IV. Limits on the strength $\lambda_N^{\rm SI}$ of $V_N^{\rm SI}(r)$ inferred from measurements of radiative transitions in exotic atoms and Eq. (2.22) of the text. The column labeled δ_N shows the change in level splitting which would arise for $\lambda_N^{\rm SI}=1$.

Bound particle	Atom	Transition	$\Delta E_{\rm th} - \Delta E_{\rm exp}$ (eV)	N	δ_N (eV)	Limit on $\lambda_N^{\rm SI}$
K^-	$Z=28$, $A=59$	$n=5$ to $n=4$	-170 ± 60 [a]	3	4.4×10^5	5×10^{-4}
				4	2.4×10^4	10^{-2}
				5	1.7×10^3	10^{-1}
				6	1.6×10^2	1
				7	22	8
$\bar{p}$	$Z=16$, $A=32$	$n=5$ to $n=4$	-60 ± 40 [b]	3	3×10^5	2×10^{-4}
				4	1.8×10^4	3×10^{-3}
				5	1.4×10^3	5×10^{-2}
				6	1.4×10^2	0.5
				7	21	3
π^-	$Z=67$, $A=165$	$4f$ [d]	350 ± 80 [c,e]	3	5×10^5	7×10^{-4}
				4	1.8×10^4	2×10^{-2}
				5	7.8×10^2	0.5
				6	46	8
				7	4	100

[a] P. D. Barnes *et al.*, Ref. 24.
[b] A. Bamberger *et al.*, Ref. 24.
[c] P. Ebersold *et al.*, Ref. 24.
[d] For the pionic atom the total energy rather than the transition energy is measured.
[e] This number refers to the difference between the calculated and measured energy of the $4f$ state.

III. THEORETICAL ASPECTS

A. Dispersion-theoretic approach

We review briefly the dispersion-theoretic approach to the analysis of the asymptotic behavior of interparticle potentials.[3,4] Apart from the advantages mentioned in Sec. I, this approach also avoids the need to deal with some of the divergences which one may encounter in Feynman diagrams, because one only needs the absorptive part of Feynman amplitudes.[25] Consider the elastic scattering of spinless particles A and B with initial four-momenta p_A and p_B and final four-momenta p'_A and p'_B, respectively. Let $F=F(s,t)$ denote the invariant Feynman amplitude for the scattering with

$$s=(p_A+p_B)^2\,,\quad t=(p_A-p'_A)^2 \qquad (3.1)$$

and let $F_{\mathfrak{D}}=F_{\mathfrak{D}}(s,t)$ denote the contribution to F arising from any set $\mathfrak{D}$ of Feynman diagrams each of which is irreducible with respect to A and B, i.e., does not involve an intermediate state containing only A and B. We assume that $F_{\mathfrak{D}}$ is an analytic function of t with a nearest right-hand branch point $t_{\mathfrak{D}}$ and left-hand branch point $t^L_{\mathfrak{D}}$, both of which lie outside the physical region of t. For s above the threshold for scattering

$$s\geq s_0\equiv(m_A+m_B)^2\,, \qquad (3.2)$$

this region is given by

$$t_0(s)\leq t\leq 0\,, \qquad (3.3)$$

where $t_0(s)=-4k^2(s)$ with k the magnitude of the c.m. momentum. Ignoring subtractions, one can write

$$F_{\mathfrak{D}}(s,t)=F^R_{\mathfrak{D}}(s,t)+F^L_{\mathfrak{D}}(s,t) \qquad (3.4)$$

with

$$F^R_{\mathfrak{D}}(s,t)=\frac{1}{\pi}\int_{t_{\mathfrak{D}}}^{\infty}\frac{\rho^R_{\mathfrak{D}}(s,t')}{t-t'}dt' \qquad (3.5a)$$

and a similar equation holding for $F^L(s,t)$:

$$F^L_{\mathfrak{D}}(s,t)=\frac{1}{\pi}\int_{-\infty}^{t^L_{\mathfrak{D}}}\frac{\rho^L_{\mathfrak{D}}(s,t')dt'}{t-t'}\,. \qquad (3.5b)$$

Here $2i\rho^R_{\mathfrak{D}}$ and $2i\rho^L_{\mathfrak{D}}$ are the discontinuities of $F_{\mathfrak{D}}$ across the branch cuts extending from $t_{\mathfrak{D}}$ to $+\infty$ and $t^L_{\mathfrak{D}}$ to $-\infty$, respectively, and according to our assumptions we have

$$t_{\mathfrak{D}}\geq 0\,,\quad t^L_{\mathfrak{D}}\leq t_0(s)\,. \qquad (3.6)$$

One can now associate a *local* potential with $F_{\mathfrak{D}}$ by the definition

$$V_{\mathfrak{D}}(r)\equiv\frac{1}{32\pi^3 m_A m_B}\int d\vec{q}\,e^{-i\vec{q}\cdot\vec{r}}F^R_{\mathfrak{D}}(s_0,t=-\vec{q}^{\,2})\,. \qquad (3.7)$$

Note that $F^R_{\mathfrak{D}}$ is uniquely defined for all $t<0$ by (3.5a), so that the definition (3.7) of $V_{\mathfrak{D}}$ is unambiguous. On using (3.5a) in (3.7) and reversing the order of integration one gets, with $N_{AB}=(16\pi^2 m_A m_B)^{-1}$,

$$V_{\mathfrak{D}}(r)=N_{AB}\int_{t_{\mathfrak{D}}}^{\infty}dt\,\frac{e^{-\sqrt{t}\,r}}{r}\rho^R_{\mathfrak{D}}(s_0,t)\,, \qquad (3.8)$$

which expresses $V_{\mathfrak{D}}(r)$ directly in terms of the spectral function $\rho^R_{\mathfrak{D}}$, given by

$$\rho^R_{\mathfrak{D}}(s_0,t)\equiv\frac{1}{2i}[F_{\mathfrak{D}}(s_0,t+i\epsilon)-F_{\mathfrak{D}}(s_0,t-i\epsilon)]\quad(t\geq t_{\mathfrak{D}})\,. \qquad (3.9)$$

One may similarly associate an exchange potential $V^{\text{ex}}_{\mathfrak{D}}$ with $F_{\mathfrak{D}}$, corresponding to the $F^L_{\mathfrak{D}}$ part of $F_{\mathfrak{D}}$, but this is not of interest here.

The main virtue of the representation (3.8) of $V_{\mathfrak{D}}(r)$ is that it relates the asymptotic behavior of $V_{\mathfrak{D}}$ for large r to the behavior of $\rho^R_{\mathfrak{D}}$ for small $t-t_{\mathfrak{D}}$ and this latter behavior can often be established on the basis of rather general considerations.[3,4]

It follows immediately from (3.8) that if $t_{\mathfrak{D}}\neq 0$, $V_{\mathfrak{D}}(r)$ will decrease exponentially for large r, whereas if $t_{\mathfrak{D}}=0$, $V_{\mathfrak{D}}(r)$ will normally fall off like an inverse power of r. For example, if $t_{\mathfrak{D}}\neq 0$ and

$$\rho^R_{\mathfrak{D}}(s_0,t)=(t-t_{\mathfrak{D}})^P\phi(t) \qquad (3.10)$$

with $P>0$ and $\phi(t_{\mathfrak{D}})\neq 0$, one finds from (3.8) that[26]

$$V_{\mathfrak{D}}(r)\sim K_P e^{-m_{\mathfrak{D}}r}/r^{P+2}\,, \qquad (3.11a)$$

where $K_P=(2m_{\mathfrak{D}})^{P+1}P!\,\phi(t_{\mathfrak{D}})$ and

$$m_{\mathfrak{D}}\equiv\sqrt{t_{\mathfrak{D}}} \qquad (3.11b)$$

is the minimal mass of any intermediate state contributing to $F_{\mathfrak{D}}$, as viewed from the t channel $A+\bar{A}\to B+\bar{B}$. In contrast, if $t_{\mathfrak{D}}=0$ in (3.10), use of Eq. (3.8) yields

