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Z dependence of coherent μe conversion rate in anomalous neutrinoless muon capture

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The coherent muon-electron conversion rate in anomalous neutrinoless muon capture by nuclei is calculated in terms of phenomenological coupling constants defined by effective Hamiltonian densities. The possibilities of muon-number violation occurring through lepton mixing, through muon-number-violating gauge couplings, or through scalar-Higgs-boson exchange are considered. The variation of the coherent conversion rate with atomic number Z is discussed for different values of the coupling constants. The present results are compared with a previous approximate formula given by Weinberg and Feinberg. The following limits on the phenomenological coupling constants are found: $|g_V^{(0)}| < 10^{-6}$ and $|g_V^{(1)}| < 2.9\times10^{-4}$, or $|g_S^{(0)}| < 10^{-6}$ and $|g_S^{(1)}| < 3.2\times10^{-4}$.

In this paper I present some calculations for the coherent muon conversion rate in the process $\mu^- + (A,Z) \to e^- + (A,Z)$ in terms of phenomenological coupling constants. In particular, I consider the dependence of this rate on A and Z for different combinations of the phenomenological coupling constants. In experimental searches for this process, negative muons are stopped in some material and quickly cascade down to the 1s orbit. From this orbit the muon is either captured by the nucleus with the emission of a neutrino, or it decays into an electron and two neutrinos. If muon number is not exactly conserved, the muon conversion process could occur some of the time.

The separate conservation of muon and electron numbers has been the subject of many theoretical and experimental investigations. The theoretical understanding of muon-number conservation made rapid progress after the discovery of spontaneously broken renormalizable gauge theories. Many models are now known which predict a small violation of muon number.[1-6] Some of these models[3-6] reproduce the phenomenology of the standard Weinberg-Salam model[7] for all present experiments. If muon-number violation does occur, then it has been suggested that muon conversion is the best place to look for it.[2,8] Experimentally, a limit of $R_{eN} < 1.5\times10^{-10}$ has been set at SIN for sulphur in 1978,[9] and this limit will be improved further by new experiments.[10] (R_{eN} is the ratio of the muon-number-violating rate to the ordinary capture rate.) Before 1977, the limit on R_{eN} was 1.6×10^{-8} for copper, set in 1972.[11]

In most theories, muon-number violation occurs due to (a) lepton mixing which arises if the lepton mass matrix is not diagonal with respect to the weak eigenstates,[1,2] (b) existence of exotic gauge bosons which have muon-number-violating fermion couplings,[4-6] or (c) scalar Higgs bosons with muon-number-violating fermion couplings.[3] In the first two cases, to a good approximation the matrix element for muon conversion can be calculated from a local effective Hamiltonian density involving vector and axial-vector fermion currents.[1,12] For the Higgs-scalar case, the effective Hamiltonian density involves scalar and pseudoscalar fermion currents, and I discuss this case separately.

For the case involving vector and axial-vector currents, the effective Hamiltonian density can be parametrized as

$$H_{eff} = \frac{G_F}{\sqrt{2}}\left[\sum_{i=0,1} g_V^{(i)}\bar{e}\gamma^\lambda(1-\alpha_V^i\gamma_5)\mu V_\lambda^{(i)} + g_A^{(i)}\bar{e}\gamma^\lambda(1-\alpha_A^i\gamma_5)\mu A_\lambda^{(i)}\right], \quad (1)$$

where e, μ, etc., represent the corresponding particle fields, the index i represents the isospin transformation properties of the quark currents, $g_V^{(i)}$, $g_A^{(i)}$, α_V^i, etc., are phenomenological coupling constants to be determined from different theories, and $V_\lambda^{(0)}$, $V_\lambda^{(1)}$, etc., are the quark currents

$$V_\lambda^{(0)} = (\bar{u}\gamma_\lambda u + \bar{d}\gamma_\lambda d)/2, \text{ etc.} \quad (2)$$

In many theories, only the left- or right-handed electron is emitted in muon conversion in the limit $m_e = 0$.[1,3] In that case, all the α's become equal and are ±1.

