Transverse-momentum distributions in lepton-hadron scattering from quantum chromodynamics

Howard Georgi and Jon Sheiman
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
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We use quantum-chromodynamic perturbation theory to calculate $d\sigma/dx_H dy d\Pi_T$ in electromagnetic lepton-hadron scattering, where Π_T is the sum over all produced hadrons of the absolute value of the component of three-momentum perpendicular to the virtual-photon direction.

I. INTRODUCTION

There are two well-known methods for extracting information about the fundamental constituents of hadrons from scattering data. These are summarized in the concepts of jets[1,2,3] and partons.[4] Recently, both concepts have been put on a firmer theoretical basis in the context of an explicit model of the strong interactions, quantum chromodynamics (QCD), so that they can be combined to give a description of jet structure in lepton-hadron scattering.

A. Inclusive lepton-hadron scattering and the parton model

It has been shown how the parton-model analysis of inclusive lepton-hadron scattering can be extended to include the effects of QCD interactions, neglecting effects of the order $1/Q$.[2,5] One treats the process as incoherent scattering of the virtual photon (or W boson) with the individual partons (light quarks and gluons) which compose the hadron. The fundamental virtual-photon–parton scattering process is computed in QCD perturbation theory. Logarithmic singularities associated with taking the (massless) incoming partons to be on-shell can be absorbed into the parton distribution functions. Singularities associated with the outgoing partons vanish when the sum over final states is taken. This analysis gives the same results as the standard operator-produce-expansion analysis.[6]

B. Final-state jets

The attempt to characterize final states in QCD perturbation theory is plagued with a fundamental nonperturbative problem; one must know how the outgoing partons decay into jets of observed hadrons. This difficulty can be sidestepped by considering only variables which are independent of the details of parton decays. A quantity which depends only on the total momentum of each jet (which is simply the momentum of the decaying parton) presumably satisfies this criterion.[2] Thus certain bulk properties of the final state can be studied without a knowledge of the parton decay functions.

Another important property of variables which depend only on the total momentum of jets of particles is that they have the same value for all states which are physically equivalent in massless QCD (such as a quark with a given momentum and a quark plus a collinear gluon with the same total momentum). Sterman and Weinberg[1] have speculated that the sum over indistinguishable states will cancel out logarithmic singularities associated with the final states.

C. Transverse momentum in lepton-hadron scattering

We now combine these ideas to compute transverse-momentum distributions in lepton-hadron scattering. In the spirit of the parton model, we treat the process as a virtual-photon (W-boson)–parton collision, folding this cross section in with the parton distribution functions. Then, using the jet idea, we describe the transverse momentum of the final state in a way which is independent of the details of the parton decays. The variable we use is Π_T,

$$\Pi_T \equiv \sum_{\text{hadrons}} |\vec{p}_T| \tag{1}$$

where $\vec{p}_T$ is the component of the hadron's momentum perpendicular to the three-momentum of the virtual photon in the laboratory frame, and the sum runs over all produced hadrons. If we assume the parton decay is collinear, this is just the analogous quantity defined for the produced partons[7]

$$\Pi_T \cong \sum_{\text{partons}} |\vec{p}_T| \,. \tag{2}$$

Thus, the distribution of Π_T should have a well-behaved perturbation expansion, with all logarithmic singularities absorbed into parton distribution functions. Only the distribution functions are required as phenomenological input. We now proceed to do the lowest-order calculation.

II. PARTON CROSS SECTIONS

The lowest-order parton scattering processes which give rise to nonzero Π_T (Ref. 8) are

$$q(\bar{q})+\gamma_v \to q(\bar{q})+G\,, \tag{3}$$

$$G+\gamma_v \to q+\bar{q}\,, \tag{4}$$

where q ($\bar{q}$) is a quark (antiquark), G is gluon, and γ_v is the virtual photon.[9] The Feynman diagrams for these processes are shown in Figs. 1 and 2, respectively.