$$V_{\mathfrak{D}}(r)\sim D_P/r^N\,, \qquad (3.12a)$$

where $D_P=2(2P+1)!\,\phi(0)$ and

$$N=2P+3\,. \qquad (3.12b)$$

B. Theoretical implications

Let us suppose that the potential describing the low-energy scattering of hadrons A and B has a part $V_N=V_N(r)$ with inverse-power behavior for large r:

$$V_N(r)\sim C_N/r^N\,. \qquad (3.13)$$

It follows from Eqs. (3.11) that in ordinary quantum field theories, in which perturbation theory is a reliable guide at least to the analyticity properties of scattering amplitudes, there must be one or more diagrams in $\mathfrak{D}$ for which the smallest value for a t singularity is zero, so that we can have $t_{\mathfrak{D}}=0$. However, in such theories a singularity of the physical-sheet amplitude at $t=0$ is normally present only if a physical state of zero rest mass is accessible in the t-channel reaction $A+\bar{A}'\to\bar{B}+B'$, corresponding to a threshold in an unphysical region. The existence of such a physical state would imply, in turn, the existence of zero-mass particles in the theory.

Alternatively and more directly, we may consider the contribution $F_N^{\rm Born}(t)$ to $F(s,t)$ arising from a potential $V_N(r)$ with asymptotic behavior described in (3.14):

$$F_N^{\rm Born}(t)=\text{const}\times\int d\vec{r}\,e^{i\vec{q}\cdot\vec{r}}V_N(r)\,. \qquad (3.14)$$

A short calculation then shows that $F_N^{\rm Born}(t)$ is not analytic in any neighborhood of $t=0$. To be precise, let us distinguish between two cases:

(I) N is an odd integer larger than unity.

(II) N is real and larger than unity but is not an odd integer.

In case (I) one finds, for $t<0$,

$$\text{(I)}:\ F_N^{\rm Born}(t)=\text{const}\times(-t)^{(N-3)/2}\log(-t)+A(t)\,, \qquad (3.15a)$$

where $A(t)$ denotes a function analytic at $t=0$. In case (II) one gets

$$\text{(II)}:\ F_N^{\rm Born}(t)=\text{const}\times(-t)^{(N-3)/2}+A(t)\,. \qquad (3.15b)$$

Thus, in either case we get a function which has a branch point at $t=0$. Note that the discontinuity across the cut, taken along the positive real axis, is proportional to $t^{(N-3)/2}$, regardless of the value of N.

From the above discussion we conclude that if the effective potential describing the low-energy scattering of A and B has a long-range part $V_N\sim r^{-N}$, then $F(s,t)$ will be singular in t at $t=0$, with s fixed in the neighborhood of the threshold s_0. The singularity will be a brach point of either a logarithmic or an algebraic type, depending on whether N is an odd integer or not. Ordinarily, when scattering amplitudes are calculated from covariant Feynman diagrams, such a singularity in $F(s,t)$ arises because of the existence of physical zero-mass particles in the theory. Obvious candidates for these zero-mass particles are the color gluons of QCD. However, these gluons are supposedly confined and so cannot propagate from one hadron to another if the hadrons are far apart. This would suggest that in the standard picture of QCD long-range forces would not occur between hadrons. We shall see below, in Sec. IV, that this conclusion may not hold in that while massless gluons do not occur as real particles, some of their effects as virtual particles may still generate long-range forces.

It should be emphasized that the connection between long-range forces and zero-mass particles is in fact stronger than any argument based only on perturbation theory might imply. Consider any quantum field theory, satisfying the usual axioms, in which there is also a mass gap, i.e., in which in the physical spectrum of the mass-squared operator $P_\mu P^\mu$ there is a gap between the zero eigenvalue associated with the vacuum and the values for other physical states. Then as shown long ago by Lehmann,[27] $F(s,t)$ has the following analyticity property:

Theorem. For $s>s_0$, $F(s,t)$ is analytic in a domain which includes the disk

$$|t|<2k^2|\cos\theta_s-1|\,, \qquad (3.16)$$

where

$$\cos\theta_s=\left[1+\frac{(m_A'^2-m_A{}^2)(m_B'^2-m_B{}^2)}{k^2[s-(m_A'-m_B')^2]}\right]^{1/2}. \qquad (3.17)$$

Here m_A' is the lowest mass of any physical state $|A'\rangle$ such that $\langle A'|j_A(0)|\text{vac}\rangle\neq0$, with $j_A(x)$ the source current for the local field associated with particle A and m_B' is similarly defined.

Since $k\neq0$ for $s>s_0$, the region of analyticity described by (3.16) is an open disk which includes the origin $t=0$. It follows that as long as the mass-gap condition is fulfilled, in a relativistic quantum field theory satisfying the axioms of the Wightman type long-range forces are excluded.

However, the usual axioms include the assumption of a Hilbert space of physical states equipped with a positive-definite metric and the existence of a set of local fields which have manifestly Lorentz-covariant transformation properties and which are complete, i.e., such that smeared polynomials in the fields acting on the (unique) vacuum generate a dense set. Since present formulations of QCD have not, to our knowledge, been shown to be equivalent to one in which such axioms hold, we cannot draw a firm conclusion regarding the incompatibility of long-range forces with QCD. Since there apparently is a mass gap in the physical spectrum of hadrons, the discovery of a strong long-range force between hadrons might have a significant bearing on the field-theoretic foundations of QCD.

C. Color polarizability

Another handle on the long-range force question can be obtained from analogy with Eq. (1.5a) giving the potential in QED arising from two-photon exchange between electrically neutral particles in terms of their electric and magnetic polarizabilities, α^e and α^m. Suppose for the moment that an analogous formula holds in QCD, with α^e and α^m replaced by "color polarizabilities" $\alpha^{e\text{ col}}$, $\alpha^{m\text{ col}}$. Analogs of such polarizabilities might in principle be definable in terms of Fourier transforms of quantities such as

$$W^{ab}_{\mu,\nu}(x-y)=\langle p'|T(A^a_\mu(x)A^b_\nu(y))|p\rangle\,, \tag{3.18}$$

where $A^a_\mu(x)$ is an octet gluon field, in analogy with the definition of the off-shell Compton amplitude for $\gamma+A\to\gamma'+A'$.

However, if we adopt as a consequence of confinement the idea that all octet states are at infinite energy, then a heuristic argument suggests that, on-shell and off-shell, the polarizabilities would vanish. For if we consider a QCD analog of the formula (1.4c) for α^e_{nr}, with the bound particle in an "external color-electric field," the intermediate states would have to be color-octet states; such states would presumably be very high up in energy, if confinement is only partial, and so in the limit of perfect confinement would have infinite energy: $E_n\to\infty$. Because of the energy denominator in (1.4c), it seems likely, on the one hand, that $\alpha^{e\text{ col}}_{\text{nr}}$ vanishes in this limit. On the other hand, it is conceivable that there is an analog in QCD of the diamagnetic polarizability $\alpha^{m\text{ dia}}_{\text{nr}}$ in nonrelativistic quantum theory which arises from a direct term in the Lagrangian $\sim e^2\phi^\dagger(x)\phi(x)\vec{A}_T{}^2(x)$, where $\phi(x)$ is a charged matter field. In the nonrelativistic picture there are no "intermediate states" associated with this part of α^m_{nr}, so that it will not vanish when the intermediate-state energies go to infinity. Of course, for an elementary spin-$\frac{1}{2}$ particle, such a term arises only as a nonrelativistic approximation to a contribution to α^m which does involve intermediate states, namely those containing an extra particle-antiparticle pair, which may have infinite energy in the corresponding QCD case. Thus, although the concept of color polarizability may play a role in future investigations along these lines, at present we are unable to use it to reach any sharp conclusions concerning the existence of long-range forces in QCD.