With the above Hamiltonian, both coherent and incoherent muon conversion processes are possible. The coherent muon conversion process, in which the final nucleus is in its ground state, is expected to dominate the rate if $g_V^{(0)}$ is not very small or zero. Experimentally, it is easier to search for, because it involves detecting a single monoenergetic electron of energy

$$E_e = E_\mu - E_\mu^2/(2M_A) \approx E_\mu, \quad (3)$$

where E_μ is the muon energy (muon mass minus the ground-state binding energy) and M_A is the mass of the recoiling nucleus. At this energy the background (due to electrons coming from boundmuon decay and from conversion of γ rays produced in radiative muon and pion capture) is negligible. The rate for the coherent μe conversion process can be calculated in a straightforward manner. Neglecting the bottom components of the nucleon spinors and terms of order $1/A^2$ resulting from the process where the nuclear spin changes, the coherent rate is

$$\omega(\mu N \to eN) = G_F^2 \frac{E_e^2}{16\pi^2} \left\{ \left[\left(\frac{3g_V^{(0)} + g_V^{(1)}}{2} \right) ZR_Z + \left(\frac{3g_V^{(0)} - g_V^{(1)}}{2} \right) NR_N \right]^2 + \left[\left(\frac{3\alpha_V^0 g_V^{(0)} + \alpha_V^1 g_V^{(1)}}{2} \right) ZR_Z + \left(\frac{3\alpha_V^0 g_V^{(0)} - \alpha_V^1 g_V^{(1)}}{2} \right) NR_N \right]^2 \right\}, \tag{4}$$

where

$$R_{Z(N)} = \int (g_e g_\mu + f_e f_\mu) \rho_{Z(N)} 4\pi r^2 dr, \tag{5}$$

and ρ_Z, ρ_N are the proton and neutron densities (normalized to 1), g_μ, f_μ are the top and bottom components of the 1s muon wave function, g_e, f_e are the top and bottom components of the Coulomb-modified spherical electron wave function.[13] R_Z, R_N should be corrected for the nucleon form factors. The corrections are small (of the order of 2%) and vary slowly as Z varies. Also, from isospin and from the fact that the neutron charge radius is negligible, it follows that R_Z and R_N are modified by similar factors. Hence, these corrections can be absorbed in the phenomenological coupling constants, and I do not consider them further.

Present evidence indicates that the proton and neutron densities are very similar. If we assume that $\rho_Z = \rho_N = \rho$ and that all the α's are either +1 or −1, then using (4) we can write for R_{eN}

$$R_{eN} = \omega_G(Z)[g_V^{(0)} + g_V^{(1)}(Z-N)/(3A)]^2/\omega(\mu N \to \nu N'), \tag{6}$$

where

$$\omega_G(Z) = (G_F^2/2) 9 E_e^2 A^2 \left[\int (g_e g_\mu + f_e f_\mu) \rho r^2 dr \right]^2. \tag{7}$$

$\omega(\mu N \to \nu N')$ is the rate for ordinary muon capture. For the light elements, $\omega_G(Z)$ varies approximately as $A^2 Z^3$, since the muon wave function at the nucleus varies approximately as $Z^{3/2}$. Finite-size effects cause $\omega_G(Z)$ to increase less slowly as Z increases. For heavy elements, $\omega_G(Z)$ will decrease because of the effect of g_e. If $g_V^{(0)} = 0$, R_{eN} becomes small for the light atoms since $Z \approx N$. To set limits on $g_V^{(0)}$ and $g_V^{(1)}$ one has to make measurements on at least two elements. It is best to choose elements with significantly different N/Z ratios.

Assuming a Fermi distribution[14] for ρ, i.e.,

$$\rho = \rho_0/[1 + \exp((r-c)/a)], \tag{8}$$

with $c = 1.07\ A^{1/3}$ fm and a constant surface thickness t (defined in Ref. 14) of 2.4 fm, I have calculated ω_G for a number of elements and Table I gives $\omega_G(Z)$ and $\omega_G/\omega(\mu N \to \nu N')$. The latter has been calculated using experimental values of

