PHYSICAL REVIEW D VOLUME 20, NUMBER 6 15 SEPTEMBER 1979

Neutrino viscosity and isotropization of anisotropic-curvature cosmological models

N. Caderni
Institute of Astronomy, University of Cambridge, United Kingdom

R. Fabbri
Istituto di Fisica Superiore, Università di Firenze, Italy
(Received 21 July 1978; revised manuscript received 17 April 1979)

We investigate the evolution of Bianchi spaces of type VII_h, VIII, and IX during the lepton era, and find that a large entropy amplification occurs due to neutrino viscosity for large initial anisotropies. The curvature has a weak influence on the dissipative phenomena. However, following the adiabatic evolution in subsequent epochs we find that models with large anisotropic curvature and high dissipation are not consistent with the isotropy of the Hubble expansion inferred from the observations of the cosmic background radiation. We conclude that our universe is either remarkably flat or nondissipative, the only exception being the particular case of isotropically curved open spaces.

I. INTRODUCTION

The existence of dissipative processes in the early stages of the cosmological expansion has been invoked by several authors[1] in order to explain the high regularity in the structure of the universe at large scales. Moreover, it was proposed that it could explain also the large amount of radiation entropy carried by the cosmic background. These ideas have developed to form the "philosophy" of chaotic cosmology, according to which it should be possible to prove that, whatever the initial conditions may be, the universe evolves toward regular configurations.[2,3] Any primordial large-scale anisotropy of the expansion rate and part of the matter inhomogeneities are smoothed out by radiation viscosity[4-7] or by quantum processes near the singularity[8-10]; during the damping process the photon number in the background radiation increases up to the high value observed at the present epoch (10^8-10^9 photons per baryon; this number gives also the order of magnitude of the radiation entropy per baryon in units of the Boltzmann constant).

The idea of Misner,[4] that neutrino viscosity suffices to damp homogeneous anisotropies, was proved to be very successful either in flat (i.e., Bianchi type-I) or in isotropically curved (type-V) spaces. It turns out that in flat spaces the present-day expansion anisotropy must be less than $\sim 10^{-6}$, and the dissipation of the primordial anisotropy energy occurring in the lepton era is associated with a huge production of the radiating energy[11-13]; thus the large photon entropy per baryon which is observed today is no longer surprising.[14]

However, since both type-I and type-V spaces are a very special subset of homogeneous spaces, one should consider more general situations in order to check what neutrino viscosity can really offer to cosmologists. The most generic sets of homogeneous spaces are Bianchi types VI, VII_h, VIII, and IX.[15] In the present paper we consider the last three types. Type IX constitutes the largest homogeneous generalization of the closed Friedmann model; type VIII (semiclosed topology) does not include strictly isotropic solutions, but contains models which are quite plausible on observational grounds, and have many properties similar to type IX.[16] Both types admit diagonal metrics in the canonical frame, and here we limit ourselves to this case. Type VII_h is the largest generalization of the open Friedmann model, containing also type V as a special subset, and one may expect *a priori* that it is favored by the available evidence for a low-density universe.[17] Since this type does not admit diagonal metrics we shall consider a canonical frame with rotating axes.

We investigate the production of radiation entropy in these spaces by following the evolution of cosmological models throughout the lepton era (temperatures ranging between 6×10^9 K and 1.5×10^{12} K) by numerical integration of the Einstein equations. In our calculations we assume a lepton plasma in thermal equilibrium, and make use of the neutrino viscosity coefficients derived from the Weinberg-Salam interaction through the relativistic kinetic theory.[12] In spite of its shortcomings, this approximate description allows us to display clearly the main features of the dissipative process during the lepton era; moreover, it makes feasible the investigation of a very large number of models with a reasonable computer time. A brief discussion of the viscous fluid approximation is given in Sec. III B in connection with

Stewart's theorem.[18]

Within our approximations we find that, if the curvature of space is small throughout the lepton era, the radiation entropy is enhanced by a factor which depends only on the initial anisotropy of the expansion rate and may be as large as to account for the present ratio of cosmic radiation to matter. The entropy production is not hampered by curvature. On the contrary, in high-curvature spaces it can even be enhanced with respect to flat spaces if bounces[19,20] from one Kasner epoch to another occur during the lepton era. However, models with high dissipation and large anisotropic curvature are ruled out, because we shall see that they do not lead to isotropic configurations at the present epoch.

As far as isotropization is concerned, fundamental results were given in two classic papers by Collins and Hawking.[15,21] These authors show that Friedmann models are unstable under the most general homogeneous perturbations, and that the expansion anisotropy does not become arbitrarily small for times $t \to \infty$ in type-VII$_h$ and type-IX models. We can ask, however, a different question, namely: For which sets of initial conditions do homogeneous models exhibit a level of anisotropy at $t \simeq 10^{10}$ yr compatible with the available experimental data? In our view the expression of chaotic cosmology can be put on more empirical grounds if we simply require the anisotropy of the expansion rate (i.e., of the Hubble parameter) to be small today, with no regard for either the ultimate fate of the universe or the equalization of the cosmic scale factors (defined in Sec. II). In this connection we must consider the work of Doroshkevich *et al.*,[16] according to which models possessing a largely anisotropic-curvature tensor eventually become cylindrically symmetric from the point of view of the (unmeasurable) scale factors, but "almost" isotropic in terms of the Hubble parameter. We investigate such models, which are somewhat more general than those of Collins and Hawking, in order to have a picture as complete as possible of the role of neutrino viscosity in chaotic cosmology. We checked their physical acceptability following also their evolution in adiabatic epochs, up to zero red-shift.

Our calculations show that the analytic results of Doroshkevich *et al.* are not correct since a coupling between expansion anisotropy and the anisotropic part of curvature makes the former increase at late epochs; on the other hand, our results are consistent with those of Collins and Hawking, and those of Liang,[22] who considers inhomogeneous perturbations in a Friedmann background. We show that the set of physically plausible models includes solutions where the curvature tensor is strongly anisotropic; however, in such models the requirement of low-expansion anisotropy implies a small magnitude for curvature.

