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Ground-state baryons in a quark model with hyperfine interactions

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We discuss the ground-state baryons N, Λ, Σ, Ξ, Δ, Σ^*, Ξ^*, and Ω in a quark model with flavor-independent confinement and color hyperfine interactions. We include the effects of wave-function distortions for unequal quark masses as well as interband mixing via the hyperfine interactions and find good agreement with the observed masses.

In a series of recent articles[1-6] we have shown that a model of approximately nonrelativistic pointlike quarks moving in a flavor-independent confinement potential perturbed by color hyperfine interactions is capable of explaining reasonably well the properties of excited baryons. These calculations were based to some extent on our prior investigations of the ground-state baryons which we belatedly present here.

The model we have been exploring is based on the quantum-chromodynamics-motivated Hamiltonian

$$H=\sum_i \left(m_i+\frac{p_i^2}{2m_i}\right)+\sum_{i<j} V^{ij}+\sum_{i<j} H^{ij}_{\rm hyp}\,, \qquad (1)$$

in which (after color averaging in a baryon)

$$H^{ij}_{\rm hyp}=\frac{2\alpha_s}{3m_im_j}\left[\frac{8\pi}{3}\vec{S}_i\cdot\vec{S}_j\,\delta^3(\vec{r}_{ij})+\frac{1}{r_{ij}^3}\left(\frac{3\vec{S}_i\cdot\vec{r}_{ij}\,\vec{S}_j\cdot\vec{r}_{ij}}{r_{ij}^2}-\vec{S}_i\cdot\vec{S}_j\right)\right]\,, \qquad (2)$$

and

$$V^{ij}=\tfrac{2}{3}V(r_{ij})\,, \qquad (3)$$

where $V(r_{ij})$ is the spin-independent color potential introduced in Ref. 5.

Most of the machinery we have introduced previously for the excited baryons does not come into play in the ground states so that here our discussion can be considerably simplified. We will use the masses of the N and Δ as input to fix the two completely free parameters of the model: the unperturbed position E_0 of the nonstrange sector and an overall strength parameter δ of the hyperfine interaction. In addition we take the quark masses to be $m_u \simeq m_d = 0.33$ GeV and $m_s = 0.55$ GeV to give $x=m_d/m_s=0.6$ and reasonable values for baryon magnetic moments[7] and, approximating the ground-state wave function by a Gaussian,[8] we take its shape parameter α to be 0.32 GeV in the nonstrange sector to give a reasonable value for the proton rms charge radius.[9] [We choose α in this way since with $V(r_{ij})$ nonharmonic it is improper to determine it from the excitation energy of the P-wave baryons.[2,3,5]]

We perform all of our calculations in the "*uds* basis" introduced in Ref. 2. As pointed out there, when $m_3 \neq m_1 = m_2$, this is a more appropriate basis than the totally antisymmetrized "SU(6) basis"; moreover, all results can be obtained from the *uds* sector by taking appropriate limits. In the *uds* basis $m_1=m_2=m$ and $m_3=m'$ so that the $S=0$ sector is the case $m'=m=m_d$, the $S=-1$ sector is the case $m=m_d$, $m'=m_s$, the $S=-2$ sector corresponds to $m=m_s$, $m'=m_d$, and of course the $S=-3$ sector has $m=m'=m_s$.

There are three steps in calculating the mass of a given baryon:

(1) Calculate its zeroth-order energy by perturbing E_0, if necessary, by the operator

$$\Delta Z=-\sum_i (1-x_i)\frac{p_i^2}{2m_d}+\sum_i \Delta m_i\,, \qquad (4)$$

where $x_i=m_d/m_i$ and $\Delta m_i=m_i-m_d$.

(2) Calculate the hyperfine perturbation using the zeroth-order wave function given by the harmonic-oscillator model

$$\psi_{00}=\frac{\alpha_\rho^{3/2}\alpha_\lambda^{3/2}}{\pi^{3/2}}\exp(-\tfrac{1}{2}\alpha_\rho^2\rho^2)\exp(-\tfrac{1}{2}\alpha_\lambda^2\lambda^2)\,, \qquad (5)$$

where

$$\alpha_\rho^4=3Km\,, \qquad (6)$$

$$\alpha_\lambda^4=3Km_\lambda\,, \qquad (7)$$

with

$$m_\lambda=\frac{3mm'}{2m+m'}\,, \qquad (8)$$

and where

$$\vec{\rho}=\frac{1}{\sqrt{2}}(\vec{r}_1-\vec{r}_2), \tag{9}$$

$$\vec{\lambda}=\frac{1}{\sqrt{6}}(\vec{r}_1+\vec{r}_2-2\vec{r}_3) \tag{10}$$

are two appropriate relative coordinates.

