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Radiation corrections to quantum processes in an intense electromagnetic field

N. B. Narozhny*
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627

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A derivation of an asymptotic expression for the mass correction of order α to the electron propagator in an intense electromagnetic field is presented. It is used for the calculation of radiation corrections to the electron and photon elastic scattering amplitudes in the α^3 approximation. All proper diagrams contributing to the amplitudes and containing the above-mentioned correction to the propagator were considered, but not those which include vertex corrections. It is shown that the expansion parameter of the perturbation theory of quantum electrodynamics in intense fields grows not slower than $\alpha\chi^{1/3}$ at least for the electron amplitude, where $\chi = [(eF_{\mu\nu}p_\nu)^2]^{1/2}/m^3$, p is a momentum of the electron, and F is the electromagnetic field tensor.

I. INTRODUCTION

It is a fundamental problem of quantum electrodynamics in intense fields to find the true expansion parameter of the perturbation theory as respects the radiation field when the external field strength or the energy of a particle initiating a process are extremely high. This is one of the reasons a lot of recently published papers[1-5] are dealing with radiation corrections to quantum processes in very strong electromagnetic fields, i.e., in fields whose intensity is close to the critical value $F \sim F_0 = m^2c^3/e\hbar = 4.4\times 10^{13}$ Oe.

Such field strengths are not available experimentally at the present time. However, for ultrarelativistic particles with a momentum $p \sim F_0 m/F \gg m$, the field strength will be of the order of F_0 in the proper frame. In addition, independent of the form of the field in the laboratory system, it will be nearly a plane-wave field in the proper frame, and if its characteristic wavelength and period are large in comparison with the characteristic length and time of the formation of the process, this field can be regarded as a constant crossed field $\vec{E}\perp\vec{H}, E=H\equiv F$. We shall consider below precisely such a field.

Quantum processes initiated by an electron with a momentum p in a crossed field are determined by a single invariant parameter $\chi = [(eF_{\mu\nu}p_\nu)^2]^{1/2}/m^3$ (the same quantity for a photon with a momentum l we shall denote as κ). It is equal to $\chi = p_- F_0/mF$ in a "special" coordinate system with the 1, 2, 3 axes along the directions $\vec{E}$, $\vec{H}$, and $\vec{E}\times\vec{H}$, and $p_- = p_0 - p_3$. Thus, the limit of a very strong field is the limit $\chi \gg 1$ (or $\kappa \gg 1$). At the same time one can regard the limit $\chi \gg 1$ as the high-energy ($p_- \gg m$) limit for an incident particle.

We shall investigate below radiation corrections to electron and photon elastic scattering amplitudes in a crossed field in the limit $\chi \gg 1$. These quantities in the α approximation were found by Ritus[6] and the author.[7] Radiation corrections of order α^2 have been partially investigated in Refs. 1–3. Unfortunately these papers do not answer the question about the dependence of the expansion parameter on χ when $\chi \gg 1$ because of a rather unusual situation.

The polarization correction to the electron elastic scattering amplitude, which is determined by Fig. 1(a), increases as[1] $\chi \ln\chi$ when $\chi \gg 1$, while the mass correction, which is determined by Fig. 1(b), is of order $\chi^{2/3}\ln\chi$.[2] The mass correction to the photon elastic scattering amplitude [Figs. 2(a) and 2(b)] increases as $\kappa^{2/3}\ln\kappa$.[3] Though vertex corrections to both amplitudes [Figs. 1(c) and 2(c)] have not been calculated yet, it is unlikely that they could increase more rapidly than the mass corrections. Taking into account now that the amplitudes of order α increase as $\chi^{2/3}$ or $\kappa^{2/3}$, respectively,[6,7] one can notice two unusual features of the expansions for the amplitudes.

First, the expansion parameters for the electron and photon amplitudes are different in the high-energy limit. It is $\alpha\chi^{1/3}\ln\chi$ for the electron's amplitude and $\alpha \ln\kappa$ for the photon's. Second, the expansion parameter for the electron amplitude increases with energy more rapidly than in conventional quantum electrodynamics where it is equal to $\alpha \ln(\epsilon/m)$, as is well known. The same statements are true, of course, for mass and polarization operators in an intense crossed field since the considered amplitudes are their matrix elements on the mass shell.

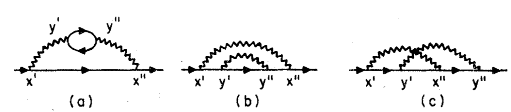

FIG. 1. The diagrams representing the elastic electron scattering amplitude in the α^2 approximation.

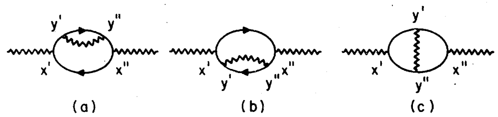

FIG. 2. The diagrams representing elastic photon scattering amplitude in the α^2 approximation.

