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Hadron-jet production in high-energy electron-positron annihilation

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A simple model is developed for the conversion of an initially produced quark-antiquark pair into hadron jets in electron-positron annihilation. The model assumes that only two quark flavors (u and d) are produced, both quarks have the same mass, and all final-state hadrons are pions. In the model, hadrons are produced in two oppositely directed jets in the center-of-momentum (c.m.) frame of the annihilating electron and positron, and the average hadron momentum transverse to the jet direction is constant. The average multiplicity of charged hadrons and the average energy per charged hadron rise as $s^{1/4}$ at high energies, where s is the square of the total c.m. energy. If the component of the final-state hadron momentum parallel to the jet direction is denoted by $p_{\parallel}$, and the average value of the momentum component perpendicular to the jet axis is calculated for hadrons with different values of $x_{\parallel} = 2p_{\parallel}/s^{1/2}$, a seagull effect is observed. Particle-density distributions of the charged final-state hadrons can also be calculated as a function of $x_{\parallel}$. These distributions rise to a peak as $x_{\parallel} \to 0$ and are not strongly dependent on the c.m. energy.

INTRODUCTION

Experimental data on hadron production in high-energy electron-positron data have been reviewed by Hanson[1] and by Wiik and Wolf.[2] The data are consistent with the assumption that hadrons are produced in two jets developing from a single quark-antiquark pair (each member of the pair carrying spin $\frac{1}{2}$) generated from the one-photon intermediate state resulting from electron-positron annihilation. This paper is an initial investigation of a simple model for the metamorphosis of the parent quark and antiquark into two jets of hadrons.

The purpose of this study is to demonstrate that a simple model for the conversion of the initially produced quarks into hadron jets is capable of reproducing the general features of the experimental data. Consequently, it is assumed for simplicity that only two flavors of quarks (u and d) are produced, both with an effective mass m_Q equal to one-third of the proton mass, and that all final-state hadrons are pions. At least two flavors of quarks are necessary to allow for the production of charged hadrons.

PHYSICAL PICTURE

The model is based on the ideas of colored-quark theory, as sketched, for example, in a recent popular article by Nambu.[3] The initial stages of hadron production in high-energy electron-positron annihilation are assumed to proceed through processes such as those schematically diagrammed in the example shown in Fig. 1. In Fig. 1 an electron and positron annihilate to create a one-photon intermediate state, with time increasing to the right in the sketch. The photon subsequently creates a quark-antiquark ($Q\bar{Q}$) pair. The force between this initial quark and antiquark is negligible when the distance between them is small and grows when the distance between them gets larger.

If E is the center-of-momentum (c.m.) energy of the colliding electron-positron system, the momentum p_i of the initial quark and antiquark resulting from the annihilation process is determined from

$$E = 2(p_i^2 + m_Q^2)^{1/2}. \quad (1)$$

Since the model aims at describing interactions where E is greater than about 2 GeV and m_Q is taken as one-third of the proton mass, the initial quark and antiquark move apart with a velocity approximately equal to the velocity of light and a relative momentum of magnitude $2|\vec{p}_i|$.

When the initial quark and antiquark have moved apart to a distance such that the potential energy of the interaction between them exceeds the energy needed to create a $Q\bar{Q}$ pair, another quark-antiquark pair is created out of the potential energy

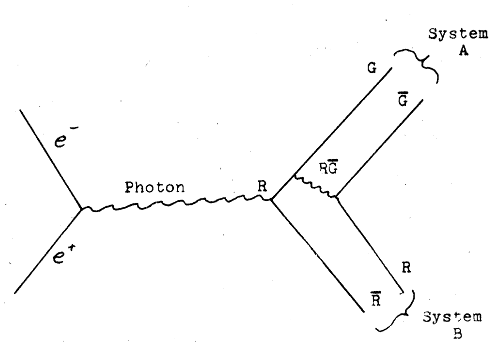

FIG. 1. Creation of hadron-jet precursors in electron-positron annihilation. The color labels R (red) and G (green) are chosen only as an example.

contained in the gluon field mediating the force between the initial $Q\bar{Q}$ pair. After this happens, the final-state hadron jets develop from the two color-neutral pairs which fly apart in opposite directions in the c.m. system and no longer interact via the superstrong color forces. Each of these two $Q\bar{Q}$ pairs is therefore the parent or precursor of all the hadrons formed in one of the hadronic jets.

