PHYSICAL REVIEW D VOLUME 20, NUMBER 5 1 SEPTEMBER 1979

Intermediate inelastic states in *pd* scattering and the triple-Regge couplings

M. Baig and C. Pajares
Departament de Física Teòrica, Universitat Autònoma de Barcelona, Bellaterra, Barcelona, Spain
(Received 26 June 1978; revised manuscript received 27 April 1979)

The high-energy real and imaginary parts of the *pd* forward amplitude are compared with those predicted by the Glauber theory. The differences are qualitatively explained including the intermediate inelastic states in the framework of the triple-Regge formalism. A Pomeron with intercept larger than one is clearly required. In a quantitative way the *RRR* and *RRP* couplings extracted from inclusive data cannot explain the difference of the real parts. We study the possibility of changing the values of these couplings in order to explain such a difference without spoiling the fit to inclusive data.

I. INTRODUCTION

Hadron-nucleus scattering has been revealed as a good laboratory to test several ideas and models on particle-particle interactions. In this sense, extensive work has been performed in the past few years.[1] In particular, the role of the intermediate inelastic states in hadron-nucleus collision has been studied in connection with triple-Regge ideas.[2-6] The Glauber model[7] predicts total cross sections which grow faster than the experimental data.[8] The inclusion of the intermediate states reduces the Glauber cross section, and using the triple-Regge couplings determined from the inclusive data, one obtains good qualitative agreement[5,6] with the experimental results. In this paper we go further in this type of analysis, looking at the real part of the *pd* scattering amplitude. This real part, extracted from the data by means of derivative analyticity relations[9] (DAR), is compared with the Glauber model[10] showing some differences which are discussed in terms of the triple-Regge approach. We find qualitative agreement if the Pomeron has intercept larger than one, although some quantitative discrepancies appear due mainly to the strength of the *RRR* and *RRP* couplings. We also show that in the imaginary part of the *pd* scattering amplitude some quantitative differences occur. We study the possibility of changing the values of those couplings in order to explain such a difference without spoiling the fit to inclusive data. We conclude that the introduction of intermediate inelastic states represents an improvement to the Glauber-model predictions, and, moreover, a good place to study and test triple-Regge ideas.

The paper is organized as follows: In the next section we present the Glauber-model predictions for the real and imaginary parts of *pd* scattering amplitude; in Sec. III the experimental data on the *pd* total cross section and the DAR are presented; in Sec. IV we introduce the contribution of the intermediate inelastic states in terms of the triple-Regge formalism, and we present the results for the different contributions. Finally, in Sec. V the discussion and conclusions are presented.

II. THE GLAUBER MODEL

In the context of the Glauber model,[6,7] the amplitude for *pd* scattering can be given by the sum of single- and double-scattering terms

$$T^G_{pd}(k,\vec{q}) = T^s_{pd}(k,\vec{q}) + T^d_{pd}(k,\vec{q})\,, \qquad (1)$$

where k is $|\vec{p}_{\text{lab}}|$ and $\vec{q}$ is the momentum transfer. Normalizing in such a way that $\sigma_T(k) = \operatorname{Im} T(k,0)$, the expressions of these two terms are

$$T^s_{pd}(k,\vec{q}) = [\,T_{pp}(k,\vec{q}) + T_{pn}(k,\vec{q})]G(q^2/4)\,,$$

$$T^d_{pd}(k,\vec{q}) = \frac{i}{8\pi^2}\int d^2p\, G(p^2)\, T_{pp}(k, \tfrac{1}{2}\vec{q}+\vec{p}) \times T_{pn}(k, \tfrac{1}{2}\vec{q}-\vec{p})\,, \qquad (2)$$

where T_{pp} and T_{pn} are the amplitudes corresponding to pN processes and $G(q^2)$ stands for the deuteron form factor.