One may doubt the validity of these conclusions which are based on the analysis of the α^2-order diagrams. There is still a possibllity that the diagrams of order α^2 contain some anomaly. The existence of such an anomaly seems to be very probable due to the fact that the leading term in the asymptotic expanison of the electron amplitude of order α^2 is caused by Fig. 1(a) which contains the polarization correction to the photon Green's function while the diagrams determining the correction to the photon amplitude of order α^2 do not contain it.

In the next order cf the perturbation theory both electron and photon amplitudes "enjoy equal rights" and one could expect that they increase with χ in a similar way. To verify it one should calculate radiation corrections to both amplitudes in the α^3 approximation. Unfortunately it would be too cumbersome to make explicit calculations. Therefore approximate methods for direct calculations of the asymptotics of diagrams of high orders in the limit $\chi \gg 1$ should be developed.

In the present paper we have found the high-energy asymptotic for the mass correction to the electron Green's function in a crossed field which allows one in a relatively simple way to calculate asymptotics of some diagrams of order α^3 in the limit of very large χ. Figure 3 contains the most important diagram among the considered diagrams since it increases as $\chi^{4/3}$ when $\chi \gg 1$, while other diagrams are of the order $\chi^{2/3}\ln^2\chi$. This fact confirms the idea that the diagrams containing polarization corrections to the photon Green's function give the main contribution to the asymptotics of radiation corrections in the limit $\chi \gg 1$ and make it possible to assert that the expansion parameter of the perturbation theory increases with χ not slower than $\chi^{1/3}$, at least for the mass operator.

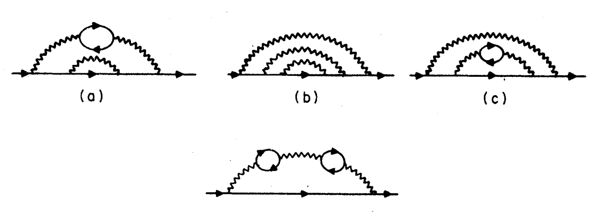

FIG. 3. Some diagrams representing elastic electron scattering amplitude in the α^3 approximation.

A derivation of the high-energy asymptotic for the mass correction to the electron propagator is presented in the next section of the paper and it is used then for calculating diagrams of order α^2. In Secs. III and IV we calculate some diagrams of order α^3 and the obtained results are discussed in Sec. V.

II. MASS CORRECTION TO THE ELECTRON GREEN'S FUNCTION IN THE α APPROXIMATION

It will be convenient to use the so-called E_p representation introduced by Ritus[6] with basic functions $E_p(x)$, which for the particular case of a crossed field with 4-potential $A_\mu(x) = a_\mu(k \cdot x)$, $k^2 = a \cdot k = 0$ may be written in the form

$$E_p(x) = \left[1 + e\frac{(k\cdot\gamma)(a\cdot\gamma)}{2k\cdot p}(k\cdot x)\right] \times \exp\left[i\frac{ep\cdot a}{2p\cdot k}(k\cdot x)^2 - i\frac{e^2a^2}{6(p\cdot k)}(k\cdot x)^3 - ip\cdot x\right] \qquad (1)$$

[we use the units $\hbar = c = 1$, $\alpha = e^2/4\pi = \frac{1}{137}$ and the notation $p_\mu = (\vec{p}, ip_0)$, $p\cdot q = \vec{p}\cdot\vec{q} - p_0q_0$].

A remarkable property of the E_p representation is the fact that in this representation the electron propagation function $S^c(p)$ reduces to the vacuum one and the mass operator is diagonal[5]:

$$M_F(q,p) = (2\pi)^4\delta(q-p)M_F(p)\,, \qquad (2)$$

$$M_F(p) = mS(\chi,\nu) + V_1(\chi,\nu)(m + ip\cdot\gamma) + V_2(\chi,\nu)\frac{ie^2\gamma_\mu F_{\mu\nu}F_{\nu\sigma}p_\sigma}{m^4\chi^2} + T(\chi,\nu)\frac{e\sigma_{\mu\nu}F_{\mu\nu}}{m\chi} + A(\chi,\nu)\frac{ie\gamma_5\gamma_\mu F^*_{\mu\nu}p_\nu}{m^2\chi}\,. \qquad (3)$$