Let system A be the system of hadrons which evolves from the initial quark as shown in Fig. 1. Similarly, let system B denote the system of hadrons evolving from the initial antiquark. Since the u and d quarks have the same mass and are equally likely to be produced, in 50% of the events both members of the $Q\bar{Q}$ pair which is the precursor of jet A will have the same flavor; the same will be true of jet B, and neither jet will have a net electric charge. In the remaining 50% of the events the members of the $Q\bar{Q}$ pair which is the precursor of jet A will have different flavors and the same will be true for jet B, resulting in one jet with net charge $+e$ and one jet with net charge $-e$.

The potential energy V_{AB} of the interaction between systems A and B as a function of the distance r_{AB} between their centers of momentum will behave roughly as shown in Fig. 2. The linear growth of the potential with distance which is shown in Fig. 2 is only for illustration, and the exact shape of the increase of the potential energy with distance has little effect on the argument. As the initially created colored quark and antiquark move apart, the potential energy arising from the interaction between them rises until it reaches a value V_F, when an additional $Q\bar{Q}$ pair is created by a mechanism similar to that shown in Fig. 1, resulting in two color-neutral $Q\bar{Q}$ pairs which no longer interact through the superstrong color force. Consequently, when systems A and B are separated by distances greater than the distance r_F at which the second $Q\bar{Q}$ pair is formed, the interaction between the two systems can be neglected in comparison with the superstrong color force. Since the force is essentially zero after systems A and B are separated by a distance greater than r_F, the force between systems A and B acts for only a finite time as the systems fly apart. The change in relative momentum Δp_{AB} experienced during this time is given by

$$\Delta p_{AB} = \int_0^{t_F} F_{AB}\, dt\,, \tag{2}$$

where F_{AB} is the force between systems A and B and t_F is the elapsed time between the formation of the initial $Q\bar{Q}$ pair at $t = 0$ and the formation of the second $Q\bar{Q}$ pair when the force between systems A and B becomes negligible. Since the relative velocity of systems A and B is approximately equal to the velocity of light, $dr_{AB} = c\,dt$, and (in units where $c = 1$)

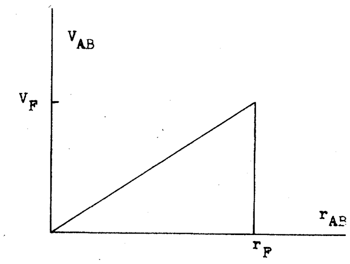

FIG. 2. Schematic representation of the potential V_{AB} between the two hadronic-jet systems (A and B) produced in electron-positron annihilation as a function of the distance r_{AB} between their centers of momenta.

$$\Delta p_{AB} = \int_0^{t_F} F_{AB}\, dt = -\int \frac{dV_{AB}}{dr_{AB}}\, dr_{AB} = -\int_0^{V_F} dV_{AB} = -V_F\,. \tag{3}$$

Since the $Q\bar{Q}$ pair formed out of the potential energy of interaction between the initial quark and antiquark is formed within a volume with the (small) characteristic dimension r_F, the uncertainty principle demands that the members of the $Q\bar{Q}$ pair formed from the potential energy of interaction have a minimum momentum denoted p_Q. It can therefore be assumed that

$$V_F = 2(p_Q^{\,2} + m_Q^{\,2})^{1/2}\,. \tag{4}$$

Consequently,

$$\Delta p_{AB} = -2(p_Q^{\,2} + m_Q^{\,2})^{1/2}\,, \tag{5}$$

and the magnitude p' of the final momentum of systems A and B after the color force between those two systems ceases to act (i.e., after the second $Q\bar{Q}$ pair is formed, producing two color-neutral pairs) is given by

$$p' = p_i - (p_Q^{\,2} + m_Q^{\,2})^{1/2}\,. \tag{6}$$

The invariant mass m^* of systems A and B is then given by

$$E = 2(p'^2 + m^{*2})^{1/2}\,. \tag{7}$$

p' is the total momentum of each of the two hadron jets resulting from electron-positron annihilation and m^* is the invariant mass of each jet of hadrons.