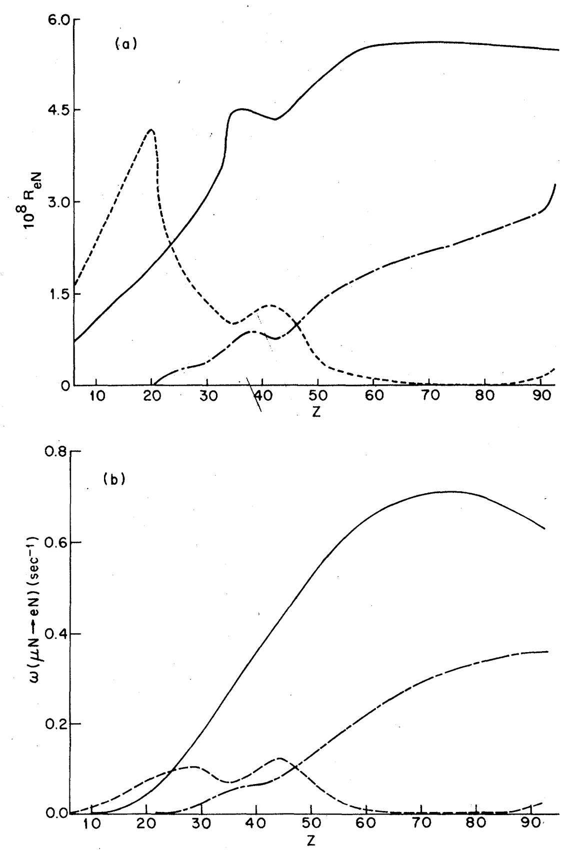

FIG. 1. Qualitative variation of: (a) R_{eN} and (b) $\omega(\mu N \to eN)$, with atomic number Z, for the vector-current case. For most of the elements in Table I R_{eN} lies within 10% of the smooth curves shown. The solid curve is for $g_V^{(0)} = 10^{-5}$, $g_V^{(1)} = 0$; the dash-dot curve is for $g_V^{(0)} = 0$, $g_V^{(1)} = 10^{-4}$; the dashed curve is for $g_V^{(0)} = 1.5 \times 10^{-5}$, $g_V^{(1)} = 2.25 \times 10^{-4}$.

TABLE I. Constants useful in calculating the coherent μe conversion rate for some elements. ω_G is defined by Eq. (7) and ω_H is defined by Eq. (20).

Z	A	ω_G (10^8 sec^{-1})	$\frac{\omega_G}{\omega(\mu N\to\nu N')}$	ω_H (10^8 sec^{-1})	$\frac{\omega_H}{\omega(\mu N\to\nu N')}$
6.0	12.00	0.03	72.78	0.03	70.45
12.0	24.30	0.65	136.64	0.61	127.41
16.0	32.10	2.04	152.67	1.85	138.11
20.0	40.10	4.63	188.94	4.06	165.52
22.0	47.90	6.91	265.64	5.94	228.31
25.0	54.96	10.45	267.21	8.74	223.51
29.0	63.60	16.32	287.84	13.14	231.79
31.0	69.74	19.89	350.26	15.69	276.16
33.0	74.95	23.06	380.58	17.83	294.30
34.0	79.02	25.53	448.76	19.48	342.43
38.0	87.64	32.62	449.95	23.91	329.83
42.0	95.90	39.72	430.77	27.95	303.15
46.0	106.39	46.79	467.87	31.46	314.56
50.0	118.80	53.22	498.35	34.01	318.43
58.0	140.15	63.22	552.65	36.48	318.91
64.0	157.30	67.93	561.88	36.01	297.82
67.0	165.00	69.64	537.78	35.37	273.16
74.0	183.80	71.55	544.91	32.63	248.50
80.0	200.70	70.96	557.01	29.18	229.01
82.0	207.30	70.05	538.01	27.67	212.52
83.0	209.00	70.19	529.33	27.33	206.08
90.0	232.20	65.24	498.04	21.56	164.59
92.0	238.10	63.97	581.54	20.18	183.43

$\omega(\mu N\to\nu N')$ given by Eckhause.[15] With these values, the Z dependence of R_{eN} can be studied for different values of the coupling constants. Figure 1 shows the Z dependence of R_{eN} and $\omega(\mu N\to eN)$ for some combinations of $g_V^{(0)}$ and $g_V^{(1)}$. Table II gives R_{eN} for some elements as a fraction of the maximum R_{eN}, for different values of $g_V^{(1)}/g_V^{(0)}$. The limits on R_{eN} for sulphur and copper give $|g_V^{(0)}|<10^{-6}$ and $|g_V^{(1)}|<2.9\times10^{-4}$. To improve the limit on $g_V^{(1)}$, it is best to measure R_{eN} for the heavy elements.