(3) Take into account second-order effects in $H_{\rm hyp}$ by calculating the mixing between the ground states and the positive-parity excited states associated with the $N=2$ level of the harmonic-oscillator model[10] using the masses and compositions of these excited states from Ref. 3; then diagonalize the resulting mixing matrix to find the lowest eigenvalue.

While obviously simple in principle, in practice these calculations, especially step three, tend to become rather numerical; consequently we do not show them all here. Rather, in the Appendix we explicitly display a sample calculation and list the relevant matrix elements and parameters so that the reader may, if so inclined, reproduce the results which we show in Table I. Our results for charmed baryons, including excited states, are presented elsewhere.[11]

The agreement shown is such that the deviations are practically at the level of electromagnetic corrections. That the agreement is so good is undoubtedly because some of the inadequacies of the model are hidden in the fitting of our parameters; nevertheless, we believe that the model is describing several real effects. Of course the reduced chromomagnetic moment of the strange quark plays a central role in these results, and there are some manifestations of this which are rather unusual. Consider, for example, the Σ^- and the Ξ^- with quark content *dds* and *ssd*, respectively. In each case the two identical quarks must be in a spin-one state and so repel each other by roughly $+\frac{1}{6}(\Delta-N)$ and $+\frac{1}{6}x^2(\Delta-N)$, respectively. The remaining pairs attract each other in each case by roughly $-\frac{2}{3}x(\Delta-N)$; thus we see that in the Ξ^- the presence of an extra strange quark *increases* the total hyperfine interaction and this fact is reflected in the spectrum. We believe this point to be worth mentioning because, although a hyperfine interaction that decreases monotonically with the number of strange quarks (as in the decuplet) can be confused with a quark mass difference, this sort of effect cannot be. Second-order effects also play a significant role in the spectrum. The naive $\Sigma-\Lambda$ mass difference[12]

$$\Sigma-\Lambda=\tfrac{2}{3}(1-x)(\Delta-N), \tag{11}$$

which of course also arises from the reduced chromomagnetic moment of the strange quark and which is numerically correct, is modified by two competing effects. Wave-function distortion brings the strange quark in closer to the other quarks and tends to compensate for $x<1$.[13] This effect reduces $\Sigma-\Lambda$ by ~30 MeV. On the other hand, the Λ is much more strongly mixed with $N=2$ levels than the Σ and this tends to open up the $\Sigma-\Lambda$ gap.[14] These two contributions in practice nearly cancel so that the "naive" result is numerically accurate. In general, of course, these two kinds of effects may cooperate instead of compete in which case they would not be negligible.

Finally we mention that the $N=2$ mixing not only causes significant energy shifts in these states, but it also mixes non-$N=0$ configurations into the ground states. We have recently discussed the effects of the admixture of ^2S_M (i.e., $[70,0^+]$) components into the nucleon and shown that there is good evidence for these admixtures from SU(6)-violating decays and moments.[4]

TABLE I. The masses of the ground-state baryons in MeV.

	E_0	ΔZ	$H_{\rm hyp}^{(1)}$	$H_{\rm hyp}^{(2)}$	Predicted	Observed[a]
N	1135	0	−130	−65	940	940
Λ	1135	160	−130	−55	1110	1115
Σ	1135	160	−80	−25	1190	1195
Ξ	1135	320	−100	−30	1325	1320
Δ	1135	0	+130	−25	1240	1240
Σ^*	1135	160	+105	−10	1390	1385
Ξ^*	1135	320	+85	−10	1530	1535
Ω	1135	480	+70	−10	1675	1670

[a] We compare to the most negatively charged state.

APPENDIX

The relevant hyperfine matrix elements are

$$\langle \Lambda\,{}^2S\,\tfrac{1}{2}^+|H_{\rm hyp}|\Lambda\,{}^2S\,\tfrac{1}{2}^+\rangle=-\tfrac{1}{2}\,\delta\,, \tag{A1}$$

$$\langle \Lambda\,{}^2S_{\lambda\lambda}\,\tfrac{1}{2}^+|H_{\rm hyp}|\Lambda\,{}^2S\,\tfrac{1}{2}^+\rangle=0\,, \tag{A2}$$

$$\langle \Lambda\,{}^2S_{\rho\lambda}\,\tfrac{1}{2}^+|H_{\rm hyp}|\Lambda\,{}^2S\,\tfrac{1}{2}^+\rangle=+\frac{\sqrt{3}}{4}x\,\delta\,, \tag{A3}$$