Let K_1 (K_2) be the initial (final) lepton momentum, so that $q \equiv K_1 - K_2$ is the virtual photon's momentum. Further, let p_1 be the initial parton's momentum and p_2 be the outgoing quark (antiquark) momentum for process (3), or either outgoing momentum for process (4). We will describe the process by the following variables[10,11]:

$$Q^2=-q^2,\quad x=Q^2/2p_1\cdot q,\quad 0<x\leq 1$$

$$y=\frac{p_1\cdot q}{p_1\cdot K_1},\quad z=\frac{p_1\cdot p_2}{p_1\cdot q}\,,\quad 0\leq z\leq 1\,,$$

$$p_T^{\,2}=|\vec{p}_{2T}|^2, \text{ and } \phi\,,$$

where ϕ is the azimuthal angle of $\vec{p}_{2T}$ measured from $\vec{K}_{1T}$; the subscript T denotes the component orthogonal to $\vec{q}$ in the lab frame.

Energy conservation in processes (3) and (4) gives a constraint among the variables. The invariant phase-space integral over p_3, the momentum of the second outgoing parton, gives

$$\delta^{(4)}(q+p_1-p_2-p_3)\frac{d^3p_3}{2p_3^0} \to z\,\delta\!\left(p_T^{\,2}-\frac{Q^2z}{x}(1-x)(1-z)\right). \tag{6}$$

The parton invariant cross section (integrated over the azimuthal angle about the incident lepton's direction) is given by

$$\frac{d\sigma}{dx\,dy\,dz\,dp_T^{\,2}d\phi}=\frac{\alpha^2}{32\Pi^2Q^4}y\,\delta\!\left(p_T^{\,2}-\frac{Q^2z}{x}(1-x)(1-z)\right)\times L^{\mu\nu}M_{\mu\nu}\,, \tag{7}$$

where $L^{\mu\nu}$ ($M_{\mu\nu}$) is the square of the leptonic (partonic) current and α is the electromagnetic fine-structure constant. $L^{\mu\nu}$, averaged (summed) over initial (final) spin, is given by

$$L^{\mu\nu}=4K_1^\mu K_1^\nu - Q^2g^{\mu\nu} \tag{8}$$

up to terms proportional to q^μ or q^ν. The general form of $M_{\mu\nu}$, averaged (summed) over initial (final) spins, is, by considerations of gauge, Lorentz, and parity invariance,

$$\begin{aligned}M^{\mu\nu}=&A\left[-g^{\mu\nu}+\frac{p_1^\mu q^\nu}{p_1\cdot q}+\frac{q^\mu p_1^\nu}{p_1\cdot q}+\frac{Q^2}{(p_1\cdot q)^2}p_1^\mu p_1^\nu\right]\\&+B\frac{Q^2}{(p_1\cdot q)^2}\left(p_1^\mu+\frac{p_1\cdot q}{Q^2}q^\mu\right)\left(p_1^\nu+\frac{p_1\cdot q}{Q^2}q^\nu\right)\\&+C\frac{Q}{p_1\cdot q}\left[e_2^\nu\left(p_1^\mu+\frac{p_1\cdot q}{Q^2}q^\mu\right)+\mu\leftrightarrow\nu\right]+De_2^\mu e_2^\nu\,,\end{aligned} \tag{9}$$

where, in the laboratory frame, $e_2^0=0$ and $\vec{e}_2=\hat{p}_{2T}$ so that $e_2\cdot e_2=-1$. Combining this with $L^{\mu\nu}$ gives

$$\begin{aligned}L^{\mu\nu}M_{\mu\nu}=&A\frac{2Q^2}{y^2}[1+(1-y)^2]+B\frac{4Q^2}{y^2}(1-y)\\&+C\frac{4Q^2}{y^2}(y-2)(1-y)^{1/2}\cos\phi+DQ^2\\&\times\left(4\frac{1-y}{y^2}\cos^2\phi+1\right).\end{aligned} \tag{10}$$

A computation of the graphs of Fig. 1 gives, averaging (summing) over initial (final) parton spin and color,

$$\begin{aligned}A_{\text{quark}}&=\tfrac{8}{3}g^2\frac{2(1-x-z)+(x+z)^2}{(1-x)(1-z)}\,,\\B_{\text{quark}}&=D_{\text{quark}}=\tfrac{32}{3}g^2xz\,,\\C_{\text{quark}}&=\tfrac{16}{3}g^2\left[\frac{zx}{(1-z)(1-x)}\right]^{1/2}[(1-x)(1-z)+xz]\,,\end{aligned} \tag{11}$$

