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Baryonium and nonexotic hadron trajectories from a color-dependent potential

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A natural explanation is presented for the apparent increase in Regge slope with the number of quarks in a bound state. The framework used is that of a nonrelativistic harmonic-oscillator potential with a color dependence suggested by quantum chromodynamics. Ratios of modes of orbital excitations in baryonium, baryon, and meson states which are in good agreement with experiment are derived, and the modes of excitation within the baryonium states are discussed.

INTRODUCTION

The spectrum of the four-quark baryonium states has been the subject of extensive investigation.[1-8] From an analysis based on the diquark structure, two different color "isomers" are possible: Either two quarks bind to form a diquark in color 3 which binds with a similar antidiquark in color $\overline{3}$, or a diquark in color 6 binds with an antidiquark in color $\overline{6}$. The rich spectroscopy of the S-wave baryonium states where the 3-$\overline{3}$ and 6-$\overline{6}$ states mix has been treated by Jaffe[1] by considering the effects of the color-magnetic spin-spin interaction acting in an SU(6) color-spin group. With orbital excitations between the diquark and antidiquark the 3-$\overline{3}$ and 6-$\overline{6}$ states no longer mix and two separate series form.[2-4] To treat the spectrum of the orbital excitations some assumptions are needed about the slopes of the trajectories.[2-5] From bag-model considerations, it has been argued that the slopes of trajectories associated with an orbital excitation between an aggregate of quarks and a quark or another aggregate of quarks which together form a color singlet is proportional to $(C_3)^{-1/2}$ where C_3 is the color Casimir operator associated with the aggregate.[9] Thus the slopes of the 3-$\overline{3}$ baryonium series, baryons, and mesons should all be the same, as a 3-$\overline{3}$ bond is involved in each case. The narrow widths of the $S(1.936)$, $T(2.15)$, $U(2.31)$, and $V(2.48)$ (Refs. 10,11) resonances in the $p\overline{p}$ reactions which lie on a reasonably straight trajectory suggest that these states may be good candidates for baryonium states. The slope of this trajectory is 1.26 GeV^{-2}, thus greater than the natural-parity nucleon trajectory, $N(0.939)$-$N(1.68)$-$N(2.22)$,[12,13] which in turn is greater than the ρ-A_2-g trajectory.[13] This tendency for slopes to increase with quark number[14] is illustrated in Fig. 1.

In this paper we provide a straightforward treatment which unifies the slopes of these states, and which provides an explanation of this phenomenon of increasing slopes. We proceed by taking a harmonic-oscillator confining potential and include a color dependence suggested by quantum chromodynamics (QCD). The oscillator potential has the advantage of giving analytic solutions and has been extensively used in quark models, for example, by Horgan and Dalitz[15] and Isgur and Karl[16] in baryon spectroscopy and by Novikov *et al.*[17] and Jackson[18] in charmonium spectroscopy. We assume then that the potential associated with an N-quark system is given by a sum of two-body terms and has the form

$$V=-\sum_{i>j}^{N}\sum_{a}\tfrac{1}{2}(\lambda_i^a)\tfrac{1}{2}(\lambda_j^a)g_s^2 f(m_i,m_j)(r_i-r_j)^2\,. \quad (1.1)$$

The eight Gell-Mann SU(3) generators, $\frac{1}{2}\lambda_i$, are the color charges associated with the ith quark. For the antiquarks, the replacement $\lambda_i \to -\lambda_i^*$ is made. This form of color dependence, when it can be factored from the radial and mass dependence, leads to color singlets having the lowest energy.[19] The strong coupling constant g_s is assumed to be flavor independent. The form of $f(m_i,m_j)$ in Eq. (1.1) determines the scaling properties of the Schrödinger equation.[20] We have assumed

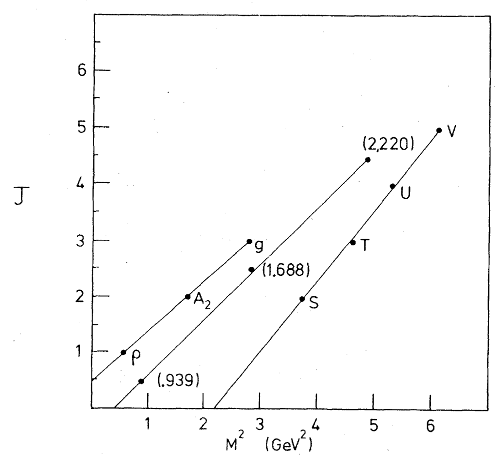

FIG. 1. Trajectories for the meson, baryon [ground state N(0.939 GeV)], and baryonium resonances.