The scalar functions S, $V_{1,2}$, T, and A may be written in the α approximation in the following way:

$$S^{(2)}(\chi,\nu) = -\frac{\alpha}{2\pi}\int_0^\infty \frac{dv}{(1+v)^3}(1+2v) \times\left[f_1(y) - \ln\left(1+\frac{\nu}{v}\right)\right], \qquad (4)$$

$$V_1^{(2)}(\chi,\nu) = \frac{\alpha}{\pi}\int_0^\infty \frac{dv}{(1+v)^3}\left\{\frac{1+2v}{v_\lambda} + \frac{1}{2}\left[f_1(y) - \ln\left(1+\frac{\nu}{v}\right)\right]\right\}, \qquad (5)$$

$$V_2^{(2)}(\chi,\nu) = -\frac{\alpha}{\pi}\int_0^\infty \frac{dv}{(1+v)^2}\left(1+\frac{v}{3}\right)\left(\frac{\chi}{v}\right)^{2/3} f'(y)\,, \qquad (6)$$

$$T^{(2)}(\chi, \nu) = -\frac{\alpha}{\pi}\int_0^\infty \frac{dv}{(1+v)^2}\left(\frac{\chi}{v}\right)^{1/3} f(y), \tag{7}$$

$$A^{(2)}(\chi, \nu) = -\frac{\alpha}{2\pi}\int_0^\infty \frac{dv}{(1+v)^2}(2+v)\left(\frac{\chi}{v}\right)^{1/3} f(y). \tag{8}$$

Functions $f(y)$ and $f_1(y)$ in (4)–(8) are defined by relations

$$f(y) = i\int_0^\infty d\sigma \exp(-iy\sigma - i\sigma^3/3),$$
$$f_1(y) = \int_0^\infty dx[f(x) - 1/x], \tag{9}$$

and

$$y = z\left(1 + \frac{\nu}{v}\right), \quad z = \left(\frac{v}{\chi}\right)^{2/3}, \quad \nu = \frac{p^2 + m^2}{m^2},$$
$$v_\lambda = v + \lambda_0\frac{1+v}{v^2}, \quad \lambda_0 = \left(\frac{m_{\text{ph}}}{m}\right)^2. \tag{10}$$

A small photon mass m_{ph} was introduced in (3) to eliminate the infrared divergence which appeared in the mass operator after renormalization.

As one can see, the mass operator (3) is expanded into five real γ-matrix operators m, $m + i\gamma\cdot p$, $ie^2\gamma_\mu F_{\mu\nu}F_{\nu\sigma}p_\sigma/m^4\chi^2$, $e\sigma_{\mu\nu}F_{\mu\nu}/m\chi$, and $ie\gamma_5\gamma_\mu F^*_{\mu\nu}p_\nu/m^2\chi$. To find the asymptotic behavior of the mass operator (3) in the limit $\chi \gg 1$ we will first compare the orders of magnitudes of the terms of the expansion. This is easy to do if we notice that we deal, in fact, with matrix elements $\bar{u}(p)M(p)u(p)$ of the mass operator, where $u(p)$ is a free Dirac spinor, while calculating radiation corrections described by diagrams including the mass operator as an inner part. Such matrix elements of all γ-matrix operators in (3), except the operator $m + i\gamma\cdot p$, are of order m and therefore the magnitudes of all terms of the expansion (3), except the second one, are determined by the magnitudes of scalar functions. For the second term the situation is different because matrix elements $\bar{u}(m + i\gamma\cdot p)u \sim m\nu$. The role of the variable ν is to form causal θ functions in the integrands of the integrals representing radiation corrections and the whole region of integration over ν, $-\infty < \nu < \infty$, is essential. Since it is impossible to point out the effective values of the variable ν, we can compare the contribution of this term to the asymptotics of radiation corrections with the contributions of other terms only after performing the integration over ν.

It is important that the effective values of the variable v, which has a simple physical meaning (that the ratio of virtual photon and electron momenta $v = l_-/p_-$), is of order 1 in all integrals (4)–(9) except the infrared-divergent one in (5). The effective values of the variable σ, which determine the f functions [see integral representation of the f function (9)], are of one and the same order in integrals (6)–(8), and since the function $V_2^{(2)}$ has the factor $\chi^{2/3}$ while the functions $T^{(2)}$ and $A^{(2)}$ $\sim\chi^{1/3}$ we may omit the latter two in the high-energy asymptotic of the mass operator.

To compare the magnitudes of functions $S^{(2)}$ and $V^{(2)}$ we will get a simple asymptotic expression for the function $S^{(2)}$. There is the following integral representation for the function $f_1(y) - \ln(1 + \nu/v)$:

$$f_1(y) - \ln\left(1 + \frac{\nu}{v}\right) = \times\int_0^\infty \frac{d\sigma}{\sigma}e^{-iz\sigma}\left[\exp\left(-i\frac{z\nu}{v}\sigma - i\frac{\sigma^3}{3}\right) - 1\right] \tag{11}$$

and $\sigma \sim (v/\chi)^{1/3}(eF/m)\Delta y_-$, where Δy_- is the duration of interaction of the electron with the radiation field. In any diagram including mass operator $M_F^{(2)}(p)$ the term $\sim e^{-iz\nu\sigma/v}$ in (11) will lead to the appearance of a causal θ function after integration over ν, which will limit Δy_- by the duration of the external interaction (see, for example, Figs. 1 and 2). Thus the formula (11) may be effectively written down as