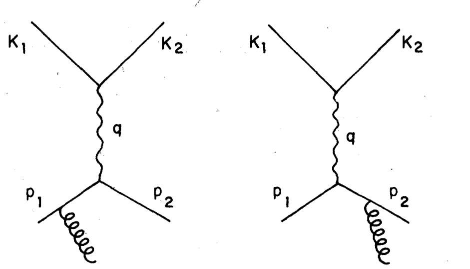

FIG. 1. Feynman diagrams for electromagnetic quark (antiquark)-lepton scattering. The wavy line is a virtual photon, the curly line is a gluon.

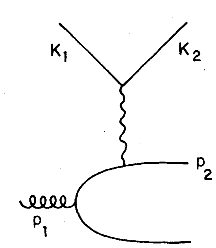

FIG. 2. Feynman diagram for electromagnetic gluon-lepton scattering.

and for Fig. 2,

$$A_{\text{gluon}} = \frac{g^2}{z(1-z)}[z^2 + (1-z)^2 - 2x(1-x)]\,,$$

$$B_{\text{gluon}} = D_{\text{gluon}} = 8g^2 x(1-x)\,, \tag{12}$$

$$C_{\text{gluon}} = 2g^2(1-2x)(1-2z)\left[\frac{x(1-x)}{z(1-z)}\right]^{1/2}.$$

The subscripts in Eqs. (11) and (12) indicated which type of parton is struck in the process.

Equation (7) gives the cross section to produce a quark with momentum P_2. We can convert Eq. (7) into a jet cross section by reinterpreting the variables z, P_T, and ϕ as jet variables [given by Eq. (5) where p_2 is the total jet momentum] and by adding the contribution of the other jet (gluon in Fig. 1 or antiquark in Fig. 3) as follows:

$$d\sigma^{\text{jet}} = d\sigma + d\sigma(z \to 1 - z, \phi \to \phi + \pi)\,. \tag{13}$$

This gives the following jet cross sections[12]:

$$\frac{d\sigma^{\text{jet}}_{\text{quark (gluon)}}}{dx\,dy\,dz\,dP_T^2\,d\phi} = \frac{\alpha^2\alpha_s}{3\pi Q^2}\, y\,\delta(P_T^2 - Q^2(z/x)(1-x)(1-z))K_{\text{quark (gluon)}}\,, \tag{14a}$$

$$K_{\text{quark}} = \frac{2}{y^2}[1+(1-y)^2]\frac{z(1-z)(1-4x)+1+x^2}{z(1-z)(1-x)} + 4x\left[4\frac{1-y}{y^2}(1+\cos^2\phi)+1\right]$$
$$+8\frac{(1-y)^{1/2}}{y^2}(y-2)\cos\phi\,\frac{x^{3/2}(2z-1)}{[(1-x)z(1-z)]^{1/2}}\,,$$

$$K_{\text{gluon}} = \frac{3}{2y^2}[1+(1-y)^2]\frac{z^2+(1-z)^2-2x(1-x)}{z(1-z)} + 6\left[4\frac{(1-y)}{y^2}(1+\cos^2\phi)+1\right]x(1-x)$$
$$+6\frac{(y-2)}{y^2}(1-y)^{1/2}\left(\frac{x(1-x)}{z(1-z)}\right)^{1/2}(1-2x)(1-2z)\cos\phi \tag{14b}$$

and $\alpha_s = g^2/4\pi$. Again, the subscripts indicate whether a quark or gluon is struck.

III. FOLDING IN THE PARTON DISTRIBUTIONS

Let P be the initial hadron's momentum. We define the scaling variable via

$$p_1 = \xi P\,, \tag{15}$$

so that ξ is the fraction of the hadron's momentum carried by the struck quark. All of the variables in Eq. (5) are unchanged if P_1 is replaced by P with the exception

$$x_H = \frac{Q^2}{2P\cdot q} = \xi x\,,$$

where X_H is the usual Bjorken scaling variable.