It is also possible that the picture of confinement used above is wrong and that gluons and quarks are not observed because of different mathematical properties of Green's functions. We discuss one such alternative in the next section.

IV. MULTIGLUON EXCHANGE

We remarked earlier that in the usual treatment of long-range forces in QED, higher-order electromagnetic effects are neglected. This includes radiative corrections to the Compton amplitude and to the photon propagator as well as the scattering of one photon by another. Such neglect is not obviously justified for the case of gluon exchange in QCD. Therefore we present a formalism that is appropriate for the analysis of long-range forces arising from the exchange of strongly interacting quanta.[28]

A. General considerations

We wish to analyze the effects of multigluon exchange on hadron-hadron scattering amplitudes at low-momenta transfer. We shall write

$$F(s,t)=F^{(0)}(s,t)+\sum_{n=2}^{\infty}F^{(n)}(s,t)\,, \tag{4.1}$$

where $F^{(n)}(s,t)$ denotes a part of F to be associated with the exchange of n gluons ($n=2,3,\ldots$) and $F^{(0)}$ is the remainder; $F^{(1)}$ vanishes because the hadrons are color singlets. To proceed, we must define the $F^{(n)}$ more precisely. Let $M^{(n)}_{g;A}$ denote the amplitude for the emission of n (off-shell) gluons by A and $M^{(n)}_{B;g}$ denote the corresponding absorption amplitude for B. To avoid double counting, we also introduce the quantity $M^{(n)\text{irr}}_{g;A}$, which is the part of $M^{(n)}_{g;A}$ which is irreducible with respect to the exchange of any number of gluons, i.e., is the sum of all those graphs which contribute to $M^{(n)}_{g;A}$ which are such that they cannot be divided into two disjoint parts by cutting only gluon-propagator lines. We now define the $F^{(n)}$ by writing, in an obvious symbolic notation and with vector and color indices suppressed,

$$F^{(2)}(s,t)=M^{(2)}_{B;g}D_gD_gM^{(2)\text{irr}}_{g;A} \tag{4.2}$$

and similarly for the higher $F^{(n)}$, e.g.,

$$F^{(3)}(s,t)=M^{(3)}_{B;g}D_gD_gD_gM^{(3)\text{irr}}_{g;A}\,, \tag{4.3}$$

with D_g denoting the full or "dressed" gluon propagator.[29] We concentrate on $F^{(2)}$, the contribution to F from two-gluon exchange. Some examples of relevant Feynman diagrams are shown in Fig. 1. Equation (4.2) may be written more explicitly, with $Q=p_A-p_A'$, as

$$F^{(2)}(s,t)=\text{const}\times\int M^{(2)}_{B;g}(p_B',p_B;k_1k_2)D_g(k_1{}^2)D_g(k_2{}^2)M^{(2)\text{irr}}_{g;A}(p_A',p_A;k_1k_2)\delta(Q-k_1-k_2)d^4k_1d^4k_2\,. \tag{4.4}$$

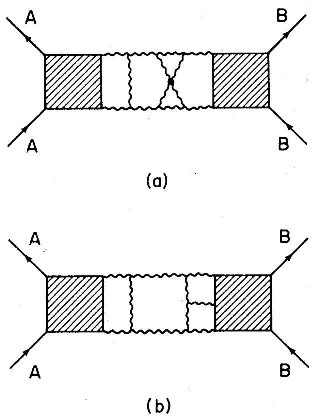

FIG. 1. Examples of Feynman diagrams contributing to $F^{(2)}(s,t)$, the part of the scattering amplitude arising from two-gluon exchange, as defined by Eq. (4.2) of the text. The shaded boxes represent irreducible gluon-emission amplitudes and the wavy lines denote gluon propagators. The diagram obtained by interchange of particles A and B in (a) also contributes to $F^{(2)}$ but the one obtained from (b) by interchange does not.

If the gluon propagator $D_g(k^2)$ has a simple pole at $k^2=0$, i.e., if $D_g(k^2)\sim(k^2)^{-1}$ for $k^2\to 0$, then ordinarily $F^{(2)}$ would have a branch-point singularity at $Q^2=t=0$, arising from the poles in $D_g(k_1^2)$ and $D_g(k_2^2)$. However, one way of implementing the confinement hypothesis, applied to gluons rather than quarks, is to assume that gluon emission or absorption amplitudes vanish when the gluons are on their mass shell. If this vanishing is, on the one hand, simply accomplished by the presence of a factor k^2 for any external gluon line, e.g., if

$$M^{(2)}_{B;g}=k_1^2k_2^2\hat{M}^{(2)}_{B;g}(p_B',p_B;k_1k_2) \qquad (4.5)$$

with $\hat{M}$ regular at $k_i^2=0$, the integrand of (4.4) will be analytic at $k_1^2=0, k_2^2=0$. There will be no singularity at $t=0$ arising from this mechanism and hence no long-range force. Similar remarks hold for the contributions from the exchange of three or more gluons.

On the other hand, it is possible for there to be a long-range force even if the $M^{(n)}_{B;g}$ vanish on the gluon-mass shell. As an example, let us suppose that the two-gluon amplitudes entering (4.4) have the form

$$M^{(2)}_{B;g}=(k_1^2)^b(k_2^2)^b\hat{M}^{(2)}_{B;g} \qquad (4.6)$$

and

$$M^{(2)\,\mathrm{irr}}_{g;A}=(k_1^2)^a(k_2^2)^a\hat{M}^{(2)\,\mathrm{irr}}_{g;A}, \qquad (4.7)$$

where the careted amplitudes are assumed to be regular at $k_i^2=0$ $(i=1,2)$, with $b>0$, $a\geq 0$, and ϵ, defined by

$$\epsilon\equiv a+b, \qquad (4.8)$$

is *not* an integer.[30] Then, as we shall show below, $F^{(2)}$ will have a singularity at $t=0$ despite the fact that $M^{(2)}_{B;g}$ vanishes for $k_i^2=0$ and even if D_g has *no* pole at $k^2=0$. Note that behavior such as (4.6) for the $M^{(n)}$, in contrast to the simple zeros in (4.5), would not be at all unexpected within the framework of QCD. One possible origin of such fractional powers of k^2 or of other functions of k^2 which have a branch point at $k^2=0$ but vanish there, e.g., $k^{2n}\ln k^2$, is in the behavior of the vertex function $\Gamma^{(1)}(k^2)$ describing the emission of a virtual gluon by a quark.[31] This function certainly has a branch point at $k^2=0$, in any order of perturbation theory beyond the first, and so the exact $\Gamma^{(1)}$ is likely to have such a singularity also. If, for example, $\Gamma^{(1)}(k^2)\propto(k^2)^\delta$ near $k^2=0$, with $0<\delta<1$, then with our definition of gluon irreducibility one would expect that $b=\delta$ and $a=0$ so that $\epsilon=\delta$.