Restrictions are found also for the production of entropy: If we assign a finite curvature at the beginning of the lepton era (compatible with a small curvature at the present epoch) we may pose a significant upper limit on the entropy amplification. A limit case is represented by Friedmann models where the curvature may be large but the specific entropy remains practically constant. However, a large entropy amplification is not excluded, provided the universe is sufficiently flat or its curvature is isotropic. Note that, in the context of homogeneous cosmologies including the Friedmann models, an isotropic curvature is possible only for an open universe, even if the expansion rate is anisotropic. The same does not apply to a closed universe where the anisotropy of the expansions is coupled to the anisotropy of the curvature tensor. Our results show that most of the photons of the background radiation may have been produced in the lepton era of the universe, but limit the range of validity of chaotic cosmology.

II. THE EINSTEIN EQUATIONS IN THE VISCOUS REGIME

In this section we write the equations that govern the evolution of models of types VII$_h$, VIII, and IX with matter at rest in the homogeneity frame. At each point of spacetime we take a local orthonormal tetrad where the metric can be written as

$$ds^2 = -(\omega^0)^2 + \sum_{i=1}^{3} (\omega^i)^2 .$$

The differential one-forms ω^μ will be taken as

$$\begin{aligned} \omega^0 &= dt , \\ \omega^1 &= R_1(t)(\cos\phi\Omega^1 - \sin\phi\Omega^2) , \\ \omega^2 &= R_2(t)(\sin\phi\Omega^1 + \cos\phi\Omega^2) , \\ \omega^3 &= R_3(t)\Omega^3 , \end{aligned} \tag{1}$$

where $R_i(t)$ are the cosmic scale factors, and Ω^i are the canonical, time-independent one-forms obeying the relations

$$d\Omega^i = -\tfrac{1}{2}C^i_{kn}\Omega^k\Omega^n . \tag{2}$$

The C^i_{kn} are the canonical structure constants and can be written as

$$C_{ik}{}^{l} = -\epsilon_{ikl}c_l + (\delta_{il} + \delta_{kl})(\delta_{k3} - \delta_{i3})a \tag{3}$$

with the constants c_l and a listed in Table I.

Although the ansatz (1) is not the most general one that one can choose, it is sufficient to display all the main features of the Bianchi types considered here.

The cosmological expansion is described by the Hubble parameters

$$H_i = \frac{d}{dt}\ln R_i \tag{4}$$

with, in general, $H_i \neq H_k$ for $i \neq k$. The Einstein equations involve the time derivatives of H_i; after long manipulations we obtain

TABLE I. Values of the constants c_l and a for Bianchi-type VII_h, VIII, and IX cosmological models.

	c_1	c_2	c_3	a
VII_h	1	1	0	$a \neq 0$
VIII	1	1	−1	0
IX	1	1	1	0

$$\frac{dH_k}{dt} = -3HH_k + 2\left(\frac{a}{R_3}\right)^2 + \frac{1}{4}\left[\frac{1}{R_k^2} - c_k\left(\frac{d\phi}{dt}\right)^2\right]\sum_{i\neq k\neq l}\left[\left(\frac{c_l R_l}{R_i} - \frac{c_i R_i}{R_l}\right)^2 - \left(\frac{c_k R_k^2}{R_i R_l}\right)^2\right] + \tfrac{1}{2}(\epsilon - p) + \tfrac{3}{2}\eta_v H + 2\eta_s(H - H_k)\,, \tag{5}$$

where $H = \frac{1}{3}\sum_i H_i$ is the average Hubble parameter, ϵ and p are the energy density and pressure, respectively, of the cosmological fluid, and η_s and η_v are the shear and volume viscosity. We have $\phi = 0$ (and therefore $d\phi/dt = 0$) for types VIII and IX. For type VII_h we have

$$\frac{d\phi}{dt} = 2a(2H_3 - H_1 - H_2)\left(\frac{R_1}{R_2} - \frac{R_2}{R_1}\right)^{-2}. \tag{6}$$

The set of Einstein equations is completed by the constraint equation

$$\epsilon = \frac{1}{2}\sum_{i,k} H_i H_k + \frac{1}{2}R^* - \frac{1}{4}\left(\frac{d\phi}{dt}\right)^2\left(\frac{R_1}{R_2} - \frac{R_2}{R_1}\right)^2, \tag{7}$$

with R^* the curvature of three-space,

$$R^* = c_3 \sum_l \frac{c_l}{R_l^2} - \frac{1}{4}\sum_{i\neq l\neq u}\left(\frac{R_i}{R_l R_u}\right)^2$$

(types VIII and IX),

$$R^* = -6\left(\frac{a}{R_3}\right)^2 - \frac{1}{R_3^2}\left(\frac{R_1}{R_2} - \frac{R_2}{R_1}\right)^2 \tag{8}$$

(type VII_h). However, we found that Eq. (7) is not easy to handle in computer calculations because ϵ is often given by the difference between larger quantities. It is preferable instead to use the energy-balance equation, namely

$$\frac{d\epsilon}{dt} = -3H(\epsilon + p) + 9\eta_v H^2 + 4\eta_s(3H^2 - \epsilon) + 2\eta_s R^*. \tag{9}$$

Thus Eq. (7) only gives a constraint for the parameters specifying the initial conditions.