Let $f_i(\xi)$ be the distribution function for type i partons, i.e., $f_i(\xi)\,d\xi$ is the probability of finding a parton of type i with momentum between P and $(\xi + d\xi)P$ in the initial hadron. Then, if $d\sigma_i^{\text{jet}}$ is the jet cross section for type i partons,

$$\frac{d\sigma^{\text{jet}}}{dx_H\,dy\,dz\,dP_T^2\,d\phi} = \sum_i \int_0^1 d\xi\, f_i(\xi)\frac{d\sigma_i^{\text{jet}}}{\xi\,dx\,dy\,dz\,dP_T^2\,d\phi}\bigg|_{x = x_H/\xi}\,, \tag{16}$$

where $d\sigma^{\text{jet}}$ is the jet cross section for the initial hadron. We use the δ function to collapse the ξ integration. Then integrating over ϕ and using

$$\Pi_T = 2P_T \tag{17}$$

gives

$$\frac{d\sigma}{dx_H\,dy\,dz\,d\Pi_T} = \frac{4}{3}\frac{\alpha^2\alpha_s}{Q^3}\left(\frac{x(1-x)}{z(1-z)}\right)^{1/2}\frac{1}{y}\times\left[A_q\sum_j Q_j^2 q_j\left(\frac{x_H}{x}\right) + \tfrac{3}{4}A_g\left(\sum_j Q_j^2\right)g\left(\frac{x_H}{x}\right)\right]\,, \tag{18}$$

where

$$A_q = [1+(1-y)^2]\frac{z(1-z)(1-4x)+1+x^2}{z(1-z)(1-x)} + 2x[6(1-y)+y^2]\,,$$

$$A_g = [1+(1-y)^2]\frac{z^2+(1-z)^2-2x(1-x)}{z(1-z)} + 4x(1-x)[6(1-y)+y^2]\,,$$

and

$$x = \frac{z(1-z)}{z(1-z)+\Pi_T^2/4Q^2}\,,$$

and q_j is the distribution function for quarks plus antiquarks of flavor j, g is the gluon distribution function, and j runs over flavors.

The jet cross section refers to one jet characterized by z and one characterized by $1-z$; thus the appropriate phase space can be chosen as $\frac{1}{2} \leq z \leq 1$. For given Π_T and x_H, the constraint $x \geq x_H$

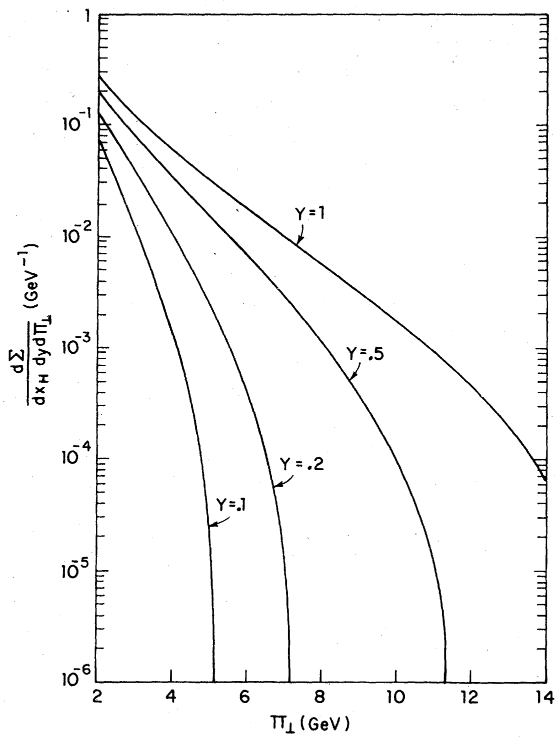

FIG. 3. Perturbative contribution to $d\Sigma/dx_H dy\, d\Pi_T$ for $x_H=0.2$, $E_{\text{beam}}=200$ GeV.

implies

$$z \leq z_{\max}=\frac{1}{2}\left[1+\left(1-\frac{\Pi_T^{\,2}}{Q^2}\,\frac{x_H}{(1-x_H)}\right)^{1/2}\right], \tag{19}$$

which can be satisfied only if

$$\Pi_T \leq \Pi_T^{\max}=Q\left(\frac{1-x_H}{x_H}\right)^{1/2}. \tag{20}$$

Integrating over z gives the desired hadronic cross section

$$\frac{d\sigma}{dx_H dy\, d\Pi_T}=\int_{1/2}^{z_{\max}}\frac{d\sigma}{dx_H dy\, dz\, d\Pi_T}. \tag{21}$$

