$SU(4)_F \times SU(4)_C$ synthesis of strong, weak, and electromagnetic interactions

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The gauge group $[SU(2)_L \times SU(2)_R]_F \times SU(4)_C$ is extended to $SU(4)_F \times SU(4)_C$ and the emergence of the low-energy symmetry $SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$ is discussed.

I. INTRODUCTION

The extension of the standard theory of weak and electromagnetic interactions based on the gauge group[1] $SU(2)_L \times U(1)$ to the left ⟷ right symmetric group (Ref. 2) $SU(2)_L \times SU(2)_R \times U(1)_{L+R}$ was to accommodate whatever possible outcome of the experimental results on parity in heavy atoms (bismuth). In the light of the recent SLAC experiment[3] that the neutral current is parity violating to the extent predicted by the standard theory, such an extension seems unnecessary. However, if the possibility of a "light" neutral gauge boson is to be realized, at the same time taking into account the SLAC experiment, then the introduction of $SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$ gauge structure[4] on the left ⟷ right symmetry principle is only natural. On the other hand, if the motivation is the aesthetic desire to have a superunified gauge theory of strong, weak, and electromagnetic interactions with one coupling constant, e.g., chiral flavor × chiral color based on the gauge group[5] $[SU(4)]^4$, then the emergence of the low-energy symmetry $SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$ occurs as an inevitable by-product of the spontaneous symmetry breaking. The noteworthy features of this low-energy symmetry are as follows:

(i) All neutrino phenomenology is identical to that of the standard Weinberg-Salam model.

(ii) Neutral currents are necessarily parity violating as required by the recent SLAC experiment.

(iii) It offers the attractive possibility of a light neutral gauge boson detectable at PEP and PETRA.

The above features are entertained in the $SU(4)_{\text{flavor}} \times SU(4)_{\text{color}}$ gauge theory[6,7] of strong, weak, and electromagnetic interactions. This gauge group is an extension of the $SU(2)_L \times SU(2)_R \times SU(4)^c_{L+R}$ Pati-Salam group[8] and an economic version of the superunified theory $[SU(4)]^4$ with one gauge coupling constant.[5] In passing, it is to be noted that grand unified schemes are far from complete. Experimental data are still inadequate to suggest any hard core clues to the superunifying structure. The possibilities are several; SO(10), E_7, SU(5), and SU(16) have been proposed.[9]

We proceed with a detailed description of the mode.

II. THE MODEL

Gauge bosons

The model is based on the gauge group $G = SU(4)_F \times SU(4)_C$ (F = flavor, C = color) and is a minimal extension of the Pati-Salam model based on the gauge group $[SU(2)_L \times SU(2)_R]_F \times SU(4)_C$ with a discrete symmetry flavor ⟷ color such that the theory has only one coupling constant. This single coupling constant is denoted by g and f in the flavor and color sectors, respectively, since the single coupling constant gets renormalized differently in the two sectors. In this sense, $G = SU(4)_F \times SU(4)_C$ serves to unify the basic forces of nature.

The gauge bosons mediating flavor interactions are, for convenience, represented by the following matrix in flavor space:

$$\frac{\lambda \cdot W}{2} = \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{1}{2}(\sqrt{2}W^0_{\text{I}} + U^0) & W^-_{\text{I}} & W^-_K & W^0_X \\ W^+_{\text{I}} & \frac{1}{2}(-\sqrt{2}W^0_{\text{I}} + U^0) & W^0_K & W^+_X \\ W^+_K & W^0_K & \frac{1}{2}(-\sqrt{2}W^0_{\text{II}} - U^0) & W^+_{\text{II}} \\ W^0_X & W^-_X & W^-_{\text{II}} & \frac{1}{2}(\sqrt{2}W^0_{\text{II}} - U^0) \end{bmatrix}. \quad (2.1)$$

The block diagonal matrices represent the subgroup $G_{\rm sub} = SU(2)^{\rm I} \times SU(2)^{\rm II} \times U(1)^{\rm I+II}$, where $(W^+_{\rm I,II}, W^-_{\rm I,II}, W^0_{\rm I,II}) \equiv SU(2)^{\rm I,II}$ and $U^0 \equiv U(1)^{\rm I+II}$. Similarly, the gauge bosons mediating color interactions are represented by the matrix

$$\frac{\lambda \cdot V}{2} = \frac{1}{\sqrt{2}} \begin{bmatrix} V_{11} & V_\rho^- & V_K^- & \overline{X}^0 \\ V_\rho^+ & V_{22} & V_K^0 & X^+ \\ V_K^+ & V_K^0 & V_{33} & X'^+ \\ X^0 & X^- & X'^- & V_{44} \end{bmatrix}, \tag{2.2}$$

where

$$\begin{aligned} V_{11} &= -\frac{V_3}{\sqrt{2}} - \frac{V_8}{\sqrt{6}} - \frac{V_{15}}{\sqrt{12}}, \\ V_{22} &= \frac{V_3}{\sqrt{2}} - \frac{V_8}{\sqrt{6}} - \frac{V_{15}}{\sqrt{12}}, \\ V_{33} &= 2\frac{V_8}{\sqrt{6}} - \frac{V_{15}}{\sqrt{12}}, \\ V_{44} &= 3\frac{V_{15}}{\sqrt{12}}; \end{aligned} \tag{2.3}$$

V_{15} is the 15th generator of $SU(4)_C$. In what follows V_{15} will be identified with S^0 of Ref. 8. S^0 is a singlet of the usual $SU(3)_C$ space. The group structure $G_W = G_{\rm sub} \times U(1)_C$, $G_{\rm sub}$ from $SU(4)_F$, and $U(1)_C$ $[\equiv S^0]$ from $SU(4)_C$, is postulated to be the low-energy symmetry of weak and electromagnetic interactions.

