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Low-energy meson-nucleon scattering analysis in the P-matrix formalism

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We analyze the low-energy S-wave scattering of pseudoscalar mesons by nucleons in the P-matrix formalism recently proposed by Jaffe and Low. We find from the data clear evidence for the predicted bag-model $Q^4\bar{Q}$ states in all the exotic and nonexotic channels we analyze.

I. INTRODUCTION

Quarks have now become well established as important phenomenological and theoretical entities. The main success of the quark model is meson and baryon spectroscopy: Quark and antiquark make a meson, three quarks make a baryon. Multiquark hadrons ($Q^2\bar{Q}^2$ mesons, $Q^4\bar{Q}$ baryons$\cdots$) are apparently absent from the spectroscopy. This elusiveness is somewhat puzzling because quantum chromodynamics, the only available candidate theory of interquark forces, seems to promise a much richer spectrum than has yet been observed. In particular, in the MIT bag model[1] where confinement is built in, Jaffe showed[2] that $Q^2\bar{Q}^2$ mesons and $Q^4\bar{Q}$ baryons are not qualitatively different from $Q\bar{Q}$ and Q^3 hadrons which are successfully accounted for.[1]

It has always been thought that quark states should show up as conventional resonances in partial-wave analyses. But recently Jaffe and Low[3] have shown that this need not be true for multiquark states. They developed a method for analyzing low-energy hadron-hadron scattering. They introduced a quantity, called the P matrix, which serves as a link between the discrete states of the quark model and the scattering states in which quarks do not appear. They describe the hadron-hadron scattering in the center-of-mass system by making a division between an inside where quarks and gluons are the dynamical variables and an outside where the interactions between hadrons are negligible. The P matrix is constructed to join the two regions.[3] Bag-model eigenstates correspond to poles in P with approximately calculable residues. The exterior scattering wave function is parametrized with the P matrix which thus is related to the S matrix and may be extracted from measured phases and elasticities. Jaffe and Low studied the S-wave scattering of pseudoscalar mesons in this manner and found clear evidence for internal $Q^2\bar{Q}^2$ states.[3]

In this paper we apply their formalism to low-energy S-wave meson-nucleon scattering. We extract the P matrix from the experimental phase-shift analyses of the pion-nucleon system in the $I=\frac{1}{2}, \frac{3}{2}$ channels and the kaon-nucleon, antikaon-nucleon systems in the $I=0,1$ channels. We find that the P-matrix description of S-wave meson-nucleon scattering reproduces the qualitative features expected in a bag model. In particular, we find poles corresponding closely to bag-model predictions of multiquark $Q^4\bar{Q}$ states in all the channels we analyze.

This paper is divided into three parts. The first part (Sec. II) outlines the P-matrix formalism and the specialization to the bag model. The spectrum and decay properties of $Q^4\bar{Q}$ baryons are studied in Sec. III in the framework of the MIT bag model. In Sec. IV we apply the P-matrix formalism to the $I=0, \frac{1}{2}, 1, \frac{3}{2}$ pseudoscalar-meson–nucleon S-wave system and compare the results with bag-model predictions.

II. REVIEW OF THE P-MATRIX FORMALISM

Here we wish only to review the main features of the P-matrix formalism applied to the meson-nucleon system in order to make this paper self-contained. For a detailed discussion the reader is referred to Ref. 3. Outside a relative separation b in the center-of-mass system, the n-channel meson-nucleon system is supposed to be free. The S-wave scattering wave function is then parametrized by

$$\psi_{ij}(r_j) \propto \delta_{ij}\cos k_j(r_j-b)+\frac{P_{ij}}{k_j}\sin k_j(r_j-b)\,, \quad (2.1)$$

with

$$k_j=[(E^2-m_j{}^2-M_j{}^2)^2-4m_j{}^2M_j{}^2]^{1/2}/2E\,. \quad (2.2)$$

(E is the total energy in the center-of-mass system m_j and M_j are respectively the masses of the meson and the baryon produced in channel j, and i labels the state.)

A pole of P corresponds to a state whose wave function vanishes at $r=b$; this is the kind of boundary conditions which is imposed on the internal wave function in a bag model.[4] The P matrix is simply related to the S matrix:

$$P = -ik \frac{1 + e^{ikb}(1/\sqrt{k}) S \sqrt{k}\, e^{ikb}}{1 - e^{ikb}(1/\sqrt{k}) S \sqrt{k}\, e^{ikb}}\,. \tag{2.3}$$

In the one-channel case, $S = e^{2i\delta}$ and P reduces to $k\cot(kb+\delta)$. When two channels are open S is given by

$$S = \begin{pmatrix} \eta e^{2i\delta_1} & i(1-\eta^2)^{1/2} e^{i(\delta_1+\delta_2)} \\ i(1-\eta^2)^{1/2} e^{i(\delta_1+\delta_2)} & \eta e^{2i\delta_2} \end{pmatrix}, \tag{2.4}$$

and the different elements of P are

$$P_{11} = \frac{k_1}{D}[\eta \sin(\theta_1 - \theta_2) - \sin(\theta_1 + \theta_2)]\,, \tag{2.5}$$

$$P_{12} = P_{21} = \frac{(k_1 k_2)^{1/2}}{D}(1-\eta^2)^{1/2}\,, \tag{2.6}$$

$$P_{22} = -\frac{k_2}{D}[\eta \sin(\theta_1 - \theta_2) + \sin(\theta_1 + \theta_2)]\,, \tag{2.7}$$

$$D = \cos(\theta_1 + \theta_2) - \eta \cos(\theta_1 - \theta_2)\,, \tag{2.8}$$

$$\theta_i = k_i b + \delta_i\,. \tag{2.9}$$

The P matrix is symmetric, real for real energies and can be expanded uniquely as a sum of poles with factorizing residues and a single matrix subtraction constant provided the interaction vanishes outside b. The poles are the eigenvalues of the internal Hamiltonian subject to the boundary condition $\psi = 0$ at $r = b$. Because P depends on b a pole in $P \approx r(b)/[s - s_0(b)]$ has a residue which satisfies