The proton-nucleon amplitudes have been parametrized in different forms. The form adopted here is

$$T_{pp}(s,t) = T_{pn}(s,t) = \sigma_T^{pp}(s)[i + \rho_{pp}(s)]\exp[B_0(s)t/2]\,, \qquad (3)$$

and its value will constitute the input of our calculations in this section. We have introduced

$$\rho_{pp}(s) = \operatorname{Re} T_{pp}(s,0)/\sigma_T^{pp}(s)$$

and

$$B_0(s) = \frac{d}{d\ln s}\left(\ln\frac{d\sigma}{dt}\right)\bigg|_{t=0}.$$

The phase of the amplitude has been approximated by a constant, corresponding to its value at $t=0$. This seems a reasonable approximation since only the small t values are relevant, owing to the presence of the d form factor. A fit to pp data leads to the following numerical results:

 © 1979 The American Physical Society

$$\sigma_T^{pp}(s) = 25.25 + 50.48 s^{-0.58} + 2.07 \ln s ,$$
$$B_0(s) = 8.32(1 + 0.068 \ln s) , \qquad (4)$$
$$\rho_{pp}(s) = 0.612 + 0.098 \ln s ,$$

where σ_T, B_0, and s are given in mb, GeV^{-2}, and GeV^2, respectively. The deuteron form factor is given by

$$G(q^2) \equiv G(t) \simeq \exp(37t) , \qquad (5)$$

which is sufficiently accurate for the range of energies considered.

The real and imaginary parts of $T^G_{pd}(s,0)$ in the forward direction predicted by the Glauber model can be expressed in terms of the pN parameters as follows:

$$\mathrm{Re}T^G_{pd}(s,0) = 2\sigma_T^{pp}\left(\frac{s}{2}\right)\rho_{pp}\left(\frac{s}{2}\right) - \frac{1}{4\pi}\,\sigma_T^{pp2}\left(\frac{s}{2}\right)\rho_{pp}\left(\frac{s}{2}\right)\frac{1}{37 + B_0(s/2)} ,$$
$$\mathrm{Im}T^G_{pd}(s,0) = 2\sigma_T^{pp}\left(\frac{s}{2}\right) + \frac{1}{8\pi}\left[\rho_{pp}{}^2\left(\frac{s}{2}\right) - 1\right]\sigma_T^{pp\,2}(s/2) \times \frac{1}{37 + B_0(s/2)} , \qquad (6)$$

where, as before, $\mathrm{Im}T(s,0) = \sigma_T(s)$. Using the values quoted in Eqs. (4), one can immediately evaluate $\mathrm{Re}T^G_{pd}$ and $\mathrm{Im}T^G_{pd}$. The corresponding results for $100 \le s \le 1200\ \mathrm{GeV}^2$ have been plotted in Figs. 1 and 2.

III. EXPERIMENTAL DATA ON pd AND DAR

The real part of the forward pd scattering amplitude can be extracted from the experimental data on pd and $\bar{p}d$ total cross sections by means of derivative analyticity relations (DAR). From the usual dispersion relations, one can deduce the following expressions[9] on rather general grounds:

$$\mathrm{Re}T_+ = -\sigma_T^+ \cot\left(\frac{\pi\alpha}{2}\right) + \frac{s^{\alpha-1}}{2}\,\pi \csc^2\left(\frac{\pi\alpha}{2}\right)\frac{d}{d\ln s}\left(\frac{\sigma_T^+}{s^{\alpha-1}}\right) + \cdots ,$$
$$\mathrm{Re}T_- = \sigma_T^- \tan\left(\frac{\pi\beta}{2}\right) + \frac{s^{\beta-1}}{2}\,\pi \sec^2\left(\frac{\pi\beta}{2}\right)\frac{d}{d\ln s}\left(\frac{\sigma_T^-}{s^{\beta-1}}\right) + \cdots , \qquad (7)$$

where $T_\pm$ are the definite-signature amplitudes

$$T_\pm = T(\bar{p}d) \pm T(pd) ,$$
$$\sigma_T^\pm = \sigma_T(\bar{p}d) \pm \sigma_T(pd) . \qquad (8)$$

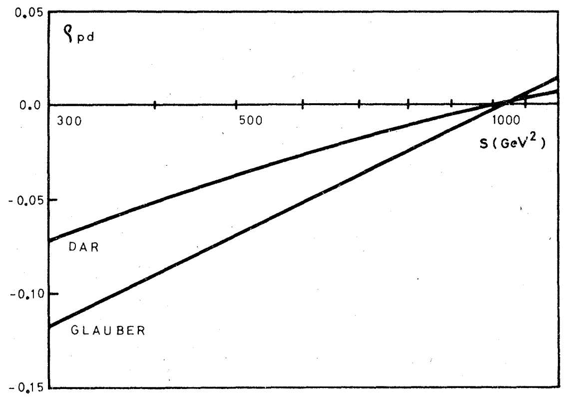

FIG. 1. Ratio ρ_{pd} for DAR and Glauber theory. In this ratio the imaginary part is taken from data.

