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Generation of u- and d-quark masses by weak interactions

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The masses of the u and d quarks are calculated as finite contributions from one-loop diagrams in an $SU_L(4) \times SU_R(4) \times U(1)$ gauge model of the weak and electromagnetic interactions. The parameters in our model can be chosen in such a way that the low-energy phenomenology resembles closely that of the Weinberg-Salam model.

I. INTRODUCTION

The advent of renormalizable gauge theories of the weak and electromagnetic interactions presents us with the possibility to understand the long-standing problem of the proton-neutron mass difference. It has been emphasized by Weinberg and by Georgi and Glashow[2] that when masses are subject to the zeroth-order mass relation for all values of the parameters in the Lagrangian, corrections to this relation can be calculated as finite higher-order effects. Explicit calculations of the proton-neutron mass difference were performed along these lines by Freedman and Kummer and by Duncan and Schattner[3] in a semirealistic model based on the $SU_L(2) \times SU_R(2) \times U(1)$ gauge group. The actual value of $m_p - m_n$ obtained in this model, however, was shown to have the wrong sign due to the dominant photon contribution. In realistic models, therefore, the contribution of the photon must be suppressed. One way out of this difficulty is to assume a zeroth-order mass relation $m_p = m_n = 0$, since the contribution of the photon is proportional to the zeroth-order proton or neutron mass in perturbation theory.

The assumption of this zeroth-order mass relation also offers a natural explanation of the chiral $SU(2) \times SU(2)$ symmetry[4] of the strong interactions, if we accept quantum chromodynamics[5] as a model of the strong interactions. The spontaneous breakdown of this hadronic symmetry gives a flavor-independent mass of order $m_p/3 \approx 300$ MeV to the constituent quarks.[6] The bulk of the proton and neutron masses is then attributed to this process. In this scheme, the small p-n mass difference would be introduced by the exchange of gauge bosons other than the photon and in principle it can have either sign.

In this paper, we will work with quarks instead of p and n. We will then assume that the u- and the d-quark masses satisfy the zeroth-order relation $m_u = m_d = 0$.

We shall look for a realistic model of the weak and electromagnetic interactions such that these quarks acquire small masses as finite higher-order effects of these interactions, rather than from bare masses or vacuum expectation values (VEV's) of scalar fields.

A straightforward extension of the model considered in Ref. 3 is a gauge model based on $SU_L(3) \times SU_R(3)$ with a left-handed triplet $(u,d,b)_L$ and a right-handed triplet $(u,d,b)_R$, where b denotes the newly discovered heavy quark with charge $-\frac{1}{3}$. It turns out, however, that the u quark remains massless while the d quark acquires a finite mass from higher-order corrections, when we require the zeroth-order mass relations $m_u = m_d = 0$ and $m_b \neq 0$. Therefore, in this paper, we introduce another heavy quark t with charge $\frac{2}{3}$ and enlarge the gauge group[7] to $SU_L(4) \times SU_R(4) \times U(1)$ in order to generate the u-quark mass. A left-handed quartet $(u,d,b,t)_L$ couples to a right-handed quartet $(u,d,b,t)_R$ through a Higgs scalar field Φ. The VEV's of Φ are arranged in such a way that the b and t quark acquire mass in the lowest order while the u and d quark remain massless in this order. The masses of the u and d quark are generated as finite second-order effects. Since the photon does not contribute to the masses of the u and the d quark, it is possible to get the right sign for the u-d mass difference. Although no reliable quark masses are available at the present because of the unknown renormalization effects of the strong interactions, we try to compare our calculated values with the standard values of the current quark masses[6] obtained from the PCAC (partial conservation of axial-vector current) relations and baryon mass splittings.

There are quite a few parameters in our model. However, low-energy weak-interaction phenomenology sets severe constraints on them. Within these limits, we find that reasonable u- and d-quark masses (~ a few MeV) can be obtained.

In Sec. II, we present details of our model based on the $SU_L(4) \times SU_R(4) \times U(1)$ group. In Sec. III, finite expressions for the u- and d-quark masses are given. In Sec. IV, some phenomenological aspects of our model are discussed briefly. In particular, the weak interactions of the b quark are analyzed. Section V is devoted to a discussion of our results and a summary. A brief account of this work has been published elsewhere.[8]

II. GAUGE MODEL BASED ON $SU_L(4)\times SU_R(4)\times U(1)$

In this section, we shall establish the notation and give a detailed description of our model. The symmetry group $SU_L(4)\times SU_R(4)\times U(1)$ has 31 generators T^0, T^α_L, and T^α_R ($\alpha=1,2,\ldots,15$), and is characterized by the commutation relations

$$[T^\alpha_L, T^\beta_L]=if^{\alpha\beta\gamma}T^\gamma_L,\qquad [T^\alpha_R, T^\beta_R]=if^{\alpha\beta\gamma}T^\gamma_R, \tag{2.1}$$

and

$$[T^\alpha_L, T^\beta_R]=[T^0, T^\alpha_L]=[T^0, T^\alpha_R]=0,$$

where $f^{\alpha\beta\gamma}$ are the totally antisymmetric structure constants of SU(4). The representations of $SU_L(4)\times SU_R(4)\times U(1)$ are denoted by $(\underline{n},\underline{m},h)$, $n[m]$ being the dimensionality under $SU_L(4)\,[SU_R(4)]$ and the U(1) charge h being defined by

$$[T^0,\psi]=-h\psi \tag{2.2}$$

for any field ψ. Gauge fields corresponding to T^0, T^α_L, and T^α_R are represented by A^0_μ, $A^\alpha_{L\mu}$, and $A^\alpha_{R\mu}$, respectively. Then the covariant derivative for a field ψ is defined by

$$D_\mu\psi\equiv\partial_\mu\psi+\tfrac{1}{2}ig_0A^0_\mu[T^0,\psi]+ig_LA^\alpha_{L\mu}[T^\alpha_L,\psi]+ig_RA^\alpha_{R\mu}[T^\alpha_R,\psi], \tag{2.3}$$

and gauge-covariant curls for gauge fields are given by

$$F^\alpha_{L\mu\nu}\equiv\partial_\mu A^\alpha_{L\nu}-\partial_\nu A^\alpha_{L\mu}+g_Lf^{\alpha\beta\gamma}A^\beta_{L\mu}A^\gamma_{L\nu},\qquad F^\alpha_{R\mu\nu}\equiv\partial_\mu A^\alpha_{R\nu}-\partial_\nu A^\alpha_{R\mu}+g_Rf^{\alpha\beta\gamma}A^\beta_{R\mu}A^\gamma_{R\nu}, \tag{2.4}$$

and

$$F_{\mu\nu}\equiv\partial_\mu A^0_\nu-\partial_\nu A^0_\mu.$$

The quarks and Higgs scalars are assigned to the following representations of the gauge group:

(a) Quarks

$$q_L\equiv\Lambda_L\begin{pmatrix}u\\d\\b\\t\end{pmatrix}\sim(\underline{4}^*,\underline{1},\tfrac{1}{3}),\quad q_R\equiv\Lambda_R\begin{pmatrix}u\\d\\b\\t\end{pmatrix}\sim(\underline{1},\underline{4}^*,\tfrac{1}{3}), \tag{2.5}$$

where $\Lambda_L=\frac{1}{2}(1-\gamma_5)$, $\Lambda_R=\frac{1}{2}(1+\gamma_5)$. The commutation relations are

$$[T^\alpha_L,q_L]=-t^\alpha_Lq_L,\quad [T^\alpha_R,q_R]=-t^\alpha_Rq_R,\qquad [T^\alpha_L,q_R]=[T^\alpha_R,q_L]=0, \tag{2.6}$$

where $t^\alpha_L\equiv\Lambda_L\frac{1}{2}\lambda^\alpha$ and $t^\alpha_R\equiv\Lambda_R\frac{1}{2}\lambda^\alpha$ are the fundamental representations of generators T^α_L and T^α_R, respectively, and λ matrices being normalized according to $\mathrm{Tr}(\lambda^\alpha\lambda^\beta)=2\delta^{\alpha\beta}$.

(b) Higgs scalars

$$\Phi\sim(\underline{4}^*,\underline{4},0),\quad D^\alpha_L\sim(\underline{15},\underline{1},0),\quad D^\alpha_R\sim(\underline{1},\underline{15},0),\quad \Xi_L\sim(\underline{4}^*,\underline{1},-1),\quad \Xi_R\sim(\underline{1},\underline{4}^*,-1).$$

If we define $\hat{D}_L$ and $\hat{D}_R$ by

$$\hat{D}_L\equiv\frac{1}{\sqrt{2}}\sum_{\alpha=1}^{15}\lambda^\alpha D^\alpha_L,\qquad \hat{D}_R\equiv\frac{1}{\sqrt{2}}\sum_{\alpha=1}^{15}\lambda^\alpha D^\alpha_R, \tag{2.7}$$

the commutation relations for the Higgs scalars are

$$\begin{aligned}&[T^\alpha_L,\Phi]=-(\tfrac{1}{2}\lambda^\alpha)\Phi,\quad [T^\alpha_R,\Phi]=\Phi(\tfrac{1}{2}\lambda^\alpha),\\&[T^\alpha_L,\Xi_L]=-(\tfrac{1}{2}\lambda^\alpha)\Xi_L,\quad [T^\alpha_R,\Xi_R]=-(\tfrac{1}{2}\lambda^\alpha)\Xi_R,\\&[T^\alpha_L,\Xi_R]=[T^\alpha_R,\Xi_L]=0,\\&[T^\alpha_L,\hat{D}_L]=-[\tfrac{1}{2}\lambda^\alpha,\hat{D}_L],\quad [T^\alpha_R,\hat{D}_R]=-[\tfrac{1}{2}\lambda^\alpha,\hat{D}_R],\\&[T^\alpha_L,\hat{D}_R]=[T^\alpha_R,\hat{D}_L]=0.\end{aligned} \tag{2.8}$$