$$r(b) = \left(-\frac{\partial s_0}{\partial b}\right) Q\,, \tag{2.10}$$

where Q is a projection operator given, in the absence of degeneracy, by a single unit vector ξ $(\sum_i \xi_i^2 = 1)$:

$$Q_{ij} = \xi_i \xi_j\,. \tag{2.11}$$

The connection with the bag model is made in the following way. The eigenenergies of the spherical bag calculation for four quarks and one antiquark are interpreted as the energies for which the meson-baryon wave function vanishes at some radius b, i.e., as the poles of P. To estimate the relation between b and the bag radius R, we follow the recipe of Jaffe and Low adapted to the $Q^4\overline{Q}$ system. The mean value of $\vec{r}^{\,2}$, where $\vec{r}$ is the relative separation of three quarks and a quark-antiquark pair inside the bag is

$$
\begin{aligned}
{}^2 &= \int d^3r_1 d^3r_2 d^3r_3 d^3r_4 d^3r_5 \left(\frac{\vec{r}_1+\vec{r}_2+\vec{r}_3}{3} - \frac{\vec{r}_4+\vec{r}_5}{2}\right)^2 \\
&\quad \times \rho(\vec{r}_1,\vec{r}_2,\vec{r}_3,\vec{r}_4,\vec{r}_5) \qquad (2.12) \\
&= \tfrac{5}{6}\int d^3\vec{r}\, r^2 |\psi_q(r)|^2\,, \qquad (2.13)
\end{aligned}
$$

where $\rho(\vec{r}_1,\vec{r}_2,\vec{r}_3,\vec{r}_4,\vec{r}_5)$ is the five-quark density function and $\psi_q(r)$ the lowest S-wave bag eigenstate wave function.

Equating $\overline{r}^2$ with the mean value of r^2 obtained from a free meson-nucleon wave function $\psi(r)$ which vanishes at $r = b$,

$$\psi(r) = \frac{1}{\sqrt{2\pi b}}\,\frac{\sin(\pi r/b)}{r} \tag{2.14}$$

gives

$$b = 1.25R\,. \tag{2.15}$$

This relation is not directly suited for extracting the P matrix from the data. But the radius R is related to the mass M of the bag by a virial theorem:

$$R = \left(\frac{3M}{16\pi B}\right)^{1/3}. \tag{2.16}$$

Therefore, to extract the P matrix we shall use the following dependence of b upon the total energy in the center-of-mass system:

$$b(E) = 6.4E^{1/3}\,. \tag{2.17}$$

It is also possible to extract information about the pole residues from the bag-model calculations. In the next section we shall see that a $Q^4\overline{Q}$ state can be expanded uniquely in a (Q^3)-$(Q\overline{Q})$ basis which contains both singlet subunits which are open channels and octet subunits which are closed channels:

$$
\begin{aligned}
|Q^4\overline{Q}\rangle &= \sum_o \zeta_o |(Q^3)_1 (Q\overline{Q})_1\rangle \\
&\quad + \sum_c \zeta_c |(Q^3)_8 (Q\overline{Q})_8\rangle\,, \qquad (2.18)
\end{aligned}
$$

where the subscripts denote open (o) and confined (c) channels $(\sum_o \zeta_o^2 + \sum_c \zeta_c^2 = 1)$. The channel space labeled by i in Eq. (2.11) contains only physical open channels, and Jaffe and Low identified the unit vector ξ in the following way:

$$\xi_o = \frac{\zeta_o}{(\sum_o \zeta_o^2)^{1/2}}\,. \tag{2.19}$$

They also estimated the slope ds_0/db to be approximately given by

$$\frac{ds_0}{db} = -\frac{3}{2}\frac{s_0}{b}\left(\sum_o \zeta_o^2\right). \tag{2.20}$$

The approximations involved in Eq. (2.20) are rather crude. Since the radius R of a bag eigenstate minimizes its energy, ds_0/db would have to be always zero. Equation (2.20) corrects this simply by switching off the bag pressure B by the quantity $(\sum_o \zeta_o^2)B$ when open channels are present. It will turn out that the predicted residues are in general too small.

III. PHENOMENOLOGY OF $Q^4\bar{Q}$ BARYONS

This section is concerned with phenomenology—spectrum and decay properties—of $Q^4\bar{Q}$ baryons in their ground state, in the framework of the MIT bag model. Jaffe[2] developed general techniques for estimating the masses and decay couplings of multiquark hadrons. Strottman[5] has recently studied multiquark baryons in this framework. His work overlaps ours, and we shall refer to his paper for many group-theoretical results. However, our approach to the phenomenology is essentially different.

A. Spectrum

In the MIT bag model $Q^4\bar{Q}$ baryons in their ground state occur when all quarks and antiquarks are in S-wave modes. Therefore, the possible spin-parity assignments of such baryons are $J^P = \frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-$. The phenomenological Hamiltonian of the bag model[1] includes a volume energy, a zero-point energy, a kinetic-energy term diagonal in eigenstates of the strange-quark number (n_s) and independent of color and spin, and a gluon interaction given by

$$H_g = -\frac{\alpha_c}{R}\sum_{a=1}^{8}\sum_{i>j}\vec{\sigma}_i\cdot\vec{\sigma}_j\lambda_i^a\lambda_j^a M(m_iR, m_jR)\,, \quad (3.1)$$

where $\alpha_c = g^2/4\pi$ is the strong coupling constant ($\alpha_c = 0.55$), R is the bag radius, a labels color, and i labels quarks. M is the magnetic interaction strength determined by an integral over bag wave functions.[1] Jaffe[2] approximated Eq. (3.1) by replacing $M(m_iR, m_jR)$ by

$$M\left(\frac{n_s}{N}m_s, \frac{n_s}{N}m_s\right),$$

where n_s is the number of strange quarks and N the total number of quarks in the state ($N=5$), m_s is the strange quark mass ($m_u = m_d = 0$). With this linear interpolation M may be removed from the summation in Eq. (3.1) which then can be expressed in terms of the quadratic Casimir operators of SU(2), $SU(3)_c$, and $SU(6)_{cs}$[2]:

$$\begin{aligned}-\sum_a\sum_{i>j}\vec{\sigma}_i\cdot\vec{\sigma}_j\lambda_i^a\lambda_j^a &= 8N + \tfrac{1}{2}C_6(\text{tot}) - \tfrac{4}{3}S_{\text{tot}}(S_{\text{tot}}+1)\\ &\quad + C_3(Q) + \tfrac{8}{3}S_Q + 1) - C_6(Q)\\ &\quad + C_3(\bar{Q}) + \tfrac{8}{3}S_{\bar{Q}}(S_{\bar{Q}}+1) - C_6(\bar{Q})\,, \end{aligned} \quad (3.2)$$

where the labels Q, $\bar{Q}$, and tot refer to the representations of the quarks, antiquarks, and the total system, respectively; S stands for the spin value. The SU(6) and SU(3) irreducible representations for Q^4 and $Q^4\bar{Q}$ sectors have been listed by Hogassen and Sorba[6] together with the values of the Casimir operators $C_6(R)$ and $C_3(R)$.

From Eq. (3.2) the usefulness of labeling the hadronic states by representations of $SU(6)_{cs} \supset SU(3)_{\text{color}} \times SU(2)_{\text{spin}}$ is apparent. However, there is a price to pay: States with the same number of strange quarks and belonging to the same $SU(6)_{cs}$ multiplet, such as Σ and Λ, are degenerate in Jaffe's approximation. Since $M_\Sigma - M_\Lambda = 75$ MeV, we expect the masses of $Q^4\bar{Q}$ states obtained in this approximation to be correct only within 50–100 MeV.

In the following we use the notations introduced by Jaffe[2]:

$[d_{cs}]$ denotes $SU(6)_{cs}$ multiplets labeled by their dimension;

$(d_c, 2j+1)$ denotes $SU(3)_c \times SU(2)$ multiplets labeled by their color and spin dimensions;

d_f denotes flavor multiplet by their dimension.

For example, $\bar{Q}$ is $[\bar{6}]$ in $SU(6)_{cs}$ and $\underline{\bar{3}}$ in $SU(3)_F$. Because of the Pauli principle, the Q^4 state must be completely antisymmetric in color × spin × flavor. There are four antisymmetric combinations for $Q^4\bar{Q}$:

$$[210]\underline{3}\otimes[\bar{6}]\underline{\bar{3}} = ([56]\oplus[70]\oplus[1134];\underline{1}\oplus\underline{8}=\underline{9})\,, \quad (3.3a)$$

$$[105]\underline{\bar{6}}\otimes[\bar{6}]\underline{\bar{3}} = ([70]\oplus[560];\underline{8}\oplus\underline{\overline{10}}\equiv\underline{\overline{18}})\,, \quad (3.3b)$$

$$[105']\underline{15}\otimes[\bar{6}]\underline{\bar{3}} = ([20]\oplus[70]\oplus[540];\underline{8}\oplus\underline{10}\oplus\underline{27}\equiv\underline{45})\,, \quad (3.3c)$$

$$[\overline{15}]\underline{15'}\otimes[\bar{6}]\underline{\bar{3}} = ([20]\oplus[\overline{70}];\underline{10}\oplus\underline{35}\equiv\underline{45'})\,. \quad (3.3d)$$

The flavor multiplets mix to diagonalize the strange quark content. This is the so-called magic mixing. Since $Q^4\bar{Q}$ states must be color singlets, they come from the combination of four quarks making up a color triplet. But $Q^4\bar{Q}$ states with definite $SU(6)_{cs}$ symmetry and total spin can be linear combinations of different spin representations of the four quarks. If so, these $SU(6)_{cs}$ multiplets will be mixed by gluon interactions to form eigenstates of H_g. The appropriate SU(6) ⊃ SU(3) × SU(2) Wigner coefficients and mixing angles have been calculated by Strottman[5] together with the masses of all $J^P = \frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-$ $Q^4\bar{Q}$ baryons. We reproduce in Table I the masses of the $J^P = \frac{1}{2}^-$ states, which are those of interest for us in the next section. States are labeled as follows:

Exotics (E) carry a subscript denoting the flavor channel to which they couple and their isospin. They are also labeled by the SU(3) flavor multiplet ($\underline{9}, \underline{\overline{18}}, \underline{45}, \underline{45'}$) to which they belong.

Cryptoexotics have the same name as the Q^3 baryon with the same flavor quantum numbers. A subscript 5 recalls that they are $Q^4\bar{Q}$ states. They are also labeled by their $SU(3)_f$ multiplet. The presence of a hidden $s\bar{s}$ pair is denoted by a

TABLE I. Predicted masses of all $Q^4\bar{Q}$ $\frac{1}{2}^-$ baryons (Ref. 5) listed according to their $SU(3)_F$ multiplet [see Eqs. (3.3a)–(3.3d)] and name. Masses are quoted to the nearest 50 MeV.

$SU(3)_F$ multiplet	State	Mass (GeV)
$\underline{9}_A$	Λ_5, Σ_5	1.40
	N_5^s, Ξ_5	1.65
	Λ_5^s	1.90
$\overline{\underline{18}}$	N_5	1.50
	$\Sigma_5, \Lambda_5, E_{(KN)^0}$	1.70
	$N_5^s, \Xi_5, E_{(\pi\Xi)^{3/2}}$	1.90
	Σ_5^s	2.10
$\underline{45}_A$	Δ_5, N_5	1.70
	$E_{(KN)^1}, \Lambda_5, \Sigma_5^{1/2}, \Sigma_5^{3/2}, E_{(\bar{K}\Delta)^2}$	1.90
	$E_{(\bar{K}\Sigma)^{3/2}}, \Xi_5^0, \Xi_5^1, N_5^s, \Delta_5^s$	2.05
	$E_{(\bar{K}\Xi)^1}, \Omega_5, \Lambda_5^s, \Sigma_5^s$	2.25
	Ξ_5^s	2.45
$\underline{9}_B$	Λ_5, Σ_5	1.85
	N_5^s, Ξ_5	2.05
	Λ_5^s	2.25
$\underline{45}_B$	Δ_5, N_5	1.95
	$E_{(KN)^1}, \Lambda_5, \Sigma_5^{1/2}, \Sigma_5^{3/2}, E_{(\bar{K}\Delta)^2}$	2.10
	$E_{(\bar{K}\Sigma)^{3/2}}, \Xi_5^0, \Xi_5^1, N_5^s, \Delta_5^s$	2.30
	$E_{(\bar{K}\Xi)^1}, \Omega_5, \Lambda_5^s, \Sigma_5^s$	2.45
	Ξ_5^s	2.60
$\underline{45}'$	$\Delta_5, E_{(\pi\Delta)^{5/2}}$	2.25
	$E_{(K\Delta)^2}, \Sigma_5, E_{(\bar{K}\Delta)^2}$	2.35
	$\Xi_5, E_{(\bar{K}\Sigma)^{3/2}}, \Delta_5^s$	2.50
	$\Omega_5, E_{(\bar{K}\Xi)^1}, \Sigma_5^s$	2.65
	$E_{(\bar{K}\Omega)^{1/2}}, \Xi_5^s$	2.80
	Ω_5^s	2.95