The maximum value of R_{eN} does not necessarily occur for Z around 29, as was stated in Ref. 16 and as is generally believed.[17] This is because one has to consider contributions from the exchange of heavy gauge bosons,[1,2,12] in addition to the photon exchange considered in Ref. 16. As a result, the ratio $g_V^{(1)}/g_V^{(0)}$ becomes model-dependent, and the atom with maximum R_{eN} also becomes model dependent. Even if one considers only photon exchange, the maximum does not occur for Z around 29. This is because of the Prim-

TABLE II. Coherent-rate branching ratio as a function of the coupling constants for the vector-current case. The first column gives $g_V^{(1)}/g_V^{(0)}$ and the other columns give R_{eN} for the elements indicated, as a fraction of the maximum value of R_{eN} for each value of $g_V^{(1)}/g_V^{(0)}$.

$g_V^{(1)}/g_V^{(0)}$ \ Z	16.0	20.0	29.0	34.0	64.0	80.0	82.0	92.0
−10.0	0.09	0.11	0.27	0.54	0.82	0.87	0.86	1.00
−5.0	0.14	0.17	0.34	0.62	0.87	0.90	0.88	1.00
0.0	0.26	0.32	0.49	0.77	0.97	0.96	0.93	1.00
5.0	0.54	0.67	0.75	0.94	0.95	0.87	0.81	0.80
10.0	0.80	1.00	0.77	0.69	0.49	0.31	0.27	0.18
15.0	0.80	1.00	0.49	0.22	0.01	0.00	0.01	0.06
20.0	0.80	1.00	0.27	0.01	0.18	0.38	0.45	0.84
25.0	0.31	0.39	0.04	0.03	0.37	0.57	0.64	1.00
30.0	0.15	0.19	0.00	0.07	0.44	0.63	0.68	1.00
35.0	0.09	0.11	0.00	0.11	0.49	0.66	0.70	1.00
∞	0.00	0.00	0.07	0.29	0.65	0.76	0.78	1.00

akoff factor mentioned in the erratum to Ref. 16, and because of other correction factors which I consider later on.

The lepton and quark currents in the effective Hamiltonian density (1) may contain terms involving the lepton momentum transfer q_λ. In general, these terms will be suppressed relative to the terms included in Eq. (1) by powers of some heavy mass M, where M is the mass of new heavy leptons or gauge bosons which lead to muon-number violation in different theories. When muon conversion occurs due to photon exchange, this suppression may not take place. One can use electromagnetic-current conservation and Lorentz invariance in discussing the photon-exchange case.[1,2,16] The matrix element of the electromagnetic current operator between muon and electron states is of the form

$$T_\lambda = \langle e(p_e)|J_\lambda(0)|\mu(p_\mu)\rangle$$
$$= \bar{u}_e(p_e)[(f_M + f_M^5\gamma_5)im_\mu\sigma_{\lambda\nu}q^\nu + (f_E + f_E^5\gamma_5)(\gamma_\lambda q^2 - q_\lambda\gamma\cdot q)]u_\mu(p_\mu)\,, \tag{9}$$

where f_M, f_M^5, f_E, f_E^5 are functions of q^2, with $q = p_\mu - p_e$ and m_μ is the muon mass. In this case, the matrix element for $\mu^- + u \to e^- + u$, for example, takes the form

$$M(p_e, p_\mu) = \tfrac{2}{3}e[\bar{u}_e(p_e)\{(f_M + f_M^5\gamma_5)im_\mu\sigma_{\lambda\nu}q^\nu + (f_E + f_E^5\gamma_5)(\gamma_\lambda q^2 - q_\lambda\gamma\cdot q)\}u_\mu(p_\mu)][\bar{u}_u(k')\gamma^\lambda u_u(k)]/q^2\,. \tag{10}$$

The initial muon and final electron are not on their mass shells, and the four-momentum transfer is not a well defined quantity. However, if f_E, f_M, etc., are not fast varying functions of q^2, then it is a good approximation to evaluate them at $q^2 = -m_\mu^2$. The uncertainty in $|q|$ is of the order of the muon binding energy. If the effective Hamiltonian density (1) represents the contributions of all processes which do not involve photon exchange, then the expression for $\omega(\mu N \to eN)$ in (4) has to be modified by replacing $g_V^{(0)}$ and $g_V^{(1)}$ with $\{g_V^{(0)} + e[f_E(-m_\mu^2) + f_M(-m_\mu^2)]/3\}$ and $\{g_V^{(1)} + e[f_E(-m_\mu^2) + f_M(-m_\mu^2)]\}$, respectively, and by replacing $\alpha_V^0 g_V^{(0)}$ and $\alpha_V^1 g_V^{(1)}$ with the corresponding expressions involving leptonic axial-vector form factors. This amounts to a redefinition of the phenomenological coupling constants in (4). Thus, the inclusion of terms involving q_λ in the effective Hamiltonian density (1) will not affect the discussion of the Z dependence of the coherent muon conversion rate.