The values of the parameters α and β in Eqs. (7) must be chosen in such a way that they minimize the s dependence of $\sigma_T^+/s^{\alpha-1}$ and $\sigma_T^-/s^{\beta-1}$, and also $\alpha,\beta \le 2$. Taking $\alpha = 1$, $\beta = 0$, Eqs. (7) reduce to

$$\mathrm{Re}T_+ = \frac{\pi}{2}\,\frac{d}{d\ln s}\,\sigma_T^+ ,$$
$$\mathrm{Re}T_- = \frac{\pi}{2}\,\frac{1}{s}\,\frac{d}{d\ln s}(s\sigma_T^-) . \qquad (9)$$

Finally, from these expressions one obtains the amplitudes for the physical processes

$$\mathrm{Re}T(pd) = \frac{\pi}{2}\,\frac{d}{d\ln s}\,\sigma_T(pd) - \frac{\pi}{4}\,\sigma_T^- ,$$
$$\mathrm{Re}T(\bar{p}d) = \frac{\pi}{2}\,\frac{d}{d\ln s}\,\sigma_T(\bar{p}d) + \frac{\pi}{4}\,\sigma_T^- . \qquad (10)$$

The experimental data[8] on $\sigma_T(pd)$, from which one can deduce $\mathrm{Im}T(pd)$ and [making use of Eqs. (10)] $\mathrm{Re}T(pd)$, may be parametrized according to

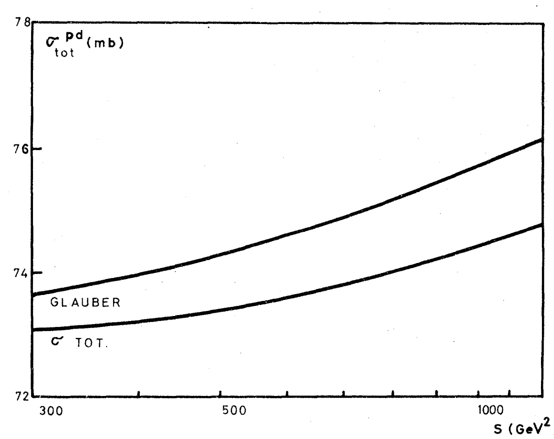

FIG. 2. σ_{tot} in pd for Glauber and the fit to experimental data.

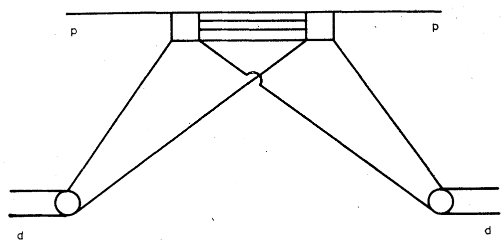

FIG. 3. Intermediate states in the double-scattering term.

$$\sigma_T^{pd}(s) = A + B\ln s + C(\ln s)^2 + Ds^{-1/2}\,,$$
$$\sigma_T^{\bar{p}d}(s) - \sigma_T^{pd}(s) = \sigma_T^- = E\,s^{-1/2}\,, \tag{11}$$

with $A = 64.65$ mb, $B = -0.92$ mb, $C = 0.29$ mb, $D = 74.62$ mb, $E = 123.6$ mb. Similar results can be obtained by means of other reasonable parametrizations, i.e., parametrizations fitting the data in the available energy range and accounting for 3.5-mb rising of the total cross section between its minimum and CERN ISR energies.

The values obtained for the real and imaginary parts of the amplitude $T(pd)$ have been plotted in Figs. 1 and 2, and can be compared with the Glauber-model predictions. Concerning $\mathrm{Re}T(pd)$, the two curves present a zero at the same energy value, $s = s_0$. Below this energy the results for $\mathrm{Re}T(pd)$ obtained through DAR are larger than those predicted by the Glauber model; the situation is reversed in the other energy range, i.e., $s > s_0$. Concerning $\mathrm{Im}T(pd)$, the prediction coming from the Glauber model turns out to be systematically larger than the experimental points.