superscript s. Occasionally another superscript is necessary to distinguish the total isospin of the four quarks.

To emphasize the fact that these $Q^4\bar{Q}$ states need not be identified with conventional resonances, we adopt in the following the name of primitives coined by Jaffe and Low.[3] A comment about the validity of masses quoted in Table I is needed. As noted earlier, within each flavor multiplet, primitives with the same number of strange quarks can be split as much as 50 to 100 MeV. The typical precision of the MIT bag model is about 2%. Therefore, we expect masses of physical primitives to be within 100 MeV of the quoted masses. This can seem rather crude, but we must recall that there are no free parameters involved in Table I, all parameters being fixed from Ref. 1.

B. Decay properties

To calculate the decay amplitude of a $Q^4\bar{Q}$ primitive into a baryon and a meson, one must decouple a quark from the Q^4 state and then recouple this quark with the antiquark to form a meson. The first step involves the expansion of a totally antisymmetric wave function of four quarks as a sum over the product of a state of three quarks and a one-quark state. The coefficients appropriate for coupling a single-quark to a many-quark wave

TABLE II. Completely antisymmetric wave functions of four quarks expanded in the basis of completely antisymmetric wave functions of three quarks (Ref. 7). Notations are as indicated in the text. For example, $|Q\rangle$ is a $(3,2)\underline{3}$ state.

$$|Q^4(3,1)\underline{3}\rangle = \frac{1}{\sqrt{3}}|Q^3(1,2)\underline{8}\rangle|Q\rangle + \frac{1}{\sqrt{3}}|Q^3(8,2)\underline{1}\rangle|Q\rangle + \frac{1}{\sqrt{3}}|Q^3(8,2)\underline{8}\rangle|Q\rangle$$

$$|Q^4(3,3)\underline{3}\rangle = -\frac{1}{\sqrt{3}}|Q^3(1,2)\underline{8}\rangle|Q\rangle + \frac{1}{\sqrt{3}}|Q^3(8,2)\underline{1}\rangle|Q\rangle + \frac{1}{\sqrt{3}}|Q^3(8,4)\underline{8}\rangle|Q\rangle$$

$$|Q^4(3,3)\underline{\bar{6}}\rangle = -\frac{1}{\sqrt{3}}|Q^3(1,2)\underline{8}\rangle|Q\rangle + \frac{1}{\sqrt{3}}|Q^3(8,2)\underline{8}\rangle|Q\rangle - \frac{1}{\sqrt{3}}|Q^3(8,4)\underline{8}\rangle|Q\rangle$$

$$|Q^4(3,1)\underline{15}\rangle = -\frac{1}{\sqrt{3}}|Q^3(1,2)\underline{8}\rangle|Q\rangle + \frac{1}{\sqrt{3}}|Q^3(8,2)\underline{8}\rangle|Q\rangle - \frac{1}{\sqrt{3}}|Q^3(8,2)\underline{10}\rangle|Q\rangle$$

$$|Q^4(3,3)\underline{15}\rangle = \frac{1}{3}|Q^3(1,2)\underline{8}\rangle|Q\rangle - \frac{\sqrt{2}}{3}|Q^3(1,4)\underline{10}\rangle|Q\rangle + \frac{2}{3}|Q^3(8,2)\underline{8}\rangle|Q\rangle + \frac{1}{3}|Q^3(8,4)\underline{8}\rangle|Q\rangle + \frac{1}{3}|Q^3(8,2)\underline{10}\rangle|Q\rangle$$

$$|Q^4(3,5)\underline{15}\rangle = \frac{1}{\sqrt{3}}|Q^3(1,4)\underline{10}\rangle|Q\rangle + \left(\frac{2}{3}\right)^{1/2}|Q^3(8,4)\underline{8}\rangle|Q\rangle$$

$$|Q^4(3,3)\underline{15'}\rangle = \frac{1}{\sqrt{3}}|Q^3(1,4)\underline{10}\rangle|Q\rangle + \left(\frac{2}{3}\right)^{1/2}|Q^3(8,2)\underline{10}\rangle|Q\rangle$$

function are known as coefficients of fractional parentage. They have been tabulated for functions of nine or fewer quarks by So and Strottman.[7] In Table II we give these expansions for the different (color, spin) content of each $SU(6)_{cs}$ representation of four quarks which appear in Eqs. (3.3a)–(3.3d). From this table we can notice a general selection rule: Only primitives belonging to the flavor multiplets $\underline{45}$ or $\underline{45}'$ can couple to a baryon decuplet.

The recombination of the quark with the antiquark to form a meson involves only SU(2) and SU(3) Racah coefficients. The spin recoupling is such that the meson and baryon are in a relative S wave. For example, a primitive of the flavor multiplet $\underline{45}'$ which always couples to a baryon decuplet cannot couple to a pseudoscalar meson because baryon and meson would have to be in a D wave. The color recoupling must be such that primitives are color singlet. Therefore, a primitive has two kinds of recoupling channels: open channels with color-singlet baryon and meson and confined channels with color-octet baryon and meson. General expressions and tables for the recoupling coefficients of $Q^4\overline{Q}$ baryons can be found in Ref. 5.