We can compare our expression with an earlier approximate expression given by Weinberg and Feinberg.[16] Essentially, their approximation consists of replacing the muon wave function by some average value, and replacing g_e, f_e by $j_0(m_\mu r)$ and zero. They use Primakoff's approximate formula[18] for the ordinary muon capture rate. To compare their formula with our result, we use experimental values for $\omega(\mu N \to \nu N')$ in their formula also, and we use the correct value E_e^2 instead of m_μ^2 in the phase-space factor. The Weinberg-Feinberg expression can be written as

$$R_{eN} = \left(\frac{G_F^2}{2}\right)\frac{m_\mu^3\alpha^3 Z_{\rm eff}^4}{Z}(E_e^2/\pi^2)\left[\left(\frac{3g_V^{(0)}g_V^{(1)}}{2}\right)Z + \left(\frac{3g_V^{(0)} - g_V^{(1)}}{2}\right)N\right]^2 F^2(-m_\mu^2)/\omega(\mu N \to \nu N')\,. \tag{11}$$

$F(q^2)$ is the form factor of the nuclear matter distribution at momentum transfer q^2. $4(m_\mu^3\alpha^3 Z_{\rm eff}^4)/Z$ is the standard notation for the square of the muon wave function averaged over the nucleus, i.e.,

$$Z_{\rm eff}^4 = \frac{Z}{4m_\mu^3\alpha^3}\int (g_\mu^2 + f_\mu^2)\rho 4\pi r^2 dr\,. \tag{12}$$

For the Fermi distribution considered by us, the form factor is given by

$$F(q^2) = 4\pi\rho_0 a^2 c\left[\frac{\pi^2\coth(\pi qa)}{qc\sinh(\pi qa)}\sin(qc) - \frac{\pi\cos(qc)}{qa\sinh(\pi qa)} - \sum_{n=1}^{\infty} 2an(-1)^n\exp(-nc/a)/c(n^2 + q^2a^2)^2\right]. \tag{13}$$

The three main corrections to the Weinberg-Feinberg formula are: (1) In calculating $Z_{\rm eff}^4$, one uses $(g_\mu^2 + f_\mu^2)$ whereas in our expression involving the square of Eq. (5), there is an interference term between g_μ and f_μ. Because this interference term is omitted, the Weinberg-Feinberg formula underestimates the coherent conversion rate.
(2) In calculating $Z_{\rm eff}^4$, one neglects the variation

TABLE III. Correction factors to the Weinberg-Feinberg formula.

Z	C_1	C_2	C_3	$C_1C_2C_3$
6.0	1.02	1.01	1.01	1.04
12.0	1.04	1.03	1.01	1.08
16.0	1.05	1.04	1.00	1.09
20.0	1.07	1.05	0.98	1.10
22.0	1.08	1.09	0.94	1.10
25.0	1.09	1.11	0.92	1.11
29.0	1.11	1.10	0.92	1.12
31.0	1.12	1.12	0.90	1.12
33.0	1.13	1.19	0.84	1.12
34.0	1.14	1.14	0.87	1.13
38.0	1.16	1.17	0.84	1.14
42.0	1.18	1.20	0.81	1.15
46.0	1.21	1.25	0.77	1.16
50.0	1.24	1.31	0.73	1.17
58.0	1.29	1.44	0.65	1.21
64.0	1.34	1.58	0.58	1.24
67.0	1.36	1.67	0.55	1.26
74.0	1.43	1.92	0.48	1.30
80.0	1.49	2.25	0.41	1.36
82.0	1.51	2.41	0.38	1.38
83.0	1.52	2.47	0.37	1.39
90.0	1.61	5.10	0.18	1.47
92.0	1.64	5.83	0.16	1.50

of the electron wave function. Since the electron wave function weights g_μ more towards the origin, using $Z_{\rm eff}{}^4$ should lead to smaller predictions for the conversion rate. (3) In calculating the form factors, one uses $q^2=-m_\mu{}^2$. However, owing to Coulomb attraction, the effective value of $|q|$ is greater than m_μ in the nucleus and this decreases the value of the form factor. Hence, using $F(-m_\mu{}^2)$ should lead to larger predictions for the conversion rate. The first two effects cause the Weinberg-Feinberg formula to make a smaller prediction for the rate, while the third effect works in the opposite direction and partially compensates for the first two. To get quantitative estimates of the three effects,[19] I define three correction factors as follows:

$$C_1=\left[\frac{\int (g_\mu g_e+f_\mu f_e)\rho 4\pi r^2 dr}{\int g_\mu g_e\,\rho 4\pi r^2 dr}\right]^2, \tag{14}$$

$$C_2=\frac{\left[\int g_\mu g_e\,\rho 4\pi r^2 dr/\int g_e\,\rho 4\pi r^2 dr\right]}{4m_\mu{}^3\alpha^3 Z_{\rm eff}{}^4/Z}, \tag{15}$$

$$C_3=\left[\frac{\int g_e\,\rho 4\pi r^2 dr}{F(-m_\mu{}^2)}\right]^2. \tag{16}$$

The product of these three factors gives the ratio of our result for R_{eN} to the value predicted by the Weinberg-Feinberg formula. Table III gives the values of C_1, C_2, C_3 and their product for some atoms. For the heavy atoms, we see that C_2 and C_3 are significant corrections, while their product is close to 1. We have seen qualitatively that these effects should be in opposite directions.

If muon-number violation occurs due to the exchange of a scalar-Higgs particle, then the effective Hamiltonian density takes the form

$$H_{\rm eff}=\left(\frac{G_F}{\sqrt{2}}\right)\left[\sum_{i=0,1} g_S^{(i)}\bar{e}(1+\alpha_S^i\gamma_5)\mu S^{(i)} + g_P^{(i)}\bar{e}(1+\alpha_P^i\gamma_5)\mu P^{(i)}\right], \tag{17}$$

where $S^{(0)}$, etc., denote the quark currents

$$S^{(0)}=(\bar{u}u+\bar{d}d)/2,\ \text{etc.} \tag{18}$$

The rate for the coherent process is given by (4), with the replacement of the vector coupling constants by the corresponding scalar coupling constants and R_Z, R_N by R_Z', R_N' where

$$R'_{Z\,(N)}=\int (g_e g_\mu - f_e f_\mu)\rho_{Z\,(N)} 4\pi r^2 dr. \tag{19}$$

If we assume that all the α's are +1 or −1 and $\rho_Z=\rho_N=\rho$, then the expression for R_{eN} is given by (6) if we replace $g_V^{(0)}$, $g_V^{(1)}$, and $\omega_G(Z)$ by $g_S^{(0)}$, $g_S^{(1)}$, and $\omega_H(Z)$ where

$$\omega_H(Z)=(G_F{}^2/2)9E_e{}^2A^2 \times\left[\int (g_e g_\mu - f_e f_\mu)\rho r^2 dr\right]^2. \tag{20}$$

Table I gives $\omega_H(Z)$ and $\omega_H/\omega(\mu N\to\nu N')$, in addition to $\omega_G(Z)$ and $\omega_G/\omega(\mu N\to\nu N')$. $\omega_H(Z)$ and $\omega_G(Z)$ differ in the sign of the contribution coming from the lower components of the lepton wave functions. This contribution becomes important for the heavy elements. For the light elements, the Z dependence of $\omega_H(Z)$ is similar to that of $\omega_G(Z)$. For the heavy elements, $\omega_H(Z)$ falls off faster than $\omega_G(Z)$. Table IV is analogous to Table II with $g_S^{(1)}/g_S^{(0)}$ replacing $g_V^{(1)}/g_V^{(0)}$. Figure 2 gives the Z dependence of R_{eN} and $\omega(\mu N\to eN)$ for some values of $g_S^{(1)}/g_S^{(0)}$. The limits on R_{eN} for sulphur and copper give $|g_S^{(0)}|<10^{-6}$ and $|g_S^{(1)}|<3.2\times10^{-4}$.

The main uncertainty in calculating the coherent muon conversion rates comes from the neutron density. If the error in R_N defined by Eq. (5) is ξ_N, then the error in the rate (ξ_r) is approximately

$$\xi_r=2\xi_N\left(\frac{3g_V^{(0)}-g_V^{(1)}}{2}\right)N\Big/\left[\left(\frac{3g_V^{(0)}+g_V^{(1)}}{2}\right)Z + \left(\frac{3g_V^{(0)}-g_V^{(1)}}{2}\right)N\right], \tag{21}$$

and this expression can easily be modified for the Higgs-particle-exchange case. Present experi-

TABLE IV. Coherent-rate branching ratio as a function of the coupling constants for the scalar-Higgs-boson case. The first column gives $g_S^{(1)}/g_S^{(0)}$ and the other columns give R_{eN} for the elements indicated, as a fraction of the maximum value of R_{eN} for each $g_S^{(1)}/g_S^{(0)}$.