It is worthwhile to note that these discrepancies are unambiguous and cannot be attributed to uncertainties associated with the curves, such as experimental errors in $\sigma_T(\bar{p}d)$, approximations introduced when dealing with DAR, uncertainties on pN data used as an input in Glauber model, etc. The main purpose of the following sections will consist of the study of these differences in the framework of Glauber theory which incorporates the contributions of inelastic intermediate states.

Notice also that the simultaneous analysis of the discrepancies appearing in the real and imaginary parts of the pd scattering amplitude is certainly of interest. Indeed, as we have shown in the Appendix, the Glauber model satisfies DAR accurately. Therefore, it is not sufficient to correct the imaginary part, and a more sophisticated treatment, correcting both parts simultaneously, seems to be required.

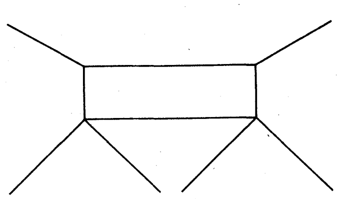

FIG. 4. Particle–two-nucleon amplitude.

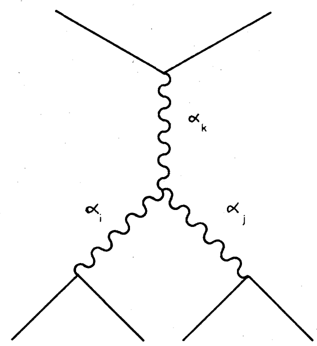

FIG. 5. Reggeization of the particle–two-nucleon amplitude.

IV. INTERMEDIATE INELASTIC STATES

The intermediate inelastic states were first introduced by Gribov in order to explain the differences between the experimental cross section and the imaginary part of the pd scattering amplitude predicted by the Glauber model.[2-5] One expects that a more complete treatment will also be able to account for the discrepancies in the real part. The diagrams corresponding to these intermediate states are shown in Fig. 3, and their contribution to $T(pd)$ can be expressed in terms of

$$\delta T = \frac{i}{\pi}\int \frac{d\vec{p}^{\,0}}{(2\pi)^3} A(k,p) G(p^2)\,, \tag{12}$$

where $A(k,p)$ is the particle–two-nucleon amplitude shown in Fig. 4. The pole originated by the exchange of the incident particle is just the double Glauber term and, consequently, it has already been taken into account.

For large values of s, the amplitude $A(k,p)$ can be Reggeized[6] as follows:

$$A(k,p) = \sum_{ij} \xi_i(t)\xi_j(t) s^{\alpha_i(t)} s^{\alpha_j(t)} g_i(t) g_j(t) \times \mathrm{Im}N_{\alpha_i\alpha_j}(M^2, t)\,, \tag{13}$$

TABLE I. Sign and energy dependence (↑rising, ↓decreasing) of the contributions of the different terms to the real and imaginary part of the scattering amplitude. PPK denotes $PPP+PPR$ and similarly for RRK and $\pi\pi K$.

	Intercept	Re T	Im T
PPK	$\alpha_p(0) = 1+\Delta$	$-$↑	$-$↑
	$\alpha_p(0) = 1$	0	$-$↑
RRK	$\alpha_R(0) = \frac{1}{2}$	$+$↓	0
$\pi\pi K$	$\alpha_\pi(0) = 0$	0	$+$↓

TABLE II. Real part at different energies. Δ_{ex} denotes the difference between DAR and the single term of the Glauber theory (Glau s). Δ_{tot} denotes the sum of the double Glauber term (Δ_{el}) and the inelastic contributions ($RRP+RRR$, $PPP+PPR$).