Production of hadrons in electron-positron annihilation is assumed to occur through isotropic decay in the rest frame of the "fireballs" of invariant mass m^* (systems A and B) into final-state hadrons through the production of additional $Q\overline{Q}$ pairs. Therefore, at least two hadrons will be produced in each jet. Since, in the average event, the final-state hadrons are formed within a volume with characteristic dimension related to r_F (the distance between a quark and an antiquark at which it becomes energetically favorable to form an additional $Q\overline{Q}$ pair out of the potential energy of interaction between the original pair) the uncertainty principle requires the final-state hadrons produced in the average event to have a minimum momentum with magnitude p_H in the rest frame of the jet system (system A or system B). In this model, a detailed description of the evolution of the fireballs (systems A and B) into final-state hadrons is not provided. Instead, it is assumed that the average number $\overline{n}$ of final-state hadrons (pions) produced in each jet in an electron-positron annihilation event is given by

$$\overline{n}=\frac{m^*}{(p_H{}^2+m_\pi{}^2)^{1/2}}\,. \tag{8}$$

The number of hadrons created in a single jet in the average event *in addition to* the minimum two hadrons is given by

$$\mu=\frac{m^*-2(p_H{}^2+m_\pi{}^2)^{1/2}}{(p_H{}^2+m_\pi{}^2)^{1/2}}\,. \tag{9}$$

It is also assumed that the probability of producing n additional hadrons in a given jet follows a Poisson distribution with mean μ, truncated at $n_{\max}=m^*/m_\pi$. The Poisson distribution is assumed because of its simplicity and because it seems that the production of final-state hadrons within the volume surrounding each fireball c.m. should satisfy the following conditions, which can be used to derive the Poisson distribution:

The probability of producing a hadron in an incremental volume ΔV is approximately $\lambda\Delta V$ for any ΔV, where λ is some small parameter.

The probability of producing two hadrons in a volume ΔV is negligible compared to $\lambda\Delta V$.

The number of hadrons produced in any volume increment is independent of the number produced in any nonoverlapping volume element.

A detailed description of the evolution of the two fireballs (systems A and B) is not incorporated into this model for the following reason. After the initial $Q\overline{Q}$ pair is created from the one-photon intermediate state, the second $Q\overline{Q}$ pair is created somewhere in the gluon field acting between the initial pair, but the exact location of the pair formation event is not known. So, although systems A and B each develop from a $Q\overline{Q}$ pair moving apart at relatively large momenta, the pairs are a finite distance apart when the process begins (unlike the initial pair). Consequently, the lower limit of an integration over r_{AB} (or V_{AB}) in Eq. (3) is unknown, and the analysis cannot be carried out as it was for the initial pair.

The invariant mass m^* of a hadron jet in this model is determined [via Eqs. (6) and (7)] by the momentum p_Q of the second $Q\overline{Q}$ pair formed in the production process. A fixed value is taken for p_Q in this paper, so the invariant mass m^* is uniquely determined by the quark mass, the c.m. energy, and p_Q. A more realistic model, with a distribution in the invariant mass of hadron jets at a given c.m. energy, could be constructed by assuming that the momenta of the members of the second $Q\overline{Q}$ pair followed a probability distribution peaked around $|p_Q|$. This would leave the essential features of the model unchanged.

MATHEMATICAL MODEL

Based on the assumptions discussed previously, the probability of producing n hadrons in a single jet with invariant mass m^* is

$$P_1(n)=\mu^{n-2}\frac{e^{-\mu}}{(n-2)!}\left(\sum_{n=2}^{n_{\max}}\mu^{n-2}\frac{e^{-\mu}}{(n-2)!}\right)^{-1},\quad 2\leq n\leq n_{\max} \tag{10}$$

where

$$\mu=[m^*-2(p_H{}^2+m_\pi{}^2)^{1/2}]/(p_H{}^2+m_\pi{}^2)^{1/2}$$

and

$$n_{\max}=m^*/m_\pi\,.$$

Because charged particles are easier to detect than neutral particles, most of the experimental information on electron-positron annihilation into hadrons involves the properties of charged-particle multiplicity and momentum distributions. Therefore, it is necessary to calculate the probability of an electron and a positron annihilating into m charged hadrons. It was noted earlier in this model that 50% of all electron-positron annihilation events producing hadrons involve hadronic jets with no net electric charge. The remaining 50% of events involve hadronic jets with a net electric charge $+e$ and $-e$, respectively. In the first case, since charge conservation requires that charged particles are produced in pairs, at least one of the jet hadrons must be neutral if an odd number of hadrons are produced in the jet. Similarly, in the second case, if an even number of particles is produced in a jet, at least one of them must be neutral, since each jet has a net electric charge. If i_{ch} takes on the values 0 and 1, denoting the absolute value of the net electric charge of each jet, the probability of finding k hadron pairs in a single jet which could potentially be additional charged-hadron pairs can be written as

$$P_1^*(k) = P_1(2k + i_{ch}) + P_1(2k + 1 + i_{ch}) . \tag{11}$$

Assuming that the probability of one of the additional pairs of hadrons being charged is $\frac{2}{3}$, the probability of producing m charged hadrons in a single hadron jet is

$$P_1^{ch}(m) = \sum_{i=j}^{n_{max}/2} \left(\frac{2}{3}\right)^j \left(\frac{1}{3}\right)^{i-j} \left(\frac{i!}{j!(i-j)!}\right) P_1^*(i) , \tag{12}$$

where $m = 2j + i_{ch}$.