$g_S^{(1)}/g_S^{(0)}$ \ Z	16.0	20.0	29.0	34.0	64.0	80.0	82.0	92.0
−10.0	0.18	0.21	0.49	0.93	0.99	0.81	0.77	0.72
−5.0	0.26	0.32	0.58	0.98	0.97	0.78	0.73	0.66
0.0	0.40	0.48	0.68	1.00	0.87	0.67	0.62	0.54
5.0	0.68	0.81	0.84	1.00	0.70	0.50	0.45	0.35
10.0	0.83	1.00	0.71	0.60	0.26	0.15	0.12	0.07
15.0	0.83	1.00	0.45	0.19	0.01	0.00	0.00	0.02
20.0	0.83	1.00	0.25	0.01	0.11	0.18	0.20	0.30
25.0	0.83	1.00	0.10	0.06	0.57	0.69	0.73	0.92
30.0	0.44	0.53	0.01	0.18	0.75	0.82	0.85	1.00
35.0	0.26	0.31	0.00	0.27	0.82	0.85	0.88	1.00
∞	0.00	0.00	0.17	0.64	1.00	0.91	0.90	0.92

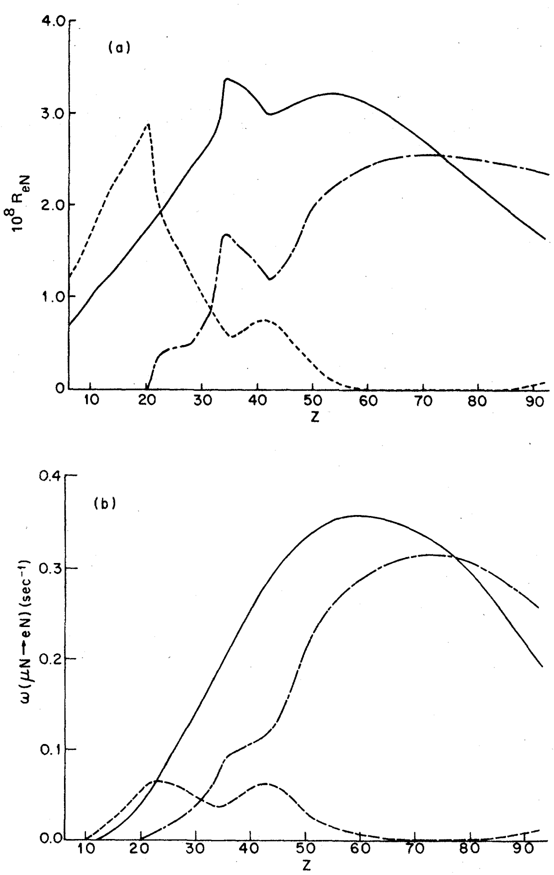

FIG. 2. Qualitative variation of: (a) R_{eN} and (b) $\omega(\mu N \to eN)$, with atomic number Z, for the Higgs-scalar case. For most of the elements shown in Table I R_{eN} lies within 10% of the smooth curves shown. The solid curve is for $g_S^{(0)}=10^{-5}$, $g_S^{(1)}=0$; the dash-dot curve is for $g_S^{(0)}=0$, $g_S^{(1)}=1.5\times10^{-4}$; the dashed curve is for $g_S^{(0)}=1.33\times10^{-5}$, $g_S^{(1)}=2\times10^{-4}$.

mental data on the neutron distribution are rather limited. However, considerable progress has been made in estimating the neutron and proton distributions, and many other nuclear properties, using realistic nucleon forces.[20] The proton distribution calculated by this method agrees well with experiments. The calculated neutron density also agrees with the experimental density, to the extent that the latter is determined. Also, the neutron and proton properties calculated from many-body theory are expected to be comparable in reliability. Hence, such calculations provide the most reliable neutron densities, and correspondingly the most reliable estimates of ξ_N.