The coupling to confined channels is ignored in Eq. (2.20). This approximation was suggested[2] by the fact that a primitive $Q^4\overline{Q}$ decaying into an open channel simply dissociates as illustrated in Fig. 1(a). If the Zweig rule is interpreted as an inhibition associated with the creation or annihilation of quark lines, dissociation decays of primitives are "OZI (Okubo-Zweig-Iizuka) super-

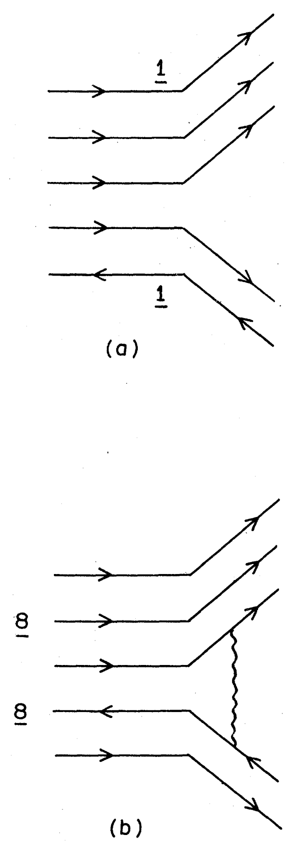

FIG. 1. $Q^4\overline{Q}$ primitive decay diagrams. (a) Dissociation decay of a S-wave $Q^4\overline{Q}$ primitive into a color-singlet baryon and meson in relative S wave. (b) Confined channel with color-octet baryon and meson becoming open after a gluon exchange.

allowed" as compared to "OZI allowed" decays of Q^3 states such as $\Delta \to p\pi$, which require a quark pair creation. However, the very existence of the latter decays implies that confined channels can be important. In the MIT bag model $Q\overline{Q}$ or Q^3 states such as ρ and Δ are stable and all hadronic decay channels are closed. Decays like $\Delta \to p\pi$ or $\rho \to \pi\pi$ could be accounted for only when quark pair creation is included in the bag Hamiltonian. This is illustrated by Jaffe and Low's study of the P-wave $\pi\pi$ scattering in the P-matrix formalism. They found a pole near the ρ mass with a residue corresponding to a projection onto open channels $\langle \Lambda \rangle = (\xi_{\pi\pi}{}^2) = 0.23$. Since $\langle \Lambda \rangle \approx \alpha_s{}^2$ ($\alpha_s = 0.55$ in the MIT bag model), this suggests that the $\pi\pi$ channel becomes open via a first-order process. If we assume that a perturbative approach can be consistent, a very crude refinement of Eq. (2.20) would be

$$\frac{ds_0}{db} = -\frac{3}{2}\frac{s_0}{b}\left(\sum_o \zeta_o{}^2 + \alpha_s{}^2 \times O(1) \sum_c \zeta_c{}^2\right). \qquad (3.4)$$

In Eq. (3.4) all confined channels which can become an open channel by a further exchange of a gluon [Fig. 1(b)] are to be included. The net effect is simply that primitives are more strongly coupled to existing open channels. At this order, new channels become open only for flavor multiplets $\underline{45}$ and $\underline{45}'$. For these multiplets, 0^--meson–$\frac{3}{2}^+$-baryon channels become open via P-wave coupling to the gluon.

We now turn to a comparison with experimental data in the P-matrix formalism.

IV. P-MATRIX ANALYSIS OF LOW-ENERGY S-WAVE MESON-NUCLEON SCATTERING

A theoretical limitation on the range of application of the P-matrix formalism is its inability to parametrize multibody channels. In meson-nucleon scattering multibody channels with two pions in the final state do occur, but below 2 GeV they can be considered as quasi-two-body channels with a good approximation. A practical limitation is the profusion of quasi-two-body channels which are already expected below 2 GeV and the lack of experimental data to construct the full P matrix when inelastic channels are present.

There exist sufficient data to make a single-channel analysis for the $I = \frac{1}{2}, \frac{3}{2}$ pion-nucleon system and the $I = 0, 1$ exotic kaon-nucleon system and a two-channel analysis for the $I = 0$ antikaon-nucleon system. The range of energies for which such analyses are valid will be discussed below. We restrict the analysis to meson-nucleon scattering in s wave. For this reason there is no mention in this work of $Q^4\overline{Q}$ primitives with $J^P = \frac{3}{2}^-, \frac{5}{2}^-$. We expect they will be more difficult to identify. First, they lie higher, and several channels are open barring the construction of the full P matrix. Second, because of the success of Regge trajectories, the first pole found in a D-wave analysis would probably correspond to a three-quark spinning bag.[8]

Available experimental data are sufficient to test the lower part of the $J^P = \frac{1}{2}^-$ $Q^4\overline{Q}$ spectrum for which the MIT bag model makes definite predictions. In the following, we make a detailed discussion channel by channel, considering separately the exotic channels from the nonexotic ones. Finally, in a third part we study the two-channel $I = 0$ $\overline{K}N$ system.

A. Exotic channels

Partial-wave analyses of the $I = 0$ elastic KN channel give solutions for the S-wave phase shift which often differ according to the type of analysis. In Fig. 2 we reproduce three solutions of different analyses to show this uncertainty. Two are solutions C and D from Giacomelli *et al.*[9] who found by different methods three main families denoted A, C, and D. Solutions C and D are the most favored. The third is from Martin[10] who found a uniformly falling negative phase shift. The latter is close to solutions C and D below 1.7 GeV, but it becomes quite different at higher energy. This difference is reflected in the elasticity which remains equal to 1 up to 2 GeV in Martin's solution, but which begins dropping around 1.6 GeV in solutions C and D. If an inelastic channel were open at 1.6 GeV, it could not be a quasi-two-body one.