With this notation, the Lagrangian density for our model is

$$\begin{aligned}\mathcal{L}=&-\tfrac{1}{4}(F^\alpha_{L\mu\nu})^2-\tfrac{1}{4}(F^\alpha_{R\mu\nu})^2-\tfrac{1}{4}(F_{\mu\nu})^2+\bar{q}_Li\gamma^\mu D_\mu q_L+\bar{q}_Ri\gamma^\mu D_\mu q_R+\mathrm{Tr}[(D_\mu\Phi)^\dagger(D^\mu\Phi)]+f(\bar{q}_L\Phi q_R+\bar{q}_R\Phi^\dagger q_L)\\&+[(D^\mu\Xi_L)^\dagger(D_\mu\Xi_L)+\tfrac{1}{2}\mathrm{Tr}(D_\mu\hat{D}_L)^2+(L\leftrightarrow R)]-P(\Phi,\Xi_L,\Xi_R,\hat{D}_L,\hat{D}_R)+(\text{gauge-fixing terms}),\end{aligned} \tag{2.9}$$

where P stands for a general fourth-order polynomial in the scalar fields, which is invariant under our gauge group. We impose manifest left-right symmetry by requiring the invariance of $\mathcal{L}$ under the following interchanges[7]:

$$\begin{aligned} &F^{\alpha}_{L\mu\nu} \longleftrightarrow F^{\alpha}_{R\mu\nu}\,, \\ &q_L \longleftrightarrow q_R\,, \\ &\Xi_L \longleftrightarrow \Xi_R\,, \\ &D_L \longleftrightarrow D_R\,, \\ &\Phi \longleftrightarrow \Phi^+\,. \end{aligned} \tag{2.10}$$

Imposing this condition leads to the requirement that the two SU(4) coupling constants are equal:

$$g_L = g_R = g\,. \tag{2.11}$$

Parity will be broken spontaneously, after the scalar fields are given nonvanishing, and asymmetric, vacuum expectation values. In addition, we impose discrete symmetries

$$R_L: \begin{cases} \Xi_L \to -\Xi_L\,, \\ \hat{D}_L \to -\hat{D}_L\,, \\ \text{other fields invariant}\,, \end{cases} \qquad R_R: \begin{cases} \Xi_R \to -\Xi_R\,, \\ \hat{D}_R \to -\hat{D}_R\,, \\ \text{other fields invariant}\,, \end{cases} \tag{2.12}$$

so that terms odd in $\hat{D}_L$, $\hat{D}_R$, Ξ_L, and/or Ξ_R are absent in P.

Corresponding to the assignment (2.5), the weak isospin I_i ($i=1,2,3$) and the electromagnetic charge are identified as follows:

$$\begin{aligned} &I_1 \equiv T^1_L\,,\quad I_2 \equiv T^2_L\,,\quad I_3 \equiv T^3_L\,, \\ &Q \equiv \tfrac{1}{2}T^0 + (T^3_L + T^3_R) + \frac{1}{\sqrt{3}}(T^8_L + T^8_R) - \left(\frac{2}{3}\right)^{1/2}(T^{15}_L + T^{15}_R)\,. \end{aligned} \tag{2.13}$$

This weak isospin together with the weak hypercharge $Y \equiv Q - I_3$ form a subgroup SU(2) × U(1):

$$[I_i, I_j] = i\epsilon_{ijk} I_k\,,\quad [I_i, Y] = 0\,. \tag{2.14}$$

This fact is very important because it makes it possible that, for appropriate VEV's of the Higgs scalar fields, the phenomenology of our model will resemble very closely that of the Weinberg-Salam (WS) model,[9] at least in the low-energy region.

Now let us discuss the symmetry-breaking scheme in more detail. Without loss of generality, we can assume that $\langle\Phi\rangle_0$ is real and diagonal:

$$\langle\Phi\rangle_0 = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{bmatrix}, \tag{2.15}$$

which gives the lowest-order mass to the quarks

$$\begin{aligned} &m_u = fa\,,\quad m_d = fb\,, \\ &m_b = fc\,,\quad m_t = fd\,. \end{aligned} \tag{2.16}$$

We assume that if the electromagnetic charge is conserved, then the VEV for the other Higgs scalar fields take the form

$$\langle\Xi_L\rangle_0 = \begin{bmatrix} 0 \\ \xi_L \\ \eta_L \\ 0 \end{bmatrix},\quad \langle\Xi_R\rangle_0 = (L \longleftrightarrow R)\,,$$

$$\langle\hat{D}_L\rangle_0 = \sqrt{2}\begin{bmatrix} u_L & 0 & 0 & v_L \\ 0 & x_L & y_L & 0 \\ 0 & y_L & s_L & 0 \\ v_L & 0 & 0 & r_L \end{bmatrix},\quad \langle\hat{D}_R\rangle_0 = (L \longleftrightarrow R)\,, \tag{2.17}$$

with

$$\begin{aligned} &u_L + x_L + s_L + r_L = 0\,, \\ &u_R + x_R + s_R + r_R = 0\,. \end{aligned}$$

With this form of the VEV, the photon remains massless and it is given by the expression

$$A_\mu = e\left[\frac{1}{2g_0}A^0_\mu + \frac{1}{g}(A^3_{L\mu} + A^3_{R\mu}) + \frac{1}{\sqrt{3}g}(A^8_{L\mu} + A^8_{R\mu}) - \frac{1}{g}\left(\frac{2}{3}\right)^{1/2}(A^{15}_{L\mu} + A^{15}_{R\mu})\right], \tag{2.18}$$

where the electromagnetic coupling constant is given by

$$\frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{4g_0^2}\,. \tag{2.19}$$

Higgs fields Φ, Ξ_L, Ξ_R, and $\hat{D}_R$ with VEV of the form (2.15) and (2.17), satisfying the additional conditions

$$a = b = 0 \tag{2.20}$$

and

$$\xi_L = 0 \tag{2.21}$$

will break $\mathrm{SU}_L(4) \times \mathrm{SU}_R(4) \times \mathrm{U}(1)$ down to SU(2) × U(1). This SU(2) × U(1) is the symmetry group defined in (2.14). If the group $\mathrm{SU}_L(4) \times \mathrm{SU}_R(4) \times \mathrm{U}(1)$ is broken down to SU(2) × U(1) in this way, then it is not hard to show that the Weinberg angle is given by

$$\sin^2\theta_W = [4 + (g/2g_0)^2]^{-1}\,. \tag{2.22}$$

It is easy to see that if

$$y_L = v_L = u_L - x_L = 0\,, \tag{2.23}$$

the $SU(2)\times U(1)$ symmetry remains unbroken even after introducing $\langle\hat{D}_L\rangle_0\neq 0$. In Appendix A, we give a plausible argument to show that for at least a finite range of parameters in $P(\Phi,\Xi_L,\Xi_R,\hat{D}_L,\hat{D}_R)$ it is possible to find a minimum of this potential for VEV of the form (2.15) and (2.17) that also satisfy Eq. (2.20). Hereafter, we always assume VEV's of this form.

It is important to notice that our mode has an approximate discrete symmetry W defined by

$$\begin{aligned} W:\ & q_L\to Wq_L\,,\quad q_R\to q_R\,,\\ & \Phi\to W\Phi\,,\\ & \hat{D}_L\to W\hat{D}_LW\,,\quad \hat{D}_R\to\hat{D}_R\,,\\ & \Xi_L\to W\Xi_L\,,\quad \Xi_R\to\Xi_R\,,\\ \hat{A}_{L\mu}\equiv & \frac{1}{\sqrt{2}}\sum_{\alpha=1}^{15}\lambda^\alpha A^\alpha_{L\mu}\to W\hat{A}_{L\mu}W\,,\\ \hat{A}_{R\mu}\equiv & \frac{1}{\sqrt{2}}\sum_{\alpha=1}^{15}\lambda^\alpha A^\alpha_{R\mu}\to\hat{A}_{R\mu}\,, \end{aligned} \tag{2.24}$$

with

$$W=\begin{bmatrix}-1&0&0&0\\0&-1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}.$$

Since $\langle\Phi\rangle_0$ is invariant under W, so are the quark parts of the Lagrangian (2.9). This symmetry W in the quark sector, which corresponds to the γ_5 transformation $u\to\gamma_5 u$, $d\to\gamma_5 d$, forbids the appearance of the u and d masses in the lowest order. Since $\langle\hat{D}_L\rangle_0$ is not invariant under W, neither is the mass matrix of the gauge bosons. Therefore, the higher-order corrections to the quark mass matrix due to the gauge boson exchange, which do not respect the W symmetry, can generate the small masses of the u and d quarks in principle. In the next section, we will calculate these masses explicitly.

III. COMPUTATION OF THE LIGHT-QUARK MASSES

As we stated in the previous section, the u and d quarks are massless in the lowest order. Since various vector bosons are mixed through the nonvanishing VEV's which are not invariant under the discrete symmetry W, they will generate masses of the light quarks in higher orders. It should be noted that these light-quark masses are finite to all orders if the zeroth-order mass relation $m_u=m_d=0$ is natural, i.e., if the forms of the VEV's given by Eqs. (2.15), (2.17), and (2.20) correspond to a minimum of the potential P for certain ranges of the parameters in P.[10] This is because divergences of the fermion self-energies are absorbed in the redefinition of the Yukawa coupling and the coefficients in P. But our zeroth-order mass relation $m_u=m_d=0$ will not be affected by this redefinition, if P is chosen to be a general fourth-order polynomial invariant under our gauge group and if (2.20) corresponds to a minimum of the potential P.

We will now discuss the generation of quark masses in the one-loop approximation. The one-loop fermion mass corrections may be grouped into three categories[10]: (1) tadpole, (2) scalar-meson exchange, and (3) vector-meson exchange. The computation will be done in the R_ξ gauge. The ξ dependence drops out after combining graphs in the three categories. At this point, we note that the Yukawa coupling f is very small compared to the gauge coupling g, so we can neglect the scalar-meson exchange. We will further assume that the Higgs mesons which couple to $\bar{u}_Lu_R+\bar{u}_Ru_L$ or $\bar{d}_Ld_R+\bar{d}_Rd_L$ are very heavy. Since the tadpole contributions are proportional to M_H^{-2}, where M_H denotes the mass of the corresponding Higgs scalar, we may neglect the tadpole contribution under this assumption. As a consequence, we need only to compute the vector-meson-exchange diagram as in Fig. 1 with the gauge-boson propagator in the Landau gauge. (In the notation of Ref. 10, we are calculating $\Sigma^{(A1)}+\Sigma^{(A\varphi)}+\Sigma^{(AT)}$.)