$$f(m_i,m_j)=\frac{m_i m_j}{m_i+m_j}\,. \tag{1.2}$$

By including the reduced mass we follow Novikov *et al.*[17] Two reasons suggest the inclusion of Eq. (1.2) in Eq. (1.1): (1) It leads to modes having an energy spacing independent of quark mass (at least for states composed of the same type of quark). Its exclusion leads to trajectory slopes proportional to $\sqrt{m_q}$, in violation of experiment. (2) The vector-meson e^+e^- decay rates suggest $|\psi(0)|\propto\mu^{-1.8}$ where $\psi(0)$ is the wave function of the q-$\bar{q}$ system at the origin, and μ the two-body reduced mass.[18] The harmonic-oscillator potential with the reduced mass present gives $|\psi(0)|^2\propto\mu^{-1.5}$ and without the reduced mass $|\psi(0)|^2\propto\mu^{-0.75}$. In the next section the color factors $\sum_a \frac{1}{2}\lambda_i^a\frac{1}{2}\lambda_j^a$ are evaluated for the cases of relevance. In the third section solutions to the Schrödinger equation are considered for two-, three-, and four-quark systems and the relativistic ansatz $m\to m^2$ introduced and the slopes of these states tabulated. We conclude in the last section with a discussion.

COLOR DEPENDENCE

The color factor can be readily evaluated once the color wave function is known.[18] For mesons, the use of the color-singlet wave function gives

$$-\left\langle\sum_a \tfrac{1}{2}\lambda_q^a\tfrac{1}{2}\lambda_{\bar{q}}^a\right\rangle=\tfrac{4}{3}\equiv\eta_M\,. \tag{2.1}$$

For baryons in a color singlet

$$-\left\langle\sum_a \tfrac{1}{2}\lambda_q^a\tfrac{1}{2}\lambda_q^a\right\rangle=\tfrac{2}{3}\equiv\eta_B \tag{2.2}$$

for all q-q pairs. The color wave function for the 3-$\bar{3}$ baryonium states (labeled T_4) has the form

$$\frac{1}{2\sqrt{3}}\sum_{\substack{i,j,p\\k,l}}\epsilon_{ijp}\epsilon_{pkl}\,|q_iq_j\bar{q}_k\bar{q}_l\rangle\,. \tag{2.3}$$

If we label the quarks by 1 and 2, and the antiquarks by 3 and 4, then

$$-\left\langle\sum_a \tfrac{1}{2}\lambda_i^a\tfrac{1}{2}\lambda_j^a\right\rangle=\tfrac{2}{3}\equiv\eta_{T_4}\,,\quad (i,j)=(1,2)\text{ and }(3,4) \tag{2.4}$$

and

$$-\left\langle\sum_a \tfrac{1}{2}\lambda_i^a\tfrac{1}{2}\lambda_j^a\right\rangle=\tfrac{1}{3}\equiv\eta_{T_4}'\,,\quad (i,j)=(1,3),(1,4),(2,3),(2,4)\,. \tag{2.5}$$

With the 6-$\bar{6}$ baryonium states (labeled by M_4) the color wave function is

$$\frac{1}{2\sqrt{6}}\sum_{\substack{i,j,\\k,l}}(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk})\,|q_iq_j\bar{q}_k\bar{q}_l\rangle\,, \tag{2.6}$$

with

$$-\left\langle\sum_a \tfrac{1}{2}\lambda_i^a\tfrac{1}{2}\lambda_j^a\right\rangle=-\tfrac{1}{3}\equiv\eta_{M_4}\,,\quad (i,j)=(1,2)\text{ and }(3,4) \tag{2.7}$$

and

$$-\left\langle\sum_a \tfrac{1}{2}\lambda_i\tfrac{1}{2}\lambda_j\right\rangle=\tfrac{5}{6}\equiv\eta_{M_4}'\,,\quad (i,j)=(1,3),(1,4),(2,3),(2,4)\,. \tag{2.8}$$