Equations (5), (6), and (9) can be integrated to give the complete evolution of the cosmological models. Our main purpose is to calculate the entropy amplification and the isotropization of the expansion rate; the radiation entropy amplification is given by

$$\Sigma = \frac{R_1 R_2 R_3}{R_{1\mathrm{in}} R_{2\mathrm{in}} R_{3\mathrm{in}}}\left(\frac{\epsilon_{\mathrm{rad}}}{\epsilon_{\mathrm{rad,in}}}\right)^{3/4}, \tag{10}$$

where the subscript "in" denotes the initial conditions (in our case the beginning of the lepton era). It is also useful to define the anisotropy parameter[11]

$$A_1 = \frac{1}{3}\sum_k \left(\frac{H_k - H}{H}\right)^2. \tag{11}$$

However, in a metric with a nonvanishing $d\phi/dt$, shear originates not only by the anisotropy of the Hubble parameters, but also from the rotations of the principal axes. We therefore define a second shear parameter

$$A_2 = \left(\frac{d\phi}{dt}\right)^2 (6H^2)^{-1}\left(\frac{R_1}{R_2} - \frac{R_2}{R_1}\right)^2 \tag{12}$$

such that the shear energy density[4] is

$$\epsilon_{\mathrm{shear}} = \tfrac{3}{2}H^2(A_1 + A_2) \equiv \tfrac{3}{2}H^2 A. \tag{13}$$

Entropy production and anisotropy damping depend on the magnitude of the viscosity coefficients. For the lepton era we use the shear viscosity coefficient calculated in Ref. 12. However, in quasi-isotropic models where the role of bulk viscosity is dominant, the coefficient η_v calculated in Ref. 12 does not allow us to calculate Σ accurately. In this special case it is better to use the expansion

$$\eta_v = \eta_0 X^5(1 - 3.37X^2 + 8.27X^4 - 9.65X^6 + 4.03X^8)\,,$$

where $X = 0.51$ MeV/kT and $\eta_0 = 7.0\times 10^{-14}$ cm^{-1}. For the subsequent plasma era we set $\eta_s = \eta_v = 0$. This is a very good approximation, since the entropy enhancement at this stage is quite negligible.

Some care must be taken in defining the limits of the dissipative period. In the standard Friedmann models the lepton era is comprised between the earlier hadron era and the subsequent plasma era. The starting and ending points are defined by the hadron annihilation and the electron-positron pair recombination, respectively. The limiting temperatures can be taken as $T_{\mathrm{in}} = 1.5\times 10^{12}$ K and $T_f = 5.9\times 10^9$ K(kT set equal to the pion and electron rest-mass energy, respectively). These results are useful for our purposes. As a matter of fact, in the hadron era the short mean free paths of particles do not allow a large dissipation, and in the plasma era neutrinos are decoupled from matter, whereas photon viscosity gives a negligible dissipation in homogeneous models.

However, in anisotropic models one is led to consider two classes of solution: class A, where the temperature decreases monotonically with time, and class B where the "isotropic" temperature (see below) increases for some part of the lepton era before dropping down. For class A we can take $T_{\rm in}$ and T_f as in Friedmann models; for class B we start the dissipative epoch at $T_{\rm in} = 5.9 \times 10^9$ K, but we are aware that the beginning of the lepton era is not so well defined as in class A for the additional difficulty of treating a collisionless regime in the first steps of the integration.

III. PROPERTIES OF THE SOLUTIONS

The study of the general properties of models of type VII$_h$, VIII, and IX is complicated by the fact that one can choose a large variety of initial conditions for the integration of the Einstein equations. As a matter of fact only the temperature (and consequently, the radiation density) is prescribed for the beginning of the lepton era, after fixing $\epsilon_{\rm in}$ (where the subscript "in" means initial, referring to the beginning of the lepton era) several more parameters are to be chosen since anisotropic-curvature spaces allow a large number of degrees of freedom. In this connection we notice that all the models which present the cosmic background temperature $T_0 = 3$ K, an acceptable matter density ($3H^2 \gtrsim \epsilon_{\rm matter} \gtrsim 0.2\, H^2$) and an expansion anisotropy smaller than $10^{-3} - 10^{-4}$ when $H = 50-100$ km sec^{-1} Mpc^{-1} are compatible with observational constraints. It can be shown that anisotropic-curvature models can be made compatible with the observed specific entropy at $t \simeq 10^{10}$ yr by simply adjusting the baryon density at the beginning of the integration. Then, during the cosmic evolution, the baryon density adjusts itself to become $\simeq 3H^2$ in the quasi-isotropic stage, so that a quasi-critical density is provided automatically.

In order to dispose of the remaining degrees of freedom we adopted the following procedure. For each world model we fixed the ratios $R^*_{\rm in}/H_{\rm in}{}^2$, $(R_1/R_3)_{\rm in}$, $(R_2/R_3)_{\rm in}$, $(H_3/H)_{\rm in}$, and $(H_2 - H)_{\rm in}/(H_1 - H)_{\rm in}$, and finally $H_{\rm in}$. Then Eq. (8) determined the initial value of R_3, whereas the initial shear parameter $A = A_1 + A_2$ was determined by the constraint (7). (In type VII$_h$ we had also to choose the constant a whose value determines, for a given A, the parameters A_1 and A_2.) Then we allowed $H_{\rm in}$ to vary by keeping the other parameters fixed. Thus we were able to observe the influence of the expansion anisotropy on the properties of the models. As a matter of fact, varying $H_{\rm in}$ for a given temperature simply reflects on the amount of shear, larger values of $H_{\rm in}$ corresponding to more anisotropic models.