It is convenient to introduce the eigenfields of the vector-meson mass matrix μ^2. Since μ^2 is a real, symmetric matrix, it can be diagonalized by an orthogonal transformation C:

$$\begin{aligned} &\mu^2=C^{-1}\bar{\mu}^2C\,,\\ &C^{-1}=C^T\,,\quad C^*=C\,, \end{aligned} \tag{3.1}$$

where $\bar{\mu}^2$ denotes a diagonal matrix with eigenvalues μ_N^2. Then the gauge fields A_i^μ ($i=0$, $L\alpha$, or $R\alpha$ with $\alpha=1,2,\ldots,15$) in the original basis can be expressed in terms of the mass eigenfields W_N^μ,

$$A_i^\mu=\sum_N C_{Ni}W_N^\mu\equiv\sum_N\langle N|i\rangle W_N^\mu\,. \tag{3.2}$$

Now the interaction Lagrangian between quarks and gauge bosons reads

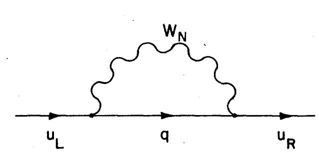

FIG. 1. Contribution to u-quark mass from vector-meson exchange.

$$\mathcal{L}_{\text{int}} = -\tfrac{1}{2}g_0\bar{q}\gamma^\mu[T_0,q]A^0_\mu - g\bar{q}\gamma^\mu[T^\alpha_L,q]A^\alpha_{L\mu} - gq\gamma^\mu[T^\alpha_R,q]A^\alpha_{R\mu} \equiv \sum_i \bar{q}\gamma^\mu t_i q A_{i\mu}\,, \tag{3.3}$$

where

$$t_i = \begin{cases} \tfrac{1}{2}hg_0 \text{ for } i=0 \ \ (h=\tfrac{1}{3}),\\ \tfrac{1}{2}g\Lambda_L\lambda^\alpha \text{ for } i=L\alpha\,,\\ \tfrac{1}{2}g\Lambda_R\lambda^\alpha \text{ for } i=R\alpha\,. \end{cases} \tag{3.4}$$

By introducing

$$\bar{t}_N \equiv \sum_n C_{Ni}t_i \equiv \sum_i \langle N|i\rangle t_i\,, \tag{3.5}$$

we can express $\mathcal{L}_{\text{int}}$ in terms of W^μ_N:

$$\mathcal{L}_{\text{int}} = \sum_N \bar{q}\gamma^\mu \bar{t}_N q W_{N\mu}\,. \tag{3.6}$$

It is also convenient to introduce the following notation for the gauge bosons in the original basis:

$$(\hat{A})^L_\mu \equiv \frac{1}{\sqrt{2}}\sum_{\alpha=1}^{15}\lambda^\alpha A^\alpha_{L\mu} = \begin{bmatrix} \frac{N_1}{\sqrt{2}}+\frac{N_2}{\sqrt{6}}+\frac{N_3}{\sqrt{12}} & X^+ & Y^+ & \bar{U}^0 \\ X^- & -\frac{N_1}{\sqrt{2}}+\frac{N_2}{\sqrt{6}}+\frac{N_3}{\sqrt{12}} & Y^0 & U^- \\ Y^- & \bar{Y}^0 & -\left(\frac{2}{3}\right)^{1/2}N_2+\frac{N_3}{\sqrt{12}} & V^- \\ U^0 & U^+ & V^+ & -\frac{\sqrt{3}}{2}N_3 \end{bmatrix}^L_\mu ,$$

$$(\hat{A})^R_\mu = (L \longleftrightarrow R). \tag{3.7}$$

The contribution of Fig. 1, where q denotes any member of the quark quartet, is represented by

$$\Sigma(p) = \sum_N \int \frac{d^4k}{(2\pi)^4 i}\gamma^\mu \bar{t}_N \frac{m+(\not{p}-\not{k})}{m^2-(p-k)^2}\gamma^\nu \bar{t}_N \times\left(\frac{1}{\mu_N^{\,2}-k^2}\right)\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right), \tag{3.8}$$

where $\Sigma(p)$ is related to the quark mass correction δm by

$$\delta m = \Sigma(p)\big|_{\not{p}=m}\,. \tag{3.9}$$

In Eq. (3.8), the $(\not{p}-\not{k})$ part of the quark propagator gives a contribution proportional to $\not{p}$ after integration. Since we are interested in the u and d masses, we put $\not{p}=0$ in (3.9), so that the $(\not{p}-\not{k})$ part does not contribute to these masses. Because m is of the form

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & m_b & 0 \\ 0 & 0 & 0 & m_t \end{bmatrix},$$

$\bar{t}_N$ in Eq. (3.8) should connect between $u(d)$ and b, or $u(d)$ and t for the $u(d)$-quark mass. In particular, this means that the photon γ does not contribute. From (3.5) we see that the couplings at the vertices $\bar{t}_N$ are proportional to $(1/\sqrt{2})gC_{Ni}$, with A^μ_i being $Y^\mu_{L,R}$ or $U^\mu_{L,R}$. In the computation, the numerator of the vector-meson propagator $g_{\mu\nu}-k_\mu k_\nu/k^2$ can be replaced by $\frac{3}{4}g_{\mu\nu}$, as can be easily verified. The contribution of W^μ_N then becomes

$$\tfrac{3}{2}g^2 m_{b,t}\int \frac{d^4k}{(2\pi)^4 i}\frac{1}{m_{b,t}^{\,2}-k^2}\frac{C_{Ni}C_{Ni}}{\mu_N^{\,2}-k^2}, \tag{3.10}$$

where, for instance, $C_{Ni}C_{Nj}=\langle Y^+_L|N\rangle\langle N|Y^+_R\rangle$ in the case of the b-quark contribution to the u-quark mass. After making a Wick rotation, the integral is evaluated without difficulty. The divergent parts cancel after summing up with respect to N because of the orthogonality of C: $\sum_N C_{Ni}C_{Nj}=0$ for $i\neq j$. The remaining convergent results are

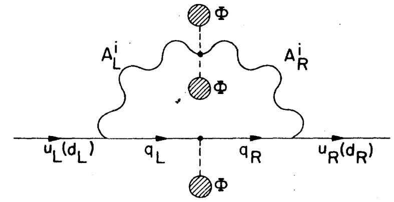

FIG. 2. Contribution to u- (d-) quark mass that, if present, would necessitate an infinite counterterm in Lagrangian density.

$$m_u=-\frac{3g^2}{32\pi^2}\sum_{N\neq\gamma}\left[m_b\langle Y_L^+|N\rangle\langle N|Y_R^+\rangle\left(\ln\mu_N{}^2+\frac{m_b{}^2}{\mu_N{}^2-m_b{}^2}\ln\frac{\mu_N{}^2}{m_b{}^2}\right)+m_t\langle \overline{U}_L^0|N\rangle\langle N|\overline{U}_R^0\rangle\left(\ln\mu_N{}^2+\frac{m_t{}^2}{\mu_N{}^2-m_t{}^2}\ln\frac{\mu_N{}^2}{m_t{}^2}\right)\right],$$

$$m_d=-\frac{3g^2}{32\pi^2}\sum_{N\neq\gamma}\left[m_b\langle Y_L^0|N\rangle\langle N|Y_R^0\rangle\left(\ln\mu_N{}^2+\frac{m_b{}^2}{\mu_N{}^2-m_b{}^2}\ln\frac{\mu_N{}^2}{m_b{}^2}\right)+m_t\langle U_L^-|N\rangle\langle N|U_R^-\rangle\left(\ln\mu_N{}^2+\frac{m_t{}^2}{\mu_N{}^2-m_t{}^2}\ln\frac{\mu_N{}^2}{m_t{}^2}\right)\right]. \tag{3.11}$$

When the gauge bosons are much heavier than quarks, we can neglect the term $[m^2/(\mu_N{}^2-m^2)]\ln \times \ln(\mu_N{}^2/m^2)$. Since the photon γ does not contribute to the u or d mass, it is possible to get the right sign for the u-d mass difference, as we have discussed in the Introduciton. What we have shown so far is that, within our framework, it is indeed possible to generate finite, calculable masses for the u and d quarks. The crucial ingredients in this calculation are the existence of the right-handed currents and the existence of at least two heavy quarks.

At this point, it is important to note that in our model we do not have a contribution to the u- (d-) quark mass of the form represented by Fig. 2. The tadpoles in this diagram represent the VEV of the Higgs scalar fields. The absence of such diagrams is very important because they would cause the Higgs scalar coupled to the u (d) quark to develop a zeroth-order VEV.[11] This would necessitate an infinite u- (d-) quark-mass counterterm and it would make the u- (d-) quark mass a free parameter. In our model, contributions to the u- (d-) quark mass come from diagrams such as Fig. 3, which represents the charged contribution to the u-quark mass. Again, such diagrams give rise to a VEV for the Higgs scalar coupled to the u and d quark. However, this time the contribution is finite and calculable and does not necessitate an infinite counterterm. It is this finite contribution that was obtained in this section.