$$f_1(y) - \ln\left(1 + \frac{\nu}{v}\right) = \int_0^{\sigma_0}\frac{d\sigma}{\sigma}e^{-iz\sigma}\left[\exp\left(-i\frac{z\nu}{v}\sigma - i\frac{\sigma^3}{3}\right) - 1\right] + \int_{\sigma_0}^\infty \frac{d\sigma}{\sigma}e^{-iz\sigma}, \tag{12}$$

where σ_0 is always of order 1 if $z \lesssim 1$ ($v_{\text{eff}} \sim 1$, $\chi \gtrsim 1$). In the limit $\chi \gg 1$ ($z \ll 1$) the second integral in (12) grows logarithmically while the first one is of order 1. Therefore, independent of σ_0,

$$f_1(y) - \ln\left(1 + \frac{\nu}{v}\right) \approx \ln z \approx -\tfrac{2}{3}\ln\chi, \quad \chi \gg 1 \tag{13}$$

and we see that the function $S^{(2)}$ may also be omitted in the high-energy asymptotic of the mass operator.

We may now regard the expression

$$M_F^{(2)}(p) \approx V_1^{(2)}(\chi, \nu)(m + i\gamma\cdot p) + V_2^{(2)}(\chi, \nu)\frac{ie^2\gamma_\mu F_{\mu\nu}F_{\nu\sigma}p_\sigma}{m^4\chi^2} \tag{14}$$

as the asymptotic of the mass operator in the limit $\chi \gg 1$ with the asymptotic expression for $V_1^{(2)}(\chi, \nu)$,

$$V_1^{(2)}(\chi, \nu) \approx -\frac{\alpha}{2\pi}\ln(\lambda_0\chi^{1/3}), \tag{15}$$

which one can easily derive from (5) with the help of formula (13) after separating the infrared divergence.

Unfortunately it is impossible to find a simple universal asymptotic expression like (15) for the function $V_2^{(2)}(\chi, \nu)$. But we shall see that for the leading terms of asymptotics of radiation corrections it reduces to a very simple form.

One should remember that the leading term of the asymptotic of $V_1^{(2)}$ originates from the vacuum part of the mass operator (though it is $\sim \ln\chi$ and depends on F) and can lead to the appearance of ultraviolet divergences which should be removed by the standard renormalization procedure.

For the mass correction of order α to the electron propagation function in the E_p representation

$$\Delta S^{c(2)}(p) = -iS^c(p)M_F^{(2)}(p)S^c(p)\,, \tag{16}$$

we may get, using formula (14),

$$\Delta S^{c(2)}(p) \approx \frac{\alpha}{2\pi}\ln(\lambda_0\chi^{1/3})S^c(p) - i\frac{2}{\nu}V_2^{(2)}(\chi,\nu) \times\left[iS^c(p) - \frac{ie^2\gamma_\mu F_{\mu\nu}F_{\nu\sigma}p_\sigma}{2m^6\chi^2}\right],\ \chi\gg 1\,. \tag{17}$$

We shall now use the expression (17) for $\Delta S^{c(2)}(p)$ for calculating the asymptotics of the mass corrections to the electron and photon elastic scattering amplitudes in the intense crossed field, which are described by Figs. 1(b) and 2(a) and 2(b), respectively. We will follow the method introduced by Ritus in his paper.[6] The usage of expression (11) for $\Delta S^{c(2)}(p)$ is justified by the fact that the charged particles created in a process initiated by an ultrarelativistic particle ($\chi\gg 1$) are always ultrarelativistic.

We shall consider the electron amplitude without a change of spin direction first. The contribution of the first term from (17) to the asymptotic is obvious and is equal to

$$T_b^{(4)\prime}(p,F) \approx \frac{\alpha}{2\pi}\ln(\lambda_0\chi^{1/3})T^{(2)}(p,F)\Big|_{\chi\gg 1}\,, \tag{18}$$

where $T^{(2)}(p,F)|_{\chi\gg1}$ denotes the asymptotic of the amplitude in the α approximation[6]

$$T^{(2)}(p,F)|_{\chi\gg1} = -\frac{\alpha m^2}{p_0}(3\chi)^{2/3}\frac{7}{27\sqrt{3}} \times\Gamma(\tfrac{2}{3})(1-i\sqrt{3})\,. \tag{19}$$

The contribution of the second term in (17) to the amplitude may be written to read in the limit $\chi\gg1$

$$T_b^{(4)\prime\prime}(p,F) = -\frac{2ie^2m^2}{(2\pi)^4p_0\chi}\int d\kappa\int_{-\infty}^{\infty}\frac{d\mu}{(\mu-i\epsilon)^2}\int_{-\infty}^{\infty}\frac{d\lambda}{\lambda+i\epsilon}V_2^{(2)}(\chi',\mu)\left[(1+\tfrac{1}{2}\lambda-\tfrac{1}{2}u\mu)\phi_1(t)+\frac{2+2u+u^2}{1+u}\left(\frac{\chi}{u}\right)^{2/3}\phi'(t)\right]. \tag{20}$$

We have dropped spin terms in formula (20) since they are obviously small compared to the remaining ones in the limit $\chi\gg1$.