The assumption $\rho_N=\rho_Z$ is quite good for the light nuclei, and the largest errors from this assumption will be for the heavy nuclei. For these nuclei, the neutron density has a larger root-mean-square radius and hence R_N will be smaller than R_Z. Using the neutron density for ^{208}Pb calculated by Negele,[21] I estimate that R_N (R_N') is smaller than R_Z (R_Z') by about 10%, where R_N, R_Z are defined by Eq. (5) and R_N', R_Z' are defined by Eq. (19). Hence the assumption $\rho_Z=\rho_N$ leads to ξ_r of about −12% for lead if the phenomenological coupling is purely isoscalar. For the case in which the isovector coupling is much larger than the isoscalar, the error ξ_r may be much greater. For the heavy elements, the actual isovector curves will be lower than shown in Figs. 1 and 2. The detailed shape will depend on the nuclear model chosen to estimate the neutron density.

We can now consider the nuclear matrix elements contributing to incoherent capture. Such matrix elements have been calculated for oxygen,[12] and are also being calculated for sulphur,[22] using nuclear-model wave functions. The nuclear matrix elements for the isovector Lorentz vector and axial-vector currents are very similar to those that occur in ordinary muon capture. From the measurements of the partial rates for ordinary muon capture leading to different excited nuclear states, one can determine the required matrix elements by isospin rotations. The isoscalar Lorentz vector matrix elements can be determined from nuclear electromagnetic transitions and electron scattering experiments.[23] The main uncertainty comes from having to correct for the Coulomb effects on the electron wave function and from the variation of the muon wave function within the nucleus. If other matrix elements are needed, then the best approach would be to use nuclear wave functions determined from optical potentials.

To illustrate the approach of isospin rotations, we consider the special case $g_V^{(1)} = -g_A^{(1)} = g$ and $g_V^{(0)} = g_A^{(0)} = 0$. In this case, the total rates for the two processes are related by isospin and for isospin zero nuclei we have

$$R_{\text{incoh}} = \omega(\mu N \to eN')/\omega(\mu N \to \nu N') = \tfrac{1}{2} g^2 . \qquad (22)$$

Since the capture rate involves an incoherent sum over protons and neutrons, this equation can be modified for the nonzero isospin nuclei. For these nuclei, normal capture has an extra inhibition factor because of the Pauli exclusion effect of the excess neutrons. If we estimate this factor using the Primakoff formula we find

$$R_{\text{total}} \approx \tfrac{1}{8} g^2 \left\{ 1 - 3\frac{A-Z}{2A} \right\}^{-1} (A/2Z) . \qquad (23)$$

For the heavy atoms, the coherent and incoherent rates might be comparable if the coupling is isovector.

CONCLUSION

I have discussed coherent muon conversion and have considered the possibility of muon-number violation occurring by lepton mixing, gauge bosons or scalar-Higgs bosons. Tables II and IV and Figs. 1 and 2 show the Z dependence of R_{eN} for different values of the coupling constants. If the isoscalar and isovector couplings are comparable in magnitude, the Z dependence of R_{eN} does not depend strongly on the isovector coupling constant. For purely isoscalar couplings, the Z dependence is rather flat for the heavy elements, for the gauge-boson case. R_{eN} for sulphur, for which we have some experimental information, is about 25% of the maximum R_{eN} for this case. The limit on R_{eN} for sulphur implies $|g_V^{(0)}| < 10^{-6}$. For the Higgs exchange case, the maximum R_{eN} occurs for $Z = 34$. R_{eN} does not vary much between $Z = 34$ and $Z = 64$, and begins to drop for the heavy elements. R_{eN} for sulphur is 40% of the maximum R_{eN}, in this case. The limit on R_{eN} for sulphur implies $|g_S^{(0)}| < 10^{-6}$. Present experimental bounds on R_{eN} do not provide significant limits for the isovector coupling constants. The limits on R_{eN} for sulphur and copper imply $|g_V^{(1)}| < 2.9 \times 10^{-4}$ or $|g_S^{(1)}| < 3.2 \times 10^{-4}$. The isovector coupling constants can be constrained by experiments on R_{eN} for the medium and heavy elements. R_{eN} does not vary significantly for these elements when the coupling is purely isovector. For Z between 50 and 92, we estimate that R_{eN} can vary by a maximum factor of about 2 for the gauge-boson case and about 4 for the Higgs-scalar exchange case, even when the calculation is done with the correct neutron distribution. Also, the coherent and incoherent conversion rates may be comparable in magnitude for these elements, if the coupling is purely isovector.

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