The u- and d-quark masses in Eq. (3.11) are of the order $\alpha\epsilon$ compared with m_b or m_t, where ϵ depends on the specific VEV's. This dependence

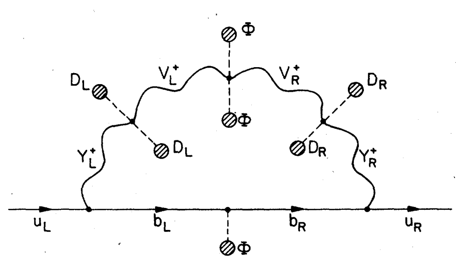

FIG. 3. Finite charged contribution to u-quark mass.

is nonlinear and complicated, and *a priori* there is no guarantee that there is a realistic solution. By "realistic" we mean a solution such that the low-energy behavior of our model conforms to the WS model, that the u- and d-quark masses are of the order of a few MeV (given that $m_b \simeq 5$ GeV), and that the ratio m_d/m_u is ~1.8.[6] To study this problem we use a computer search, with arbitrary VEV's as input. Then we diagonalize the vector-meson mass matrix μ^2 numerically, obtaining both the coefficients C_{Ni} and eigenvalues μ_N^2. We require that there is one light charged boson W_1 and, apart from the photon ($\gamma=Z_1$), one light neutral boson Z_2, which may be identified with the W and Z bosons in the WS model, except that the heavier bosons $W_2, \ldots$ and $Z_3, \ldots$ will make small corrections in the low-energy domain. Details of our computer search are given in Appendix B. We have found some sets of parameters that satisfy these criteria. One of the satisfactory choices of parameters is the following (in GeV):

$$\begin{aligned}
&c=3.6\times10^2,\quad d=1.6\times10^3,\\
&\xi_R=2.7\times10^3,\quad \xi_L=1.0\times10,\\
&\eta_R=3.2\times10^3,\quad \eta_L=2.3\times10^2,\\
&u_R=6.2\times10^3,\quad u_L=-6.0,\\
&x_R=-1.3\times10^3,\quad x_L=5.5\times10,\\
&s_R=2.5\times10^3,\quad s_L=7.6\times10^2,\\
&r_R=-7.5\times10^3,\quad r_L=-8.0\times10^2,\\
&v_R=-3.9\times10^3,\quad v_L=-1.2\times10^2,\\
&y_R=8.9\times10^3,\quad y_L=2.1\times10^2,
\end{aligned} \tag{3.12}$$

and

$$g=0.63,\quad g_0=0.55,\quad f=1.4\times10^{-2}.$$

With these VEV's, b and t acquire zeroth-order masses

$$m_b=fc=5\text{ GeV},\quad m_t=fd=23\text{ GeV}, \tag{3.13}$$

whereas u and d remain massless to zeroth order. Equation (3.11) gives u and d masses to the second order

$$m_u=1.1\text{ MeV},\quad m_d=2.0\text{ MeV}, \tag{3.14}$$

with the ratio $m_d/m_u \sim 1.8$. Thus, in our model, it is possible to get a reasonable order of magnitude for the u- and d-quark masses.

So far, we have neglected the effects of the strong interactions. The quark masses calculated in this work come entirely from the weak interactions. Although there is no reliable method to include the effects of the strong interactions, we can include at least part of them by using the renormalization-group equations. When we turn on the strong interactions, the (bare) quark masses are renormalized by the effects of the strong interactions. These renormalized quark masses depend on the renormalization point, where the renormalized parameters of the theory are defined.[12] Now, the standard values of the light-quark masses have been obtained by using the PCAC relation.[13,6] This means that these standard values are renormalized around $m_\pi/2$. On the other hand, in our computation, we have used the b-quark mass at about 5 GeV, which may be interpreted as renormalized around $m_\Upsilon/2$. Therefore, the quark masses m_u and m_d which we have calculated may be regarded as renormalized around $m_\Upsilon/2$. If we denote the quark masses, renormalized at $m_\pi/2$, by m_u^* and m_d^*, they are related to our quark masses m_u and m_d, multiplicatively,[12] i.e.,

$$m_{u,d}^* = Z_m(\alpha_s, m_\pi/m_\Upsilon) m_{u,d}\,, \tag{3.15}$$

where $Z_m(\alpha_s, m_\pi/m_\Upsilon)$ is a finite renormalization factor. If we apply perturbation theory and improve the result by using a renormalization-group equation, Z_m is obtained in Appendix C:

$$Z_m(\alpha_s, m_\pi/m_\Upsilon) = \left(1 + \frac{33-2n_f}{6\pi}\alpha_s \ln\frac{m_\pi}{m_\Upsilon}\right)^{-12/(33-2n_f)}, \tag{3.16}$$

where α_s is the strength of the strong interaction, renormalized at $m_\Upsilon/2$, and n_f is the effective number of flavors. By choosing $n_f = 3$ and $\alpha_s(m_\Upsilon/2) = 0.15$ (0.16),[14] we obtain $Z_m \sim 3$ (5), and correspondingly,

$$\begin{aligned} m_u^* &= 3.3\ (5.5)\ \text{MeV} \\ m_d^* &= 6.0\ (10)\ \text{MeV}. \end{aligned} \tag{3.17}$$

Though these numbers are in good agreement with standard values, one should not take these too seriously, since Eq. (3.16) may not be applicable in this region of very small masses.

IV. EFFECTIVE LAGRANGIAN OF THE WEAK INTERACTIONS

Since we have chosen small values for $|v_L|$, $|y_L|$, and $|u_L - x_L|$ and ξ_L in (3.12), the predictions of our model in the low-energy region are similar to the predictions obtained from an $SU_L(2) \times U(1)$ model, as we discussed in Sec. II. In Tables I and II we list the masses of the vector mesons as well as the expansion of these vector mesons in the original basis for the values in (3.12). If we identify W_1 and Z_2 with the W and Z bosons of the WS model ($Z_1 = \gamma$ = photon), the mass ratio $\mu_W/\mu_Z = 0.89$ is in conformity with the WS model with $\sin^2\theta_W = 0.21$. Corrections due to W_2, $W_3, \ldots$ and $Z_3, Z_4, \ldots$ are discussed in this section.

In order to derive the effective Lagrangian of the weak interactions, we tentatively assign leptons to the following representation:

$$l_{1L} \equiv \begin{bmatrix} \nu_e \\ e^- \\ E^- \\ E^0 \end{bmatrix}_L, \quad l_{2L} \equiv \begin{bmatrix} \nu_\mu \\ \mu^- \\ M^- \\ M^0 \end{bmatrix}_L, \quad l_{3L} \equiv \begin{bmatrix} \nu_\tau \\ \tau^- \\ T^- \\ T^0 \end{bmatrix}_L \sim (\underline{4}^*, \underline{1}, -1)$$
$$l_{2R} \equiv \begin{bmatrix} \nu_e \\ e^- \\ E^- \\ E^0 \end{bmatrix}_R, \quad l_{2R} = \begin{bmatrix} \nu_\mu \\ \mu^- \\ M^- \\ M^0 \end{bmatrix}_R, \quad l_{3R} \equiv \begin{bmatrix} \nu_\tau \\ \tau^- \\ T^- \\ T^0 \end{bmatrix}_R \sim (\underline{1}, \underline{4}^*, -1)\,, \tag{4.1}$$

where E^-, E^0, M^-, M^0, T^-, and T^0 denote heavy leptons. The interaction Lagrangian between fermions and gauge bosons is obtained from (3.6):

$$\mathcal{L}_{\text{int}} = \sum_N (\bar{q}\gamma^\mu \bar{t}_N q + \sum_i \bar{l}_i \gamma^\mu \bar{t}_N l_i) W_{N\mu}\,, \tag{4.2}$$

where $\bar{t}_N$ are defined by Eqs. (3.4) and (3.5). In Eq. (3.4), $h = \frac{1}{3}$ for quarks and $h = -1$ for leptons. For notational simplicity, we represent quarks and leptons by ψ. Then the second-order interactions of (4.2) are described by the following effective Lagrangians.

Charged currents:

$$-\mathcal{L}_{\text{eff}}^{\text{cc}} = 2 \sum_N^{\text{charged}} \frac{1}{\mu_N^2} J_{W_N}^{\mu\dagger} J_{\mu W_N}\,, \tag{4.3}$$

where

$$J_{W_N}^\mu = \frac{g}{4} \sum_{a,b=1}^4 [g_N^L(ab)\bar{\psi}_a \gamma^\mu (1-\gamma_5)\psi_b + g_N^R(ab)\bar{\psi}_a\gamma^\mu(1+\gamma_5)\psi_b]\,, \tag{4.4}$$

with

$$\begin{aligned} g_N^L(ab) &\equiv \sum_{L\alpha}^{\text{charged}} \langle N|L\alpha\rangle (\lambda_\alpha)_{ab}, (Q_a - Q_b = 1)\,, \\ g_N^R(ab) &\equiv \sum_{L\alpha}^{\text{charged}} \langle N|R\alpha\rangle (\lambda_\alpha)_{ab}, (Q_a - Q_b = 1)\,. \end{aligned} \tag{4.5}$$

Neutral currents:

TABLE I. Neutral vector bosons (mass-matrix eigenstates): mass in GeV/c^2, coefficients of the expansion in the original basis.

Neutral vector boson	Mass	N_1^L	N_1^R	N_2^L	N_2^R	N_3^L	N_3^R
Z_1	0	4.8×10^{-1}	4.8×10^{-1}	2.8×10^{-1}	2.8×10^{-1}	-3.9×10^{-1}	-3.9×10^{-1}
Z_2	8.5×10	-8.4×10^{-1}	2.7×10^{-1}	1.5×10^{-1}	1.6×10^{-1}	-1.8×10^{-1}	-1.9×10^{-1}
Z_3	1.6×10^{2}	6.6×10^{-2}	-2.4×10^{-1}	3.6×10^{-1}	4.8×10^{-2}	1.6×10^{-1}	1.6×10^{-1}
Z_4	2.3×10^{2}	-9.8×10^{-2}	-6.7×10^{-2}	-7.1×10^{-1}	5.2×10^{-2}	-2.6×10^{-1}	-2.3×10^{-1}
Z_5	3.1×10^{2}	-8.8×10^{-3}	5.5×10^{-1}	-2.2×10^{-1}	3.3×10^{-1}	4.4×10^{-1}	4.0×10^{-1}
Z_6	5.5×10^{2}	0	0	0	0	0	0
Z_7	5.5×10^{2}	-2.3×10^{-1}	-2.8×10^{-2}	4.7×10^{-1}	-2.0×10^{-2}	7.2×10^{-4}	-9.6×10^{-3}
Z_8	8.7×10^{2}	4.9×10^{-2}	-1.5×10^{-3}	3.2×10^{-2}	9.8×10^{-4}	-7.7×10^{-3}	7.8×10^{-2}
Z_9	8.9×10^{2}	0	0	0	0	0	0
Z_{10}	1.2×10^{3}	-6.6×10^{-3}	-2.8×10^{-2}	1.3×10^{-2}	3.8×10^{-4}	-7.3×10^{-1}	6.1×10^{-1}
Z_{11}	4.0×10^{3}	-6.9×10^{-6}	-1.9×10^{-1}	-4.8×10^{-4}	-2.2×10^{-1}	-9.8×10^{-3}	2.0×10^{-1}
Z_{12}	9.9×10^{3}	0	0	0	0	0	0
Z_{13}	9.9×10^{3}	2.9×10^{-7}	-2.4×10^{-1}	-7.5×10^{-7}	-1.4×10^{-1}	3.2×10^{-3}	-4.0×10^{-1}
Z_{14}	1.2×10^{4}	0	0	0	0	0	0
Z_{15}	1.2×10^{4}	4.6×10^{-8}	-4.9×10^{-1}	-2.1×10^{-4}	8.5×10^{-1}	7.2×10^{-5}	4.1×10^{-4}