Charge operator

The charge operator in the fundamental representations (4,1) and (1,4) takes the respective forms

$$Q_F(4,1) = W^0_{\rm I} + W^0_{\rm II} = \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}, \tag{2.4}$$

and either

$$Q_C(1,4) = V_3 + \frac{V_8}{\sqrt{3}} - (\tfrac{2}{3})^{1/2} S^0 = \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix} \tag{2.5}$$

for integer charges, or

$$Q_C(1,4) = -(\tfrac{2}{3})^{1/2} S^0 = \begin{bmatrix} -\frac{1}{6} & 0 & 0 & 0 \\ 0 & -\frac{1}{6} & 0 & 0 \\ 0 & 0 & -\frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix} \tag{2.6}$$

for fractional charges.

The $Q_C(1,\bar{4})$ is simply the negative of the $Q_C(1,4)$. The structure of the photon is easily written down once Q is known. It is

$$\frac{A}{e} = \frac{1}{g}(W^0_{\rm I} + W^0_{\rm II}) - \left(\frac{2}{3}\right)^{1/2} \frac{S^0}{f_{15}}, \quad \frac{1}{e^2} = \frac{2}{g^2} + \frac{2}{3 f_{15}{}^2}, \tag{2.7}$$

for fractionally charged quarks, and

$$\begin{aligned} \frac{A}{e} &= \frac{1}{g}(W^0_{\rm I} + W^0_{\rm II}) + \frac{1}{f_s}\left(V_3 + \frac{V_8}{\sqrt{3}}\right) - \left(\frac{2}{3}\right)^{1/2} \frac{S^0}{f_{15}}, \\ \frac{1}{e^2} &= \frac{2}{g^2} + \frac{2}{3 f_{15}{}^2} + \frac{4}{3 f_s{}^2}, \end{aligned} \tag{2.8}$$

for integer-charged quarks. [f_s = strong coupling constant, f_{15} = hyperchargelike coupling constant identified with the V_{15} generator (see Sec. IV).]

Fermions

The fermions of the theory can be grouped into either the (4,4) or the $(4,\bar{4})$ representation of $G = SU(4)_F \times SU(4)_C$. Within each representation quarks can be either integer charged or fractionally charged. These charge assignments can be deduced from Eqs. (2.7) and (2.8). Physical considerations, i.e., charge of the proton, rule out the assignment of fermions in the (4,4) representation. However, the assignment of scalar Higgs fields in the (4,4) representation is particularly useful in the process of spontaneous symmetry breaking. The (4,4) representation[10] allows vacuum expectation values not available in the $(4,\bar{4})$ thus providing different modes which a particular symmetry can break into (see below). Additional constraints such as freedom from anomalies[11] require the following multiplets:

The electronic multiplet

$$F^e_L = \begin{bmatrix} p_r & p_y & p_b & \nu_e \\ n_r & n_y & n_b & e^- \\ \Lambda_r & \Lambda_y & \Lambda_b & E^- \\ C_r & C_y & C_b & E^0 \end{bmatrix}_L, \quad F^e_R = \begin{bmatrix} C_r & C_y & C_b & E^0 \\ \Lambda_r & \Lambda_y & \Lambda_b & E^- \\ n_r & n_y & n_b & e^- \\ p_r & p_y & p_b & \nu_e \end{bmatrix}_R. \tag{2.9}$$

In such a scheme, where the right-handed fermions are flipped relative to the left-handed ones, the fermion mass matrix takes the form

$$\overline{F}^e_R M^e F_L = \overline{F}^e_R \begin{bmatrix} 0 & 0 & 0 & m_C \\ 0 & 0 & m_\Lambda & 0 \\ 0 & m_n & 0 & 0 \\ m_p & 0 & 0 & 0 \end{bmatrix} F^e_L. \tag{2.10}$$

Its origin in a Yukawa coupling will be discussed in the section on spontaneous symmetry breaking (Sec. III).

The muonic multiplet

$$F_L^\mu=\begin{bmatrix} c_r & c_y & c_b & \nu_\mu \\ \lambda_r & \lambda_y & \lambda_b & \mu^- \\ \mathfrak{N}_r & \mathfrak{N}_y & \mathfrak{N}_b & M^- \\ \mathcal{P}_r & \mathcal{P}_y & \mathcal{P}_b & M^0 \end{bmatrix}_L, \quad F_R^\mu=\begin{bmatrix} \mathcal{P}_r & \mathcal{P}_y & \mathcal{P}_b & M^0 \\ \mathfrak{N}_r & \mathfrak{N}_y & \mathfrak{N}_b & M^- \\ \lambda_r & \lambda_y & \lambda_b & \mu^- \\ c_r & c_y & c_b & \nu_\mu \end{bmatrix}_R . \tag{2.11}$$

Possibly the τ multiplet

$$F_L^\tau=\begin{bmatrix} \Upsilon_r & \Upsilon_y & \Upsilon_b & \nu_\tau \\ b_r & b_y & b_b & \tau^- \\ B_r & B_y & B_b & L^- \\ T_r & T_y & T_b & L^0 \end{bmatrix}_L, \quad F_R^\tau=\begin{bmatrix} T_r & T_y & T_b & L^0 \\ B_r & B_y & B_b & L^- \\ b_r & b_y & b_b & \tau^- \\ \Upsilon_r & \Upsilon_y & \Upsilon_b & \nu_c \end{bmatrix}_R , \tag{2.12}$$

if τ has its own lepton number.

In nature the right-handed current interactions of observed hadrons and leptons are very much suppressed relative to the left-handed ones. If $(W_{\rm I}^0, W_{\rm I}^+, W_{\rm I}^-)$ bosons mediate the known left-handed current interactions and the $(W_{\rm II}^0, W_{\rm II}^+, W_{\rm II}^-)$ gauge bosons mediate the as yet unknown right-handed current interactions, then the known left-handed fermionic doublets $(pn)_{r,y,b}(\nu_e e^-)$ are required to be "flipped" relative to the right-handed ones. Hence the arrangement of the fermions into the $F_{L,R}^e$, $F_{L,R}^\mu$, and $F_{L,R}^\tau$ multiplets.