IV. NUMERICAL COMPUTATION

We have evaluated the integral in Eq. (21) numerically for various x_H and y for a fixed beam energy of 200 GeV. The following parton distribution functions for the proton were used:

$$\begin{aligned} q_u(\xi) &= 2q_d(\xi)=\frac{4}{3}\,\frac{(1-\xi)^3}{\xi},\\ q_s(\xi) &= 0,\\ g(\xi) &= \frac{5}{2}\,\frac{(1-\xi)^4}{\xi}. \end{aligned} \tag{22}$$

The form of the quark distribution functions is compatible with the SLAC electroproduction data.[6] The form of the gluon distribution function (which is difficult to determine phenomenologically) is chosen to keep gluons away from $\xi=1$ and to have the proper parton-model behavior as $\xi\to 0$ [i.e., $f(\xi)\sim 1/\xi$ as $\xi\to 0$]. The normalizations are determined by requiring an even division of energy-momentum between quark and gluon sectors,[13] and by requiring $q_u=2q_d$, $q_s=0$ as in the naive quark model. Our final result is not very sensitive to the shape of the distribution functions, so these crude forms are adequate.

We found that for $x_H>0.05$ the contribution of struck gluons was always less than 20%, falling rapidly with increasing x_H. Indeed, using a gluon distribution of the form $(1-\xi)^5/\xi$ produced changes of at most a few percent. The dominance of struck quarks over struck gluons is easily understood. For Π_T near $\Pi_T^{\max}$, only partons near $\xi=1$ contribute. Thus quarks dominate due to the absence of hard gluons. For small Π_T the propagators in Figs. 1 and 2 become infinite, but the singularity is worse in Fig. 1 than in Fig. 2, so quarks dominate again.

We define the cross section per event as

$$\frac{d\Sigma}{dx_H dy\, dz\, d\Pi_T}\equiv\frac{d\sigma}{dx_H dy\, d\Pi_T}\Big/\frac{d\sigma}{dx_H dy}, \tag{23}$$

where $d\sigma/dx_H dy$ is the inclusive hadronic cross section. To lowest order in α_s, this is given by the standard parton-model result

$$\frac{d\sigma}{dx_H dy}=\frac{4}{3}\,\frac{\pi\alpha^2}{Q^2 y}\,[1+(1-y)^2]\sum_j Q_j^{\,2}q_j(x_H), \tag{24}$$

which depends only on the quark distribution functions. To the extent that gluons make a negligible contribution to $d\sigma/dx_H dy\, d\Pi_T$, there is a cancellation of the overall normalization of quark distribution functions between the numerator and denominator of Eq. (23).

Furthermore, the effect of using Q^2-dependent distribution functions (as required by the renormalization group),[14] is also small, less than 10%.[15] Thus the major nonscaling Q dependence of $d\Sigma$ is in the factor $\alpha_s(Q^2)$,[16] for which we use the standard three-flavor result

$$\frac{\alpha_s(Q^2)}{\pi}=\frac{4}{9}\,\frac{1}{\ln(Q^2/\Lambda^2)}, \tag{25}$$

with Λ taken to the 500 MeV. The results of our computation are shown in Figs. 3 and 4.

We believe that the dominant nonperturbative effect is the nonzero transverse momentum of the zeroth-order process.[17,18] We can make a rough estimate of these effects by assuming that the produced hadrons have identical independent P_T distributions which are independent of the jet multiplicity n. This gives for the nonperturbative correction to Eq. (23)

$$\frac{d\Sigma_{\rm np}}{dx_H dy\, dz\, d\Pi_T} = \sum_n P_n D_n(\Pi_T)\,, \qquad (26)$$

where P_n is the probability of the jet being composed of n hadrons and $D_n(\Pi_T)$ is a convolution of n P_T distributions. For large Π_T, the sum is dominated by large n, so that D_n can be approximated by