Neutral vector boson	Y^{0L}	Y^{0R}	$Y^{\overline{0L}}$	$Y^{\overline{0R}}$	$U^{\overline{0L}}$	$U^{\overline{0R}}$	U^{0L}	U^{0R}	A^0
Z_1	0	0	0	0	0	0	0	0	2.8×10^{-1}
Z_2	-2.0×10^{-1}	-1.5×10^{-2}	-2.0×10^{-1}	-1.5×10^{-2}	3.7×10^{-2}	-1.0×10^{-2}	3.7×10^{-2}	-1.0×10^{-2}	1.6×10^{-1}
Z_3	-1.1×10^{-1}	-5.5×10^{-1}	-1.1×10^{-1}	-5.5×10^{-1}	-1.9×10^{-2}	-8.4×10^{-3}	-1.9×10^{-2}	-8.4×10^{-3}	3.6×10^{-1}
Z_4	2.6×10^{-1}	-2.6×10^{-1}	2.6×10^{-1}	-2.6×10^{-1}	3.4×10^{-2}	8.1×10^{-2}	3.4×10^{-2}	8.1×10^{-2}	2.6×10^{-1}
Z_5	1.1×10^{-1}	-3.0×10^{-2}	1.1×10^{-1}	-3.0×10^{-2}	-2.3×10^{-2}	-2.7×10^{-1}	-2.3×10^{-2}	-2.7×10^{-1}	1.4×10^{-1}
Z_6	7.1×10^{-1}	0	-7.1×10^{-1}	0	0	0	0	0	0
Z_7	6.0×10^{-1}	1.2×10^{-2}	6.0×10^{-1}	1.2×10^{-2}	-2.5×10^{-3}	1.1×10^{-2}	-2.5×10^{-3}	1.1×10^{-2}	-1.9×10^{-2}
Z_8	4.9×10^{-4}	-5.2×10^{-3}	4.9×10^{-4}	-5.2×10^{-3}	7.0×10^{-1}	-2.5×10^{-2}	7.0×10^{-1}	-2.5×10^{-2}	-1.5×10^{-2}
Z_9	0	0	0	0	-7.1×10^{-1}	0	7.1×10^{-1}	0	0
Z_{10}	7.0×10^{-4}	-4.7×10^{-2}	7.0×10^{-4}	-4.7×10^{-2}	-4.7×10^{-2}	-1.9×10^{-1}	-4.7×10^{-2}	-1.9×10^{-1}	-1.3×10^{-1}
Z_{11}	5.7×10^{-5}	3.2×10^{-1}	5.7×10^{-5}	3.2×10^{-1}	-2.8×10^{-5}	-3.0×10^{-3}	-2.8×10^{-5}	-3.0×10^{-3}	8.2×10^{-1}
Z_{12}	0	0	0	0	0	7.1×10^{-1}	0	-7.1×10^{-1}	0
Z_{13}	0	-3.8×10^{-5}	0	-3.8×10^{-5}	1.4×10^{-6}	-6.2×10^{-1}	1.4×10^{-6}	-6.2×10^{-1}	-9.6×10^{-5}
Z_{14}	0	-7.1×10^{-1}	0	7.1×10^{-1}	0	0	0	0	0
Z_{15}	-9.3×10^{-8}	1.5×10^{-1}	-9.3×10^{-8}	1.5×10^{-1}	0	1.3×10^{-6}	0	1.3×10^{-6}	2.1×10^{-3}

TABLE II. Charged vector bosons (mass-matrix eigenstates): mass in GeV/c^2, coefficients of the expansion in the original basis.

Charged vector boson	Mass	X^{+L}	X^{+R}	Y^{+L}	Y^{+R}	U^{+L}	U^{+R}	V^{+L}	V^{+R}
W_1^+	7.5×10	9.7×10^{-1}	-6.6×10^{-4}	-2.4×10^{-1}	4.4×10^{-4}	-4.8×10^{-2}	-2.3×10^{-3}	1.1×10^{-2}	1.9×10^{-3}
W_2^+	5.4×10^{2}	2.4×10^{-1}	1.3×10^{-2}	9.7×10^{-1}	-1.0×10^{-2}	-2.0×10^{-2}	4.9×10^{-2}	-9.2×10^{-2}	-4.0×10^{-2}
W_3^+	6.9×10^{2}	9.7×10^{-3}	-2.0×10^{-1}	7.6×10^{-2}	1.6×10^{-1}	-8.2×10^{-2}	-7.4×10^{-1}	1.4×10^{-1}	6.0×10^{-1}
W_4^+	8.8×10^{2}	4.8×10^{-2}	-2.1×10^{-2}	-7.1×10^{-3}	2.0×10^{-2}	9.6×10^{-1}	-8.5×10^{-2}	-2.4×10^{-1}	6.8×10^{-2}
W_5^+	1.3×10^{3}	2.2×10^{-2}	1.8×10^{-2}	8.3×10^{-2}	-2.6×10^{-2}	2.5×10^{-1}	9.1×10^{-2}	9.6×10^{-1}	-6.9×10^{-2}
W_6^+	2.5×10^{3}	3.7×10^{-5}	6.1×10^{-1}	1.9×10^{-4}	7.5×10^{-1}	3.8×10^{-4}	-1.6×10^{-1}	9.9×10^{-3}	-2.0×10^{-1}
W_7^+	9.9×10^{3}	-7.6×10^{-8}	7.5×10^{-1}	-4.2×10^{-7}	-6.1×10^{-1}	-7.8×10^{-7}	-2.0×10^{-1}	-3.8×10^{-4}	1.6×10^{-1}
W_8^+	1.2×10^{4}	-1.8×10^{-7}	1.6×10^{-1}	-1.0×10^{-6}	2.0×10^{-1}	-1.9×10^{-6}	6.1×10^{-1}	-1.3×10^{-3}	7.5×10^{-1}

$$-\mathcal{L}_{\text{eff}}^{\text{nc}} = \frac{1}{2} \sum_{N \neq \gamma}^{\text{neutral}} \frac{1}{\mu_N^2} J_{Z_N}^{\mu} J_{\mu Z_N}, \tag{4.6}$$

where

$$J_{Z_N}^{\mu} = \frac{1}{4} g \sum_{a,b=1}^{4} [g_N^L(ab)\bar{\psi}_a \gamma^\mu (1-\gamma_5)\psi_b + g_N^R(ab)\bar{\psi}_a\gamma^\mu(1+\gamma_5)\psi_b], \tag{4.7}$$

with

$$g_N^L(ab) \equiv \frac{2g_0 h}{g}\langle N|0\rangle + \sum_{L\alpha}^{\text{neutral}} \langle N|L\alpha\rangle(\lambda_\alpha)_{ab},$$
$$g_N^R(ab) \equiv \frac{2g_0 h}{g}\langle N|0\rangle + \sum_{R\alpha}^{\text{neutral}} \langle N|R\alpha\rangle(\lambda_\alpha)_{ab}. \tag{4.8}$$

In the remaining part of this section, we will look at some specific examples.

A. β decay of d quark and muon

From Eqs. (4.3), (4.4), and (4.5), we get the effective Lagrangian for β decay:

$$\begin{aligned}-\mathcal{L}_{\text{eff}}^{\beta\text{ decay}} = \frac{G_F}{\sqrt{2}}\{&[\bar{e}^-\gamma^\mu(1-\gamma_5)\nu_e][\bar{u}\gamma_\mu(1-\gamma_5)d] + \epsilon_{LR}[\bar{e}^-\gamma^\mu(1-\gamma_5)\nu_e][\bar{u}\gamma_\mu(1+\gamma_5)d] \\ &+\epsilon_{RL}[\bar{e}^-\gamma^\mu(1+\gamma_5)\nu_e][\bar{u}\gamma_\mu(1-\gamma_5)d] + \epsilon_{RR}[\bar{e}^-\gamma^\mu(1+\gamma_5)\nu_e][\bar{u}\gamma_\mu(1+\gamma_5)d] \\ &+(e \leftrightarrow \mu) + (u \leftrightarrow \nu_\mu, d \leftrightarrow \mu^-)\} + \text{H.c.},\end{aligned} \tag{4.9}$$

where

$$\frac{1}{\sqrt{2}} G_F = \frac{g^2}{8} \sum_{N}^{\text{charged}} \frac{1}{\mu_N^2} |\langle X_L^- | N\rangle|^2, \tag{4.10}$$

$$\epsilon_{LR} = \epsilon_{RL} = \frac{1}{\sigma} \sum_{N}^{\text{charged}} \frac{\mu_W^2}{\mu_N^2} \langle N|X_L^-\rangle\langle N|X_R^-\rangle, \tag{4.11}$$

$$\epsilon_{RR} = \frac{1}{\sigma} \sum_{N}^{\text{charged}} \frac{\mu_W^2}{\mu_N^2} |\langle X_R^-|N\rangle|^2, \tag{4.12}$$

with

$$\sigma = \sum_{N}^{\text{charged}} \frac{\mu_W^2}{\mu_N^2} |\langle X_L^-|N\rangle|^2. \tag{4.13}$$

Using the parameters we have chosen in (3.12), we have

$$\epsilon_{LR} = \epsilon_{RL} = -6.5 \times 10^{-4}, \quad \epsilon_{RR} = 9.1 \times 10^{-4}, \tag{4.14}$$

which means that the charged-current interactions of the ordinary quarks and leptons are almost purely $V-A$ at low energies. In particular, neutrinos produced in β decay of ordinary hadrons are almost purely left-handed.