One should notice that $H_{\rm in}$ is not completely arbitrary: For class A, $H_{\rm in}$ is limited by the obvious condition $3\,H^2 \gtrsim \epsilon - \frac{1}{2} R^*$, and by the requirement $d\epsilon/dt < 0$ at $T = 1.5 \times 10^{12}$ K. For class B we require $d\epsilon/dt > 0$ at the beginning of the lepton era; on the other hand, we impose that the temperature should be always less than 1.5×10^{12} K in order to avoid a hadron era that we are not able to treat at present. So for low curvatures we set

$$\begin{aligned} &3.4 \times 10^{-7}\ \text{cm}^{-1} < H_{\rm in} \lesssim 15\ \text{cm}^{-1}\ \ (\text{class } A), \\ &10^{-11}\ \text{cm}^{-1} \lesssim H_{\rm in} \lesssim 10\ \text{cm}^{-1}\ \ (\text{class } B). \end{aligned} \tag{14}$$

After covering the above ranges of $H_{\rm in}$ we repeated the procedure for many values of $R^*_{\rm in}/H_{\rm in}{}^2$, ranging in magnitude from 10^{-50} to 10^{-16}. (In realistic models the initial curvature must be very small, as we will show below.) Fixing the sign of the initial curvature limits the choice of the ratios R_2/R_1 and R_3/R_1 for type-IX models. We adopted $R_2/R_1 = \frac{3}{2}$, $R_3/R_1 = 2$ for positive curvatures, and $R_2/R_1 = 3$, $R_3/R_1 = 5$ for negative curvatures. We considered also the effect of systematically changing the above ratios, but it is small except for very special models (cylindrically symmetric models in type IX).

In most calculations we also posed $(H_1 - H_3)_{\rm in} = -(H_2 - H_3)_{\rm in}$ and $(H_3/H)_{\rm in} = 1$. However, the values of such parameters turn out to be irrelevant for the isotropization and entropy amplification (for a given value of $A_{\rm in}$).

We integrated about 400 models throughout the lepton era. For part of them we followed also the evolution in the subsequent plasma era, mainly in order to check which models exhibit an acceptable anisotropy level at the present epoch. For type VII$_h$ we also checked the possibility of switching to the Milne epoch claimed in Refs. 16 and 23.

In the rest of this section we shall describe the results of our computations and discuss the approximations involved.

A. The entropy amplification

In spite of the large variety of solutions the entropy production turns out to be described by a few simple laws. Let us denote by Σ_f the total entropy amplification, namely, the value of Σ at the end of the lepton era. (Since no other dissipative epoch occurs, the specific entropy at the present epoch is $S = S_{\rm in}\, \Sigma_f$, with $S_{\rm in}$ the specific entropy at the beginning of the lepton era.)

We find that the final entropy ratio Σ_f depends mainly on $H_{\rm in}$ and on the solution class. The situa-

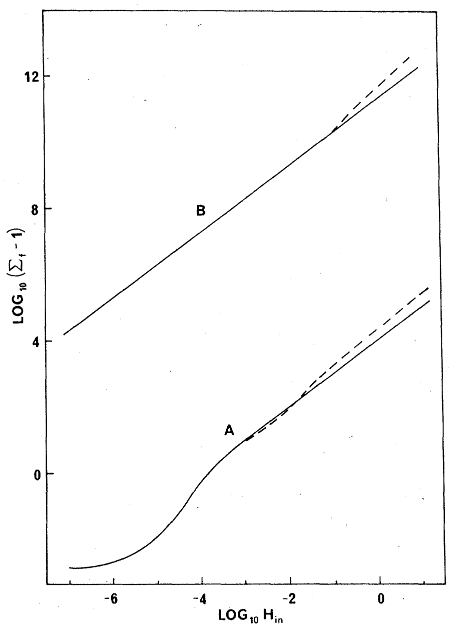

FIG. 1. The entropy amplification Σ_f versus the initial Hubble parameter H_{in}. The full lines refer to quasiflat spaces. The dashed lines refer to class-A models with $R^*_{in}=10^{-32}H_{in}^2$ and class-B models with $R^*_{in}=10^{-36}H_{in}^2$. Note that larger values of H_{in} simply correspond to higher shear parameter A_{in}.

tion is especially simple for low-curvature models. The full lines in Fig. 1 report Σ_f as a function of H_{in} for models where $|R^*/H^2| \lesssim 10^{-1}$ throughout the lepton era. It is easy to check that Σ_f depends linearly on H_{in} for all of (low-curvature) class-B models, and also for the class-A models satisfying $H > 10^{-4}$ cm^{-1}. We find

$$\Sigma_f^{(0)} = 1.2\frac{H_{in}}{H_f} \quad (\text{class } B),$$
$$\Sigma_f^{(0)} = 0.6\times 10^{-7}\frac{H_{in}}{H_f} \quad (\text{class } A), \tag{15}$$

where the superscript (0) denotes the low-curvature limit and $H_f = 0.6\times 10^{-11}$ cm^{-1}. For $H_{in} \sim 10^{-4}$ cm^{-1} the class-A curve in Fig. 1 exhibits a bump which represents the transition to quasi-isotropic models. For small anisotropies ($A_{in} \lesssim 1$, or equivalently $H_{in} \lesssim 10^{-6}$ cm^{-1}) the entropy production is due to bulk viscosity, and $\Sigma_f^{(0)} = 1.0016$.[24]

Since we must set the limits (14) on H_{in}, we find that $\Sigma_f^{(0)} \lesssim 10^5$ for class-A models and $\Sigma_f^{(0)} \lesssim 3\times 10^{12}$ for class-B models.

The models to which Eqs. (15) apply have a rather simple behavior. The lepton era begins in a vacuum regime which is described very accurately by the simple Kasner model since $(\epsilon/\epsilon_{shear})_{in} \lesssim 10^{-5}$ for $H_{in} \gtrsim 10^{-4}$ cm^{-1}. The condition $\epsilon \ll \epsilon_{shear}$ remains valid for part of the lepton era, but the radiation content always becomes important before the end of the lepton era and then the universe evolves toward a quasi-isotropic stage.