In view of the recent experimental developments, the idea of new quantum numbers $(\mathcal{P}, \mathfrak{N}, T, B, b, \Upsilon, E^0, E^-, M^0, M^-, L^0, L^-)$, which are required to fill the above fermionic multiplets, is perhaps not so intolerable after all. That their interactions are not felt at present energies is attributed to their being much heavier than ordinary quarks and leptons (>5 GeV).

III. PARTICLES AND MASSES

Here we demonstrate a scheme of spontaneous symmetry breaking for generating masses of particles and leading to eigenstates that give rise to currents describing interactions consistent with present-day phenomenology. For convenience, the scheme of spontaneous symmetry breaking is divided into two stages, i.e., primary stage and secondary stage, which are discussed below.

Primary stage

In this stage of symmetry breaking, $G=\mathrm{SU}(4)_F \times \mathrm{SU}(4)_C$ descends to the intermediate symmetry $G_{\rm int}=[\mathrm{SU}(2)_{\rm I}\times \mathrm{SU}(2)_{\rm II}\times \mathrm{U}(1)_{\rm I+II}]_F\times[\mathrm{SU}(3)\times \mathrm{U}(1)]_C$. Two multiplets of scalar Higgs fields that transform as the adjoint representation of SU(4) are needed.[12]

In the flavor sector, the scalar Higgs multiplet H_F with vacuum expectation values $\langle H_F\rangle$,

$$H_F\sim(15,1), \quad \langle H_F\rangle=\frac{h_f}{2\sqrt{2}}\begin{bmatrix} 1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{bmatrix}, \tag{3.1}$$

breaks $\mathrm{SU}(4)_F$ down to $\mathrm{SU}(2)_{\rm I}\times\mathrm{SU}(2)_{\rm II}\times \mathrm{I}(1)_{\rm I+II}$. The particles $W_K(W_K^\pm, W_K^0, W_{\bar K}^0)$ and $W_X(W_X^\pm, W_X^0, W_{\bar X}^0)$ acquire masses while the particles in the block diagonal matrices in (2.1) remain massless. In the color sector, the scalar Higgs multiplet H_C with vacuum expectation values $\langle H_C\rangle$,

$$H_C\sim(1.15), \quad \langle H_C\rangle=\frac{h_c}{4\sqrt{2}}\begin{bmatrix} 1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-3\end{bmatrix}, \tag{3.2}$$

breaks $\mathrm{SU}(4)_C$ down to $\mathrm{SU}(3)\times\mathrm{U}(1)$. The particles $X\,(X^\pm, X'^\pm, X^0, \bar X^0)$ acquire masses, while the octet of gluons and a singlet S^0 remain massless.

Secondary stage

In this state of symmetry breaking the residue symmetry

$$G_{\rm int}=[\mathrm{SU}(2)_{\rm I}\times\mathrm{SU}(2)_{\rm II}\times\mathrm{U}(1)_{\rm I+II}]_F\times[\mathrm{SU}(3)\times\mathrm{U}(1)]_C \tag{3.3}$$

descends further to $\mathrm{U}(1)_{\rm em}$ or to $\mathrm{U}(1)_{\rm em}\times\mathrm{SU}(3)_C$ depending on taste, i.e., whether quantum chromodynamics (QCD) is liberated (integer-charged quarks and light-mass gluons) or confined (fractionally charged quarks, massless and neutral gluons). The set of scalar Higgs fields with their transformation properties and postulated vacuum expectation values are displayed below:

$$\Phi\sim(6,1), \quad \langle\Phi\rangle=\frac{\phi}{2}\begin{bmatrix} 0&1&0&0\\-1&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}, \tag{3.4}$$

$$\Psi\sim(1,6), \quad \langle\Psi\rangle=0 \quad (4\times4 \text{ matrix}), \tag{3.5}$$

$$S\sim(4,4), \quad \langle S\rangle=\begin{bmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&\sigma\\0&0&0&0\end{bmatrix}. \tag{3.6}$$

$$R \sim (4,4), \quad \langle R\rangle = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.7}$$

($C=0$ for the case of fractionally charged quarks)

$$C \sim (4,\overline{4}), \quad \langle C\rangle = \begin{bmatrix} c & 0 & 0 & 0 \\ 0 & c & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & c_4 \end{bmatrix}, \tag{3.8}$$

$$A \sim (15,1), \quad \langle A\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 0 & a_1 \\ 0 & 0 & a_2 & 0 \\ 0 & a_3 & 0 & 0 \\ a_4 & 0 & 0 & 0 \end{bmatrix}. \tag{3.9}$$

These fields play the following roles:

(a) The scalar Higgs multiplet $\phi \sim (6,1)$ gives masses to W_K, W_X, and U^0. This breaks $G_{\rm int}$ down to $G^1_{\rm int} = [SU(2)_{\rm I} \times SU(2)_{\rm II}]_F \times [SU(3)\times U(1)]_C$.

(b) The scalar Higgs multiplet $S \sim (4,4)$ gives masses to $W^{\pm}_{\rm II}$ and one linear combination of neutral gauge fields. This breaks $G^{(1)}_{\rm int}$ down to $G^{(2)}_{\rm int}$

$$G^{(2)}_{\rm int} = SU(2)_{IF} \times U(1)_{F+C} \times SU(3)_C .$$

(c) The scalar Higgs multiplet $R \sim (4,4)$ gives masses to $W^{\pm}_{\rm I}$ and another linear combination of the neutral gauge fields to break the leftover symmetry from (b) down to $G^{(3)}_{\rm int}$,

$$G^{(3)}_{\rm int} = U(1)_{\rm em} \times SU(4)_C . \tag{3.10}$$

(d) Depending on taste, if massless gluons are desired, then the scalar Higgs field $C \sim (4,\overline{4})$ is not needed, but if gluons have a light mass ($m_V \ll m_W$), then C not only gives masses to gluons but also mixes the weak and strong gauge bosons leading to unstable quarks

$$G^{(3)} \Rightarrow G^{(4)} = U(1)_{\rm em} . \tag{3.11}$$

(e) The role of the scalar Higgs multiplet A is to mix the $W_{\rm I}$ and $W_{\rm II}$ gauge bosons and provide masses for the fermions while still preserving the U(1) electromagnetism attained in (d).