B. Elastic $\nu_\mu e^-$ scattering

The effective Lagrangian that describes $\nu_\mu e^- \to \nu_\mu e^-$ and the neutral-current contribution to $\nu_e e^- \to \nu_e e^-$ is obtained from Eqs. (4.6), (4.7), and (4.8):

$$\begin{aligned}-\mathcal{L}_{\text{eff}}^{\nu e} = \frac{G_F}{\sqrt{2}}\{&\epsilon_{LL}[\bar{\nu}_\mu\gamma^\lambda(1-\gamma_5)\nu_\mu][\bar{e}^-\gamma_\lambda(1-\gamma_5)e^-] + \epsilon_{LR}[\bar{\nu}_\mu\gamma^\lambda(1-\gamma_5)\nu_\mu][\bar{e}^-\gamma_\lambda(1+\gamma_5)e^-] \\ &+\epsilon_{RL}[\bar{\nu}_\mu\gamma^\lambda(1+\gamma_5)\nu_\mu][\bar{e}^-\gamma_\lambda(1-\gamma_5)e^-] + \epsilon_{RR}[\bar{\nu}_\mu\gamma^\lambda(1+\gamma_5)\nu_\mu][\bar{e}^-\gamma_\lambda(1+\gamma_5)e^-] + (\nu_\mu \to \nu_e)\},\end{aligned} \tag{4.15}$$

where

$$\epsilon_{LL} = \frac{1}{2\sigma}\frac{\mu_W^2}{\mu_Z^2}\sum_{N=2}^{15}\frac{\mu_Z^2}{\mu_{Z_N}^2} g_N^L(\nu) g_N^L(e^-), \quad \epsilon_{LR} = \frac{1}{2\sigma}\frac{\mu_W^2}{\mu_Z^2}\sum_{N=2}^{15}\frac{\mu_Z^2}{\mu_{Z_N}^2} g_N^L(\nu) g_N^R(e^-),$$
$$\epsilon_{RL} = \frac{1}{2\sigma}\frac{\mu_W^2}{\mu_Z^2}\sum_{N=2}^{15}\frac{\mu_Z^2}{\mu_{Z_N}^2} g_N^R(\nu) g_N^L(e^-), \quad \epsilon_{RR} = \frac{1}{2\sigma}\frac{\mu_W^2}{\mu_Z^2}\sum_{N=2}^{15}\frac{\mu_Z^2}{\mu_{Z_N}^2} g_N^R(\nu) g_N^R(e^-), \tag{4.16}$$

with

$$g_N^L(\nu)=-\frac{2g_0}{g}\langle N|0\rangle+\langle N|3L\rangle+\frac{1}{\sqrt3}\langle N|8L\rangle+\frac{1}{\sqrt6}\langle N|15L\rangle\,,\quad g_N^R(\nu)=(L\to R)\,, \tag{4.17}$$

$$g_N^L(e^-)=-\frac{2g_0}{g}\langle N|0\rangle-\langle N|3L\rangle+\frac{1}{\sqrt3}\langle N|8L\rangle+\frac{1}{\sqrt6}\langle N|15L\rangle\,,\quad g_N^R(e^-)=(L\to R)\,.$$

Equation (3.12) gives

$$\epsilon_{LL}=-2.0\times10^{-1}\,,\quad \epsilon_{LR}=2.9\times10^{-1}\,,\quad \epsilon_{RL}=6.5\times10^{-2}\,,\quad \epsilon_{RR}=3.3\times10^{-2}\,. \tag{4.18}$$

Experimentally, the initial neutrinos are almost purely left-handed because they are decay products of ordinary hadrons. Therefore, we can discard the term $\bar\nu_\mu\gamma^\lambda(1+\gamma_5)\nu_\mu$ in Eq. (4.15). Then (4.15) reduces to

$$-\mathcal{L}_{\text{eff}}^{\nu e}=\frac{G_F}{\sqrt2}[\bar\nu_\mu\gamma^\lambda(1-\gamma_5)\nu_\mu+(\nu_\mu\to\nu_e)][\bar e^-\gamma_\lambda(g_V-\gamma_5 g_A)e^-]\,, \tag{4.19}$$

where

$$g_V\equiv\epsilon_{LL}+\epsilon_{LR}\,,\quad g_A\equiv\epsilon_{LL}-\epsilon_{LR}\,. \tag{4.20}$$

The WS model gives

$$g_V=-\tfrac12+2\sin^2\theta_W\,,\quad g_A=-\tfrac12\,,$$

whereas the predictions of our model are obtained from (4.18)

$$g_V=0.09\,,\quad g_A=-0.48\,. \tag{4.21}$$

These are consistent with the axial-vector dominant solution

$$g_V=0.0\pm0.1\,,\quad g_A=-0.5\pm0.1\,, \tag{4.22}$$

which is obtained by combining the Aachen-Padova and Gargamelle (PS) data with those from the reactor experiment.[15]

C. Neutral currents in neutrino reactions

The effective Lagrangian for these processes are obtained from Eqs. (4.6), (4.7), and (4.8):

$$-\mathcal{L}_{\text{eff}}^{\nu N}=\frac{G_F}{\sqrt2}\{\epsilon_{LL}(u)[\bar\nu_\mu\gamma^\lambda(1-\gamma_5)\nu_\mu][\bar u\gamma_\lambda(1-\gamma_5)u]+\epsilon_{LR}(u)[\bar\nu_\mu\gamma^\lambda(1-\gamma_5)\nu_\mu][\bar u\gamma_\lambda(1+\gamma_5)u]$$
$$+\epsilon_{RL}(u)[\bar\nu_\mu\gamma^\lambda(1+\gamma_5)\nu_\mu][\bar u\gamma_\lambda(1-\gamma_5)u]+\epsilon_{RR}(u)[\bar\nu_\mu\gamma^\lambda(1+\gamma_5)\nu_\mu][\bar u\gamma_\lambda(1+\gamma_5)u]+(u\to d)\}\,, \tag{4.23}$$

where $\epsilon_{LL}(u)$ $[\epsilon_{LL}(d)]$. $\epsilon_{LR}(u)$ $[\epsilon_{LR}(d)]$, etc. are obtained from Eq. (4.16) by replacing $g_N^{L,R}(e^-)$ with $g_N^{L,R}(u)$ $[g_N^{L,R}(d)]$. $g_N^{L,R}(u)$ and $g_N^{L,R}(d)$ are defined by

$$g_N^L(u)=\frac{2g_0}{3g}\langle N|0\rangle+\langle N|3L\rangle+\frac{1}{\sqrt3}\langle N|8L\rangle+\frac{1}{\sqrt6}\langle N|15L\rangle\,,\quad g_N^R(u)=(L\to R)\,,$$
$$g_N^L(d)=\frac{2g_0}{3g}\langle N|0\rangle-\langle N|3L\rangle+\frac{1}{\sqrt3}\langle N|8L\rangle+\frac{1}{\sqrt6}\langle N|15L\rangle\,,\quad g_N^R(d)=(L\to R)\,. \tag{4.24}$$

If we drop the terms including the right-handed neutrino by the same arguments as in the preceding example, the effective Lagrangian (4.23) reduces to

$$-\mathcal{L}_{\text{eff}}^{\nu N}=\frac{G_F}{\sqrt2}[\bar\nu_\mu\gamma^\lambda(1-\gamma_5)\nu_\mu]\{\epsilon_{LL}(u)[\bar u\gamma_\lambda(1-\gamma_5)u]+\epsilon_{LR}(u)[\bar u\gamma_\lambda(1+\gamma_5)u]$$
$$+\epsilon_{LL}(d)[\bar d\gamma_\lambda(1-\gamma_5)d]+\epsilon_{LR}(d)[\bar d\gamma_\lambda(1+\gamma_5)d]\}\,. \tag{4.25}$$

These chiral couplings were determined as[15,16]

$$\epsilon_{LL}(u)=0.32\pm0.03\,,\quad \epsilon_{LR}(u)=-0.17\pm0.05\,,\quad \epsilon_{LL}(d)=-0.43\pm0.04\,,\quad \epsilon_{LR}(d)=0.0\pm0.12\,. \tag{4.26}$$

In our model, they are calculated to be

$$\epsilon_{LL}(u)=0.35\,,\quad \epsilon_{LR}(u)=-0.18\,,\quad \epsilon_{RL}(u)=-0.04\,,\quad \epsilon_{RR}(u)=\pm0.01\,, \tag{4.27}$$

and

$$\epsilon_{LL}(d) = -0.44\,,\quad \epsilon_{LR}(d) = 0.05\,,\quad \epsilon_{RL}(d) = -0.03\,,\quad \epsilon_{RR}(d) = -0.06\,,$$

in good agreement with the experimental values (4.26). They are consistent with the WS model with $\sin^2\theta_W = 0.21$.