B. Analysis

To see the implications for long-range forces of implementing gluon confinement through a behavior such as (4.6) for the hadron-gluon amplitudes, we first write the propagator in the form

$$D_g(k^2)=d_g(k^2)/k^2 \qquad (4.9)$$

and leave $d_g(k^2)$ unspecified for the moment. We substitute (4.6), (4.7), and (4.9) into (4.4) and use the spectral representation

$$\frac{1}{(k^2)^{1-\epsilon}}=\frac{1-e^{2\pi i(1-\epsilon)}}{2\pi i}\int_0^\infty d\zeta\,\frac{\zeta^{-1+\epsilon}}{\zeta-k^2}, \qquad (4.10)$$

valid for $0<\epsilon<1$. We may then rewrite (4.4) in the form

$$F^{(2)}=\text{const}\times\int_0^\infty\int_0^\infty d\zeta_1 d\zeta_2\, g(\zeta_1,\zeta_2)\hat{F}^{(2)}(s,t;\zeta_1,\zeta_2), \qquad (4.11)$$

where $g(\zeta_1,\zeta_2)$ is a weight factor defined by

$$g(\zeta_1,\zeta_2)=d_g(\zeta_1)d_g(\zeta_2)/\zeta_1^{1-\epsilon}\zeta_2^{1-\epsilon} \qquad (4.12)$$

and the careted amplitude $\hat{F}^{(2)}$ is defined by

$$\hat{F}^{(2)}=\text{const}\times\int\int d^4k_1 d^4k_2\,\delta(Q-k_1-k_2)\times\hat{M}^{(2)}_{B;g}\hat{D}_g(k_1^2;\zeta)\hat{D}_g(k_2^2;\zeta)\hat{M}^{(2)\,\mathrm{irr}}_{g;A} \qquad (4.13)$$

with

$$\hat{D}_g(k^2;\zeta)=\frac{d_g(k^2)/d_g(\zeta)}{k^2-\zeta}. \qquad (4.14)$$

The quantity $\hat{F}^{(2)}$ may be interpreted as the amplitude which would arise if quanta 1 and 2 of mass $\sqrt{\zeta_1}$ and $\sqrt{\zeta_2}$ with propagator $\hat{D}_g(k^2;\zeta_1)$ and

$\hat{D}_g(k^2; \zeta_2)$, respectively, were exchanged between A and B and with, e.g., $\hat{M}_{B;g}$ representing the amplitudes for absorption of these two quanta by B. From the viewpoint of the t channel, the process giving rise to $\hat{F}^{(2)}$ is $A+\bar{A}'\to 1+2\to \bar{B}+B'$ so that $\hat{F}^{(2)}$ will have a branch point at $t=\tau$, the threshold for "production" of the quanta 1 and 2:

$$\tau=\tau(\zeta_1, \zeta_2)=(\sqrt{\zeta_1}+\sqrt{\zeta_2})^2 . \tag{4.15}$$

According to (4.11), $F^{(2)}$ is a superposition of the $\hat{F}^{(2)}(s, t; \zeta_1, \zeta_2)$ in which the range of (ζ_1, ζ_2) extends to $(0, 0)$. Since $\tau(0, 0)=0$, $\hat{F}^{(2)}$ itself will get contributions from the exchange of quanta of effective mass zero and will therefore ordinarily be singular at $t=0$. This will lead to a long-range part in the corresponding potential $V_{AB}^{(2)}(r)$.

To find the large-r dependence of $V_{AB}^{(2)}(r)$ we need to compute the discontinuity $[F^{(2)}]$ of $F^{(2)}$ across the cut starting at $t=0$. This is given, near $t=0$, by

$$[F^{(2)}]=\text{const}\times\int_0^\infty\int_0^\infty d\zeta_1 d\zeta_2\, g(\zeta_1, \zeta_2)[\hat{F}^{(2)}], \tag{4.16}$$

where $[\hat{F}^{(2)}]$ is the discontinuity of $\hat{F}^{(2)}$ across the cut starting at $t=\tau(\zeta_1, \zeta_2)$. To find the latter discontinuity we may apply the techniques of generalized unitarity to Eq. (4.13). We assume for the moment that $d_g(k^2)$ is regular in the neighborhood of $k^2=0$, so that the absorptive part of $\hat{D}_g(k^2; \zeta)$ is just $-2\pi i\delta(k^2-\zeta)$ for small enough ζ. On replacing $\hat{D}_g(k_i^2; \zeta_i)$ by $-2\pi i\delta(k_i^2-\zeta_i)\theta(k_i^0)$ $(i=1, 2)$ in (4.14) and replacing also $\hat{M}_{g;A}^{(2)\,\text{irr}}$ by the full amplitude $\hat{M}_{g;A}^{(2)}$ we then get

$$[\hat{F}^{(2)}]=\text{const}\times\int\int d\Phi_2\, \hat{M}_{B;g}^{(2)}\hat{M}_{g;A}^{(2)}+\cdots, \tag{4.17}$$

where

$$d\Phi_2=d^4k_1 d^4k_2\,\delta(k_1^2-\zeta_1)\theta(k_1^0)\times\delta(k_2^2-\zeta_2)\theta(k_2^0)\delta(Q-k_1-k_2) \tag{4.18}$$

is the Lorentz-invariant volume element in the phase space of the pseudogluons 1 and 2. The dots in (4.17) include the contributions to $[\hat{F}^{(2)}]$ from processes involving the virtual emission of three or more gluons.[32] The total phase-space volume Φ_2 is given by

$$\Phi_2(t; \zeta_1, \zeta_2)\equiv\int\int d\Phi_2=(\pi k/W)\theta(t-\tau_+), \tag{4.19}$$

where k and W are the c.m. momentum and energy,

$$k=\frac{(t-\tau_+)^{1/2}(t-\tau_-)^{1/2}}{2t^{1/2}}, \quad W=t^{1/2} \tag{4.20a}$$

and

$$\tau_\pm=(\sqrt{\zeta_1}\pm\sqrt{\zeta_2})^2, \tag{4.20b}$$

with τ_+ coinciding with τ, defined by (4.15). It follows from (4.17) and (4.19) that the integral in (4.17) may be written in the form

$$[\hat{F}^{(2)}]_{2g}=\text{const}\times\Phi_2(t; \zeta_1, \zeta_2)f(t; \zeta_1, \zeta_2), \tag{4.21}$$

where

$$f(t; \zeta_1, \zeta_2)=(\hat{M}_{B;g}^{(2)}\hat{M}_{g;A}^{(2)})_{\text{av}} \tag{4.22}$$

is the average of the indicated product over the direction of the three-momentum $\vec{k}$ of, say, the pseudogluon 1 in the c.m. system of 1 and 2.

We only need the function $f(t; \zeta_1, \zeta_2)$ for small values of its arguments. In the case of QED one encounters, instead of (4.22), the angular average of $M_{B;\gamma}^{(2)}M_{\gamma;A}^{(2)}$ where, e.g., $M_{\gamma;A}^{(2)}$ is the amplitude for emission of two photons by an electrically neutral particle A. We note that because the photons are spin-1 or vector particles, $M_{\gamma;A}^{(2)}$ is a tensor amplitude of the form $F_E T_E^{\mu\nu}+F_M T_M^{\mu\nu}$, where the T's are covariant tensors constructed from the available four-momenta. Current conservation and the neutrality of A then imply that these tensors each involve two powers of the momentum components, which behave like $\sqrt{t}$ for $t\to 0$. As a consequence it turns out that[3]

$$(M_{B;\gamma}^{(2)}M_{\gamma;A}^{(2)})_{\text{av}}\sim\text{const}\times t^2 \tag{4.23}$$

for $t\to 0$. Because the gluons are vector quanta like the photons, a similar behavior might be expected for $f(t; \zeta_1, \zeta_2)$ when ζ_1 and ζ_2 are small, of order t. That is, if we set

$$\zeta_1=ty_1, \quad \zeta_2=ty_2, \tag{4.24}$$

in f and let $t\to 0$ with the y_i fixed in (4.22), the resulting function will have a behavior in t similar to (4.23). To be safe, we shall write

$$f(t; ty_1, ty_2)\sim t^s h(y_1, y_2) \tag{4.25}$$

for $t\to 0$ and assume only that $s\geq 0$. If we now introduce the variables y_1 and y_2 in (4.16), we get, on using (4.10), (4.21), and the fact that $\Phi_2(t; ty_1, ty_2)$ is independent of t, viz.,

$$\Phi_2(t; ty_1, ty_2)=\Phi_2(1; y_1, y_2), \tag{4.26}$$

the result

$$[F^{(2)}]_{2g}\sim\text{const}\times t^{2\epsilon+s}\int_0^\infty\int_0^\infty\frac{dy_1 dy_2}{y_1^{1-\epsilon}y_2^{1-\epsilon}}d_g(ty_1)d_g(ty_2)\Phi_2(1; y_1, y_2)h(y_1, y_2). \tag{4.27}$$

Allowing for a zero of integer order c in $d_g(k^2)$, i.e., $d_g(k^2) \sim (k^2)^{c}$, we get finally

$$[F^{(2)}]_{2g} \sim \text{const} \times t^{2\epsilon + s + 2c} \quad (t \sim 0). \tag{4.28}$$

Since $[F] \propto t^Q$ implies $V \propto r^{-(2Q+3)}$ for large r, we conclude that if the assumptions made in arriving at (4.28) are valid there will be a long-range potential arising from two-gluon exchange given by

$$V_{AB}^{(2)} \sim \text{const} \times r^{-N}, \quad N = 4(\epsilon + c) + 2s + 3. \tag{4.29}$$

We note that a long-range force occurs even if $c = 1$, that is, even if there is no pole in the gluon propagator $D_g = d_g/k^2$. This force arises from the singularities in $M_{g;B}^{(2)}$ that are imposed by Eq. (4.6). Thus the very ansatz that confines the gluons acts to generate the long-range force.