$$\langle \Lambda\,{}^2S_{\rho\rho}\,\tfrac{1}{2}^+|H_{\rm hyp}|\Lambda\,{}^2S\,\tfrac{1}{2}^+\rangle=+\frac{\sqrt{6}}{4}\,\delta\,, \tag{A4}$$

$$\langle \Sigma\,{}^2S\,\tfrac{1}{2}^+|H_{\rm hyp}|\Sigma\,{}^2S\,\tfrac{1}{2}^+\rangle=-\frac{1}{2}\left(\frac{4\hat{x}-1}{3}\right)\,\delta\,, \tag{A5}$$

$$\langle \Sigma\,{}^2S_{\lambda\lambda}\,\tfrac{1}{2}^+|H_{\rm hyp}|\Sigma\,{}^2S\,\tfrac{1}{2}^+\rangle=+\frac{\sqrt{6}}{4}x\,\delta\,, \tag{A6}$$

$$\langle \Sigma\,{}^2S_{\rho\lambda}\,\tfrac{1}{2}^+|H_{\rm hyp}|\Sigma\,{}^2S\,\tfrac{1}{2}^+\rangle=\frac{\sqrt{3}}{4}x\,\delta\,, \tag{A7}$$

$$\langle \Sigma\, {}^2S_{\rho\rho}\tfrac{1}{2}^+ | H_{\rm hyp} | \Sigma\, {}^2S\tfrac{1}{2}^+\rangle = -\frac{\sqrt{6}}{12}(1-x)\,\delta\,, \quad \text{(A8)}$$

$$\langle \Sigma\, {}^4S\tfrac{3}{2}^+ | H_{\rm hyp} | \Sigma\, {}^4S\tfrac{3}{2}^+\rangle = +\frac{1}{2}\left(\frac{1+2\hat{x}}{3}\right)\delta\,, \quad \text{(A9)}$$

$$\langle \Sigma\, {}^4S_{\lambda\lambda}\tfrac{3}{2}^+ | H_{\rm hyp} | \Sigma\, {}^4S\tfrac{3}{2}^+\rangle = -\frac{\sqrt{6}}{8}\,x\,\delta\,, \quad \text{(A10)}$$

$$\langle \Sigma\, {}^4S_{\rho\rho}\tfrac{3}{2}^+ | H_{\rm hyp} | \Sigma\, {}^4S\tfrac{3}{2}^+\rangle = -\frac{\sqrt{6}}{8}\left(\frac{2+x}{3}\right)\delta\,, \quad \text{(A11)}$$

where δ is defined below and where

$$\hat{x} = \frac{x}{(\frac{3}{4}y^2+\frac{1}{4})^{3/2}} \text{ and } y^4 = \frac{2x+1}{3}\,. \quad \text{(A12)}$$

In the above we have neglected $\alpha_\lambda \neq \alpha_\rho$ effects in the $N=0 \leftrightarrow N=2$ matrix elements. As mentioned in the text and discussed in Ref. 2, the matrix elements relevant to the sectors N, Δ, Ξ^*, and Ω follow from these by simple replacements. We have not included tensor matrix elements here because their effect is negligible.

The parameters of the fit are, aside from m_d, m_s, and α which we have previously mentioned, $E_0 = 1135$ MeV and

$$\delta \equiv \frac{4\alpha_s\alpha^3}{3\sqrt{2\pi}\,m_d^2} = 260 \text{ MeV} \simeq \Delta - N\,.$$

This value of δ differs slightly from values we have used elsewhere as the result of our inclusion of second-order effects; it corresponds to $\alpha_s/\pi \simeq \frac{1}{2}$ (see, however, Refs. 7 and 9).

Finally, as an illustration, we explicitly display the calculation for the $\Lambda\frac{1}{2}^+$. Following the steps outlined in the text, we begin with $E_0 = 1135$ MeV and add

$$\langle \Delta Z\rangle = \langle -(1-x)p_3^2/2m_d\rangle + m_s - m_d$$
$$= m_s - m_d - 0.4(\alpha^2/2m_d) = 160 \text{ MeV}\,.$$