(f) The Higgs scalar multiplet $\psi \sim (1,6)$ preserves discrete flavor $\leftrightarrow$ color symmetry in the Higgs sector prior to spontaneous symmetry breaking. ψ develops zero or vanishingly small vacuum expectation values for global SU(3) color symmetry to emerge.

Figure 1 summarizes the effect of the various scalar Higgs fields employed in the spontaneous-symmetry-breaking process.

The full Lagrangian of the theory, for completeness, is

$$\mathcal{L} = {\rm Tr}[i\overline{F}^e_L\gamma_\mu\nabla_\mu F_L + i\overline{F}^e_R\gamma_\mu\nabla_\mu F^e_R + (e\to\mu) + (\nabla_\mu H_F)^2 + (\nabla_\mu H_C)^2 + \nabla_\mu\Phi\nabla_\mu\Phi^\dagger + (\nabla_\mu A)^2 + \nabla_\mu R^\dagger\nabla_\mu R$$
$$+ \nabla_\mu S^\dagger\nabla_\mu S + \nabla_\mu C^\dagger \nabla_\mu C - V(H_F, H_C, \Phi, A, R, C, S) + (Y^e\overline{F}^e_R A F^e_L + {\rm H.c.}) + (e\to\mu)] ,$$

where

$$\begin{aligned}
&\nabla_\mu F^e_{L,R} = \partial_\mu F^e_{L,R} + igW_\mu F^e_{L,R} - ifF^e_{L,R}V_\mu , \\
&\nabla_\mu H_F = \partial_\mu H_F + ig[W_\mu, H_F] , \\
&\nabla_\mu H_C = \partial_\mu H_C + if[V_\mu, H_C] , \\
&\nabla_\mu \Phi = \partial_\mu\Phi + igW_\mu\Phi + ig\Phi W^* , \\
&\nabla_\mu \Psi = \partial_\mu\Psi + ig\Psi_\mu + igV^*_\mu\Psi , \\
&\nabla_\mu R = \partial_\mu R + igW_\mu R + ifRV_\mu , \\
&\nabla_\mu S = \partial_\mu S + igW_\mu S + ifSV_\mu , \\
&\nabla_\mu C = \partial_\mu C + igW_\mu C - ifCV_\mu .
\end{aligned} \tag{3.12}$$

The Higgs part of the Lagrangian leads to the particle masses in the usual way. Here it is assumed that the self-interaction among the various scalar fields in V can be arranged to get the fields $\{H_F, H_C, \phi, \psi, A, R, C, S\}$ to develop the vacuum expectation values in (3.12). The above Lagrangian is not only left $\leftrightarrow$ right symmetric but flavor $\leftrightarrow$ color symmetric prior to spontaneous symmetry breaking.

Also the discrete symmetry $H_F \to H_F$ and $H_C \to H_C$ forbids Yukawa couplings ${\rm Tr}(\overline{F}_R H_F H_L)$ and ${\rm Tr}(\overline{F}_R F_L H_C)$ so as not to generate unreasonable fermion masses. The interaction mass Lagrangian leads to the following eigenstates and masses after diagonalization. Here we have treated the case of fractionally charged quarks, i.e., $c = c_4 = 0$:

(i) States $X^0, \overline{X}^0, X^{\pm}, X'^{\pm}$, $\quad m_X{}^2 = (f_{15}{}^2/2)(h_c{}^2 + \rho^2 + \sigma^2)$, $\quad$ (3.13)

(ii) $K^{\pm}_W = W^{\pm}_K \cos\alpha + W^{\pm}_X \sin\alpha$,

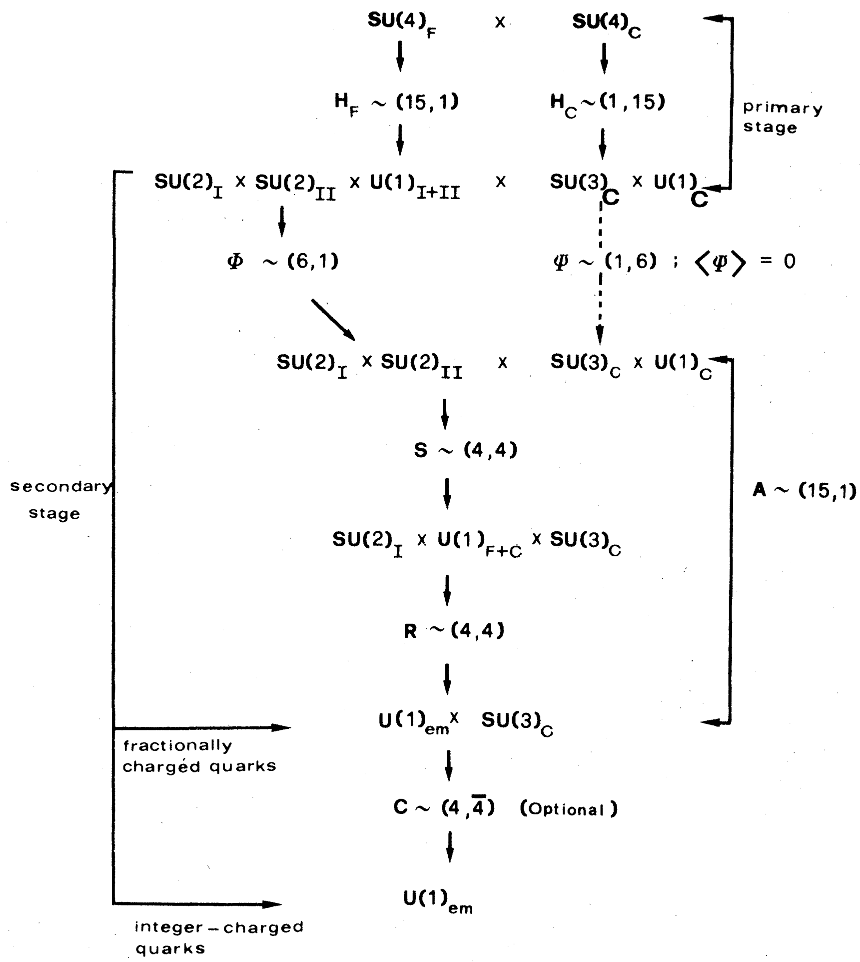

FIG. 1. Spontaneous-symmetry-breaking scheme. The Higgs multiplet $A \sim (15.1)$ provides additional masses to the weak gauge boson and also mixes the W_{I} and W_{II} gauge particles. Neutrino phenomenology restricts $\langle A\rangle \sim 0$ if the model has to have the same predictions as the Weinberg-Salam model.