More complicated situations arise when R^*/H^2 is not small; in particular, at $|R^*/H^2| \sim 1$ the Kasner regime is temporarily replaced by a curvature-dominated regime, which preludes to a new Kasner epoch[19,20]; the process is known as a curvature-driven "bounce", and is favored by larger values of both R^*_{in} and H_{in}. For $|R^*_{in}| = 10^{-16} H_{in}^2$ and $H_{in} \sim 1$ cm^{-1} we observed up to three bounces and four distinct Kasner epoches within the lepton era. The effect of bounce processes on the entropy production is moderate. In Fig. 2 we show the total entropy production Σ_f (compared with the curvature and anisotropy at the end of the lepton era) as a function of H_{in}. We can check that the entropy amplification is only slightly smaller than $\Sigma_f^{(0)}$ for those models that are just bouncing at neutrino decoupling. However, when the universe has enough time to enter a new Kasner epoch a larger dissipation is allowed (see also dashed lines in Fig. 1). For models that show two fully developed Kasner epochs, the production of entropy is larger than $\Sigma_f^{(0)}$ by a factor $\simeq 2.7$. This is interpreted as the entropy enhancement due to one complete curvature bounce. The interpretation is confirmed by the analysis of models with N complete bounces, where we found $\Sigma_f \simeq (2.7)^N \Sigma_f^{(0)}$. We could verify this rule up to $N=3$ (four distinct Kasner epochs).

If one considers the upper limits on H_{in} already discussed, one finds the following upper limits on

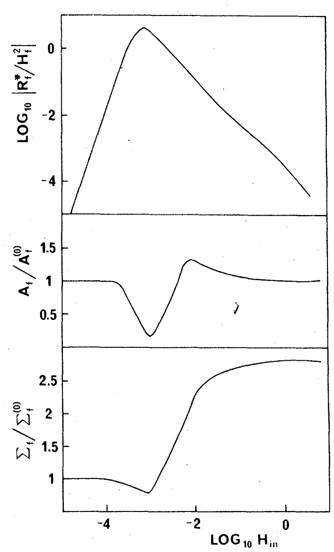

FIG. 2. Comparative behavior of R^*_f, A_f, and Σ_f functions of H_{in}. The curves refer to type-IX class-B models with $R^*_{in}=10^{-28}H_{in}^2$, but their qualitative features are quite general.

the entropy ratio:

$$\Sigma_f \leq 3\ 10^{12}\ (2.7)^N \quad \text{(class } B\text{)},$$
$$\Sigma_f \leq 10^5\ (2.7)^N \quad \text{(class } A\text{)}. \tag{16}$$

However, we shall see below that arbitrary curvatures are not allowed in realistic models.

B. Discussion

The results exposed in Sec. III A are based on some assumptions that we shall discuss here. One of them is the validity of the viscous fluid approximation. The objection to the viscosity scheme is that large viscosity coefficients may formally give negative signs for the diagonal spatial stresses, which are not realistic for a gaseous system. A criterion for the validity of the viscous approximation was given by Stewart,[18] and can be written in the following form:

$$\sigma(t) \equiv 3\,\frac{d\ln\Sigma(t)}{d\ln(R_1R_2R_3)} \leq \tfrac{1}{2}. \tag{17}$$

In order to check the reliability of our calculation we calculated σ as a function of time throughout the lepton era.

In all class-A models we find that the dissipation proceeds rather uniformly in the lepton era, in the sense that σ is a slowly varying function of time. In the most dissipative models of this class we have $0.1 \lesssim \sigma \lesssim 0.5$ throughout the lepton era. In class-B models σ may be larger by several orders of magnitude in the first steps of integration, but it drops to $\sigma \lesssim 0.5$.

From these results it follows that we should discuss separately the accuracies we can obtain for the two classes of models. For class A we can be confident that a good accuracy is achieved for σ, and therefore for $\log_{10}\Sigma_f$. This confidence is also strengthened by the fact that the neutrino collision time t_c turns out to be smaller than the hydrodynamic time t_H for $T \gtrsim 2\times10^{10}$ K, namely for the largest part of the lepton era. We note that the condition $t_c < t_H$ is *not* necessary for efficient dissipation[25] but gives further evidence for small errors. For class B the situation is worse, for the violation of Stewart's criterion. The meaning of the temporary violation of Stewart's theorem was discussed in Ref. 11, following the ideas of Matzner and Misner.[25] A consistent kinetic treatment of the dissipative process shows that in a collisionless regime most of the shear energy density resides in the radiation density, so that

$$T^{00} = \epsilon_{\text{iso}} + \epsilon_{\text{aniso}}\ ,$$

with $\epsilon_{\text{aniso}} \gg \epsilon_{\text{iso}}$. In the viscous approximation we formally ascribe ϵ_{aniso} to the shear energy density, so that ϵ appearing in Eqs. (5), (7), and (9) is to be identified with ϵ_{iso}. The sharp increase of ϵ and Σ and the negative stresses refer only to the isotropic part of the stress-energy tensor. However, it seems reasonable to take only ϵ_{iso} into account when we calculate the entropy (no entropy being associated with an ordered motion) so that Eq. (10) remains the best estimate we can make of the parameter Σ. However, we admit that large errors may be implied by this naive procedure; such errors are larger for the most dissipative models ($\Sigma_f \sim 10^{12}$) where Stewart's theorem is violated for an appreciable part of the lepton era. We can give a *lower* limit to the entropy enhancement by neglecting the contribution during the violation of Stewart's theorem; if we do so we find that in the extreme cases we reduce Σ_f by ~4 orders of magnitude.

Another question is the accurate identification of the initial and final point for the dissipative epoch. As already remarked for class A these points are defined by pion annihilation and lepton pair recombination. Although they are not pointlike events, the temperatures T_{in} and T_f are well defined, being uncertain at most by a factor 2. In our calculations we checked that this implies an uncertainty by a factor <3 on Σ_f. However, the beginning of the lepton era is not so well defined for class B, where neutrino dissipation starts in a collisionless regime and gradually leads the cosmological plasma to a collision-dominated regime.