$$D_n(\Pi_T) \approx \frac{1}{(2\Pi n \sigma_{P_T}{}^2)^{1/2}} \times \exp\left[\frac{-(\Pi_T - n\langle P_T\rangle)^2}{2n\sigma_{P_T}{}^2}\right], \qquad (27)$$

where $\langle P_T\rangle$ (σ_{P_T}) is the mean (variance) of the single-particle P_T distribution. Using the data of Ref. 19, we get typical values of $\langle P_T\rangle = 0.45$ GeV and $\sigma_{P_T} = 0.42$ GeV. To estimate P_n, we use the Koba-Nielsen-Olesen scaling hypothesis[20]

$$P_n = \frac{1}{\langle n\rangle}\Psi(n/\langle n\rangle)\,, \qquad (28)$$

where $\langle n\rangle$ is the mean multiplicity. We use the fit of Ref. 21 to the hadron-hadron scattering data[22]

$$\Psi(z) = (1.89z + 16.8z^3 - 3.32z^5 + 0.17z^7) \times \exp(-3.04z)\,. \qquad (29)$$

The observed value of $\langle n\rangle$ for charged hadrons in the kinematic range of interest is 2.5; we take its value for all hadrons to be 5.[23] Finally, the current experiment is insensitive to hadrons with $z_H < 0.08$.[19] Since the z_H's of all hadrons must sum to unity, this bounds the observed multiplicity at 12. We account for this (crudely) by considering only $n \leq 11$. The results are plotted in Fig. 5 along with the perturbative contribution for the particularly favorable case of $y = 1$, $x_H = 0.1$.

Based on our results, we believe that the most interesting kinematic region is characterized by large y ($y \gtrsim 0.5$), small x_H ($x_H \lesssim 0.3$), and large Π_T ($\Pi_T \gtrsim 8$ GeV). This region satisfies the following criteria:

(a) The perturbative contribution clearly dominates over the poorly known nonperturbative contribution.

(b) Perturbation theory makes sense in that we are away from the $\Pi_T \to 0$ singularity.

(c) The fraction of all events in this region is not too small.

Altarelli and Martinelli[16] have computed the Π_T moments of $d\Sigma/dx_4 dy\, d\Pi_T$ in a similar way. This is dangerous in that except for very high moments, small Π_T will dominate, leading to a large nonperturbative contribution. Thus it may be ad-

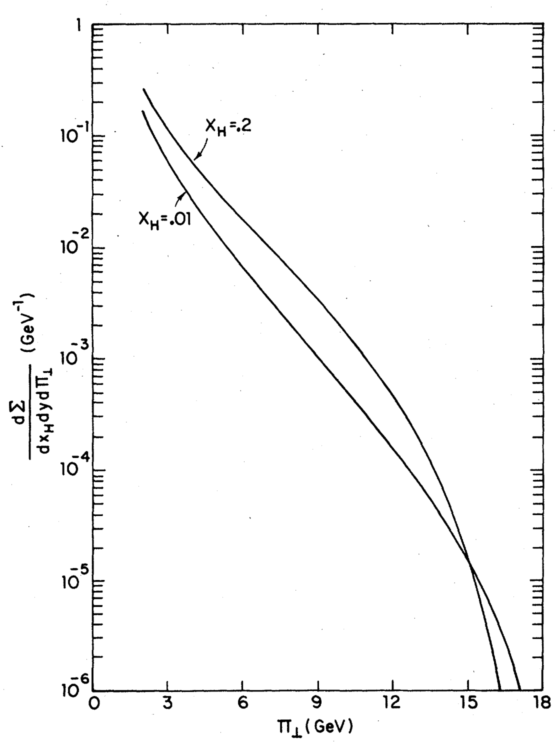

FIG. 4. Perturbative contribution to $d\Sigma/dx_H\, dy\, d\Pi_T$ for $y=1$, $E_{\rm beam} = 200$ GeV.