S_{pd}	DAR	Glau s	Δ_{el}	RRK	PPK	Δ_{tot}	Δ_{ex}
200	−7.626	−12.301	1.004	0.360	−0.065	1.299	4.674
400	−2.745	−5.491	0.449	0.295	−0.089	0.655	2.746
600	−1.972	−4.126	0.338	0.283	−0.094	0.527	2.153
800	−0.859	−1.947	0.160	0.266	−0.102	0.324	1.088
1000	−0.074	−0.234	0.019	0.255	−0.110	0.164	0.159
1200	0.520	1.183	−0.098	0.246	−0.115	0.033	−0.663

where $\xi_i(t)$, $\xi_j(t)$ are the usual signature factors

$$\xi_i(t) = \frac{1 \pm e^{-i\pi\alpha_i(t)}}{\sin\pi\alpha_i(t)},$$

g_i are the Reggeon-nucleon residua, and $N(M^2, p^2)$ is the nucleon-Reggeon amplitude. The validity of Eq. (13) requires $s/M^2 \to \infty$ and $M^2 \to \infty$, and in the last limit (large M^2) one can also Reggeize the amplitude N. One obtains

$$\operatorname{Im}N_{\alpha_1\alpha_2}(M^2, p^2) = \sum_\alpha g_\alpha(0) g_{\alpha_1\alpha_2\alpha}(p^2) \operatorname{Im}\Sigma_\alpha(0) \times (M^2)^{\alpha(0)-\alpha_1(p^2)-\alpha(p^2)}, \quad (14)$$

where $g_{\alpha_1\alpha_2\alpha}$ are the triple-Regge couplings shown in Fig. 5. The contribution of the inelastic intermediate states to the pd amplitude can be obtained introducing this last expression in Eq. (12). One has

$$\delta T = \sum_{ijk} \frac{2i}{s^2} \int dt\, dM^2\, \frac{\xi_i(t)}{\xi_j^*(t)} \left(\frac{s}{M^2}\right)^{\alpha_i(t)+\alpha_j(t)} \times G_{ijk}(t)(M^2)^{\alpha_k(0)} G(t), \quad (15)$$

where G_{ijk} are the usual triple-Regge residua defined in Ref. 11. Notice that δT is not a real function owing to the presence of the factor $\xi_i(t)/\xi_j^*(t)$, which splits the contribution of each term appearing in the sum into its real and imaginary parts.

The dominant terms in the sum of Eq. (15) can be represented by $ijk = RRR$, RRP, PPR, PPP, $\pi\pi R$, $\pi\pi P$. Interference terms in i, j do not appear since only pairs of Reggeons having the same signature and subjected to isospin and G-parity selection rules can be coupled. Following the general attitude adopted when dealing with inclusive reactions, we have also disregarded PR terms. In order to obtain the contribution to the real and imaginary parts coming from each term, we have approximated the signature factors to its value at $t=0$. This is a reasonable approximation owing to the presence of the d form factor. Terms containing R can be assumed to be dominated by f exchange, which is well known to be strongly coupled to external particles.[12] Therefore, RRR and RRP terms, owing to the value ($\frac{1}{2}$) of the Reggeon intercept, contribute exclusively to the real part of the amplitude with a positive contribution. If the intercept of P is unity, the PPR and PPP terms give a negative contribution to the imaginary part only. On the contrary, if this intercept is larger (smaller) than 1 they give an additional negative (positive) contribution to the real part. Finally, $\pi\pi R$ and $\pi\pi P$ terms only contribute to the imaginary part and with a positive sign (vanishing π intercept). The situation is summarized in Table I, where the contributions of the different terms to the real and imaginary parts, their signs, and behaviors have been indicated.

In order to get quantitative results, one has to perform the integration of Eq. (12) for each term appearing in the sum of Eq. (15) and introducing the corresponding values of triple-Regge residua. In this paper, we have used the values deduced in Ref. 13 from inclusive reactions data. The t dependence of these residua has been parametrized by

$$G_{ijk} = A_{ijk} \exp(B_{ijk} t), \quad (16)$$

TABLE III. Imaginary part at different energies (the notation is the same as that of Table II).

S_{pd}	σ_{tot}	Glau s	Δ_{el}	PPK	$\pi\pi K$	Δ_{tot}	Δ_{ex}
200	73.142	76.550	−3.045	−0.340	0.308	−3.077	−3.407
400	73.229	77.107	−3.119	−0.436	0.284	−3.271	−3.878
600	73.628	77.607	−3.179	−0.495	0.208	−3.466	−4.178
800	74.046	78.430	−3.227	−0.539	0.212	−3.544	−4.383
1000	74.442	78.974	−3.265	−0.575	0.215	−3.625	−4.532
1200	74.809	79.454	−3.297	−0.604	0.217	−3.684	−4.644

and the P intercept is taken as 1.06 as in Ref. 3. In this section, however, we shall take