When $i_{ch} = 0$, the probability of producing m^- negative hadrons in one of the fireballs is

$$P_n(m^-) = P_1^{ch}(2m^-) \tag{13}$$

since each charged pair contains one negative particle. When $i_{ch} = 1$, the probability of finding m^- negative hadrons in the jet with a net negative charge is

$$P_{n1}(m^-) = P_1^{ch}(2m^- - 1) , \tag{14a}$$

and the probability of finding m^- negative particles in the jet with a net positive charge is

$$P_{n2}(m^-) = P_1^{ch}(2m^- + 1) . \tag{14b}$$

The probability of producing a total of n charged hadrons or negative charged hadrons in an electron-positron annihilation event is obtained by convoluting the corresponding probability for the individual jets. Consequently, the probability of producing a total of n charged hadrons in an electron-positron annihilation event is given by

$$P^{ch}(n) = \tfrac{1}{2} \sum_{\substack{k=0 \\ k \text{ even}}}^{n} P_1^{ch}(n-k) P_1^{ch}(k) + \tfrac{1}{2} \sum_{\substack{k=1 \\ k \text{ odd}}}^{n} P_1^{ch}(n-k) P_1^{ch}(k) , \tag{15}$$

where n ranges over even numbers up to $2n_{max}$, while the probability of producing m^- negative particles is

$$P_n(m^-) = \tfrac{1}{2} \sum_{j=0}^{m^-} [P_n(m^- - j) P_n(j) + P_{n1}(m^- - j) P_{n2}(j)] . \tag{16}$$

Using these distributions, the average number of charged hadrons produced in an annihilation event is

$$\langle n_{ch} \rangle = \sum_{n=0}^{2n_{max}} n P^{ch}(n) . \tag{17}$$

After calculating $\langle n_{ch}{}^2 \rangle = \sum_{n=0}^{2n_{max}} n^2 P^{ch}(n)$, the correlation moment can be obtained from

$$f_2^{ch} = \langle n_{ch} \rangle^2 - \langle n_{ch} \rangle^2 - \langle n_{ch} \rangle . \tag{18}$$

If the distribution $P_n(m^-)$ is used to determine the average number of negative particles $\langle n_- \rangle$ and $\langle n_-{}^2 \rangle$, the correlation moment for negative particles can be calculated from

$$f_2^- = \langle n_-{}^2 \rangle - \langle n_- \rangle^2 - \langle n_- \rangle . \tag{19}$$

The number density of hadrons produced with different values of the component of momentum parallel to the jet direction $p_\parallel$ can be readily calculated in this model. The following discussion describes the way in which the number density of all particles is obtained as a function of $x_\parallel = 2p_\parallel / E$. The procedure for calculating the experimentally measured charged-particle density is slightly more complicated, but the idea is the same.

Suppose a system of mass m^* decays isotropically in its own rest frame into j hadrons with momentum p_j determined from $m^* = j(p_j{}^2 + m_\pi{}^2)^{1/2}$. If the momentum of the decaying system in the c.m. frame of an annihilating electron and positron is $\vec{p}'$, an analysis such as that described by Hagedorn[4] indicates that the j hadrons will be distributed evenly along the $\vec{p}'$ direction between

$$x_\parallel^{max}(j) = p_j / m^* + (2p'/m^*E)(p_j{}^2 + m_\pi{}^2)^{1/2} \tag{20}$$

and

$$x_\parallel^{min}(j) = -p_j / m^* + (2p'/m^*E)(p_j{}^2 + m_\pi{}^2)^{1/2} . \tag{21}$$

Consequently, the number density of all hadrons as a function of $x_\parallel$, with $x_\parallel$ between zero and 1, is

$$\frac{dn(x_\parallel)}{dx_\parallel} = m^* \left[\sum_j (j/p_j) P_1(j) + \sum_{j'} (j'/p_{j'}) P_1(j') \right] . \tag{22}$$