Assuming first that inelastic channels are unimportant, the position of the P-matrix pole is simply given by the energy for which $kb + \delta$ goes through π. It turns out that this position is quite

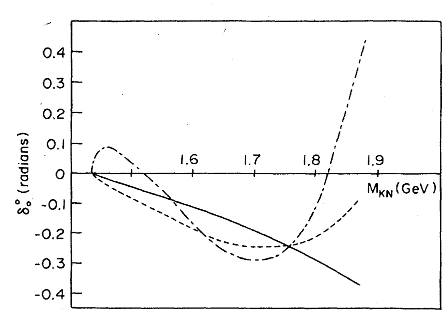

FIG. 2. KN $I = 0$ (exotic) S-wave phase shift (δ_0^0). The solid line is the solution of Ref. 10. The dashed line is solution D of Ref. 9, the dashed and dotted line is solution C of Ref. 9.

insensitive to the choice of solution:

$$E_{\text{pole}} = 1.705 \pm 0.010 \text{ GeV}. \tag{4.1}$$

At 1.7 GeV the elasticity is still around 0.85 for solutions C and D. Consequently, we expect that inelastic channels, if any, are indeed unimportant at this energy and that a one-channel P-matrix analysis is valid. The residue r of the pole is

$$r = 0.19 \pm 0.01 \text{ GeV}^3. \tag{4.2}$$

Here we must emphasize that the parameters obtained from experiment should be regarded as illustrative. We have not attempted a systematic error analysis.

Partial-wave analyses of the $I=1$ elastic KN channel are easier because K^+p is a pure $I=1$ system. In general, they are in qualitative agreement at least at low energy. Figure 3 reproduces solution H for the S-wave phase shift from the most recent K^+p scattering analysis.[11] Reference 11 did not quote errors but found two solutions H and L which are nearly identical below 1.95 GeV. The elasticity is also shown in Fig. 3. It begins dropping from 1 around the K^*N threshold.

In a one-channel P-matrix analysis the pole is found to be

$$E_{\text{pole}} = 1.78 \text{ GeV} \tag{4.3}$$

with a residue

$$r \approx 0.27 \text{ GeV}^3. \tag{4.4}$$

The pole is located just at the effective K^*N threshold. Therefore, it should be regarded as a pole in the reduced P matrix, as discussed in Ref. 3. It is possible that, in a full two-channel P-matrix analysis, the true pole lies somewhat higher, with a modified residue.

Now if we look at Table I, the following primitives are predicted: $I=0$ elastic KN channel: $E_{(KN)^0}$ $(\underline{\overline{18}})$, mass 1.70 GeV, residue 0.14 GeV^3; $I=1$ elastic KN channel: $E_{(KN)^1}$ $(\underline{45}_A)$, mass 1.90 GeV, residue 0.18 GeV^3.

The residues $-ds_0/db$ have been calculated with Eq. (2.20), and we included the K^*N channel for predicting the $E_{(KN)^1}$ (45_A) residue. Inclusion of the confined channels and the rough estimate of Eq. (3.5) would increase the residues by the right amount. For example, the complete recoupling of $E_{(KN)^0}$ $(\underline{\overline{18}})$ to meson-baryon channels is

$$E_{(KN)^0}(\underline{\overline{18}}) = -\frac{1}{2}NK - \frac{1}{\sqrt{12}}NK^* + \frac{1}{2}\vec{\text{N}}\cdot\vec{\text{K}} + \frac{1}{\sqrt{12}}\vec{\text{N}}\cdot\vec{\text{K}}^* - \frac{1}{\sqrt{3}}\vec{\text{N}}^*\vec{\text{K}}^* \tag{4.5}$$

(an arrow denotes a color octet and N^* is a configuration of three quarks with the nucleon flavor quantum number and spin $\frac{3}{2}$). The correction due to the confined channel $\vec{\text{N}}\cdot\vec{\text{K}}$ would increase $\langle\Lambda\rangle$ and the residue by about 30%.

In conclusion, as is the case for meson-meson scattering,[3] exotic channels in meson-baryon scattering are the cleanest for a one-channel P-matrix analysis. Poles have been found in both $I=0$ and $I=1$ KN channels, corresponding closely to bag-model predictions.

B. Nonexotic channels

The nonexotic channels we consider in this part are the $I=\frac{1}{2}$ and $I=\frac{3}{2}$ pion-nucleon channels. All partial-wave analyses of these channels give S-wave phases which are qualitatively identical below 2 GeV. In Figs. 4 and 5 the phase shifts δ_0^I $(I=\frac{1}{2},\frac{3}{2})$ and elasticities η_0^I are hand drawn through data taken from the partial-wave analysis of Ref. 12.

We consider first the $I=\frac{1}{2}$ πN channel. The elasticity begins dropping sharply at the ηN threshold and reaches its minimum (~0.2) around 1.535 GeV. This forces us to limit the discussion of this channel below 1.5 GeV, since there are not

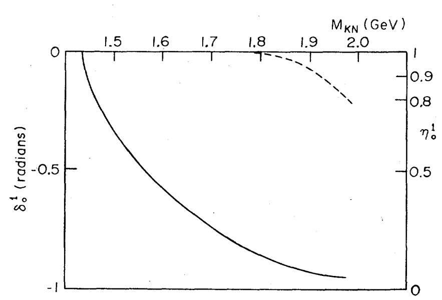

FIG. 3. KN $I=1$ (exotic) S-wave phase shift δ_0^1 and elasticity η_0^1 from the solution H of Ref. 11. The solid line denotes δ_0^1 (left-hand scale), the dashed line denotes η_0^1 (right-hand scale).

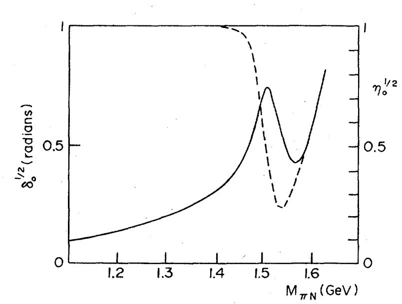

FIG. 4. πN $I=\frac{1}{2}$ S-wave phase shift $\delta_0^{1/2}$ and elasticity $\eta_0^{1/2}$ from the partial-wave analysis of Ref. 12. The solid line denotes $\delta_0^{1/2}$ (left-hand scale), the dashed line denotes $\eta_0^{1/2}$ (right-hand scale).