TRAJECTORY SLOPES

(*a*) *Mesons*. The potential in the two-body case has the form

$$V=\frac{m_1m_2}{m_1+m_2}g_s^2(\vec{r}_1-\vec{r}_2)^2\eta_M\,. \tag{3.1}$$

Introducing a relative coordinate,

$$\vec{x}=\left(\frac{2m_1m_2}{(m_1+m_2)\hbar^2}\right)^{1/2}(\vec{r}_1-\vec{r}_2)\,, \tag{3.2}$$

the Schrödinger equation separates into center-of-mass motion and relative motion. Defining $\beta^2=\frac{1}{2}\eta_M g_s^2\hbar^2$, we have

$$\left(-\frac{d^2}{d\vec{x}^2}+\beta^2\vec{x}^2\right)\psi=E\psi\,. \tag{3.3}$$

The energy is given by

$$E_{k,l}=2\beta(2k+l+\tfrac{3}{2})\,, \tag{3.4}$$

where k is the radial and l the orbital quantum number. The analysis can be carried out with a relativistic extension of the Schrödinger equation (see, for example, Feynman *et al.*[21] or Muller-Kirsten[22]), the essence of which leads to the replacement of E with $(m^2+\text{const})$ in Eq. (3.4). Implied in this replacement is a family of trajectories of slope β^{-1}. One can alternatively start with the Bethe-Salpeter equation with an oscillator potential and obtain all $[m^2,(2k+l)]$ relationship directly with a trajectory slope proportional to β^{-1} where β is defined above.[23] Note that the slope is proportional to $(\eta_m)^{-1/2}$.

(*b*) *Baryons*. Consider a set of relative coordinates,[24]

$$\vec{\rho}=\left(\frac{2m_2m_3}{(m_2+m_3)\hbar^2}\right)^{1/2}(\vec{r}_2-\vec{r}_3)\,, \tag{3.5}$$

$$\vec{\lambda}=\left(\frac{2m_1(m_2+m_3)}{(m_1+m_2+m_3)\hbar^2}\right)^{1/2}\left(\frac{m_2\vec{r}_2+m_3\vec{r}_3}{m_2+m_3}-\vec{r}_1\right). \tag{3.6}$$

The most general situation in a flavor group of SU(3) (with $m_u=m_d$) can be described with $m_2=m_3$. The potential then becomes

$$V=\beta_\rho^2(\vec{\rho})^2+\beta_\lambda^2(\vec{\lambda})^2\,, \tag{3.7}$$

with

$$\beta_\rho{}^2 = \tfrac{1}{2} g_s{}^2 \eta_B \hbar^2 \left(\frac{m_2 + 2m_1}{m_1 + m_2}\right),$$
$$\beta_\lambda{}^2 = \tfrac{1}{2} g_s{}^2 \eta_B \hbar^2 \left(\frac{m_1 + 2m_2}{m_1 + m_2}\right), \tag{3.8}$$

and the solutions yielding a spectrum

$$M^2 = 2\beta_\rho(2k_\rho + l_\rho + \tfrac{3}{2}) + 2\beta_\lambda(2k_\lambda + l_\lambda + \tfrac{3}{2}) + \text{const}. \tag{3.9}$$

The associated trajectory slopes are given in Table I.

(*c*) *Baryonium.* In this case a set of coordinates[24,25] which exhibits the diquark structure is given by

$$\vec{x} = \left(\frac{2m_1 m_2}{(m_1 + m_2)\hbar^2}\right)^{1/2} (\vec{r}_1 - \vec{r}_2), \tag{3.10}$$

$$\vec{y} = \left(\frac{2m_3 m_4}{(m_3 + m_4)\hbar^2}\right)^{1/2} (\vec{r}_4 - \vec{r}_3), \tag{3.11}$$

$$\vec{z} = \left(\frac{2(m_1 + m_2)(m_3 + m_4)}{(m_1 + m_2 + m_3 + m_4)\hbar^2}\right)^{1/2} \times \left(\frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2} - \frac{m_3\vec{r}_3 + m_4\vec{r}_4}{m_3 + m_4}\right). \tag{3.12}$$

To achieve a separation of the potential we need to make the restriction $m_1 = m_2$, $m_3 = m_4$. The potential then takes the form

$$V = \beta_x{}^2(\vec{x})^2 + \beta_y{}^2(\vec{y})^2 + \beta_z{}^2(\vec{z})^2, \tag{3.13}$$

with

$$\beta_x{}^2 = \tfrac{1}{2} g_s{}^2 \hbar^2 \left(\eta + \frac{2\eta' m_3}{m_1 + m_3}\right), \tag{3.14}$$

$$\beta_y{}^2 = \tfrac{1}{2} g_s{}^2 \hbar^2 \left(\eta + \frac{2\eta' m_1}{m_1 + m_3}\right), \tag{3.15}$$

$$\beta_z{}^2 = \eta' g_s{}^2 \hbar^2, \tag{3.16}$$

with a spectrum

$$M^2 = 2\beta_x(2k_x + l_x + \tfrac{3}{2}) + 2\beta_y(2k_y + l_y + \tfrac{3}{2}) + 2\beta_z(2k_z + l_z + \tfrac{3}{2}) + \text{const}. \tag{3.17}$$

There are now three possible modes of excitation all of which may be different if m_1 differs from m_3. The values of the color factors are given in Sec. II above. Table I contains the slopes for the T_4 and M_4 baryonium series.