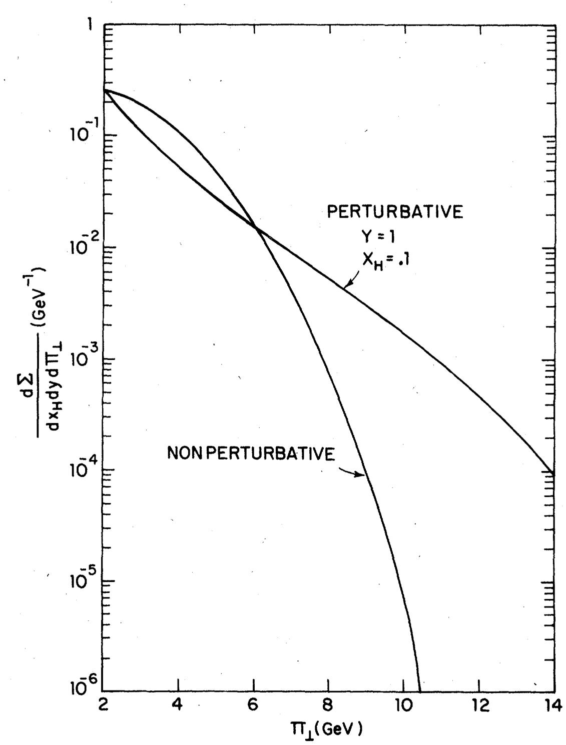

FIG. 5. Comparison of perturbative and nonperturbative contributions to $d\Sigma/dx_H dy\, d\Pi_T$ for $y=1$, $x_H = 0.1$, $E_{\rm beam} = 200$ GeV.

vantageous to consider the Π_T distribution at large values of Π_T.

The perturbative and nonperturbative effects have their own distinct signatures. Let $\vec{P}_T$ be the component of a produced hadron's momentum orthogonal to $\vec{q}$. Then, neglecting the initial hadron's fragments, nonperturbative events with large Π_T consist of many produced hadrons with $\vec{P}_T$'s distributed roughly symmetrically in the transverse-momentum plane (a characteristically one-jet event). The order-α_s perturbative events consist of hadrons with mutually parallel or antiparallel $\vec{p}_T$'s (a characteristically two-jet event). The variable Π_T does not distinguish between one- and two-jet contributions; however, it is easy to invent a variable that does,

$$\tilde{\Pi}_T \equiv \sum_{\text{hadrons}} |\vec{P}_T \cdot \hat{n}| - \sum_{\text{hadrons}} |\vec{P}_T \cdot \hat{n}_T| , \qquad (30)$$

where $\hat{n}$ and $\hat{n}_T$ are mutually orthogonal unit vectors in the transverse-momentum plane, and $\hat{n}$ is chosen to maximize $\tilde{\Pi}_T$.

For the nonperturbative events, $\tilde{\Pi}_T \ll \Pi_T$; however, for the order α_s perturbative events, $\tilde{\Pi}_T = \Pi_T$, so that to order α_s

$$\frac{d\Sigma}{dx_H dy\, d\tilde{\Pi}_T} = \frac{d\Sigma}{dx_H dy\, d\Pi_T} . \qquad (31)$$

Further, the variable $\tilde{\Pi}_T$ satisfies the criterion stated in the Introduction, so that the distribution of $\tilde{\Pi}_T$ computed in perturbation theory is independent of the parton decay functions and should be free of logarithmic singularities. Thus interpreting our results as a $\tilde{\Pi}_T$ distribution should significantly attenuate the nonperturbative effects.

V. CONCLUSIONS

QCD corrections to the naive parton model give rise to transverse momenta that scale with Q in lepton-hadron scattering. Using the variable Π_T to parametrize this transverse momentum makes a knowledge of the parton decay functions unnecessary. Further, it gives a quantity which should be free of logarithmic singularities in higher orders of perturbation theory. In the present μ-p experiment, there is a range of kinematic variables ($x_H \lesssim 0.3$, $y \gtrsim 0.5$, $\Pi_T \gtrsim 8$ GeV) for which nonperturbative effects should not spoil this prediction.

ACKNOWLEDGMENTS

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[3]E. Farhi, Phys. Rev. Lett. 39, 1587 (1977).

[4]R. P. Feynman, *Photon-Hadron Interactions* (Benjamin, Reading, Mass., 1972).

[5]H. D. Politzer, Nucl. Phys. B129, 301 (1977).

[6]H. D. Politzer, Phys. Rep. 14C, 129 (1974).

[7]We need not worry about the jets produced by the fragments of the hadron; their transverse momentum does not scale with Q.