$$\alpha_P(t)=1+\Delta+\alpha' t, \tag{17}$$

where Δ is a free parameter. The limits of the integrals are $m[\Delta(1230)] \le M^2 \le \epsilon s$, with $\epsilon \simeq 0.2$, and $0 \le |t| \le \infty$. For the real part, integration of Eq. (13) leads to

$$\mathrm{Re}\,\delta T_{RRK}=A_{RRK}\exp\left[[\alpha_K(0)-1]\eta+\alpha_K(0)\frac{37+B}{2}\right]\left[E_1\left(\left(\frac{37+B}{2}+\lambda\right)\alpha_K(0)\right)-E_1\left(\left(\frac{37+B}{2}+\eta-\Lambda\right)\alpha_K(0)\right)\right], \tag{18}$$

where $\eta=\ln s$, $\lambda=-\ln\epsilon$, $\Lambda=\ln m[\Delta(1230)]$, $K=R$ or P, and $E_1(x)$ has been defined in Ref. 14. Similarly,

$$\mathrm{Re}\,\delta T_{PPK}=-A_{PPK}\frac{1}{\alpha'}\left[\mathrm{Re}\,\frac{\xi_P(0)}{\xi_P^*(0)}\right]\exp\left[[\alpha_K(0)-1]\eta-\frac{B+37}{2\alpha'}\,[2\Delta-\alpha_K(0)+1]\right]$$
$$\times\left[E_i\left(\left(\frac{B+37}{2\alpha'}+\eta-\Lambda\right)[2\Delta-\alpha_K(0)+1]\right)-E_i\left(\left(\frac{B+37}{2\alpha'}+\lambda\right)[2\Delta-\alpha_K(0)+1]\right)\right]. \tag{19}$$

The corresponding imaginary parts have been obtained along the same lines. The numerical results corresponding to the different contributions have been collected in Tables II and III.

V. DISCUSSION AND CONCLUSIONS

Let us first discuss the real part of the amplitude. For energy values smaller than that corresponding to the zero ($s<s_0$), a positive correction is required in order to increase the real part of the Glauber prediction. Such a correction, on the other hand, should decrease with the energy. The $RRR+RRP$ term presents such a behavior. However, for $s>s_0$ the correction should have the opposite sign and increase with the energy. This can only be achieved with the presence of $PPR+PPP$ terms involving a Pomeron with intercept larger than one. Fortunately, the modulation introduced by the product of signature factors reduces the value of these contributions in such a way that at lower energies ($s<s_0$) these additional terms do not destroy the previous and successful behavior of $RRR+RRP$ terms. We can therefore conclude that from this qualitative analysis of the real part, a Pomeron with intercept larger than one is required. Such a conclusion is fully consistent with those coming from many works on two-particle scattering and multiparticle production.[12,15-18]

We now turn to the imaginary part. As is well known, the $PPR+PPP$ terms reduce the cross section coming from the Glauber model and tend to improve the agreement with the data. Here, one has also to take into account the presence of $\pi\pi R$ and $\pi\pi P$ terms contributing with a positive sign, i.e., tending to suppress the corrections introduced by PPP and PPR. If the $\pi\pi R+\pi\pi P$ term is small enough, the agreement will be good.

From a more quantitative point of view, the numerical results quoted in Table II show that the differences between DAR results and Glauber-model predictions cannot be completely explained. Triple-Regge terms are clearly insufficient, even if the large errors associated with the two curves of Fig. 1 are taken into account. The situation could only be improved with a $RRR+RRP$ contribution taking large values for small energies and then decreasing for greater energies. Such a possibility could be achieved by simply adopting a larger (smaller) residue for the RRR (RRP) vertex than those obtained from present fits to inclusive reactions data. Concerning the imaginary part quoted in Table III, triple-Regge terms are now able to explain the previous differences within the errors. Obviously, a smaller $\pi\pi R+\pi\pi P$ contribution superimposed to a more rapidly rising $PPP+PPR$ term would be highly desirable. This last possibility could be easily obtained working with a Pomeron with intercept larger than 1.06 (say 1.10) as is also required from independent analysis.[12,16-18]