In step two we add the first-order hyperfine interaction which in this case is simply $-\frac{1}{2}\delta = -130$ MeV. Finally, using Ref. 3 for the masses and compositions of the relevant $N=2$ $\Lambda\frac{1}{2}^+$ states

$$\Lambda_1(1555) = +0.75\,\Lambda\,{}^2S_{\lambda\lambda} + 0.09\,\Lambda\,{}^2S_{\rho\lambda} + 0.66\,\Lambda\,{}^2S_{\rho\rho} + \cdots\,,$$

$$\Lambda_2(1740) = -0.56\,\Lambda\,{}^2S_{\lambda\lambda} - 0.46\,\Lambda\,{}^2S_{\rho\lambda} + 0.69\,\Lambda\,{}^2S_{\rho\rho} + \cdots\,,$$

$$\Lambda_3(1860) = -0.34\,\Lambda\,{}^2S_{\lambda\lambda} + 0.85\,\Lambda\,{}^2S_{\rho\lambda} + 0.28\,\Lambda\,{}^2S_{\rho\rho} + \cdots\,,$$

and using the matrix elements (A1) to (A4) we find the mixing matrix

$$\begin{array}{cc} \begin{bmatrix} 1165 & 110 & 80 & 100 \\ 110 & 1555 & 0 & 0 \\ 80 & 0 & 1740 & 0 \\ 100 & 0 & 0 & 1860 \end{bmatrix} & \begin{array}{c} \Lambda \\ \Lambda_1 \\ \Lambda_2 \\ \Lambda_3 \end{array} \end{array}$$

which has 1110 as its lowest eigenvalue.

[1]Nathan Isgur and Gabriel Karl, Phys. Lett. 72B, 109 (1977).
[2]Nathan Isgur and Gabriel Karl, Phys. Lett. 74B, 353 (1978); Phys. Rev. D 18, 4187 (1978).
[3]Nathan Isgur and Gabriel Karl, Phys. Rev. D 19, 2653 (1979).
[4]Nathan Isgur, Gabriel Karl, and Roman Koniuk, Phys. Rev. Lett. 41, 1269 (1978).
[5]Nathan Isgur, lectures at the XVI International School of Subnuclear Physics, Erice, Italy, 1978 (unpublished).
[6]Gabriel Karl, in *Proceedings of the XIX International Conference on High Energy Phyics, Tokyo, 1978,* edited by S. Homma, M. Kawaguchi, and H. Miyazawa (Phys. Soc. of Japan, Tokyo, 1979), p. 135.
[7]If the quarks were pointlike and the ground-state baryons pure nonrelativistic S-wave states, then $m_u = m_d = M_N/\mu_p = -2M_N/3\mu_n \simeq 0.33$ GeV and $m_s = -M_N/3\mu_\Lambda \simeq 0.51$ GeV. Since one must expect deviations from these naive formulas of at least 10% due to renormalization effects alone, the masses we use are certainly acceptable. See also Ref. 9.
[8]D. Gromes and I. O. Stamatescu, Nucl. Phys. B112, 213 (1976).
[9]Assuming pointlike quarks and a pure nonrelativistic S-wave nucleon, this value for α gives

$$\left\langle \sum_i e_i r_i^2 \right\rangle^{1/2} = \alpha^{-1} = 0.6 \text{ fm.},$$

somewhat smaller than the observed rms proton charge radius. Since the quark charge is smeared out by strong interactions, this is presumably desirable. Unfortunately, this calculation alone cannot distinguish between a small decrease in α and a corresponding decrease in $m_s - m_d$, so whether such an effect exists remains moot. Incidentally, this accounts for our ability to use $m_s - m_d = 280$ MeV in Ref. 2 with no appreciable change in our results; this interplay also means that the values for α, m_d, and m_s given by the pointlike formulas of this reference and Ref. 7 are in practice acceptable.
[10]The interaction (2) is actually an illegal operator in the Schrödinger equation, and this step must be taken with some care. To make (2) bounded one must in principle consider higher-order corrections to one-gluon-exchange which, for example, smear out the δ function of the contact term. Here we take a more phenomenological view and, recognizing that such a smearing will limit the mixing to nearby states, trun-

cate the mixing series. Alternatively, one could explicitly regularize the hyperfine interactions; for example, one could introduce a finite radius for the interaction (we are indebted to Paul Fage for this suggestion). See also Refs. 7 and 9.

[11]L. A. Copley, Nathan Isgur, and Gabriel Karl, Phys. Rev. D 19, 768 (1979).

[12]A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. D 12, 147 (1975).

[13]This point has also been noted by A. Le Yaouanc *et al.*, Phys. Rev. D 18, 1591 (1978).

[14]The importance of second-order effects in baryons was noted already in Refs. 2, 3, 4, and 5. Their importance in mesons has been stressed by P. J. O'Donnell and R. H. Graham, Phys. Rev. D 19, 284 (1979); Seiji Ono, *ibid.* 17, 888 (1978); I. Cohen and H. J. Lipkin, Nucl. Phys. B151, 16 (1979); and also Ref. 5.