This expression includes contributions from both jets. The sum in the first term runs over all values of j between 2 and n_{max} which are such that

$$x_\parallel^{min}(j) \le x_\parallel \le x_\parallel^{max}(j) . \tag{23}$$

The sum in the second term, which takes account of backward hemisphere hadrons in the jet opposite the one containing the hadrons counted in the first term, runs over all values of j' such that

$$0 \le x_\parallel \le |x_\parallel^{min}(j')| . \tag{24}$$

Using Hagedorn's analysis again, the transverse momentum of hadrons produced at $x_\parallel$ by the isotropic decay of a fireball of mass m^* and momentum p' into j hadrons is

$$p_T(x_\parallel, j) = p_j{}^2 - [m^* x_\parallel - (2p'/E)(p_j{}^2 + m_\pi{}^2)^{1/2}]^2 , \tag{25}$$

where E is the total c.m. energy of the annihilating electron-positron system. Therefore, the average transverse momentum perpendicular to the jet direction of final-state hadrons as a function of $x_{\parallel}$ is

$$p_T(x_{\parallel}) = \frac{m^*}{dn(x_{\parallel})/dx_{\parallel}} \left[\sum_j p_T(x_{\parallel}, j)(j/p_j)P_1(j) + \sum_{j'} p_T(x_{\parallel}, j')(j'/p_{j'})P_1(j') \right], \tag{26}$$

where the sums over j and j' are restricted as in the inequalities (23) and (24).

RESULTS

This Section reports the results of some simple computer calculations using the model developed in this paper. Besides the pion mass, the only parameters appearing in the model are m_Q, p_H, and p_Q. The mass of the u and d quarks, m_Q, is taken as $\frac{1}{3}$ of the proton mass. The momentum of a hadron produced in the average event in the fireball rest frame p_H is chosen as 440 MeV/c. This is done to guarantee that the average momentum transverse to the jet direction of the final-state hadrons is 360 MeV/c. For simplicity, p_Q, the momentum in the electron-positron c.m. of the quark and antiquark produced out of the potential energy of the interaction between the original $Q\overline{Q}$ pair is set equal to p_H. Thus, in this calculation, Δp_{AB}, the impulse resulting from the action of the color force between system A and B has the value 1.08 GeV/c.

The total jet momentum p' and the invariant mass m^* of each jet can be obtained from Eqs. (6) and (7), respectively. According to Eq. (8), the average multiplicity of each jet is proportional to m^*. Using Eq. (7), the average multiplicity in this model grows as $p_i^{1/2} \approx s^{1/4}$ at high energies, where s is the square of the total c.m. energy. Figure 3 shows the average charged multiplicity calculated from Eq. (17), together with the experimental data points and error bars provided by Albini et al.[5]

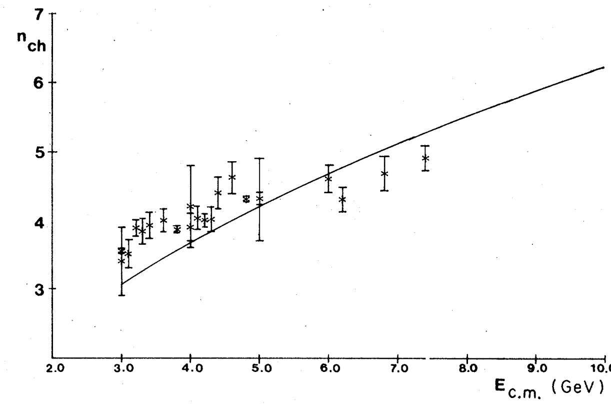

FIG. 3. The solid line is a cubic spline curve fitted to the calculated values of the average charged multiplicity for c.m. energies from 3 to 10 GeV. The data points and error bars are taken from Ref. 5.

The average c.m. energy per secondary hadron is

$$\overline{E} \approx [(p'/\langle n\rangle)^2 + \tfrac{2}{3}p_H^2 + m_\pi^2]^{1/2}.$$

TABLE I. Results of computer calculations for c.m. energies ($E_{c.m.}$) between 2.5 and 10.0 GeV. The quantities calculated are m^*, the invariant mass of each hadron jet; p', the total momentum of each hadron jet; $\langle n_{ch}\rangle$, the average charged multiplicity; $\overline{E}$, the average c.m. energy carried by each final-state hadron; f_2, the correlation moment for all charged final-state hadrons; and f_2^-, the correlation moment for negative charged hadrons.