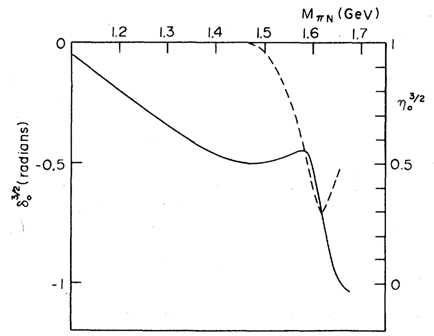

FIG. 5. πN $I=\frac{3}{2}$ S-wave phase shift $\delta_0^{3/2}$ and elasticity $\eta_0^{3/2}$ from the partial-wave analysis of Ref. 12. The solid line denotes $\delta_0^{3/2}$ (left-hand scale), the dashed line denotes $\eta_0^{3/2}$ (right-hand scale).

sufficient data to make a two-channel P-matrix analysis. As already mentioned a one-channel P-matrix pole is characterized by $kb+\delta$ going through π. We find a pole whose energy is

$$E_{\text{pole}}=1.43 \text{ GeV} \tag{4.6}$$

and residue

$$r\approx 0.14 \text{ GeV}^3 . \tag{4.7}$$

This pole is located below the ηN threshold. There is possibly some small inelasticity at the pole energy, owing to multibody $N\pi\pi$ channels. However, we expect that the parameters quoted in Eqs. (4.6), (4.7) have the right order of magnitude. The lightest primitive predicted by the MIT bag model in the $I=\frac{1}{2}$ πN channel is $N_5(\underline{18})$ with a mass $M=1.50$ GeV and a residue $r=0.09$ [Eq. (2.20)]. Again it can be seen that the agreement is very good.

If we perform the same analysis for the $I=\frac{3}{2}$ πN channel, we find that $kb+\delta$ goes through π at $E=1.56$ GeV. There is a long extrapolation in the inelastic region which begins around 1.50 GeV, and this pole should be considered only as approximate. Nothing firm can be said about its parameters. The main inelastic channel is $\pi N \to \pi\Delta$ with π and Δ in a relative D wave. Such a coupling is always difficult for $\frac{1}{2}^-$ $Q^4\overline{Q}$ primitives since they couple only to hadrons in relative S wave in zeroth order and since only primitives of the flavor multiplet $\underline{45}$ can couple to a baryon decuplet. However, the lightest primitive predicted by the bag model in the $I=\frac{3}{2}$ πN channel is $\Delta_5(\underline{45}_A)$ with a mass $M=1.70$ GeV which does belong to the right multiplet. Therefore, $\Delta_5(\underline{45}_A)$ can couple to $\pi\Delta$ in a first-order process in the same way as ρ couples to $\pi\pi$.

C. $I=0$ $K\overline{N}$ channel

The difficulties of determining low-energy $\overline{K}N$ amplitudes are well known. Even at threshold there is a multichannel ($\overline{K}N, \pi\Sigma, \pi\Lambda$) problem. Since in general no information is available on elastic channels $\pi\Sigma\to\pi\Sigma$ or $\pi\Lambda\to\pi\Lambda$, partial-wave analyses of the $\overline{K}N$ system have to be model dependent. Most published studies analyze data on single channels ($\overline{K}N\to\overline{K}N$, $\pi\Sigma$, or $\pi\Lambda$) and generally use simple resonance-plus-background forms for the partial-wave amplitudes. This procedure ignores the constraints of multichannel unitarity and such analyses are, at best, only qualitative.

Data at very low energy ($k_{\text{lab}}\lesssim 300$ MeV/c) have been extensively studied using the multichannel K-matrix formulation of Dalitz and Tuan.[13] In this region S-wave scattering is dominant, and good fits to the data can be achieved in the zero-range approximation, where the K-matrix elements are taken to be energy independent. Of course, when no information is used about the $\pi Y\to\pi Y$ channels, only six combinations of the nine K-matrix elements are well determined. Although the K-matrix elements of the various published solutions are often very different, the parameters are highly correlated and the predicted $K\overline{N}$ S-wave amplitudes in the physical region are usually similar to each other.

Because of this, we tentatively construct the two-channel P matrix for the $I=0$ $\overline{K}N$ system between threshold and $E=1.5$ GeV in the center-of-mass system, using the parametrizations of the amplitudes obtained by Martin[14] with a multichannel K matrix formalism. His fit both reproduces the low-energy data and requires consistency with a forward-dispersion-relation analysis. However, he found it necessary to allow the $I=0$ S-wave M matrix elements to be energy dependent:

$$M^{I=0}=A+Rk^2 , \tag{4.8}$$

where $M=K^{-1}$ and k is the $\overline{K}N$ c.m. momentum.

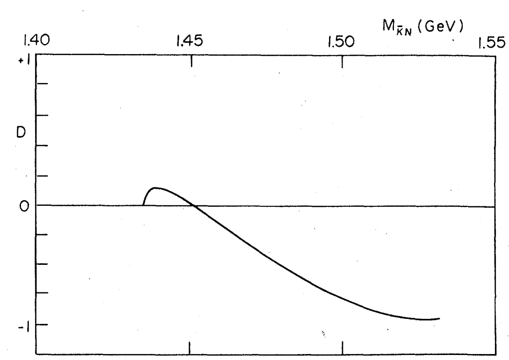

FIG. 6. Denominator D [Eq. (2.8)] of the two-channel P matrix for the $I=0$ ($\overline{K}N$) system.