Although in our derivation we have assumed that $d_g(k^2)$ is regular at $k^2 = 0$, this restriction is easily removed. For example, if $d_g(k^2)$ has an algebraic singularity, i.e., $d_g(k^2) \propto (k^2)^c$ near $k^2 = 0$ with c *not* an integer, (4.29) continues to hold because the fractional part of c can be absorbed by a redefinition of ϵ. Similarly, the restriction $0 < \epsilon < 1$ made in order to avoid the appearance of subtraction terms in the spectral representation (4.10) for $(k^2)^{-1+\epsilon}$ is also inessential. If $\epsilon = n' + \epsilon'$ with n' a positive integer and $0 < \epsilon' < 1$, the factor $(k^2)^{n'}$ can be absorbed in $d_g(k^2)$ and the discussion proceeds as before. Of course, if, e.g., c is an integer and $n' + c > 0$, so that there are no poles at $k_i^2 = 0$, there is no long-range force in the limit $\epsilon \to 0$ (or $\epsilon \to 1$). This is because the factor $1 - e^{2\pi i(1-\epsilon)}$ entering (4.10), which has been absorbed in the symbol "const" in Eq. (4.11), vanishes in this limit.

As examples of the use of the result (4.29) note that if $s = 2$ as in the case of QED and D_g has a simple pole at $k^2 = 0$, so that $c = 0$, one gets

$$N = 7 + 4\epsilon. \tag{4.30}$$

If D_g is regular and nonzero at $k^2 = 0$ so that $c = 1$, we get instead

$$N = 11 + 4\epsilon, \tag{4.31}$$

a high-powered result indeed.

The exchange of more than two gluons will of course also give rise to a long-range force if the mechanism considered in this section is operative, but the associated potential will fall off more rapidly than that from two-gluon exchange. The phase-space integral now gives a factor of t^{n-2} so that one finds,[33] from the n-gluon analog of (4.27),

$$[F^{(n)}]_{ng} \sim \text{const} \times t^{n(\epsilon + c) + s_n + n - 2}, \tag{4.32}$$

where s_n is the analog of the power s entering (4.25). Then

$$V_{AB}^{(n)}(r) \sim \text{const} \times r^{-N_n}, \tag{4.33}$$

where

$$N_n = 2n(\epsilon + c) + 2(s_n + n) - 1. \tag{4.34}$$

For the case of QED, a preliminary investigation indicates that $s_n = n$. If we use this value for QCD we get $N_n = 2n(\epsilon + c + 2) - 1$, which would imply, for example, that even if c is zero, N_3 is at least as large as eleven. This result is in disagreement with that of Fujii and Mima,[10] who obtain $N_3 = 7$ and $N_4 = 7$. It appears to us that their results do not come from the three- and four-gluon exchange graphs.

C. Interquark potentials and long-range forces

Some authors have adopted a potential-model approach to long-range forces between composite systems; they have taken the same potentials which are introduced to describe the interaction between the constituents of a bound system, e.g., the quarks and/or antiquarks within a hadron, and used them in higher order to discuss the nature of the potential between two such systems.[7,9-12] As already noted, this approach is inadequate even for the case of two atoms when distances large compared to inverse excitation energies are considered. For the case of hadrons, if the quark-quark potentials arise from more fundamental interactions, then iterations of these potentials may not adequately represent the full effect of these interactions. One encounters also a more exquisite danger: If the potential $V_{q\bar{q}}(r)$ between a quark and an antiquark or the potential $V_{qq}(r)$ between two quarks has a color-independent part, then the effective potential describing the interaction between two ordinary hadrons is likely to be either confining or strongly repulsive at large distances.

We first recall that the asymptotic-freedom property of QCD has been used to argue that at short distances ($r \ll 1$ F) the one-gluon exchange approximation is valid. In this approximation

$$V_{q\bar{q}}(r) \approx \vec{\lambda}_1 \cdot \vec{\lambda}_2^c \, W_{q\bar{q}}(r) \tag{4.35}$$

with

$$W_{q\bar{q}}(r) \approx \alpha_s / r \quad (r \ll 1 \text{ F}). \tag{4.36}$$

Here $\vec{\lambda}^c = -\vec{\lambda}^*$ is the conjugate of the SU(3) matrix-vector $\vec{\lambda}$ and $\alpha_s (\approx 0.2)$ is the running coupling constant of QCD. If one assumes that the potential at large r can be obtained by dressing the free gluon propagator $D^{(0)}(q^2) = (q^2)^{-1}$ then the form (4.35) is unchanged for large r—the function $W_{q\bar{q}}$ is simply no longer given by the Coulomb potential (4.36). For example, if the dressed gluon propagator $D_g(q^2)$ behaves like $(q^2)^{-2}$ for $q^2 \to 0$ one will get

$$W_{q\bar q}(r) \propto r \tag{4.37}$$

for large r, as in the linear-potential model used in the description of the psions as $c\bar c$ bound states. Now this picture also implies that the Dirac character of the potential $V_{q\bar q}(r)$ is that associated with exchange of a spin-1 quantum, corresponding to a factor $\gamma_1 \cdot \gamma_2$ [suppressed in Eq. (4.35)]. This leads to spin-dependent forces of a type which, it has been argued, are not well suited for the interpretation of details of the spectrum of the psions.

There is, however, no convincing demonstration that the confining potential does in fact arise from the exchange of a single dressed gluon. Indeed, an alternative point of view is that at large distances the exchange of many gluons or of more complicated field configurations such as those described by instantons are important; this allows $V_{q\bar q}(r)$ to have an appreciable "scalar" part (independent of Dirac matrices), which is advantageous for the phenomenology.[34] If multigluon exchange is important in determining the large-r limit of the quark-quark potential it is possible that this potential has a color-independent part, just as the multipion exchange potential has an isospin-independent part. We can easily see that this possibility is in gross disagreement with hadron physics.