Finally we must check whether entropy amplifications as large as 10^8-10^{12} are consistent with the observed specific entropy, which is of order 10^8-10^9. Fixing such a specific entropy S_f implies that the ratio of the photon number to the baryon number was initially $\sim S_f/\Sigma_f \ll 1$ in the most dissipative models. This introduces an inconsistency in our computation, since in fact we assume that the cosmological fluid was radiation dominated throughout the lepton era. We note that the baryon excess has no influence on the geometrical evolution of the universe at early times because the class-B models with $H_{\text{in}} > 10^{-3}$ cm^{-1} (as required in order to have the baryon excess) are very largely dominated by the shear energy density. The equation of state of matter ($p=0$ or $p=\epsilon/3$) has no relevance. Later, when the energy density ϵ becomes comparable to ϵ_{shear} the radiation density dominates the rest-mass density of baryons. However, the baryon excess decreases the viscous dissipation, since the neutrino number N_ν becomes smaller than the number of scatters N_s, and $\eta_s \propto N_\nu/N_s$. As a result, the entropy amplification should be overrated when we find $\Sigma_f > 10^8-10^9$, and we can suspect that the maximum allowable enhancement of entropy Σ_f cannot be much larger than S_f. It is certainly attractive to speculate on an initial specific entropy $S_{in} \sim 1$.

C. Limits posed by the isotropy requirement

We have already remarked that the cosmological models can be made consistent with a present matter density $\epsilon_{matter} \sim 10^{-29}$ g/cm^3 and a specific entropy $S \sim 10^8$ with a suitable choice of $(\epsilon_{matter}/\epsilon_{rad})_{in}$; a significant selection arises instead from the isotropy requirements. The available limits on the quadrupole anisotropy of the cosmic background radiation imply that, if the universe is represented by a type-VIII or type-IX model, then[26]

$$|[A(0)]^{1/2} - [A(z_l)]^{1/2}| \lesssim 10^{-4}, \tag{18a}$$

where $A(z)$ is the shear parameter at a red-shift z and z_l is the last-scattering red-shift. For type VII_h the available data on the dipole anisotropy imply that[26]

$$|[A(0)]^{1/2} - [A(z_l)]^{1/2}| \lesssim 10^{-3}. \tag{18b}$$

If the law found in isotropic curvatures or flat spaces

$$A \propto H^{-2}(R_1R_2R_3)^{-2} \tag{19}$$

were valid also for the adiabatic damping of the expansion anisotropy in generic curved spaces, the limits (18) would not be restrictive at all. In fact Eq. (19) is found to be valid when the curvature is small; in such cases we find at neutrino decoupling (see Figs. 3 and 4)

$$A_f^{(0)} \leqslant 0.8, \tag{20}$$

and a very small $A(z)$ is calculated from Eqs. (19) and (20) at low red-shifts.

Let us discuss how the coupling between anisotropy and curvature influences the cosmological evolution. The coupling is clearly observed as early as in the lepton era in those models where R^*/H^2 is not small. In such models A is no longer a monotonically decreasing function of time; more precisely, it reaches a minimum during the transition from one Kasner epoch to another, when all the Hubble parameters become temporarily positive. Afterwards "new" anisotropy is pumped by curvature, so that one may find occasionally anisotropies as large as $A \simeq 1.9$, or $\epsilon/\epsilon_{shear} \sim 10^{-1}$ near $T = 10^{10}$ K. However, in our models we always found $A_f \lesssim 1.1$.

In Fig. 3 the dashed lines give A_f as a function of H_{in} in type-IX spaces for two values of the initial curvature, namely $R^*_{in} = 10^{-32}\, H_{in}^2$ (class A) and $R^*_{in} = 10^{-36}\, H_{in}^2$ (class B). The characteristic behavior for large H_{in} is common also to types VII_h and VIII. The comparative behaviors of R_f^* and

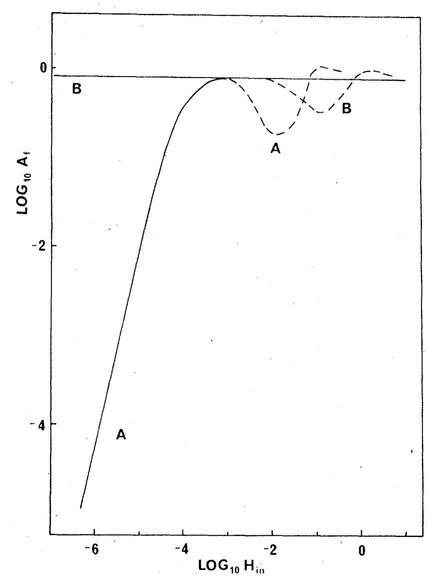

FIG. 3. The residual shear parameter A_f versus the initial Hubble parameter H_{in}. The full lines refer to quasiflat models (whatever the type, VII_h, VIII, or IX). The dashed lines refer to type-IX class-A models with $R^*_{in} = 10^{-32} H_{in}^2$, and class-$B$ models with $R^*_{in} = 10^{-36} H_{in}^2$.

A_f versus H_{in} are given in Fig. 2 for models where $R^*_{in} = 10^{-28}\, H_{in}^2$. We can see that whenever R_f^*/H_f^2 is a decreasing function of H_i, the residual anisotropy is larger than $A_f^{(0)}$. This occurs for a set of models which exhibit two distinct Kasner epochs

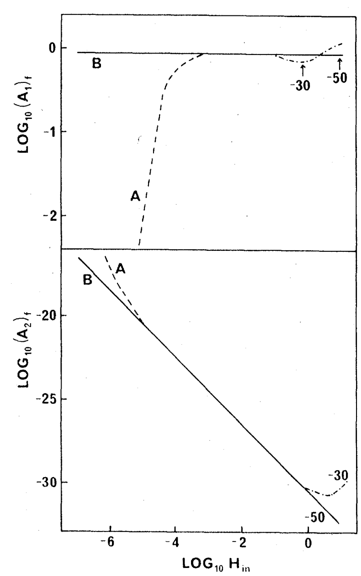

FIG. 4. The shear parameters A_{1f} and A_{2f} versus H_{in} in type-VII_h models with $a = 10^{-4}$. The curves are labeled by the corresponding values of $\log_{10}(R^*_{in}/H_{in}^2)$. Note that A_2 gives little contribution to the total shear; larger values of a tend to increase the ratio A_2/A_1, but we can have $A_1 \sim A_2$ only when $R^*/H^2 \sim -1$.

within the lepton era ($H_{in} \gtrsim 5\times 10^{-3}$ cm^{-1}). For 10^{-4} cm$^{-1} \lesssim H_{in} \lesssim 10^{-3}$ cm^{-1} the anisotropy is smaller than $A_f^{(0)}$; this refers to models where the universe is switching to a second Kasner epoch just at neutrino decoupling.