Here χ' and $\mu=(q^2+m^2)/m^2$ refer to the internal electron of momentum q; κ and $\lambda=-l^2/m^2$ refer to the external photon of momentum l; $\kappa+\chi'=\chi$, $u=\kappa/\chi'$. $\phi(t)$ is a well-known Airy function,

$$\phi(t)=\int_0^\infty d\rho\cos(t\rho+\tfrac{1}{3}\rho^3)\,,$$

function $\phi_1(t)$ is

$$\phi_1(t)=\int_t^\infty dt'\phi(t') = -\tfrac{1}{2}i\int_{-\infty}^{\infty}\frac{d\rho}{\rho-i\epsilon}e^{-it\rho-i\rho^3/3}\,,$$

and the argument of the Airy function is

$$t=z'\left(1+\lambda_0\frac{1+u}{u^2}-\mu\frac{1+u}{u}+\lambda\frac{1+u}{u^2}\right)\,, \quad z'=\left(\frac{u}{\chi}\right)^{2/3}. \tag{21}$$

In the term proportional to $\phi_1(t)$ in (20) $u_{\text{eff}}\sim1$, and it is easy to show that the contribution of this term to the amplitude is of order $\chi^{2/3}$ and is less compared to (18).

In the term proportional to $\phi'(t)$ in (20) $u_{\text{eff}}\ll1$, and it is clear after comparison of t and y that one may neglect the dependence of $V_2^{(2)}(\chi',\mu)$ on μ. This means physically that the duration of the external interaction is much larger than the duration of the internal one for this term since the emission of weak photons is essential. Thus, for this term

$$V_2^{(2)}(\chi',\mu)\approx V_2^{(2)}(\chi',0) \approx -\frac{\alpha\chi'^{2/3}}{\pi}f'(0)\int_0^\infty\frac{dv}{v^{2/3}(1+v)^2}(1+\tfrac{1}{3}v) = -\frac{\alpha(3\chi)^{2/3}}{(1+u)^{2/3}}\frac{7}{27\sqrt{3}}(1-i\sqrt{3})\Gamma(\tfrac{2}{3})\,. \tag{22}$$

The integration over μ and λ in (20) can be performed now with the help of the formula

$$\int_{-\infty}^{\infty}\frac{d\mu}{(\mu-i\epsilon)^2}\int_{-\infty}^{\infty}\frac{d\lambda}{\lambda+i\epsilon}\phi'(t)=2\pi^2 i\theta(u)z'\frac{1+u}{u}f''(z'_\lambda),$$
$$z'_\lambda=z'\left(1+\lambda_0\frac{1+u}{u^2}\right), \tag{23}$$

and we get in the limit $\chi \gg 1$

$$T_b^{(4)''}(p,F)\approx\frac{\alpha}{\pi}\left(\tfrac{3}{2}\ln\lambda_0-\ln\chi\right)T^{(2)}(p,F)\Big|_{\chi\gg1}. \tag{24}$$

Finally, for the mass correction to the α^2 amplitude

$$T_b^{(4)}(p,F)=T_b^{(4)'}(p,F)+T_b^{(4)''}(p,F)$$

we have

$$T_b^{(4)}(p,F)\approx\frac{\alpha}{\pi}\left(2\ln\lambda_0-\tfrac{5}{6}\ln\chi\right)T^{(2)}(p,F)\Big|_{\chi\gg1}. \tag{25}$$

This result coincides with the one obtained by Morozov and Ritus.[2]

The mass correction to the photon amplitude does not have infrared divergences except the one caused by the mass operator of order α. Therefore the ratio of virtual electron and positron momenta $u=\chi'/\chi\sim1$, and it is easy to show that the contribution of the second term from (17) to the amplitude is of order $\kappa^{2/3}$. Thus only the first term from (17) determines the asymptotic behavior of the photon amplitude:

$$T_a^{(4)}(l,F)+T_b^{(4)}(l,F)\approx\frac{\alpha}{\pi}\ln(\lambda_0\kappa^{1/3})T^{(2)}(l,F)\Big|_{\kappa\gg1},$$
$$T^{(2)}(l,F)\big|_{\kappa\gg1}=-\frac{1}{2l_0}\sum_{i=1,2}(e'e_i)(ee_i)\pi_i^{(2)}(\kappa)\Big|_{\kappa\gg1}, \tag{26}$$