$$
\begin{aligned}
&X_W^{\pm} = W_X^{\pm}\cos\alpha - W_K^{\pm}\sin\alpha \,, \\
&(2/g^2)m^2(K_W^{\pm}) = (h_f{}^2+\sigma^2+\phi^2+\tfrac{1}{2}a^2)\cos^2\alpha + (h_f{}^2+\rho^2+\tfrac{1}{2}a^2+\phi^2)\sin^2\alpha - \tfrac{1}{2}\sin 2\alpha(a_1a_3+a_2a_4)\,, \\
&(2/g^2)m^2(X_W^{\pm}) = (h_f{}^2+\sigma^2+\phi^2+\tfrac{1}{2}a^2)\sin^2\alpha + (h_f{}^2+\rho^2+\tfrac{1}{2}a^2+\phi^2)\cos^2\alpha + \tfrac{1}{2}\sin 2\alpha(a_1a_3+a_2a_4)\,, \\
&\tan 2\alpha = (a_1a_3+a_2a_4)/(\rho^2-\sigma^2)\,,
\end{aligned} \tag{3.14}
$$

$$
\begin{aligned}
\text{(iii)}\quad &K_W^0 = (1/\sqrt{2})(W_K^0 + W_{\bar K}^0)\,, \\
&K_{\bar W}^0 = (1/\sqrt{2})(W_K^0 - W_{\bar K}^0)\,, \\
&(2/g^2)m^2(K_W^0) = [h_f{}^2+\rho^2+\sigma^2+\phi^2+\tfrac{1}{2}(a_2-a_3)^2]\,, \\
&(2/g^2)m^2(K_{\bar W}^0) = [h_f{}^2+\rho^2+\sigma^2+\phi^2+\tfrac{1}{2}(a_2+a_3)^2]\,,
\end{aligned} \tag{3.15}
$$

$$
\begin{aligned}
\text{(iv)}\quad &X_W^0 = (1/\sqrt{2})(W_X^0 + W_{\bar X}^0)\,, \\
&X_{\bar W}^0 = (1/\sqrt{2})(W_X^0 - W_{\bar X}^0)\,, \\
&(2/g^2)m^2(X_W^0) = h_f{}^2+\phi^2+\tfrac{1}{2}(a_1-a_4)^2\,, \\
&(2/g^2)m^2(X_{\bar W}^0) = h_f{}^2+\phi^2+\tfrac{1}{2}(a_1+a_4)^2\,,
\end{aligned} \tag{3.16}
$$

(v) $W_A^{\pm} = W_{\mathrm{I}}^{\pm}\cos\beta + W_{\mathrm{II}}^{\pm}\sin\beta\,,$

$$\begin{aligned}
&W_B^{\pm} = W_{\mathrm{II}}^{\pm}\cos\beta - W_{\mathrm{I}}^{\pm}\sin\beta\,, \\
&(2/g^2)m^2(W_A^{\pm}) = (\tfrac{1}{2}a^2+\rho^2)\cos^2\beta + (\tfrac{1}{2}a^2+\sigma^2)\sin^2\beta - \tfrac{1}{2}\sin 2\beta(a_1a_2+a_3a_4)\,, \\
&(2/g^2)m^2(W_B^{\pm}) = \left(\frac{a^2}{2}+\rho^2\right)\sin^2\beta + \left(\frac{a^2}{2}+\sigma^2\right)\cos^2\beta + \tfrac{1}{2}\sin 2\beta(a_1a_2+a_3a_4)\,, \\
&\tan 2\beta = (a_1a_2+a_3a_4)/(\sigma^2-\rho^2)\,.
\end{aligned} \tag{3.17}$$

(vi) The octet of color gluons V_{ij} $(i,j=1,2,3)$. These are massless in the case we are considering.
(vii) The four neutral eigenstates; the photon

$$A/e = (1/g)(W_{\mathrm{I}}^0 + W_{\mathrm{II}}^0) - (\tfrac{2}{3})^{1/2}S^0/f_{15}\,,\quad 1/e^2 = 2/g^2 + 2/3f_{15}^{\,2}\,, \tag{3.18}$$

and the three massive eigenstates of the mass matrix $(r = f_{15}^{\,2}/g^2)$

$$\frac{4M^2}{g^2} = \begin{array}{c}
\begin{array}{ccc} \qquad Y \qquad\qquad & \qquad W_{(-)} \qquad\qquad & \qquad U_0 \qquad \end{array} \\
\left[\begin{array}{ccc}
(3r+1)(\rho^2+\sigma^2) & (3r+1)^{1/2}(\rho^2-\sigma^2) & (3r+1)^{1/2}(\sigma^2-\rho^2) \\
(3r+1)^{1/2}(\rho^2-\sigma^2) & (\rho^2+\sigma^2+a^2) & (a_1^{\,2}+a_4^{\,3})-(a_3^{\,2}+a_2^{\,2})-(\rho^2+\sigma^2) \\
(3r+1)^{1/2}(\sigma^2-\rho^2) & (a_1^{\,2}+a_4^{\,2})-(a_3^{\,2}+a_2^{\,2})-(\rho^2+\sigma^2) & (\rho^2+\sigma^2+a^2+2\phi^2)
\end{array}\right]
\end{array} \tag{3.19}$$

in the bases

$$\frac{Y}{e} = \frac{1}{\sqrt{3}f_{15}}(W_{\mathrm{I}}^0 + W_{\mathrm{II}}^0) + \frac{\sqrt{2}}{g}S^0,\quad W_{(-)} = \frac{1}{\sqrt{2}}(W_{\mathrm{I}}^0 - W_{\mathrm{II}}^0),\ \text{and } U^0\,. \tag{3.20}$$