D. Parity violation of neutral currents in electron scattering

The effective Lagrangian $\mathcal{L}_{\text{eff}}^{eN}$ for electron scatterings off the ordinary quarks is also given by (4.23) with the replacement $\nu_\mu \to e^-$. The chiral couplings $\epsilon_{LL}(eu)$ $[\epsilon_{LL}(ed)]$, $\epsilon_{LR}(eu)$ $[\epsilon_{LR}(ed)]$, etc. in this case are obtained from (4.16) by replacing $g_N^{L,R}(\nu)$ and $g_N^{L,R}(e^-)$ by $g_N^{L,R}(e^-)$ and $g_N^{L,R}(u)$ $[g_N^{L,R}(d)]$, respectively. The parity-violating parts of this Lagrangian are[17]

$$-\mathcal{L}_{\text{eff}}^{\text{PV}} = \frac{G_F}{\sqrt{2}}[\epsilon_{VA}(eu)(\bar{e}\gamma^\mu e)(\bar{u}\gamma_\mu\gamma_5 u) + \epsilon_{AV}(eu)(\bar{e}\gamma^\mu\gamma_5 e)(\bar{u}\gamma_\mu u) + \epsilon_{VA}(ed)(\bar{e}\gamma^\mu e)(\bar{d}\gamma_\mu\gamma_5 d) + \epsilon_{AV}(ed)(\bar{e}\gamma^\mu\gamma_5 e)(\bar{d}\gamma_\mu d)]\,, \tag{4.28}$$

where

$$\epsilon_{VA}(eu) = -\epsilon_{LL}(eu) + \epsilon_{LR}(eu) - \epsilon_{RL}(eu) + \epsilon_{RR}(eu)\,, \tag{4.29}$$

$$\epsilon_{AV}(eu) = -\epsilon_{LL}(eu) - \epsilon_{LR}(eu) + \epsilon_{RL}(eu) + \epsilon_{RR}(eu)\,,$$

$\epsilon_{VA}(ed)$ and $\epsilon_{AV}(ed)$ are obtained by replacing $u \to d$. Using the parameters in (3.12), we have

$$\epsilon_{VA}(eu) = 0.01\,,\quad \epsilon_{AV}(eu) = 0.16\,, \tag{4.30}$$

$$\epsilon_{VA}(ed) = -0.06\,,\quad \epsilon_{AV}(ed) = -0.35\,.$$

The asymmetry A in electron-deuteron scattering is given by[18]

$$A/Q^2 = -3G_F(10\sqrt{2}\pi\alpha)^{-1}\{[2\epsilon_{AV}(eu) - \epsilon_{AV}(dd)] + f(y)[2\epsilon_{VA}(eu) - \epsilon_{VA}(ed)]\}\,, \tag{4.31}$$

where

$$f(y) = \frac{1-(1-y)^2}{1+(1-y)^2}\,.$$

From Eqs. (4.30) and (4.31), we get

$$A/Q^2 = -7.4\times 10^{-5}\ (\text{GeV}/c)^{-2}\ \text{at}\ y = 0.21\,, \tag{4.32}$$

which is consistent with the SLAC-Yale experiment[19] on the deep-inelastic scattering of longitudinally polarized electrons on deuterium:

$$A/Q^2 = (-9.5\pm 1.6)\times 10^{-5}\ (\text{GeV}/c)^{-2}\ \text{at}\ y = 0.21\,. \tag{4.33}$$

The bismuth experiments of the optical rotation[17,20] give information on

$$Q_w = 584\epsilon_{AV}(eu) + 670\epsilon_{AV}(eu)\,. \tag{4.34}$$

Equation (4.30) gives

$$Q_w = -146\,, \tag{4.35}$$

which is consistent with the WS model and with the results from the Novosibirsk experiments[21]:

$$Q_w = -120\pm 40\,, \tag{4.36}$$

but disagrees with the smaller values obtained from the Washington[22] and Oxford[23] experiments.

E. Decay width of the b quark

The results of our model discussed in parts A–D are very close to the WS model because we have chosen small values for $|v_L|$, $|y_L|$, and $|u_L - x_L|$ in (3.12). The decay of the b quark in our model, however, can be different from that in the usual SU(2) × U(1) model. This is because in our $SU_L(4)\times SU_R(4)\times U(1)$ model the b quark belongs to a singlet with respect to the subgroup SU(2) × U(1), as is easily seen from (2.5). This means that the b quark is stable in the SU(2) × U(1) limit. The b quark decays mainly into the u or d quark via the heavy gauge bosons. However, the c and s quarks, which have not been discussed so far, also participate in the decay. To include them we will now introduce new quartets $(c,s,g,h)_{L,R}$ with the same assignments as in Eq. (2.5). We discuss the contributions from the charged and neutral currents separately.

1. Charged currents

The effective Lagrangians for the processes

$$b \to \begin{cases} u e^- \bar{\nu}_e \\ u\mu^- \bar{\nu}_\mu \\ u\tau^- \bar{\nu}_\tau \\ u d\bar{u} \\ u s\bar{c} \end{cases}$$

are defined by

$$-\mathcal{L}_{\rm eff}=\frac{G_F}{\sqrt{2}}\{\epsilon_{LL}[\bar{e}^-\gamma^\mu(1-\gamma_5)\nu_e][\bar{u}\gamma_\mu(1-\gamma_5)b]+\epsilon_{LR}[\bar{e}^-\gamma^\mu(1-\gamma_5)\nu_e][\bar{u}\gamma_\mu(1+\gamma_5)b]+\epsilon_{RL}[\bar{e}^-\gamma^\mu(1+\gamma_5)\nu_e][\bar{u}\gamma_\mu(1-\gamma_5)b]+\epsilon_{RR}[\bar{e}^-\gamma^\mu(1+\gamma_5)\nu_e][\bar{u}\gamma_\mu(1+\gamma_5)b]\}, \tag{4.37}$$

for example, for $b\to ue^-\bar{\nu}_e$. In our model we obtain

$$\epsilon_{LL}=-2.4\times10^{-1},\quad \epsilon_{LR}=4.3\times10^{-4},\quad \epsilon_{RL}=2.5\times10^{-4},\quad \epsilon_{RR}=2.6\times10^{-5}. \tag{4.38}$$

Taking the color factor into account, we get

$$\Gamma(b\to ux)=9\frac{G_F^2m_b^5}{192\pi^3}\times[|\epsilon_{LL}|^2+|\epsilon_{LR}|^2+|\epsilon_{RL}|^2+|\epsilon_{RR}|^2]=3.6\times10^{-8}\text{ MeV}. \tag{4.39}$$

for the charged-current contribution.

2. *Neutral currents*

(a) The chiral couplings defined in the effective Lagrangians similar to (4.37) are

$$\epsilon_{LL}=1.2\times10^{-1},\quad \epsilon_{LR}=5.9\times10^{-2},\quad \epsilon_{RL}=5.7\times10^{-3},\quad \epsilon_{RR}=8.4\times10^{-2}, \tag{4.40}$$

for the processes

$$b\to\begin{cases}d\nu_e\bar{\nu}_e\\ d\nu_\mu\bar{\nu}_\mu\\ d\nu_\tau\bar{\nu}_\tau\end{cases}.$$

Therefore, we have

$$\Gamma(b\to d\nu\bar{\nu})=3\frac{G_F^2m_b^5}{192\pi^3}\times[|\epsilon_{LL}|^2+|\epsilon_{LR}|^2+|\epsilon_{RL}|^2+|\epsilon_{RR}|^2]=5.4\times10^{-9}\text{ MeV}. \tag{4.41}$$

(b) For the processes

$$b\to\begin{cases}de\bar{e}\\ d\mu\bar{\mu}\\ d\tau\bar{\tau}\end{cases},$$

the chiral couplings are

$$\epsilon_{LL}=-7.5\times10^{-2},\quad \epsilon_{LR}=5.2\times10^{-2},\quad \epsilon_{RL}=5.8\times10^{-2},\quad \epsilon_{RR}=4.2\times10^{-2}, \tag{4.42}$$

which give

$$\Gamma(b\to dl\bar{l})=2.9\times10^{-9}\text{ MeV}. \tag{4.43}$$

(c) For the processes

$$b\to\begin{cases}du\bar{u}\\ dc\bar{c}\end{cases},$$

we get

$$\epsilon_{LL}=6.9\times10^{-2},\quad \epsilon_{LR}=-3.6\times10^{-2},\quad \epsilon_{RL}=-4.1\times10^{-2},\quad \epsilon_{RR}=-1.0\times10^{-2}, \tag{4.44}$$

which give

$$\Gamma(b\to du\bar{u})+\Gamma(b\to dc\bar{c})=3.4\times10^{-9}\text{ MeV}. \tag{4.45}$$

(d) Similarly, for the processes

$$b\to\begin{cases}dd\bar{d}\\ ds\bar{s}\end{cases},$$

we have

$$\epsilon_{LL}=-1.2\times10^{-1},\quad \epsilon_{LR}=-4.2\times10^{-2},\quad \epsilon_{RL}=1.1\times10^{-2},\quad \epsilon_{RR}=-5.3\times10^{-2}, \tag{4.46}$$

which give

$$\Gamma(b\to dd\bar{d})+\Gamma(b\to ds\bar{s})=8.3\times10^{-9}. \tag{4.47}$$

Therefore, the total width of the b quark is estimated to be

$$\Gamma_{\rm tot}(b)=5.6\times10^{-8}\text{ MeV}, \tag{4.48}$$

i.e.,

$$\tau(b)=1.2\times10^{-14}\text{ sec}. \tag{4.49}$$

V. SUMMARY AND DISCUSSION

We have investigated the possibility of calculating the u- and d-quark masses in a unified gauge theory of the weak and electromagnetic interactions based on the $SU_L(4)\times SU_R(4)\times U(1)$ gauge group. Starting from the zeroth-order mass relations $m_u=m_d=0$, we have generated finite masses for these light quarks of order αm_b in second-order perturbation theory. Because we started with $m_u=m_d=0$, the photon exchange does not contribute to the u-d mass difference. Thus, it is possible to overcome the wrong-sign problem which has plagued earlier calculations of the p-n mass difference. The actual values of the light-quark masses depend on quite a few parameters. At the same time, there are also a large number of constraints. As we have seen, it is indeed possible to choose these parameters so as to obtain both satisfactory agreement with neutral-current data and good agreement with the standard values of the light-quark masses.

Some improvements are necessary to make this model more realistic. First of all, we have so far neglected the computation of the masses of the leptons and the strange, charm, etc. quarks in order to make our model as simple as possible. If we assign them to quartets as we did in Sec. IV, we would have to introduce more Higgs scalars belonging to the $(\underline{4}^*, \underline{4}, 0)$ representation. This would make our model rather complicated indeed. Nevertheless, with the introduction of these new parameters, we expect that it is possible to have $m_\nu \simeq 0$ and reasonable values for m_s and m_c. Second, our minimum model has the Adler-Bell-Jackiw type anomaly. However, introduction of new leptons and/or quarks could cancel these anomalies. In this paper, again for simplicity, we have not treated this problem in detail. Our calculation of masses, of course, is independent of such considerations.