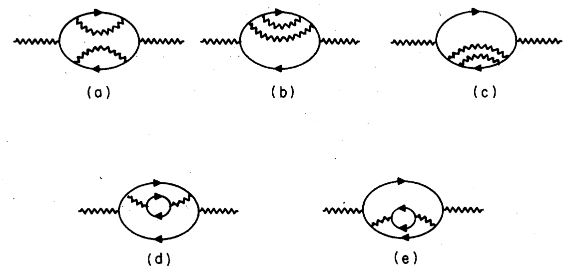

FIG. 4. Some diagrams representing elastic photon scattering amplitude in the α^3 approximation.

$$\pi_{1,2}(\kappa)\big|_{\kappa\gg1}=\frac{\alpha m^2}{\pi^2}(3\kappa)^{2/3}\sqrt{3}\,\frac{5\mp1}{28}(1-i\sqrt{3})\Gamma^4(\tfrac{2}{3}). \tag{27}$$

Unit polarization vectors e_1 and e_2 are given by

$$e_{1\mu}=\frac{eF_{\mu\nu}l_\nu}{m^3\kappa},\quad e_{2\mu}=\frac{eF^*_{\mu\nu}l_\nu}{m^3\kappa},\quad F^*_{\mu\nu}=\tfrac{1}{2}i\epsilon_{\mu\nu\lambda\sigma}F_{\lambda\sigma}$$

and describe in the "special" coordinate system the photons polarized respectively along the directions $\vec{E}$ and $\vec{H}$. This result coincides with the one obtained by Morozov and the author.[3]

The agreement between our results and those of Refs. 2 and 3 is a good check on the method. We will use it for the calculation of some diagrams of order α^3 in the next sections.

III. α^3 CORRECTIONS TO THE ELECTRON AND PHOTON ELASTIC SCATTERING AMPLITUDES A

In this section we present the calculation of asymptotics of Figs. 3(a) and 4(a). The correction to the electron amplitude of order α^3 described by Fig. 3(a) can be written with the use of expression (17) for $\Delta S^{c(2)}(p)$ in the following way:

$$\begin{aligned}T_a^{(6)}(p,F)\approx{}&\frac{\alpha}{2\pi}\ln(\lambda_0\chi^{1/3})T_a^{(4)}(p,F)\big|_{\chi\gg1}-\frac{ie^2}{(2\pi)^4p_0\chi}\int d\kappa\int_{-\infty}^{\infty}\frac{d\mu}{(\mu-i\epsilon)^2}\int_{-\infty}^{\infty}\frac{d\lambda}{(\lambda+i\epsilon)^2}V_2^{(2)}(\chi',\mu)\\&\times\left\{\left[(1-\tfrac{1}{2}u\mu)(\pi_1+\pi_2+2\pi_3)+\lambda\pi_3+\lambda\frac{2+2u+u^2}{2u^2}(\pi_1+\pi_2)-\mu\frac{1+u}{u}(\pi_1-\pi_2)\right]\phi_1(t)\right.\\&\left.+\left[\frac{2+2u+u^2}{1+u}(\pi_1+\pi_2+2\pi_3)+\pi_1-\pi_2\right]\left(\frac{\chi}{u}\right)^{2/3}\phi'(t)\right\}.\end{aligned} \tag{28}$$

Just as in formula (20) we have omitted small spin terms in (28). Here functions π_i, $i=1,2,3$, are the scalar functions which determine the polarization correction to the photon Green's function of order α:[6,7]

$$\Delta D^c_{\mu\nu}(l)=\frac{i}{(l^2-io)^2}[\pi_1e_{1\mu}e_{1\nu}+\pi_2e_{2\mu}e_{2\nu}+\pi_3(\delta_{\mu\nu}-l_\mu l_\nu/l^2)], \tag{29}$$

$$\pi_{1,2}=\frac{\alpha m^2}{3\pi}\int_0^{\infty}\frac{dv'}{(1+v')^2}\,\frac{2v'^2+(5\mp3)v'+2}{v'z''}f'(y'),$$
$$\pi_3=-\frac{2\alpha\lambda m^2}{\pi}\int_0^{\infty}dv'\frac{v'}{(1+v')^4}\left[f_1(y')-\ln\left(1-\frac{\lambda v'}{(1+v')^2}\right)\right], \tag{30}$$

$$y'=z''\left(1-\frac{\lambda v'}{(1+v')^2}\right), \quad z''=\left[\frac{(1+v')^2(1+u)}{uv'\chi}\right]^{2/3}, \tag{31}$$

and t is presented in formula (21). $T_a^{(4)}(p,F)|_{\chi\gg1}$ is the asymptotic expression for the polarization correction of order α to the electron amplitude described by Fig. 1(a) which was found by Ritus[1] and is equal to

$$T_a^{(4)}(p,F)|_{\chi\gg1}=\frac{13\alpha^2m^2\chi}{36\sqrt{3}\pi p_0}\left[\frac{\pi}{2}+i\left(\ln\frac{\chi}{2\sqrt{3}}-c-\frac{142}{39}\right)\right], \quad c=0.577.\ldots \tag{32}$$