[8]We neglect the transverse momentum of the initial parton; it does not scale with Q.

[9]We will specialize to the case of electromagnetic scattering. The weak-interaction case is somewhat more complicated, but the qualitative conclusions would be unchanged.

[10]H. Georgi and H. D. Politzer, Phys. Rev. Lett. 40, 3 (1978).

[11]These are the same variables used in Ref. 10.

[12]The $\cos\phi$ terms are of particular interest; they give rise to an azimuthal asymmetry of the jet distribution in the plane orthogonal to $\vec{q}$. However, measuring this would involve resolving the jets. This could be done only at large values of P_T where the spread of produced hadrons about the jet axes does not overcome the separation of these axes.

[13]H. D. Politzer, Nucl. Phys. B122, 237 (1977).

[14]A. De Rújula, H. Georgi, and H. D. Politzer, Ann. Phys. (N. Y.) 103, 315 (1977).

[15]This effect occurs even in the absence of gluons. The numerator of Eq. (23) depends upon $\sum_j Q_j^2 q_j(\xi)$, whereas the denominator depends upon $\sum_j Q_j{}^2 q_j(x_H)$. Thus, although changes in the overall normalization of the quark distribution functions with Q^2 cancel, changes in their shape do not. The problem is most pronounced for large Π_T. Here, the integral in Eq. (21) is dominated by the smallest possible value of ξ: $\xi_{\min} = X_H + (\Pi_T/\Pi_{T\max})^2 (1 - X_H)$. The renormalization-group improved quark distribution functions go like

$$\sum_j Q_j{}^2 q_j(\xi) \sim (1-\xi)^{A + (16/27)\ln\,\ln(Q^2/\Lambda^2)}$$

near $\xi = 1$ (see Ref. 14). Thus at small X_Π,

$$d\Sigma/dx_H dy \sim (1-\xi_{\min})^{A + (16/27)\ln\,\ln(Q^2/\Lambda^2)} .$$

At $Q^2 = 10\ \text{GeV}^2$ (a typical SLAC value), $A + \frac{16}{27}\ln\ln(Q_2/\Lambda^2) = 3$, which we use for all Q^2. However, for $x_H = 0.2$, $y = 1$, we get $Q^2 = 64\ \text{GeV}^2$, in which case the exponent becomes 3.24. Thus for $\Pi_T/\Pi_T^{\max} = 0.8$, the use of Q^2-dependent distribution functions would decrease $d\Sigma/d x_H dy$ by 22%, the effect falling rapidly with decreasing Π_T (14 GeV), $d\Sigma/dx_H dy$ is already too small to be interesting.

[16]G. Altarelli and G. Martinelli, Roma report, 1978 (unpublished).

[17]Another possible nonperturbative effect arises from the inclusion of the primordial transverse momentum of the incident partons (p_T) in the QCD processes of Figs. 1 and 2. Analogous considerations have been shown to have a significant effect on the single-jet transverse-momentum distributions in proton-proton scattering. The addition of nonzero p_T gives rise to a net transverse momentum of the two-jet system of order $\langle p_T \rangle$, altering the single-jet transverse momentum by the same order. However, the effect on Π_T (the *sum* of the jet transverse momenta) is only of order $\langle p_T \rangle \langle p_T \rangle / \Pi_T$. This is because the transverse motion of the two-jet center-of-mass adds almost the same transverse momentum to one jet as it takes from the other.

[18]The proton fragments would also have transverse momentum; however, in the parton model these hadrons would be confined to small z_H (where $z_H = p_1 \cdot p_H / p_1 \cdot q$; p_H is the hadron's momentum). Small z_H's are not detected in the present Fermilab μ-p experiment (Ref. 19).

[19]A. Loomis *et al.*, Phys. Rev. Lett. 35, 1483 (1975).

[20]Z. Koba, H. B. Nielsen, and P. Oleson, Nucl. Phys. B40, 317 (1972).

[21]P. Slattery, Phys. Rev. Lett. 29, 1624 (1972).

[22]There is no good reason to expect ψ to be the same in electroproduction as it is in hadron-hadron collisions.

[23]This number depends only weakly on kinematics. We take it to be a constant.