Consider the interaction between a meson A, composed of a quark q_1 and antiquark $\bar q_2$, and a meson B, composed of a quark q_3 and antiquark $\bar q_4$, within the framework of the nonrelativistic quark model and potential theory. The most general form of the binding potential for A and B is

$$\begin{aligned} V_{q\bar q}(r_{12}) &= U_{q\bar q}(r_{12}) + \vec\lambda_1 \cdot \vec\lambda_2^c W_{q\bar q}(r_{12}), \\ V_{q\bar q}(r_{34}) &= U_{q\bar q}(r_{34}) + \vec\lambda_3 \cdot \vec\lambda_4^c W_{q\bar q}(r_{34}), \end{aligned} \tag{4.38}$$

where $U_{q\bar q}$ is the color-independent part of $V_{q\bar q}$ and $r_{ij} = |\vec r_i - \vec r_j|$; any Dirac matrices entering U or W are suppressed. Similarly, the general form of an interaction potential between the quarks q_1 and q_3 and the antiquarks $\bar q_2$ and $\bar q_4$ is

$$\begin{aligned} V_{qq}(r_{13}) &= U_{qq}(r_{13}) + \vec\lambda_1 \cdot \vec\lambda_3 W_{qq}(r_{13}), \\ V_{\bar q\bar q}(r_{24}) &= U_{\bar q\bar q}(r_{24}) + \vec\lambda_2^c \cdot \vec\lambda_4^c W_{\bar q\bar q}(r_{24}), \end{aligned} \tag{4.39}$$

where $U_{qq}(r) = U_{\bar q\bar q}(r)$ and $W_{qq}(r) = W_{\bar q\bar q}(r)$ by C invariance. The main point now is that if the large-r behavior of $V_{q\bar q}(r)$ is determined by multigluon exchange, then, on the one hand, $U_{q\bar q}$ may well be as important as $W_{q\bar q}$ at large r and, in particular, have similar asymptotic behavior. On the other hand, if $V_{q\bar q}$ is to be confining in a color-singlet state, where $\vec\lambda_1 \cdot \vec\lambda_2^c \to -\frac{16}{3}$, then at least one of the two terms in

$$V_{q\bar q}^{\text{sing}} = U_{q\bar q} - \tfrac{16}{3} W_{q\bar q} \tag{4.40}$$

must be confining. Taken together, these statements suggest that if, say,

$$-\tfrac{16}{3} W_{q\bar q}(r) \sim k_w r^p \tag{4.41a}$$

with $p > 0$, then also

$$U_{q\bar q}(r) \sim k_u r^p \tag{4.41b}$$

and for confinement we need

$$k_u + k_w > 0\,. \tag{4.41c}$$

We note further that the nominal interaction between the constituents of A and those of B is

$$V_{\text{int}} = V_{q\bar q}(r_{14}) + V_{q\bar q}(r_{32}) + V_{qq}(r_{13}) + V_{\bar q\bar q}(r_{24})\,. \tag{4.42}$$

To lowest order in V_{int}, the effective potential describing the scattering of A and B is then given by

$$V_{AB}^{\text{eff}}(R) = \langle \psi_A \psi_B | V^{\text{int}} | \psi_A \psi_B \rangle\,, \tag{4.43}$$

where $\psi_A = \phi_A(\vec r_{12})\chi_A$, $\psi_B = \phi_B(\vec r_{34})\chi_B$ with the ϕ's the spatial wave functions of the bound states and the χ's singlet wave functions in color space. Here $\vec R$ is the separation of the c.m. coordinate of A and that of B, so that, e.g., $\vec r_{14} = (\vec r_{12}/2) + \vec R + (\vec r_{24}/2)$. Since, e.g., $\langle \chi_A | \vec\lambda_1 | \chi_A \rangle = 0$, the W-type terms in V_{int} make no contribution and

$$\begin{aligned} V_{AB}^{\text{eff}}(\vec R) = \langle \phi_A \phi_B | & U_{q\bar q}(r_{14}) + U_{q\bar q}(r_{23}) \\ & + U_{qq}(r_{13}) + U_{\bar q\bar q}(r_{24}) | \phi_A \phi_B \rangle\,. \end{aligned} \tag{4.44}$$

The contribution to $V_{AB}^{\text{eff}}(R)$ from the $U_{q\bar q}$ terms has the form

$$2k_u R^p\,, \tag{4.45}$$

which is confining if $k_u > 0$. The inequality (4.41c) allows for a negative k_u if $k_w > 0$, but then (4.45) corresponds to a long-range repulsion, which is also unacceptable.

Consideration of the contribution from the last two terms in (4.44) does not improve matters. If we assume that

$$U_{qq}(r) \sim k_u' r^p \tag{4.46}$$

for $r \to \infty$, the contribution from these terms to $V_{AB}^{\text{eff}}(R)$ is

$$2k_u' R^p \tag{4.47}$$

at large R. This will cancel the leading contribution (4.45) from the first two terms if and only if

$$k_u' = -k_u . \tag{4.48}$$

However, there seems to be no reason that the equality (4.48) should hold. Moreover, even if there were some hidden property of QCD which leads to (4.48), one would be in trouble. Let us repeat our calculation of the effective potential for the case of the scattering of two baryons C and D. Only $V_{qq}(r_{ij})$ is of interest now, and with a baryon composed of three quarks in a color-singlet state the W-type terms again make no contribution. We then get $V_{CD}^{\text{eff}}(R) \sim 9k_u' R^p$ at large R, which is again unacceptable.

For completeness let us see what the effective potential $V_{AB}^{\text{eff}}(R)$ would be if the U's the color-independent parts of the interquark potentials (4.38) and (4.39), were zero. Then the right-hand side of (4.43) vanishes and V_{AB}^{eff} is obtained by using V_{int} in higher order. With $W_{q\bar{q}}(r) = W_{qq}(r) = kr^p$ the leading contributing term in V_{int} for large separation R is

$$V_{\text{int}}^{(2)} = -pkR^{p-2}\vec{\Lambda}_A \cdot \vec{\Lambda}_B [\vec{r}_A \cdot \vec{r}_B + (p-2)\vec{r}_A \cdot \hat{R}\, \vec{r}_B \cdot \hat{R}], \tag{4.49}$$

where, e.g., $\vec{\Lambda}_A = (m_2\vec{\lambda}_1 - m_1\vec{\lambda}_2^c)/(m_1+m_2)$ and $\vec{r}_A = \vec{r}_1 - \vec{r}_2$. Second-order perturbation theory then gives

$$V_{AB}^{\text{eff}}(R) \sim -C_p/R^{4-2p}, \tag{4.50a}$$

where

$$C_p = \frac{p^2(p^2-2p+3)k_w^2 a^2 b^2}{72\Delta E} \tag{4.50b}$$

with $k_w = 16k/3$ the quantity introduced in (4.41a), $a^2 = \langle \phi_A | r_A^2 | \phi_A \rangle$, $b^2 = \langle \phi_B | r_B^2 | \phi_B \rangle$, and ΔE a mean excitation energy for octet states.[35] For a linear potential we set $p = 1$ and get

$$V_{AB}^{\text{eff}}(R) \sim \text{const}/R^2 . \tag{4.51}$$

From the analysis of Sec. IV B, it is easy to see how these results must be modified to take into account retardation. To simulate an interquark potential which goes as r^p we put $d_g(k^2) \sim (k^2)^c$ with

$$c = -(p+1)/2, \tag{4.52}$$

corresponding to a propagator $D_g(k^2)$ which behaves as $(k^2)^{-(p+3)/2}$ for $k^2 \to 0$. With $s = 2$ and $\epsilon = 0$ in (4.29) this gives, for two-gluon exchange, $V \sim R^{-N}$ with

$$N = 5 - 2p \tag{4.53}$$

rather than $N = 4 - 2p$ as found above from the potential exchange approach. Thus there is an extra power of R^{-1} coming from retardation, as one expects on intuitive grounds, regardless of the precise value of p. In particular, for $p = 1$ we get, instead of (4.50),

$$V_{AB}^{(2)}(r) \sim \text{const}/R^3, \tag{4.54}$$

in agreement with the result of Fujii and Mima,[10] who used dimensional regularization. However, our results for the exchange of more than two gluons do not reduce to those of Ref. 10 for the case considered there, $p = 1$, corresponding to a $(k^2)^{-2}$ propagator. For example, (4.34) gives, with $\epsilon = 0$ and $c = -1$, $N_n = 2s_n - 1$ which is independent of n only if s_n is independent of n—a circumstance which seems highly unlikely, since this is not the case even in QED.