The leading contribution to the asymptotic of (22) is due to the terms with $u_{\text{eff}}\sim1/\chi\ll1$. One may use formula (22) for $V_2^{(2)}$ and neglect the dependence of $\pi_{1,2}$ on λ for these terms as was done in the previous section. Integration over μ and λ may be performed then in the same way as in formula (20) and we get

$$\begin{aligned}T_a^{(6)}(p,F)\approx{}&\frac{\alpha}{2\pi}\ln(\lambda_0\chi^{1/3})T_a^{(4)}(p,F)|_{\chi\gg1}\\&-\frac{7\alpha^3m^2}{3\pi^2p_0}\frac{1-i\sqrt{3}}{27\sqrt{3}}\Gamma(\tfrac{2}{3})f(0)(3\chi)^{2/3}\int_0^\infty\frac{dv'}{(1+v')^2v'}\left[\frac{v'}{(1+v')^2}\right]^{2/3}(2v'^2+5v'+2)\int_0^\infty\frac{du}{u^{5/3}}f'(z''),\end{aligned} \tag{33}$$

where z'' may be written as

$$z''\approx\left[\frac{(1+v')^2}{uv'\chi}\right]^{2/3}$$

because of the condition $u_{\text{eff}}\ll1$.

After integration over u and v and neglecting terms $\sim\chi\ln\chi$, $\chi\ln^2\chi$ we get

$$\begin{aligned}T_a^{(6)}(p,F)\approx{}&\frac{\alpha}{2\pi}\ln\lambda_0T_a^{(4)}(p,F)\bigg|_{\chi\gg1}\\&+\frac{14\alpha^3m^2}{3\sqrt{3}\pi^2p_0}(3\chi)^{4/3}\frac{1+i\sqrt{3}}{1485}\Gamma^4(\tfrac{1}{3}), \quad \chi\gg1.\end{aligned} \tag{34}$$

The considered correction to the electron amplitude increases as $\chi^{4/3}$ when χ is large. Therefore we can conclude after comparison of expressions (34) and (32) that the expansion parameter of the perturbation theory increases with χ at least not slower than $\alpha\chi^{1/3}$ for the amplitude or for the mass operator.

The asymptotic of Fig. 4(a) is determined only by the first term in (17) and is equal to

$$T_a^{(6)}(l,F)\approx\frac{\alpha^2}{(2\pi)^2}\ln(\lambda_0\kappa^{1/3})T^{(2)}(l,F)\bigg|_{\kappa\gg1}.$$

The terms of $T_a^{(6)}(l,F)$ originated from the second term in (17) increase with κ as $\kappa^{2/3}$. Still they have infrared-divergent terms $\sim\ln\lambda_0$, but these terms increase as $\kappa^{2/3}$ too and therefore are insignificant for the amplitude in the limit $\kappa\gg1$ since there are terms $\sim\ln\lambda_0$ in expression (35) which increase as $\kappa^{2/3}\ln\kappa$.

IV. α^3 CORRECTIONS TO THE ELECTRON AND PHOTON ELASTIC SCATTERING AMPLITUDES *B*

For calculation of the asymptotics of Figs. 3(b) and Figs. 4(b) and 4(c), it is convenient to find the asymptotic expression for the mass correction of order α^2 to the electron Green's function (Fig. 5) first. It is again determined only by functions $V_1^{(4)}$ and $V_2^{(4)}$ of the mass correction to the renormalized mass operator of order α^2. Asymptotic expressions for these functions may be obtained with the help of formulas (17), (15), and (6) and are equal to

$$V_1^{(4)}(\chi,\nu)\approx\frac{\alpha}{2\pi}\ln(\lambda_0\chi^{1/3})V_1^{(2)}(\chi,\nu)\approx-\left(\frac{\alpha}{2\pi}\right)^2\ln^2(\lambda_0\chi^{1/3}), \tag{36}$$

$$V_2^{(4)}(\chi,\nu)\approx\frac{\alpha}{2\pi}\ln(\lambda_0\chi^{1/3})V_2^{(2)}(\chi,\nu)+\tilde V_2^{(4)}(\chi,\nu), \tag{37}$$

$$\begin{aligned}\tilde V_2^{(4)}(\chi,\nu)={}&\frac{\alpha^2}{\pi^2}\int_0^\infty\frac{dv'}{v'}\frac{2+2v'+v'^2}{(1+v')^{8/3}}\int_0^\infty\frac{dv}{(1+v)^2}\left(1+\frac{v}{3}\right)\left(\frac{\chi}{v}\right)^{2/3}\\&\times\int_0^\infty d\sigma\,\sigma\exp\left(-iz\sigma-i\frac{\sigma^3}{3}\right)\int_0^\infty d\rho\,\rho\exp\left(-iy_\lambda\rho-i\frac{\rho^3}{3}\right)(\rho-a\sigma)\theta(\rho-a\sigma),\end{aligned} \tag{38}$$

$$y_\lambda=z'\left(1+\lambda_0\frac{1+v'}{v'^2}+\frac{v}{v'}\right), \quad z'=\left(\frac{v'}{\chi}\right)^{2/3}, \quad z=\left[\frac{v(1+v')}{\chi}\right]^{2/3}, \quad a=\left[\frac{v'}{v(1+v')}\right]^{2/3}.$$

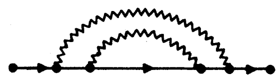

FIG. 5. Mass correction of order α^2 to the electron propagator.