$W_{(-)}$ and U^0 couple to pure axial-vector currents, while Y couples to pure vector currents. The Yukawa coupling $Y^e\overline{F}_R^e A F_L^e$ + H.c. leads to

$$\begin{aligned}
m_G &= Y^e a_1\,, \\
m_\Lambda &= Y^e a_2\,, \\
m_n &= Y^e a_3\,, \\
m_p &= Y^e a_4\,.
\end{aligned} \tag{3.21}$$

In this approximation the mass of the electron and its neutrino are the same as the n and p quarks. However, further contributions due to loop corrections will lift the degeneracy. This will be dealt with elsewhere.

IV. CONSTRAINTS FROM PHENOMENOLOGY

The $K_L \to \mu^+\mu^-$ amplitude is known to be small $\sim G_F\alpha^2$. Events of the type $K_L \to \mu^{\pm} + e^{\mp}$ are suppressed even further. The X $(X^{\pm}, X'^{\pm}, X^0, \overline{X}^0)$ particles induce events of the type $K^0 \to \mu^- + e^+$ with effective strength[13] $f_{15}^{\,2}/m_X^{\,2}$. This implies $m_X \sim 10^4$ GeV with $f_{15}^{\,2}/4\pi \sim \alpha$, which further implies $h_c \sim 10^5$ GeV.

From the absence of deep-inelastic neutrino production of neutral heavy leptons $(E^0, E^-, M^0, M^-, \ldots)$ the masses of the $W_X[W_X^{\pm}, W_X^0, \overline{W}_X]$ and $W_K[W_K^{\pm}, W_K^0, W_K^0]$ are restricted to be greater than 10^4–10^5 GeV. This sets limits on $h_f \sim 10^5$–10^6. Here $g^2/4\pi$ is taken to be of the order of α. Thus $\langle H_F\rangle$ and $\langle H_C\rangle$ are of comparable magnitude.

Since H_F and H_C are the only fields with such high vacuum expectation values, this has the effect[14] of splitting the SU(4)$_C$ gauge coupling constant into the strong coupling constants f_s [with the SU(3) color] and f_{15} (which is associated with the S^0) after renormalization,

$$f \xrightarrow{H_C} \begin{cases} f_s \equiv \text{octet of gluons} \\ f_{15} \equiv S_0 (\equiv V_{15} \text{ of SU(4)}_C) \end{cases} \tag{4.1}$$

Constraints from measurements on the Michel parameter and β decay[15] show that the right-handed current interactions mediated by the $W_B^{\pm}$ gauge boson for the p and n quarks can be as high as 13% of that of the left-handed current interactions. This in turn implies that the $W_B^{\pm}$ charged bosons could be as low as $3m_{W_A^{\pm}}$ and the mixing angle β is $\sim\frac{1}{20}$. Here $m_{W_A^{\pm}}$ is the mass of the gauge boson mediating the bulk of the left-handed current interactions, i.e.,

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_{W_A^{\pm}}^{\;2}}\,.$$

The implication of all this is that

$$\rho \sim (G_F)^{-1/2} \sim 300 \text{ GeV}\,, \tag{4.2}$$

$$\frac{\rho^2}{\sigma^2} \sim \tfrac{1}{10} \sim \tfrac{1}{15} \text{ (for small } \langle A\rangle \text{ justified later).} \tag{4.3}$$

The role of the scalar Higgs fields in neutrino neutral-current interactions has been studied extensively on general grounds by several authors. It has been found[16] that if in a gauge theory the scalar Higgs fields are chosen in such a way that there is vanishingly small or no mixing between the gauge bosons mediating left and right current interactions, then the neutrino neutral-current phenomenology of the model will be the same as that of the standard Weinberg-Salam model.

The scalar Higgs field A gives masses as well as mixing the W_{I} and W_{II} gauge bosons, among others, and thus spoils the agreement of neutrino interactions with the standard model. In the present model $\langle A\rangle$ gives masses to the fermions and they are not zero. Thus we shall limit $\langle A\rangle$ to be sufficiently small ($a^2/\rho^2 \sim \frac{1}{10}$ from $\sigma^{\nu}_{\mathrm{NC}}/\sigma^{\nu}$ cont). In this limit the neutral gauge boson mass matrix [Eq. (3.18)] is exactly the same as in Refs. 4 and 5. In this limit the angles α and β are also zero. The noteworthy features then are as follows:

(1) All the neutrino neutral-current phenomenology is identical to that of the standard $\mathrm{SU}(2)_L \times \mathrm{U}(1)$ model. The value of the "weak angle" $\sin^2\theta_W$ after renormalization[17] is

$$\sin^2\theta_W = \tfrac{1}{4} + \frac{\alpha}{3\alpha_s}\,,$$
$$\frac{\alpha_s}{\alpha} = \frac{8}{3}\left(1 - \frac{22\alpha}{3\pi}\ln\frac{M}{\mu}\right), \quad \alpha_s = f_s{}^2/4\pi \qquad (4.4)$$

where μ is an appropriately chosen subtraction mass[18] ($\sim$10 GeV) and M is the mass scale characterizing the breakdown G to $G_W \times \mathrm{SU}(3)_C$. $\sin^2\theta_W = \frac{1}{4}$, provided M is of the order of Planck mass and $\alpha_s \gg \alpha$. This estimate is somewhat more consistent with the CERN-DESY-Heidelberg-Saclay (CDHS) weak angle measurements.[19]