In concluding this section, we stress that the difficulties encountered in the potential exchange picture if the confining $q\bar{q}$ potential has a color-independent part may also occur in the multi-gluon-exchange picture of long-range forces between hadrons. This is because a major contribution to the n-gluon emission amplitude $M_{g;A}^{(n)}$ presumably comes from processes in which the hadron A decomposes into a $q_1\bar{q}_2$ pair, the quark q_1 emits n gluons, and then rejoins the antiquark to reconstitute A. These gluons can be absorbed, in a similar process, by an antiquark $\bar{q}_4$ in hadron B and there arises a contribution to the hadron-hadron amplitude $F(s,t)$ involving integration over a piece of the off-shell $(q_1, \bar{q}_4)$ scattering amplitude coming from n-gluon exchange. If this piece is required to have a confining part in the sense of dispersion theory, i.e., to have spectral function which is highly singular at $t = 0$,[36] then it may also have a confining color-independent part which will show up as a potential tending to confine two hadrons.

V. COMMENTS AND CONCLUSIONS

A. Interpretation of experimental limits

We have seen, on the one hand, that various experiments constrain the existence of strong long-range forces between hadrons and, on the other hand, that the occurrence of such forces is made difficult, although perhaps not impossible, by the apparent nonexistence of massless hadrons as observable physical states. In this final section, we summarize our conclusions about both experiments and theory, and deal with the question of what possibilities remain for long-range forces between hadrons from both standpoints.

We note first that the most plausible mechanisms for generating long-range forces, multi-gluon exchange, give potentials that behaves as r^{-N} with N greater than or equal to seven. However, the experiments described in Sec. II at present give interesting constraints on the potential only when N is six or less. This statement is based on the assumption that the length scale relevant to the hypothetical long-range interactions of hadrons is 1 F. If the length scale is significantly smaller than this, then experiments are correspondingly less sensitive even to inverse powers lower than six. Alternatively, if the length scale is greater than 1 F, existing experiments could be sensitive to long-range forces of the type given by "naive" two-gluon exchange, i.e., to an r^{-7} potential.

An estimate of the length scale for hadronic long-range forces requires a more detailed theory of hadrons then we can discuss here. If the gluons are treated like photons—we shall refer to this as the "naive theory"—and only two-gluon exchange is taken into account, the coefficient of r^{-7} in Eq. (2.1) is determined in analogy with Eq. (1.5), by the effective color polarizabilities α_A and α_B of the two hadrons via

$$r_0^6\lambda_7^{\rm SI} \simeq \alpha_A\alpha_B \,. \tag{5.1}$$

From the discussion of Sec. III C, we might expect that the "magnetic" color polarizability will be given roughly by

$$\alpha_A^m \simeq -g^2 r_0^2/m_q \,, \tag{5.2a}$$

where g is the quark-gluon coupling constant and m_q a quark mass, whereas the "electric" color polarizability α_A^e is given by

$$\alpha_A^e \simeq g^2 r_0^2/\Delta E \,, \tag{5.2b}$$

where ΔE represents some average color-octet excitation energy. These estimates suggest, but hardly prove, that the magnetic color polarizability of hadrons could be greater than the electric polarizability, unlike the case of ordinary atoms. If $g^2/\hbar c$ is of order one, the numerical value for $\lambda_7^{\rm SI}$ suggested by Eq. (5.2a) is also of order one, indicating that a modest improvement in the analysis of exotic-atom transition energies could show the effects predicted by the naive theory.

In Sec. IV we have shown that if the multigluon-emission amplitudes have mass-shell singularities, there can be a long-range force even if the gluon propagators do not have poles (or branch points) at $k^2=0$. In particular, the two-gluon-exchange potential [Eq. (4.29)] was found to fall off more rapidly than r^{-7}, as long as $\epsilon + c > 0$ and, as analogy with QED suggests, $s=2$ [see Eqs. (4.30) and (4.31)]. The experiments we have analyzed are insensitive to such rapidly decreasing forces, unless the length scale is significantly greater than 1 F. This length scale and the strength of these forces now depends on the mass scale of the zeros in the gluon emission amplitude as well as on the color polarizabilities for gluon emission. Conceivably, there is a low mass scale for these zeros, which may involve infrared singularities arising from gluon-gluon interactions. It would be worth keeping this possibility in mind if more precise analyses of radiative transitions in exotic atoms become available.

B. Does the missing-mass spectrum in hadron interactions extend to zero?

The model for gluon confinement discussed in Sec. IV avoids the emission of physical gluon pairs by hadrons. If extended to multigluon amplitudes in an obvious way, it will also avoid the emission of any number of gluons. This means that gluons do not give rise to zero-mass poles in physical amplitudes. Nevertheless, this model is not sufficient to avoid all trace of massless gluons in physical states.[37] The same discontinuities at $k^2=0$, arising from the gluon mass-shell branch points in gluon-emission amplitudes, that give the long-range forces between hadrons will also generate discontinuities in inelastic hadron scattering and in hadronic decay amplitudes arising from graphs involving virtual gluon pairs. These discontinuities, by unitarity, may require interpretation as the production of some type of state from the hadrons. If so, since the spectrum of total mass of the two gluons extends down to zero, the state in question is not composed of the known hadrons. Furthermore, if we analyze the state in terms of the mass of two individual "components," which might be possible through a missing-mass setup, we will find that the mass distribution of these components also extends down to zero. Nevertheless, there will be no pole in k^2 as would occur for physical gluon emission. That is, the missing mass cannot be understood in terms of a state containing some definite number of zero-mass particles. Instead, it would correspond to a superposition of such states, without a δ-function contribution at zero mass.[38]

Such a situation does not seem to contradict any general principles of field theory. One picture of what is happening is that the individual gluons, through their nonlinear self-interaction, couple to multigluon states, eventually containing an infinite number of gluons. The effect of this coupling is to induce the factor $(k^2)^\epsilon$ that we have introduced in Eq. (4.5). This would be somewhat analogous to what happens in QED, where the effects of soft photons is, in some gauges, to eliminate the electron-propagator pole, replacing it by a branch cut. However, there we know that the result is in most circumstances insensibly different from what is obtained by neglecting the soft photons and keeping the electron pole. In the QCD case, that is surely wrong and it is likely that the infrared effects of gluon self-interactions greatly modify the physical effects of the theory.

It would be interesting to analyze experimental data on hadron collisions to see whether the spectrum of missing matter does extend to zero, in a way that cannot be accounted for by photons. Some data on this exist in an earlier search for negative mass-squared particles, emerging from $K^- p$ and $\bar{p}p$ collisions.[39] The data have not been analyzed with this new possibility in mind, however. A determination of the spectrum of missing mass in the neighborhood of $M^2=0$ would shed light on the question of whether a naive picture of confinement such as the one presented above is plausible or not.

C. Some comments on other treatments of strong van der Waals forces

As noted in the Introduction, there have been a number of other discussions of strong van der Waals forces in the past three years. We record here a few comments on some of these other treatments.

We already remarked many years ago that a sharp division of van der Waals forces into retarded and unretarded regions, as occurs in atomic physics, is unlikely to occur elsewhere.[40,5] Since hadrons do not possess excited states with very low excitation energy ($\ll 1\ \mathrm{F}^{-1}$), it is already necessary to use the retarded expressions for the forces when the separation of two hadrons is greater than a few fermis, that is, whenever there is a significant distinction between van der Waals forces and ordinary hadron-exchange forces. This is unlike the atomic case where the unretarded London force can be used over a very wide range beyond that of the chemical forces. This implies that some treatments that rely on nonretarded expressions for strong van der Waals forces are unreliable quantitatively. This conclusion does not depend on the mechanism by which the forces arise but rather on the energy scale over which the hadronic amplitude vary. This is unlikely to be less than a few hundred MeV which implies that retarded forces must be used beyond a few fermis.

These remarks apply also to the ingenious approach of Matsuyama and Miyazawa[11] and of Gavela *et al.*[12] These authors have attempted to treat van der Waals forces in the context of quark confinement and potential exchange. They take into account the large long-range interaction between the intermediate color-octet states, which are linked to a hadron ground state by octet potential exchange, by including it in the energy denominator in the sum over intermediate states. The effect of this is to increase the inverse power in the potential by p, where r^p describes the behavior of the quark-quark potential. So for $p=1$, they obtain an r^{-3} hadron-hadron potential.