As a comparison with experiment let us compare the ratios of the slopes of the trajectories in Table I for states composed of the same type of quark. For the ratio of T baryonium to meson slopes, and the ratio of baryon to meson slopes we obtain

$$\frac{\alpha'_{T4}}{\alpha'_M} = \sqrt{2} \simeq 1.41, \quad \frac{\alpha'_B}{\alpha'_M} = (\tfrac{4}{3})^{1/2} \simeq 1.15. \tag{3.18}$$

The experimental values give $\alpha'_{T_4}/\alpha'_M = 1.42$ and $\alpha'_B/\alpha'_M = 1.13$. The experimental uncertainty in both cases is in the order of ± 0.08. These values are in good agreement with the theoretical predictions, especially considering the naivety of our model. For the ratio of the slope of the T baryonium to M baryonium we predict $(\tfrac{5}{2})^{1/2}$, thus the same result as the bag model,[9] but we predict $\alpha'_{M4} = (\tfrac{4}{5})^{1/2}\alpha'_M \simeq 0.79$; thus we are led to a higher-lying M baryonium trajectory than predicted by the bag model. The status of the M baryonium series is unclear at present; however, it is interesting to note that if we take the tentative assignment by Chan and Høgaasen[4] of 3.05, 2.85, and 2.62 as the $L = 4$, 3, and 2 resonances of M baryonium, then a slope of 0.81 is obtained. For T baryoniums composed of the same type of quark the slope associated with the z mode is $(\tfrac{3}{2})^{1/2}$ times that associated with either the x or y mode. The difficulty of lack of exchange degeneracy for the baryon trajectories and the determination of the different λ and ρ series and their slopes prevents any clear comparison with experiment.

TABLE I. Trajectory slopes (α') using the equations of Sec. III with the color factors of Sec. II.

Mode	$(2\alpha' g_s \hbar)^{-1}$
Meson	$(\frac{2}{3})^{1/2}$
Baryon	
ρ mode	$\left[\frac{1}{3}\left(\frac{m_2+2m_1}{m_1+m_2}\right)\right]^{1/2}$
λ mode	$\left[\frac{1}{3}\left(\frac{m_1+2m_2}{m_1+m_2}\right)\right]^{1/2}$
Baryonium (T_4)	
x mode	$\left[\frac{1}{3}\left(\frac{m_1+2m_3}{m_1+m_3}\right)\right]^{1/2}$
y mode	$\left[\frac{1}{3}\left(\frac{m_3+2m_1}{m_1+m_3}\right)\right]^{1/2}$
z mode	$(\frac{1}{3})^{1/2}$
Baryonium (M_4)	
x mode	$\left[\frac{1}{6}\left(\frac{4m_3-m_1}{m_1+m_3}\right)\right]^{1/2}$
y mode	$\left[\frac{1}{6}\left(\frac{4m_1-m_3}{m_1+m_3}\right)\right]^{1/2}$
z mode	$(\frac{5}{6})^{1/2}$

DISCUSSION

We have shown that a harmonic-oscillator potential with a color dependence suggested by QCD allows us to separate the Schrödinger equation for the baryonium, baryon, and meson states (provided certain constraints are placed on the masses of the constituent quarks) and to obtain the ratios of Eq. (3.18). These ratios are in good agreement with the current experimental statistics; a crucial test will be the slope of the M

baryonium trajectory. We expect that the effect of the color factors of Sec. II in more realistic potentials will follow the patterns exhibited by the harmonic-oscillator potential.[26] In one sense our analysis is limited in that the relationships such as $\alpha'_{uu\bar{u}\bar{u}} > \alpha'_{uu\bar{s}\bar{s}}$ which follow from duality arguments[27,28] cannot be obtained. It is of significance that the q-$\bar{q}$ terms in the baryonium contribute to the x and y modes. This is especially important for the M baryoniums, for while the q-q (and $\bar{q}$-$\bar{q}$) potentials are repulsive, the large q-$\bar{q}$ contributions make the x and y modes *attractive*. It is this contribution of various two-body terms to the different modes which is responsible for the slope increase with quark number. For the M baryoniums, certain mass conditions are required for stability, viz.,