An analytic description of the curvature-anisotropy coupling can be given in the adiabatic regime following the lepton era. Let us consider the most general case where the curvature tensor is strongly anisotropic, the scale factors being largely different from each other ($R_i \gg R_j \gg R_k$ for a suitable choice of the indices). Then from Eq. (5) we find

$$H_1 - H = (R_1R_2R_3)^{-1}\left[a_1(t_0) + \tfrac{4}{3}\int_{t_0}^{t} R_1R_2R_3R^*\,dt\right] ,$$

$$H_2 - H = (R_1R_2R_3)^{-1}\left[a_{2,3}(t_0) - \tfrac{2}{3}\int_{t_0}^{t} R_1R_2R_3R^*\,dt\right] ,$$

where $t+$ is an arbitrary time and $a_k(t_0)$ are constants. Thus the anisotropy can be given as an explicit function of time in the quasi-isotropic stage ($A \lesssim 0.5$), where we know that $R_k \propto t^n$. As far as the integral appearing in (20) is negligible, the anisotropy decays as in flat spaces [see Eq. (19)]. But the curvature term eventually prevails and

$$A = \tfrac{8}{9}\left(\frac{n}{1+u}\right)^2\left(\frac{R^*}{H^2}\right)^2 , \qquad (21)$$

where $n=\frac{1}{2}$ in the radiation-dominated regime and $n=\frac{2}{3}$ in the matter-dominated regime. It is important to note that R^*/H^2 increases in magnitude like $t^2\,(R_1R_2R_3)^{-2/3} \propto t^{2-2n}$ so that curvature eventually destroys the isotropy of the cosmological expansion.[27] More precisely, if the matter-dominated epoch started at a red-shift $z=10^4$, then $|R^*/H^2|$ has increased by 14 orders of magnitude since the end of the lepton era up to today.

We find similar results if at least two scale factors are fairly different from each other. In type-IX spaces we can select, as a limit case, axisymmetric models with $R_1=R_2$ and $H_1=H_2$. For $R_3 \ll R_{1,2}$ we get

$$A \simeq \frac{1}{18}\left(\frac{u}{1+u}\right)^2\left(\frac{R^*}{H^2}\right)^2 , \qquad (22)$$

whereas for $R_3 \gg R_{1,2}$ one recovers Eq. (21). The only significant exception to Eq. (21) is given by the quasi-Friedmann models, where all the scale factors are quite close to each other. Excluding this special case, we find that strong limits can be posed on $|R^*/H^2|$, which must be smaller than 10^{-3}–10^{-4} today and smaller than 10^{-17}–10^{-18} at neutrino decoupling.

For a given solution class and a given R_f^*/H_f^2, R_{in}^*/H_{in}^2 depends only on H_{in}. For class-B solutions we find

$$\left|\frac{R_f^*}{H_f^2}\right| \simeq \left|\frac{R_{in}^*}{H_{in}^2}\right|\left(\frac{H_0}{H_{in}}\right)^{\beta} \qquad (23)$$

with $\beta \simeq \frac{3}{11}$ and $H_0 \simeq 10^{-11}$ cm^{-1}. Using Eqs. (15) and (23) we derive an upper limit on the entropy enhancement

$$\Sigma_f \lesssim \frac{10^{-18}}{|R_{in}^*/H_{in}^2|^{\beta}} . \qquad (24)$$

Thus for finite curvatures significant limits on Σ_f can be derived; for instance, for $R_{in}^*/H_{in}^2 = 10^{-36}$ we get for class-B models $\Sigma_f \lesssim 10^5$, namely the overall limit that we gave for the entire class A using Eq. (14). For the same curvature, the only allowable class-A models have no appreciable dissipation $\Sigma_f \simeq 1$. Although Friedmann models may have high curvature, the entropy amplification is very small there; so we can conclude that curvature limits the entropy production *indirectly*, in the sense that a high-curvature space (with anisotropic curvature) is either unrealistic or not dissipative.

However, since we can choose arbitrarily small R_{in}^*/H_{in}^2, the only overall upper limit on Σ_f is 10^8–10^{12}. Thus Bianchi models with a strongly anisotropic curvature tensor may be highly dissipative.

We have already remarked that the models "spontaneously" give $\epsilon \simeq 3H^2$ in the quasi-isotropic stage. It is worth considering here type VII$_h$, in which one would like to find models for a low-density universe. It was claimed[16,23] that such spaces eventually enter the Milne epoch ($R^* \simeq -6H^2$, $\epsilon \ll H^2$) with an expansion anisotropy $A \simeq$ constant $\ll 1$. Such a transition being observed in Bianchi type V,[13] if we could confirm the existence of this regime in the most generic case, it would be suitable to represent the present state of the universe if $\epsilon \simeq 0.3\,H^2$. We performed calculations to check (a) whether type-VII$_h$ models are really able to reach a Milne epoch with $A \ll 1$, and (b) whether the Milne epoch has a lifetime $t \gtrsim 10^{10}$ yr against "decay" into a highly anisotropic regime.

The answer to point (a) was negative. Our models could never reach a low-density stage with $A \ll 1$ because the anisotropy-curvature coupling leads to $A \sim 1$ when $|R^*| \sim H^2$. In order to check point (b) we imposed "artificial" initial conditions at $z = 10$–100 (i.e., conditions inconsistent with the previous evolution). The Milne epoch was quickly destroyed for moderate anisotropy of the curvature tensor.