Their work is done for instantaneous, nonretarded potentials. Its extension to retardation appears nontrivial. If actual gluons are exchanged, the finite time required for them to travel from one hadron to another means that the two hadrons need not both be in octet states at the same time. Instead, an overall color singlet is obtained by combining gluons with quarks. Therefore, the quark-gluon interaction as well as the quark-quark interaction must be included in the intermediate state. This immediately goes beyond the context of potential exchange, and it is not clear how to proceed with it.

The authors in Refs. 11 and 12 conclude that the nonretarded potential they obtain is too large to agree with experiments on nulceon-nucleon scattering. We agree with their conclusion. Furthermore, from our analysis in Sec. II, we can set a limit on a $\bar{p}$-p force of the types they obtain. For example, if the quark-quark potential is taken to be linear, the authors of Ref. 12 get an inverse-cube potential $V_{\mathrm{eff}}(R)\simeq r_0^2/7R^3$ which, in our notation, implies that $\lambda_3^{\mathrm{SI}}(p,p)\simeq\frac{1}{7}$. Comparing this with the limits in Table IV, we see that a p-$\bar{p}$ potential of this magnitude would be a thousand times larger than allowed by experiment. Moreover, it is also larger by a factor of 10^{11} than the limit implied by the Cavendish experiment (Table II). A similar conclusion is also reached in Ref. 11. It will be interesting to see if the inclusion of retardation can change these conclusions.

Sawada has proposed in several papers[41,14] that an analysis of π-p and p-p scattering data reveals the existence of an R^{-7} potential between these hadrons, with values, expressed in our notation, of $\lambda_7^{\mathrm{SI}}(\pi,p)\sim 25$ and $\lambda_7^{\mathrm{SI}}(pp)\sim 100$. While these results do not directly contradict the results we give in Table IV, they do seem uncomfortably large compared with the limits obtained for $\lambda_7^{\mathrm{SI}}(\bar{p},p)$. Therefore we must be somewhat skeptical of the conclusions drawn by Sawada from scattering data. Proposals to detect R^{-7} van der Waals potential through scattering experiments have also been made by Fishbane and Grisaru.[8]

In this paper we have concentrated on models which may generate long-range forces between hadrons. Nevertheless, it should be mentioned that in some other models of quarks and gluons such forces appear to be absent. For example, in the (semiphenomenological) bag models, gluons are confined to the inside of a hadron bag and cannot propagate to another distant hadron. Therefore no long-range forces between hadrons arise from gluon exchange. Another model of hadrons withoug long-range forces has recently been proposed by Greenberg and Hietarinta.[42] We do not know if such models are consistent with quantum field theory in general or with QCD in particular. However, models in which strong long-range forces between hadrons are completely absent must be given due consideration, especially if future experiments fail to show any indication of such forces.

In summary, the situation with regard to strong long-range forces between hadrons has interesting theoretical and experimental questions associated with it and some surprises may yet result from a more detailed study of these questions.

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*On sabbatical leave from the University of Maryland, College Park, Md. 20740.

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[21a]It is also possible to detect V_1^{SI} by similar experiments using the fact that objects of equal mass but different composition will have different baryon numbers. However, the limit on λ_1^{SI} which might be obtained in this way would not be as low as that obtained from Eötvös-type experiments. It is not possible to obtain information about V_1^{SD} or V_1^{SI} from Cavendish experiments on a single pair of objects. This is because the effective force between two objects arising from these potentials has the same dependence on coordinates as the Newtonian force of gravity. However, one can get information about V_1^{SD} from Cavendish experiments by comparing the forces exerted on one reference object by two different test objects of the same mass but made of different materials. If one test object has nuclear spin zero while the other test object and the reference object both have nonzero nuclear spin then a difference in the apparent value of G will be observed in the two cases if V_1^{SD} is not zero. From Eq. (2.12) it appears that an experiment of this type in which the forces are compared to 10% accuracy could detect a value of λ_1^{SD} as small as 10^{-21}. We are unaware whether such experiments have been performed.
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[22a]In a recent paper [Physica (Utrecht) 96A, 285 (1979)] N. F. Ramsey discusses the limits derivable from these experiments on possible proton-proton tensor interactions that fall off exponentially at large distances.
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[23a]We thank Dr. Fishbane and Dr. Grisaru for informing us that, on recalculation, they agree with Eq. (2.21) of the text.
[24]See, e.g., P. D. Barnes *et al.*, Nucl. Phys. A231, 477 (1974); A. Bamberger *et al.*, Phys. Lett. 33B, (1970); P. Ebersold *et al.*, Nucl. Phys. A296, 493 (1978). For an extensive list of references see Ref. 23.
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vergent loop integral. We shall see below that the dispersion approach can also deal effectively with certain graphs involving infrared divergences.

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[28]This formalism may not be general enough to include effects such as modifications of the vacuum which can also occur in QCD.

[29]The amplitudes $F^{(n)}$ should really carry an additional subscript "A", i.e., be written as $F_A^{(n)}$, because we have organized the diagrams from the viewpoint of hadron A: $F_A^{(n)}$ is the sum of all diagrams in which n gluons are emitted irreducibly by A. The analogous amplitude $F_B^{(n)}$ is not equal to $F_A^{(n)}$ because the gluons may convert to m gluons ($m \neq n$) which may then be absorbed irreducibly by B. Thus $F_A^{(n)}$ contains diagrams not in $F_B^{(n)}$ and vice versa. This asymmetry could be removed in various ways, e.g., by taking the average of $F_A^{(n)}$ and $F_B^{(n)}$, but not much is gained thereby. Note that there is bound to be some arbitrariness in the definitions because of the possibility of a change of the number of gluons in mid-air.

[30]We know of no conflict between the assumption of such branch points in amplitudes such as $M^{(2)}$ and general principles of quantum field theory. On the other hand, we do not know if introducing zeros as in (4.6) can provide a consistent picture of confinement. We discuss some experimental implications of this assumption in Sec. V.

[31]The analyticity properties of Green's functions in QCD have recently been studied from a non-perturbation-theoretic point of view by R. Oehme and W. Zimmermann, Phys. Lett. 79B, 93 (1978).

[32]The total discontinuity of $\hat{F}^{(2)}$ can be written as a sum of two terms, the first being that obtained by replacing the $\hat{D}_g$'s by their absorptive parts with $\hat{M}_B$ and $\hat{M}_A^{\text{irr}}$ left unchanged and the second that obtained by replacing $\hat{M}_B$ by its discontinuity across the cut. Normally, on use of unitarity for $\hat{M}_B$ the overall effect of this is simply to replace $\hat{M}_A^{\text{irr}}$ by $\hat{M}_A$ in the first term apart from contributions from more complicated intermediate states. However, because of the hybrid character of the careted amplitudes, there may be additional contributions; these should also be regarded as included in the dots on the right-hand side of Eq. (4.17).

[33]The fact that only the sum $\epsilon + c$ appears in (4.32) indicates that for the purpose of computing long-range forces the decision as the whether zeros occur in the emission amplitudes or in the propagators is somewhat arbitrary. A resolution of this ambiguity might be possible if processes involving the emission of real gluons could be studied.

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[35]Although $V_{\text{int}}^{(2)}$ depends explicitly on the ratio of quark masses within each hadron, $V_{\text{int}}^{\text{eff}}$ involves the quark masses only implicitly, via ΔE, because $\langle(\vec{\Lambda}_A \cdot \vec{\Lambda}_B)^2\rangle$ is independent of these ratios, in a product color-singlet state. Formulas similar to (4.50) are given in Refs. 7 and 12.

[36]A linear potential corresponds to a spectral function $\rho_0(t) = \lim_{a\to 0} \rho_a(t)$, where $\rho_a(t) = (\partial^2/\partial a^2)\,\delta(t-a^2) = -2\delta'(t-a^2) + 4a^2\delta''(t-a^2)$.

[37]We thank A. Mueller for pointing this out.

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