(2) The polarized-electron–deuteron asymmetry parameter (A/Q^2), measured at SLAC, has the value[6]

$$\left(\frac{A}{Q^2}\right) = (-9.5 \pm 1.6)\times 10^{-5}\,, \qquad (4.5)$$

with the y value of 0.21. The quark-parton-model-based prediction for the $\mathrm{SU}(2)_L \times \mathrm{U}(1)$ model is

$$\left(\frac{A}{Q^2}\right)_0 = -9.5\times 10^{-5} \text{ at } \sin^2\theta_W = 0.20$$

and

$$\left(\frac{A}{Q^2}\right)_0 = -8.5\times 10^{-5} \text{ at } \sin^2\theta_W = 0.23\,. \qquad (4.6)$$

In the present model the asymmetry parameter (A/Q^2) in the neutral-current sector is equal to

$$\left(\frac{\sigma^2-\rho^2}{\sigma^2}\right)\left(\frac{A}{Q^2}\right)_0\,. \qquad (4.7)$$

The limits from the SLAC experiment then imply

$$\frac{\rho^2}{\sigma^2} < \tfrac{1}{6} \sim \tfrac{1}{10} \text{ for } \sin^2\theta_W = 0.2\,,$$
$$\frac{\rho^2}{\sigma^2} < \tfrac{1}{15} \sim \tfrac{1}{20} \text{ for } \sin^2\theta_W = 0.23\,. \qquad (4.8)$$

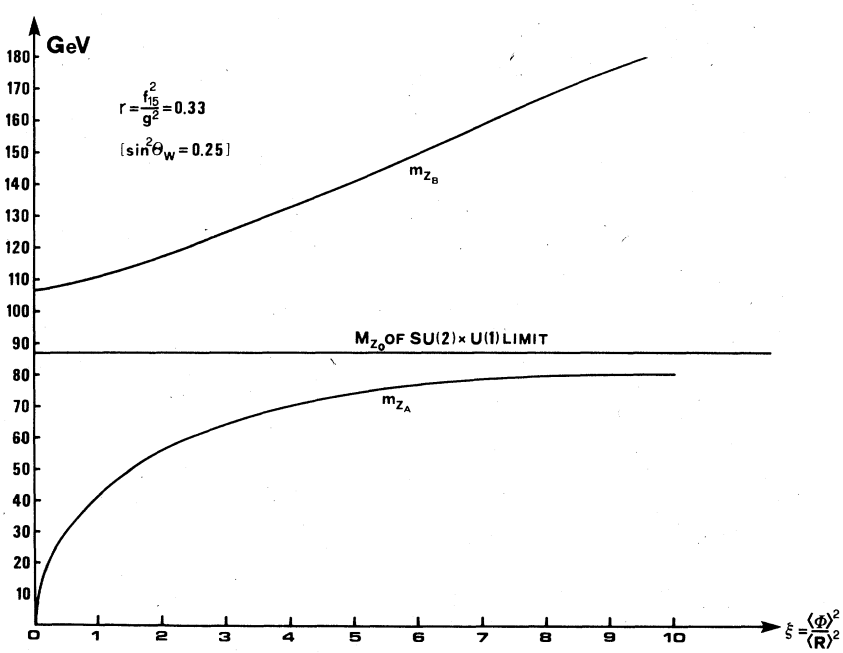

FIG. 2. Variation of masses m_{Z_A} and m_{Z_B} with $\xi = \langle\phi\rangle^2/\langle R\rangle^2$.

TABLE I. Forward-backward asymmetry in $e^+e^- \to \mu^+\mu^-$.

Masses in GeV		Forward-backward asymmetry $A^{\mu\mu}$		
m_{Z_A}	m_{Z_B}	$\sqrt{s}=7$ GeV	$\sqrt{s}=28$ GeV	$\sqrt{s}=38$ GeV
30	108.5	−8.4%	−24%	48%
41.5	111	−4.3%	−74%	−28%
50	114	−3%	−57%	−71%
60	121	−1.9%	−36%	−67%
71	137	−1.2%	−22%	−44%
80	181.5	−0.7%	−13%	−26%
SU(2)$_L$ × U(1) limit 86.5	∞	−0.33%	−6%	−12%

The last limit is consistent with the limits deduced in Eq. (4.3) from the charged-current data. In what follows we shall take $\sigma^2/\rho^2 \gg \phi^2/\rho^2$. The eigenstates and the corresponding masses of the neutral mass matrix [Eq. (3.19)] are

$$Z_A = Z_0 \cos\gamma + Z_1 \sin\gamma\,, \tag{4.9}$$

$$Z_B = Z_1 \cos\gamma - Z_0 \sin\gamma\,, \tag{4.10}$$

$$Z_C = \left(\frac{3r+1}{3r+3}\right)^{1/2} Y + \left(\frac{1}{3r+3}\right)^{1/2} (U - W_{(-)})\,, \tag{4.11}$$

$$m_{A,B}{}^2 = \frac{m_{W_L^+}{}^2}{2\cos^2\theta_W}[(P+1) \mp (P-1)\sec 2\gamma]\,, \tag{4.12}$$

$$m_C{}^2 = \frac{m_{W_L^+}{}^2}{2\cos^2\theta_W}(3r+3)\left(\frac{\sigma^2}{\rho^2}\right), \tag{4.13}$$

where

$$\tan 2\gamma = 2\left(\frac{3r+1}{3r+3}\right)^{1/2} \Big/ (P-1)\,, \tag{4.14}$$

$$P = \left(\frac{\phi^2}{\rho^2}\cos^2\theta_W + 1\right)\frac{3r+1}{3r+3}\,, \tag{4.15}$$

$$Z_0 = \left(\frac{3r+1}{3r+2}\right)^{1/2} W_{(-)} + \frac{1}{(3r+2)^{1/2}} Y\,, \tag{4.16}$$

$$Z_1 = \frac{1}{(3r+3)^{1/2}}\left(\frac{3r+1}{3r+2}\right)^{1/2} Y - \frac{1}{(3r+2)^{1/2}} W_{(-)} + (3r+2)^{1/2} U\,. \tag{4.17}$$