In conclusion, our model shows that it is consistent to regard the u- and d-quark masses as due entirely to radiative corrections. This would seem to be a natural scheme to understand the very small values of m_u and m_d. To generalize our considerations to other leptons and quarks, however, the model will become very complicated unless we have a new symmetry principle linking the various fundamental fermions. So far, we are unable to find such a scheme.

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APPENDIX A: THE HIGGS POTENTIAL

The general fourth-order polynomial P constructed from the scalar fields is of the form

$$\begin{aligned} P(\Phi,\Xi_L,\Xi_R,\hat{D}_L,\hat{D}_R) =& a_1 \mathrm{Tr}(\Phi^\dagger\Phi) + a_2 \mathrm{Tr}(\Phi^\dagger\Phi\Phi^\dagger\Phi) + a_3[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 + b_1\Xi_L^\dagger\Xi_L\Xi_R^\dagger\Xi_R + b_2\mathrm{Tr}(\hat{D}_L\hat{D}_L)\mathrm{Tr}(\hat{D}_R\hat{D}_R) \\ &+\{c_1\Xi_L^\dagger\Xi_L + c_2(\Xi_L^\dagger\Xi_L)^2 + c_3\mathrm{Tr}(\hat{D}_L\hat{D}_L) + c_4\mathrm{Tr}(\hat{D}_L\hat{D}_L\hat{D}_L\hat{D}_L) + c_5[\mathrm{Tr}(\hat{D}_L\hat{D}_L)]^2 + c_6\Xi_L^\dagger\hat{D}_L\hat{D}_L\Xi_L \\ &+ c_7\Xi_L^\dagger\Phi\Phi^\dagger\Xi_L + c_8\Xi_L^\dagger\Xi_L\mathrm{Tr}(\Phi^\dagger\Phi) + c_9\Xi_L^\dagger\Xi_L\mathrm{Tr}(\hat{D}_L\hat{D}_L) + c_{10}\Xi_L^\dagger\Xi_L\mathrm{Tr}(\hat{D}_R\hat{D}_R) \\ &+ c_{11}\mathrm{Tr}(\hat{D}_L\hat{D}_L)\mathrm{Tr}(\Phi^\dagger\Phi) + c_{12}\mathrm{Tr}(\Phi\Phi^\dagger\hat{D}_L\hat{D}_L) + (L\to R)\}, \end{aligned}$$

where terms odd in D_L and D_R are absent due to the assumed discrete symmetries R_L and R_R (2.12). The polynomial P is symmetric under the exchange $L \to R$ by construction. As was pointed out earlier, this does not mean that the VEV's have to be left-right symmetric.

To verify the choices made in Eqs. (2.15), (2.17), and (2.20), we substitute the VEV's into P and differentiate with respect to $a, b, \ldots$.

The necessary conditions for the potential P to reach a minimum value are $\partial P/\partial a = 0$, $\partial P/\partial b = 0$, etc. These equations are too complex to be recorded here. However, one may verify that the special case $a = b = 0$ is a solution. We should also note that, because of the presence of the term $(\Xi_L^\dagger\Phi\Phi^\dagger\Xi_L + \Xi_R^\dagger\Phi^\dagger\Phi\Xi_R)$, the choice $c \neq d$ is possible.

APPENDIX B:

For the search of a "realistic" solution of our model, we used a numerical computer program that, with the VEV of the Higgs scalar fields and the coupling constants as input, calculates the most relevant phenomenological aspects of the model. The program calculates the mass matrix of the vector bosons, diagonalizes it, and finds its eigenvalues and the corresponding eigenvectors. Then it calculates the masses of the u and d quark using the formulas derived in Sec. III. Finally, using the formulas of Sec. IV, it computes the most relevant phenomenological quantities that can be compared with experimental data.

In our search we were led by two considerations. First, the phenomenology at low energies must resemble that of the WS model because of the excellent agreement of this model with experimental data. The second consideration was that the calculated masses of the u and d quarks should have the right order of magnitude (a few MeV) and the ratio m_d/m_u should be approximately 1.8. To ensure the resemblance with the WS model, we made sure that the $SU(2)\times U(1)$ symmetry defined in (2.14) was broken to a much smaller degree than the rest of the $SU_L(4)\times SU_R(4)\times U(1)$ symmetry. This was accomplished by giving $\langle \hat{D}_L\rangle_0$ and ξ_L values that were approximately one order of magnitude smaller than the other VEV's. This requirement proves to be a severe constraint since it means that, except for $\langle \hat{D}_L\rangle_0$, most of the other parameters are used to push the masses of the gauge bosons (except W and Z) to large values. It turns out to be very difficult to find a set of

VEV's which give reasonable results. In addition, the complexity and nonlinearity of the problem make it hard to find a systematic method of looking for appropriate VEV's. We therefore resorted to a random-number generator for generating the VEV's. The only limitations we imposed on the randomness of the VEV's was that the values for $\langle \hat{D}_L \rangle_0$ and ξ_L were generally an order of magnitude smaller than the rest of the VEV's and that $2 < d/c < 5$. This last condition ensures that the mass of the not yet discovered t quark falls within the range 10 to 25 GeV. From approximately 10 000 trials we picked a dozen cases that gave rise to a low-energy phenomenology similar to that of the WS model and that gave approximately the right ratio for m_d/m_u. Starting from these cases, we improved the phenomenology by changing the VEV's by small amounts around the initial values. After repeating this process many times we found five cases that gave satisfactory agreement with the experimental data. The VEV's that give the "best" agreement with the experimental data are given in Sec. III. All the numerical predictions of the model that are obtained in Secs. III and IV are based on this set of VEV's.

APPENDIX C: SOLUTION OF THE RENORMALIZATION-GROUP EQUATION

In this appendix, we discuss the dependence of the renormalized mass m_R on the renormalization point μ in the framework of the zero-mass-renormalization (ZMR) scheme of 't Hooft and Weinberg.[11] Renormalized and unrenormalized quantities are specified by the subscripts R and u, respectively. In the ZMR scheme, the coupling constant and mass are renormalized multiplicatively:

$$g_R = g_u Z_g(g_u, \Lambda/\mu)\,, \quad m_R = m_u Z_m(g_u, \Lambda/\mu)\,, \tag{C1}$$

where the renormalization constants Z_g and Z_m depend only on g_u and Λ/μ, Λ and μ being the cutoff and renormalization point, respectively. Let $g_R(g_R^0)$ and $m_R(m_R^0)$ be renormalized at $\mu(\mu_0)$. Then (g_R, m_R) are related to (g_R^0, m_R^0) by

$$g_R = g_R^0 z_g(g_R^0, \mu/\mu_0)\,, \quad m_R = m_R^0 z_m(g_R^0, \mu/\mu_0)\,, \tag{C2}$$

where

$$z_g(g_R^0, \mu/\mu_0) = Z_g(g_u, \Lambda/\mu)/Z_g(g_u, \Lambda/\mu_0)\,,$$

$$z_m(g_R^0, \mu/\mu_0) = Z_m(g_u, \Lambda/\mu)/Z_m(g_u, \Lambda/\mu_0)$$

are independent of Λ because they must be finite when $\Lambda \to \infty$. Thus, g_R and m_R may be regarded as functions of g_R^0, m_R^0, and $t \equiv \ln\mu/\mu_0$, when g_u, m_u, and Λ/μ_0 are kept fixed. These finite renormalization functions z_g and z_m can be obtained by solving the renormalization-group equations that follow from (C1):

$$\frac{d}{dt} g_R(t; g_R^0) = g_R(t; g_R^0)\mu \frac{d}{d\mu} \ln Z_g\left(g_u, \frac{\Lambda}{\mu}\right)\Big|_{\mu = e^t \mu_0} \equiv \beta(g_R(t; g_R^0)) \tag{C3}$$

and

$$\frac{d}{dt} m_R(t; g_R^0, m_R^0) = m_R(t; g_R^0, m_R^0) \times \mu \frac{d}{d\mu} \ln Z_m\left(g_u, \frac{\Lambda}{\mu}\right)\Big|_{\mu = e^t \mu_0} \equiv m_R(t; g_R^0, m_R^0)\gamma_m(g_R(t, g_R^0))\,, \tag{C4}$$

with the boundary conditions

$$g_R(0; g_R^0) = g_R^0\,, \quad m_R(0; g_R^0, m_R^0) = m_R^0\,. \tag{C5}$$

Equations (C3) and (C4) give the trajectories of the renormalized coupling and mass that pass (g_R^0, m_R^0) at $\mu = \mu_0$, when g_u, m_u, and Λ are kept fixed. Solutions of these equations have been given by

$$t = \int_{g_R^0}^{g_R(t; g_R^0)} dx\, \beta^{-1}(x) \tag{C6}$$

and

$$m_R(t; g_R^0, m_R^0) = m_R^0 \exp\left[\int_0^t \gamma_m(t')dt'\right], \tag{C7}$$

where

$$\gamma_m(t) \equiv \gamma_m(g_R(t; g_R^0))\,.$$

In the perturbation theory, β and γ_m read

$$\beta(g_R) = -\tfrac{1}{2} a g_R{}^3 + \cdots\,, \tag{C8}$$

$$\gamma_m(g_R) = -c g_R{}^2 + \cdots\,. \tag{C9}$$

On substitution of Eqs. (C8) and (C9) to Eqs. (C6) and (C7), we obtain

$$m_R(t; g_R^0, m_R^0) = m_R^0[1 + a(g_R^0)^2 t]^{-c/a}\,. \tag{C10}$$

In quantum chromodynamics the coefficients a and c are given by[5,12]

$$a = \frac{1}{12\pi^2}\left(\tfrac{33}{2} - n_f\right) \tag{C11}$$

and

$$c = \frac{1}{2\pi^2}\,. \tag{C12}$$

Substitution of (C11) and (C12) into Eq. (C10) gives Eq. (3.16).

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