$$4m_1 - m_3 > 0, \quad 4m_3 - m_1 > 0 . \tag{4.1}$$

If we make the assignment $m_u = 336$ MeV, $m_s = 540$ MeV, and $m_c = 1.65$ MeV,[29] then these conditions are satisfied if $m_1 = m_u$ and $m_3 = m_s$, but not if one of the diquarks is composed of charmed quarks and the other of u-type quarks. Finally we note that if the color terms are absent from the potential (i.e., all the η factors in Sec. II are equal) the slopes *decrease* with the number of quarks per state, $\alpha'_{T_4}/\alpha'_M = (\frac{1}{2})^{1/2}$, and $\alpha'_B/\alpha'_M = (\frac{2}{3})^{1/2}$. Thus in the framework of our analysis, the result of Eq. (3.18) and the phenomenon illustrated in Fig. 1 is an implicit manifestation of color.

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[1] R. L. Jaffe, Phys. Rev. D 15, 267 (1977); 15, 281 (1977).
[2] R. L. Jaffe, Phys. Rev. D 17, 1444 (1978).
[3] Chan Hong-Mo and H. Høgaasen, Phys. Lett. 72B, 121 (1978); 72B, 400 (1978).
[4] Chan Hong-Mo and H. Høgaasen, Nucl. Phys. B136, 401 (1978).
[5] A. W. Hendry and I. Hinchliffe, Phys. Rev. D 18, 3453 (1978).
[6] G. C. Rossi and G. Veneziano, Nucl. Phys. B123, 507 (1977); Phys. Lett. 70B, 255 (1977).
[7] H. J. Lipkin, Phys. Lett. 74B, 399 (1978).
[8] R. Anderson and G. C. Joshi, J. Math. Phys. 20, 1015 (1979); Phys. Rev. D 20, 736 (1979).
[9] K. Johnson and C. B. Thorn, Phys. Rev. D 13, 1934 (1976).
[10] L. Montanet, CERN report, 1978 (unpublished).
[11] A. A. Carter *et al.*, Phys. Lett. 67B, 117 (1977).
[12] Particle Data Group, Phys. Lett. 75B (1978).
[13] P. D. B. Collins, *An Introduction to Regge Theory and High Energy Physics* (Cambridge University Press, London, 1977).
[14] The steep slope of the S, T, U, and V resonances has also been observed by P. H. Dondi, Southampton University Report No. THEP 78/9-3 (unpublished).
[15] R. Horgan and R. H. Dalitz, Nucl. Phys. B66, 135 (1973); see also O. W. Greenberg, Phys. Rev. Lett. 13, 598 (1964).
[16] N. Isgur and G. Karl, Phys. Rev. D 18, 4187 (1978).
[17] V. A. Novikov *et al.*, Phys. Rep. 41C, 1 (1978).
[18] J. D. Jackson, in *Weak Interactions at High Energy and the Production of New Particles*, proceedings of SLAC Summer Institute on Particle Physics, 1976, edited by M. Zipf (SLAC, Stanford, 1977).
[19] See, for example, H. Fritzsch, CERN Report No. TH. 2583, 1978 (unpublished).
[20] C. Quigg and J. L. Rosner, Comm. Nucl. Part. Phys. VII, 11 (1978).
[21] R. P. Feynman, M. Kislinger, and F. Ravndal, Phys. Rev. D 3, 2706 (1971).
[22] H. J. W. Müller-Kirsten, Phys. Rev. D 12, 1103 (1975).
[23] M. Böhm, H. Joos, and M. Krammer, Nucl. Phys. B51, 397 (1973).
[24] V. J. Kukulin and V. M. Krasnopol'sky, J. Phys. G 3, 795 (1977).
[25] I. M. Barbour and D. K. Ponting, Nucl. Phys. B149, 534 (1979).
[26] See, for example, the relation of the spectrum to the potential factors in J. S. Kang and H. J. Schnitzer, Phys. Rev. D 12, 841 (1975), and also Ref. 22.
[27] K. Igi and Yazaki, Phys. Lett. 74B, 257 (1978).
[28] R. Anderson and G. C. Joshi, J. Phys. G 5, 199 (1979).
[29] A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. D 12, 147 (1975).