For small values of ϕ^2/ρ^2, the masses m_A and m_B are given by

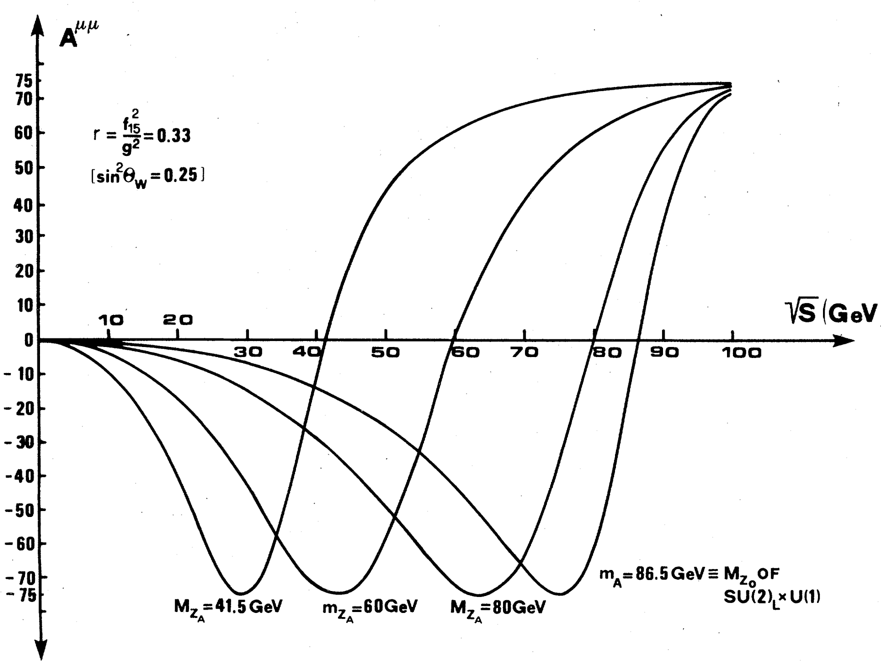

FIG. 3. Forward-backward asymmetry in $e^+e^- \to \mu^+\mu^-$ for different values of m_{Z_A}.

$$m_A{}^2 \sim \frac{m w_L^{\pm\,2}}{\cos^2\theta_W}\frac{\phi^2}{2\rho^2},$$

$$m_B{}^2 \sim \frac{m w_L^{\pm\,2}}{\cos^2\theta_W}\left(\frac{6r+4}{3r+3}\right),$$

while for large (ϕ^2/ρ^2) $(\phi^2/\rho^2 \ll \sigma^2/\rho^2)$,

$$m_A{}^2 \to \frac{m w_L^{\pm\,2}}{\cos^2\theta_W} = m_{Z_0}{}^2$$

of the $SU(2)_L \times U(1)$ theory

$$m_B{}^2 \to m_{W_L^\pm}{}^2 \frac{\phi^2}{\rho^2}.$$

In the large-ϕ^2/ρ^2 limit the eigenstate Z_A goes to Z_0, which is the neutral eigenstate of the $SU(2)_L \times U(1)$ theory. The variation of the masses m_A and m_B with (ϕ^2/ρ^2) is shown in Fig. 2. For $\rho^2/\sigma^2 \sim \frac{1}{6}$ to $\frac{1}{10}$, $m_{Z_C} \sim 250-400$ GeV. The limit $\sigma \to \infty$ corresponding to $b_4 \to \infty$ ($\langle R\rangle$ and $\langle S\rangle$ of the present case are equivalent to c_4 and b_4 of the model discussed by Elias, Pati, and Salam[5]) in Ref. 5 has been studied[20] in the light of the SLAC results with particular emphasis on the forward-backward charge asymmetry $A^{\mu\mu}$ in the reaction $e^+e^- \to \mu^+\mu^-$.

(3) As seen from Fig. 2, the mass m_{Z_A} can vary from 0 to 86.5 GeV. This leaves open the attractive possibility of having a light neutral gauge boson detectable at PETRA. Preliminary measurements[21] of the forward-backward charge asymmetry at center-of-mass energy $\sqrt{s} = 7$ GeV give $-2.5\% \leq 1.7\%$ (90% confidence). This sets a lower limit on the mass of Z_A to be ~ 50 GeV. The predictions of the forward-backward charge asymmetry $A^{\mu\mu}$ for various values of m_{Z_A} are summarized in Table I. It is seen that even if Z_A is only a few GeV lighter than the corresponding Z_0 of the standard theory ($m_{Z_A} = 80$ GeV, $m_{Z_0} = 86.5$ GeV), $A^{\mu\mu}$ is a factor of 2 larger than the standard theory. The variation of the asymmetry parameter $A^{\mu\mu}$ with the center-of-mass energy $\sqrt{s}$ is shown in Fig. 3 for various values of m_{Z_A}.

The present model based on $SU(4)_F \times SU(4)_C$ differs in several respects from that based on chiral color × chiral flavor, i.e., $[SU(4)]^4$. The flipping of the right-handed fermions relative to the left-handed fermions leads to right-handed currents involving new quantum numbers which are of strength G_F:

$$\frac{\sqrt{2}}{g} J_{W_L^\pm} = \sum_{r \to y \to b} (\bar{p}_{rL} n_{rL} + \bar{c}_{rL}\lambda_{rL} + \bar{\nu}_{eL} e_L^- + \bar{\nu}_{\mu L}\mu_L^- + \bar{C}_{rR}\Lambda_{rR} + \bar{\mathcal{P}}_{rR}\mathfrak{N}_{rR} + \bar{E}_R^0 E_R^- + \bar{M}_R^0 M_R^-).$$

Above the threshold for production of these new degrees of freedom the right-handed currents will manifest themselves with strength G_F. At asymptotic energies the theory is parity conserving.

The present treatment deals with the color degree of freedom strictly as vector current coupling to the octet of gluons. The possibility of axial-vector gluons[22] emerging naturally in the chiral $[SU(4)]^4$ theory is not available in the present scheme.

Finally, it is to be noted that the symmetry-breaking hierarchy presented here is not asymptotically free in its quartic scalar-field couplings.[13]

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