We conclude that the expansion can remain isotropic over cosmological periods in a low-density universe only if the curvature tensor is isotropic.

IV. CONCLUSION

According to the results expressed in the previous section, the entropy content of the universe increases during the lepton era by a factor that may be very large and depends on the average Hubble parameter at the beginning of the lepton era (or equivalently, on the anisotropy parameter) and on the solution class A or B). The curvature of three-space plays a role only because, for a given entropy amplification, significant upper limits can be posed on the magnitude of curvature when only realistic models are selected. The *direct* influence of curvature on the entropy production is small.

The set of plausible models showing a large dissipation has a very interesting feature: These models must be rather flat at the present epoch, $|R^*| \lesssim 10^{-4} H^2$, so that the matter density is very close to the critical density $\epsilon_c = 3H^2$. This is in agreement with the result of Collins and Hawking,[15,21] according to which only models with critical energy density isotropize for $t \to \infty$, but it is a stronger conclusion than theirs: We state that either the universe must be extremely flat at the present epoch, if it is quasi-isotropic today after being quite anisotropic at the big bang, or the expansion anisotropy was purely kinematical and the curvature tensor is then isotropic. Of course the universe may have a strong anisotropic curvature today, as would be required in particular by a low density[17]; but in such a case it was isotropic since the beginning, and the observed specific entropy is a direct imprint of the big bang. The possible existence of earlier dissipative mechanisms does not change this conclusion: No matter how small the residual anisotropy may be, new anisotropy can be produced by curvature. Only stringent limits on the magnitude of R^* can eliminate this phenomenon.

We finally observe that our work does not give support to the ideas of chaotic cosmology[28]; as a matter of fact even its weakest formulation (expressed in the Introduction) meets serious difficulties because of the anisotropy-curvature coupling.

ACKNOWLEDGMENT

N. Caderni was the recipient of an E. S. A. International Fellowship.

---

[1]For early work see the review paper by T. Criss, R. A. Matzner, M. Ryan, and L. Shepley, in *General Relativity and Gravitation*, edited by G. Shaviv and N. Rosen (Wiley, New York, 1975).

[2]C. W. Misner, in *Battelles Recontres: 1967 Lectures in Mathematics and Physics*, edited by C. DeWitt and J. Wheeler (Benjamin, New York, 1968).

[3]M. J. Rees, Phys. Rev. Lett. 28, 1669 (1972).

[4]C. W. Misner, Phys. Rev. Lett. 9, 533 (1967); Astrophys. J. 158, 431 (1968).

[5]R. A. Matzner, Ann. Phys. (N.Y.) 65, 438 (1971); Astrophys. J. 171, 433 (1972).

[6]V. A. Belinskii and I. M. Khalatnikov, Zh. Eksp. Teor. Fiz. 69, 401 (1975) [Sov. Phys.—JETP 42, 205 (1976)].

[7]E. P. Liang, Phys. Rev. D 16, 3369 (1977).

[8]V. N. Lukash, I. D. Novikov, and A. A. Starobinskii, Zh. Eksp. Teor. Fiz. 69, 1484 (1975) [Sov. Phys.—JETP 42, 757 (1975)].

[9]B. K. Berger, Ann. Phys. (N.Y.) 83, 458 (1973).

[10]C. W. Misner, Phys. Rev. D 8, 3271 (1973).

[11]N. Caderni and R. Fabbri, Lett. Nuovo Cimento 20, 185 (1977); Nuovo Cimento 44B, 228 (1978).

[12]N. Caderni, R. Fabbri, L. J. van den Horn, and Th. J. Siskens, Phys. Lett. 66A, 251 (1978).

[13]N. Caderni and R. Fabbri, Phys. Lett. 68A, 144 (1978).

[14]For alternative explanations of the large radiation entropy, see Y. B. Zeldovich, Mon. Not. R. Astron. Soc. 160, 1P (1972) and M. J. Rees, Nature 275, 35 (1978).

[15]C. B. Collins and S. W. Hawking, Astrophys. J. 180, 317 (1973).

[16]A. G. Doroshkevich, V. N. Lukash, and I. D. Novikov, Zh. Eksp. Teor. Fiz. 64, 1457 (1973) [Sov. Phys.—JETP 37, 739 (1973)].

[17]J. R. Gott III, J. E. Gunn, D. N. Schramm, and B. M. Tinsley, Astrophys. J. 194, 543 (1974).

[18]J. M. Stewart, Mon. Not. R. Astron. Soc. 145, 347 (1969).

[19]V. A. Belinskii, I. M. Khalatnikov, and E. M. Lifshitz, Adv. Phys. 19, 525 (1970).

[20]C. W. Misner, Phys. Rev. Lett. 22, 1071 (1969).

[21]C. B. Collins and S. W. Hawking, Mon. Not. R. Astron. Soc. 162, 307 (1973).

[22]E. P. Liang, Phys. Lett. 51A, 141 (1975).

[23]I. D. Novikov, in *Confrontation of Cosmological Theories with Observational Data*, edited by M. S. Longair (Reidel, Dordrecht, 1974).

[24]In the flat Friedmann model we previously found $\Sigma_f = 1.0011$ [N. Caderni and R. Fabbri, Phys. Lett. 69B, 508 (1977)]. The small discrepancy is due to the use of the $V-A$ cross sections in this paper.

[25]R. A. Matzner and C. W. Misner, Astrophys. J. 171, 415 (1972).

[26]G. F. Smoot, M. V. Gorenstein, and R. A. Muller, Phys. Rev. Lett. 39, 898 (1977).

[27]The logarithmic decrease of anisotropy given by Doroshkevich *et al*. (Ref. 16) is incorrect. Matzner (Ref. 5) gives the proper law (19) for type IX, but does not discover the subsequent anisotropy enhancement described by Eq. (21).

[28]Chaotic cosmology is criticized on different grounds by J. D. Barrow and R. A. Matzner, Mon. Not. R. Astron. Soc. 181, 719 (1977).