On the mass shell ($\nu=0$) $v'_{\rm eff} \ll 1$ in formula (38) and one can get a simple expression for $\tilde{V}_2^{(4)}(\chi, 0)$. Calculating then the matrix element

$$T_b^{(4)}(p, F) = -\bar{u}(p) M_F^{(4)}(p) u(p)\,,$$

one can easily reproduce the result (25) for the mass correction of order α^2 to the electron amplitude.

Off the mass shell the effective values of the variable v depend on ν and could be found only after the integration over ν is performed. Note that there is no infrared divergence in $\tilde{V}_2^{(4)}$ off the mass shell, but the small photon mass should be preserved in (38) since it could appear in diagrams of higher orders after integration over ν.

For the mass correction of order α^2 to the electron propagation function in the limit $\chi \gg 1$ we get

$$\Delta S^{c(4)}(p) \approx \frac{\alpha}{2\pi} \ln(\lambda_0 \chi^{1/3}) \Delta S^{c(2)}(p) - i\frac{2}{\nu} \tilde{V}_2^{(4)}(\chi, \nu)\left(iS^c(p) - \frac{ie^2 \gamma_\mu F_{\mu\nu} F_{\nu\sigma} p_\sigma}{2m^6 \chi^2} \right). \tag{39}$$

It can be shown that the contribution of the second term in (39) to the asymptotics of Figs. 3, 4(b) and 4(c) is small compared to the contributions of the first term. For the electron amplitude it is of order

$$\alpha^3 (c_1 \ln\lambda_0 + c_2 \ln\chi)\chi^{2/3}\,,$$

where c_1 and c_2 are constants and it is $\sim\alpha\kappa^{2/3}$ for the photon amplitude.

Therefore the asymptotic of $\Delta S^{c(4)}(p)$ is represented only by the first term in (39), at least while calculating these diagrams, and we get

$$T_b^{(6)}(p, F) \approx \frac{\alpha}{2\pi} \ln(\lambda_0 \chi^{1/3}) T_b^{(4)}(p, F) \Big|_{\chi \gg 1} \tag{40}$$

for the electron amplitude and

$$T_b^{(6)}(l, F) + T_c^{(6)}(l, F) \approx \frac{\alpha^2}{\pi^2} \ln^2(\lambda_0 \kappa^{1/3}) T^{(2)}(l, F) \Big|_{\kappa \gg 1} \tag{41}$$

for the photon.

V. SUMMARY

We have considered all proper diagrams of order α^3 for the electron and photon elastic scattering amplitudes which include the mass correction of order α to the electron propagation function but not those which contain vertex corrections. The method developed is based on the asymptotic expression (17) for $\Delta S^{c(2)}(p)$ and on an analysis of effective values of integration variables in exact expressions for radiation corrections. The method is valid for calculating only the leading terms of radiation corrections in the limit $\chi \gg 1$ (or $\kappa \gg 1$).

Just as in the α^2 approximation, we can distinguish two types of diagrams with different expansion parameters of the perturbation theory. The parameter grows at least as $\alpha\chi^{1/3}$ with $\chi \gg 1$ for diagrams including polarization corrections to the photon Green's function and therefore its dependence on the particle's energy is stronger than in conventional quantum electrodynamics. This result is valid at least for the electron elastic scattering amplitude or the mass operator.

For other considered diagrams the parameter is of order $\alpha \ln\chi$ (or $\alpha \ln\kappa$). Still this result must be checked by calculating vertex corrections because the considered diagrams are not gauge invariant, and it is remarkable that terms $\sim\alpha\chi^{2/3} \ln\chi$ appear in the radiation corrections as a result of procedures of renormalization and of elimination of infrared divergences by introducing a small photon mass λ_0. They always accompany terms $\sim\ln\lambda_0$ and could cancel together with infrared-divergent terms after taking into account vertex corrections.

The present paper does not, of course, answer the questions about the dependence of the true expansion parameter of the perturbation theory on χ when $\chi \gg 1$ and whether the parameters are different for mass and polarization operators. To answer these questions one still should consider the behavior of Figs. 3(c), 3(d), 4(d), and 4(c).

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*Permanent address: Moscow Engineering Physics Institute, Moscow 115409, U.S.S.R.
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