One has to remark that the RRR and RRP terms are poorly extracted from the data on inclusive reactions. Indeed, the information on these terms comes from the fit in the range $x=0.8$, 0.85 with large values of s. This information is not sufficient to separate the RRR, RRP, $\pi\pi R$, $\pi\pi P$ contributions from their global sum. In order to obtain more information, it is usual to fix the $\pi\pi R$ and $\pi\pi P$ contributions in the form[11]

$$\frac{g_{\pi pp}{}^2}{4\pi}\,\sigma_T(\pi p)\,\frac{(-t)e^{b_\pi(t-\mu^2)}}{(t-\mu^2)^2}, \tag{20}$$

taking the off-mass-shell factor with $b_\pi=0$. If one uses $b_\pi=5-10$ GeV^2, as required by other analysis, the $\pi\pi R$ and $\pi\pi P$ contributions would be smaller and a larger RRR and RRP contribution could appear when fitting the inclusive reactions data. There are additional reasons suggesting that RRR and RRP contributions are not well extracted from the data, in particular, the unconventional t dependence obtained for the RRP residua. Indeed, it turns out to increase as $|t|$ for $|t|\ge 0.3$ GeV^2 and, as has already been pointed out,[19] this suggests that contaminations from other terms are present.

On the other hand, the strength of the RRR term is strongly dependent on the P intercept, which already controls the strength of PPP.[17] Using values for the P intercept around 1.10, a much larger RRR residue is required.

The discussion in the preceding paragraph casts some doubts on the values of the triple-Regge residua RRR, RRP, $\pi\pi R$, $\pi\pi P$. It also indicates that the possibility of having larger RRR and smaller RRP, $\pi\pi R$, and $\pi\pi P$ residua is reasonable. This turns out to be in agreement with the results obtained when comparing Glauber predictions with experimental data. It has to be noticed that the real part of the pd scattering amplitude constitutes a good laboratory in order to analyze the values of RRR and RRP terms, since there are no contaminations arising from $\pi\pi R$ and $\pi\pi P$.

In conclusion, the introduction of inelastic intermediate states represents an improvement to the Glauber-model predictions concerning both the real and imaginary parts of the pd scattering amplitude. These intermediate states have been described in the framework of triple-Regge theory. Therefore, additional information on important relations of this framework can be obtained, thus complementing the usual information coming from inclusive-reactions data. In particular, the qualitative analysis on the real part implies the existence of a Pomeron with intercept larger than 1. Similarly, in order to achieve a quantitative agreement between both real and imaginary parts of the amplitude, a revision of triple-Regge residua is required. This is particularly true for the RRR and RRP terms, where some $\pi\pi R$ and $\pi\pi P$ contamination seems to be present. An experimental measurement of the real part of pd scattering amplitude at ISR energies would be of interest in order to establish the triple-Regge couplings.

ACKNOWLEDGMENTS

We thank Professor R. Pascual for discussions, and the Instituto de Estudios Nucleares (Madrid) for financial support.

APPENDIX

In this appendix we show that the real and imaginary parts of the pd scattering amplitude predicted by the Glauber model are related by means of an analyticity dispersion relations. Indeed, when one applies the DAR to the imaginary part of Glauber model, the expression of the real part of this model is obtained.

We know from Eqs. (6) that

$$\begin{aligned} \sigma_T^G(pd) &= 2\sigma_T^{pp} + (\rho_{pp}{}^2 - 1)\frac{1}{8\pi}\,\sigma_T^{pp\,2}\,\frac{1}{37+B_0^{pp}}\,, \\ \sigma_T^G(\bar{p}d) &= 2\sigma_T^{\bar{p}p} + (\rho_{\bar{p}p}{}^2 - 1)\frac{1}{8\pi}\,\sigma_T^{\bar{p}p\,2}\,\frac{1}{37+B_0^{\bar{p}p}}\,. \end{aligned} \tag{A1}$$

Introducing these expressions into Eq. (10) of the DAR, we can evaluate the real part of pd amplitude

$$\mathrm{Re}T(pd) = \frac{\pi}{2}\,\frac{d}{d\ln s}\,\sigma_T^G(pd) - \frac{\pi}{4}\,[\sigma_T^G(\bar{p}d) - \sigma_T^G(pd)]\,. \tag{A2}$$