Figure 6 shows the denominator D of the two-channel P matrix [Eq. (2.8)] constructed with the parameter values quoted in Table I of Ref. 14. We find a pole just above threshold at 1.45 GeV. The next step is to find out its parameters. We can crudely estimate them by parametrizing the P matrix between threshold and the pole energy with the equation[3]:

$$P_{ij}=a_{ij}+\frac{\lambda_i\lambda_j}{s-s_0}. \tag{4.9}$$

We get

$$a_{KK}\sim 0.14\text{ GeV},\quad a_{K\Sigma}\sim 0.10\text{ GeV},\quad a_{\Sigma\Sigma}\sim 0.06\text{ GeV};$$
$$\lambda_K{}^2\sim 0.032\text{ GeV}^3,\quad \lambda_K\lambda_\Sigma\sim 0.026\text{ GeV}^3,$$
$$\lambda_\Sigma{}^2\sim 0.020\text{ GeV}^3. \tag{4.10}$$

Using Eqs. (2.10) and (2.11) we deduce the parameters

$$-\frac{ds_0}{db}\sim 0.052\text{ GeV}^3,\quad \xi_K\sim 0.8,\quad \xi_\Sigma\sim 0.6. \tag{4.11}$$

Here again we recall that the parameters quoted here are merely illustrative. Though Ref. 14 did quote errors, we have not attempted any error analysis.

The lightest primitive predicted by the MIT bag model in the $I=0$ $\overline{K}N$ channel is $\Lambda_5(\underline{9}_A)$ with a mass $M=1.40$ GeV. Its recoupling to baryons and mesons channels is

$$\begin{aligned}\Lambda_5(\underline{9}_A)=\;&0.18\Sigma\pi-0.06\Lambda\eta_0+0.15N\overline{K}\\&-0.39\Lambda\phi_0+0.32N\overline{K}^*\\&+\text{components involving}\\&\quad\text{color-octet mesons}\\&\quad\text{and baryons.}\end{aligned} \tag{4.12}$$

(η_0 and ϕ_0 represent that part of the physical η and ϕ mesons having no strange quarks.) From Eq. (4.12) the predicted parameters [Eqs. (2.19) and (2.20)] are

$$-\frac{ds_0}{db}=0.023\text{ GeV}^3,\quad \xi_K=0.64,\quad \xi_\Sigma=0.77. \tag{4.13}$$

It can be seen that the agreement is fairly good. The bag model predicts a stronger coupling to $\Sigma\pi$ than to $N\overline{K}$ and the converse is found in this analysis, but the parameters have the right order of magnitude.

The multichannel K-matrix analyses see a narrow effect $\Lambda(1405)$ when their amplitudes are extrapolated below threshold. This effect is to be associated with the pole in P seen just above threshold at 1.45 GeV. From the general discussion of Jaffe and Low we expect to see a pole below threshold too, but in the reduced P-matrix. The reduced P matrix $\tilde{P}$ is given by[3]

$$\tilde{P}=a_\Sigma-\frac{a_{K\Sigma}{}^2}{d}+\frac{(\lambda_\Sigma-a_{\Sigma K}\lambda_K/d)^2}{s-s_0+\lambda_K{}^2/d}, \tag{4.14}$$

where $d=a_K+\kappa$ and $\kappa=-ik$ is real and positive. With the parameters of Eq. (4.10) we find a very narrow reduced P-matrix pole at $\sqrt{s_0}\sim 1.41$ GeV with a residue $r\sim 0.006$ GeV.[3] There is an appreciable shift in mass and residue due to the reduction procedure. A one-channel P-matrix analysis would find a pole in the $I=0$ $\pi\Sigma\to\pi\Sigma$ channel with approximately the parameters of the reduced P-matrix pole. This is a vivid illustration of the uncertainties in a one-channel P-matrix analysis when nearby closed channels are present.

If we attempt a three-channel analysis for the $I=1$ $\overline{K}N$ system, we find no pole in the expected range of validity of the K-matrix amplitudes, i.e., below 1.50 GeV. To find a pole we have to extrapolate the amplitudes till 1.54 GeV, which prevents any reliable analysis of the parameters. We content ourselves with noting that presumably there is a $I=1$ pole located 50 to 100 MeV higher than the $I=0$ pole. This is the expected range inferred from $M_\Sigma-M_\Lambda$.

To conclude, we find from the data clear evidence for the predicted bag-model $Q^4\overline{Q}$ primitives in all the exotic and nonexotic channels we analyze. Whenever a one-channel P-matrix analysis is valid, the agreement between the experimental and predicted parameters is good. When inelastic channels are present, lack of reliable phases makes the analysis much more difficult. The general agreement in the $I=0$ $\overline{K}N$ low-energy parameters makes this system a possible exception. A two-channel P-matrix analysis of this system is found to be in good agreement with the bag-model predictions.

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[1]T. A. DeGrand, R. L. Jaffe, K. Johnson, and J. Kiskis, Phys. Rev. D 12, 2060 (1975).

[2]R. L. Jaffe, Phys. Rev. D 15, 267, 281 (1977).

[3]R. L. Jaffe and F. E. Low, Phys. Rev. D 19, 2105 (1979).

[4]Strictly speaking, for a meson-nucleon system in relative S wave, this is true only in the nonrelativistic limit. A measure of the approximation made in Eq. (2.1) is given by the ratio $(E_N - M_N)/(E_N + M_N)$, where E_N is the nucleon energy in the center-of-mass system. This ratio remains less than 0.1 in the energy region we consider in this paper.

[5]D. Strottman, Phys. Rev. D 20, 748 (1979).

[6]H. Högassen and P. Sorba, Nucl. Phys. B145, 119 (1978). The part of this work concerned with ground state $Q^4\bar{Q}$ baryons contains wrong $SU(6) \supset SU(3) \otimes SU(2)$ Wigner coefficients and consequently wrong mixing angles and eigenvalues of the color magnetic interaction.

[7]S. I. So and D. Strottman, J. Math. Phys. 20, 153 (1979).

[8]K. Johnson and C. B. Thorn, Phys. Rev. D 13, 1934 (1976).

[9]G. Giacomelli *et al.*, Nucl. Phys. B71, 138 (1974).

[10]B. R. Martin, Nucl. Phys. B94, 413 (1975).

[11]R. A. Arndt, L. D. Roper, and P. H. Steinberg, Phys. Rev. D 18, 3278 (1978).

[12]R. Ayed, P. Bareyre, and G. Villet, Phys. Lett. 31B, 598 (1970).

[13]R. H. Dalitz and S. F. Tuan, Ann. Phys. (N.Y.) 10, 307 (1960).

[14]A. D. Martin, Phys. Lett. 65B, 346 (1976).