Taking into account that nucleon-nucleon amplitudes satisfy DAR, one obtains

$$\mathrm{Re}T(pd) = 2\sigma_T^{pp}\rho_{pp} - \frac{1}{4\pi}\,\sigma_T^{pp\,2}\,\rho_{pp}\,\frac{1}{37+B_0^{pp}} + \delta\,, \tag{A3}$$

where

$$\delta = \frac{1}{32(37+B_0)}\left\{(\sigma_T^{\bar{p}p} - \sigma_T^{pp})^2 + [\mathrm{Re}T(pp)^2 - \mathrm{Re}T(\bar{p}p)^2] + 2\,\frac{d}{d\ln s}\,[\mathrm{Re}T(pp)]^2 - \frac{2(\rho_{pp}{}^2-1)\sigma_T^{pp\,2}}{37+B_0^{pp}}\,\frac{d}{d\ln s}\,B_0^{pp}\right\}. \tag{A4}$$

The first two terms in Eq. (A3) are just $\mathrm{Re}T^G(pd)$. The last term (A4) is one order of magnitude smaller than the others, in the energy range considered. Therefore, the real and imaginary parts of the Glauber model are related by the DAR in an approximate way.

[1] K. Gottfried, Phys. Rev. Lett. 32, 957 (1974); L. Caneschi and A. Schwimmer, in *Deep Scattering and Hadronic Structure*, Proceedings of the XII Rencontre de Moriond, edited by J. Trân Thanh Vân (Editions Frontières, Paris, 1977); A. Capella and A. Krzywicki, Phys. Lett. 67B, 84 (1977).

[2] V. N. Gribov, Zh. Eksp. Teor. Fiz. 56, 892 (1969) [Sov. Phys.—JETP 29, 483 (1969)].

[3] O. V. Kancheli and S. G. Matinyan, Pis'ma Zh. Eksp. Teor. Fiz. 12, 41 (1970) [JETP Lett. 12, 30 (1970)].

[4] V. V. Anisovitch, L. G. Dakhno, and P. E. Volkovitsky, Phys. Lett. 42B, 242 (1972).

[5] C. Quigg and L. L. Wang, Phys. Lett. 43B, 314 (1973).

[6] L. Bertochi, in Proceedings of the IV GIFT Seminar, Barcelona, 1973 (unpublished).

[7] R. J. Glauber, Phys. Rev. 100, 142 (1955); V. Franco and R. J. Glauber, *ibid.* 142, 1195 (1966).

[8] A. S. Carroll *et al.*, Phys. Lett. 61B, 303 (1976).

[9] J. B. Bronzan, G. L. Kane, and U. P. Sukhatme, Phys. Lett. 49B, 272 (1974); U. P. Sukhatme, G. L. Kane,

R. Blankenbecler, and M. Davier, Phys. Rev. D 12, 3431 (1975); D. P. Sidhu and U. P. Sukhatme, *ibid.* 11, 1351 (1975).
[10]C. Pajares and R. Ruiz de Querol, Phys. Lett. 69B, 177 (1977).
[11]R. D. Field and G. C. Fox, Nucl. Phys. B80, 367 (1974).
[12]A. Capella, J. Trân Thanh Vân, and J. Kaplan, Nucl. Phys. B97, 493 (1975).
[13]S. Y. Chu, B. R. Desai, B. C. Shen, and R. D. Field, Phys. Rev. D 13, 2967 (1976).
[14]*Handbook of Mathematical Functions*, edited by M. Abramowitz and I. A. Stegun (Dover, New York, 1964).
[15]P. D. B. Collins, F. D. Gault, and A. Martin, Nucl. Phys. 80B, 135 (1974).
[16]D. R. Snider and H. W. Wyld, Phys. Rev. D 11, 2538 (1975); C. Pajares and R. Pascual, *ibid.* 16, 1359 (1977).
[17]A. M. Lapidus, V. I. Lisin, K. A. Ter-Martirosyan, and P. E. Volkovitsky, Yad. Fiz. 24, 1237 (1976) [Sov. J. Nucl. Phys. 24, 648 (1976)].
[18]C. Pajares, Phys. Lett. 69B, 101 (1977).
[19]A. Capella, H. Hogaasen, and V. Rittenberg, Phys. Rev. D 8, 2040 (1973); J. Gabarró and C. Pajares, Nucl. Phys. B64, 493 (1973).