$E_{c.m.}$ (GeV)	m^* (GeV)	p' (GeV/c)	$\langle n_{ch}\rangle$	$\overline{E}$ (GeV)	f_2	f_2^-
2.5	1.06	0.67	2.71	0.48	−1.17	−0.97
3.0	1.18	0.93	3.07	0.53	−0.99	−1.01
3.5	1.29	1.18	3.39	0.57	−0.84	−1.06
4.0	1.39	1.44	3.69	0.61	−0.71	−1.10
4.5	1.49	1.69	3.96	0.65	−0.60	−1.14
5.0	1.57	1.94	4.22	0.69	−0.51	−1.18
5.5	1.66	2.19	4.46	0.72	−0.42	−1.22
6.0	1.74	2.44	4.69	0.75	−0.33	−1.26
6.5	1.82	2.69	4.91	0.79	−0.26	−1.29
7.0	1.89	2.95	5.12	0.82	−0.19	−1.33
7.5	1.96	3.20	5.33	0.85	−0.12	−1.36
8.0	2.03	3.45	5.52	0.87	−0.05	−1.39
8.5	2.09	3.70	5.71	0.90	0.01	−1.42
9.0	2.16	3.95	5.90	0.93	0.07	−1.46
9.5	2.22	4.20	6.07	0.95	0.13	−1.49
10.0	2.28	4.45	6.25	0.98	0.19	−1.51

Since $\langle n \rangle$ grows as $p_i^{1/2}$ at high energy and p' grows as p_i, the average energy per secondary grows as $p_i^{1/2} \approx s^{1/4}$ at high energy. This is necessary since $\langle n \rangle \overline{E} = E = s^{1/2}$.

For c.m. energies between 2.5 and 10 GeV, Table I shows m^*, the invariant mass of one of the two hadron jets produced in electron-positron annihilation, the total jet momentum p', the average charged multiplicity $\langle n_{ch} \rangle$, the average energy per final-state hadron $\overline{E}$, the correlation moment for all charged final-state hadrons f_2 and the correlation moment for negative charged hadrons f_2.

Isotropic decay of a fireball with mass m^* into n hadrons (pions) leads in the fireball rest frame to a spherical decay spectrum[4] in momentum space with radius ρ^2 given by $E^2 = n^2(\rho^2 + m_\pi^2)$. In the electron-positron c.m. frame the spherical fireball decay spectra become ellipsoids of revolution. For a given decay multiplicity, $x_\parallel$ bins between the maximum and minimum longitudinal momentum along the jet axis have a uniform occupation probability.

Figure 4 shows $\log_{10}$ of the number density of charged final-state hadrons as a function of $x_\parallel = 2p_\parallel/E$. In this model the number density behaves in the same way as the experimental data shown in Fig. 11 of Ref. 1, rising slowly in each $x_\parallel$ bin as the c.m. energy increases. In Fig. 4 the asterisks denote predictions of the model for a c.m.

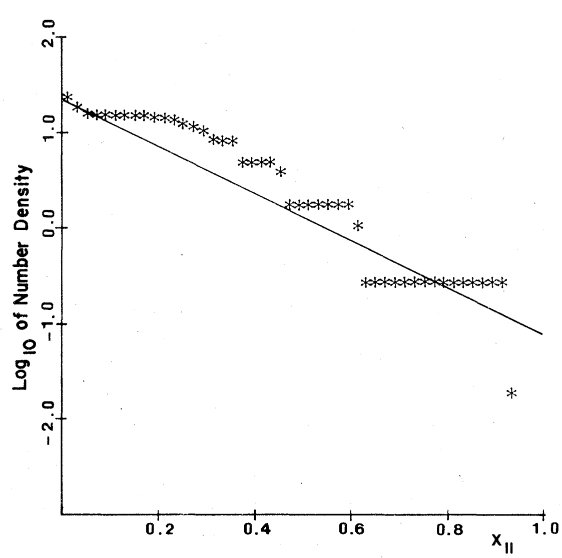

FIG. 4. $\log_{10}$ of the number density of charged final-state hadrons as a function of $x_\parallel = 2p_\parallel/E$. The asterisks denote predictions of the model for a c.m. energy of 7.5 GeV, while the straight line indicates the approximate trend of the experimental data for c.m. energies between 7.0 and 7.8 GeV as presented in Fig. 11 of Ref. 1. The flat plateaus at large $x_\parallel$ are the contributions of the forward part of the elliptical momentum decay spectra for low-multiplicity fireball decays. Bins with smaller values of $x_\parallel$, on the other hand, receive contributions from higher multiplicity fireballs.

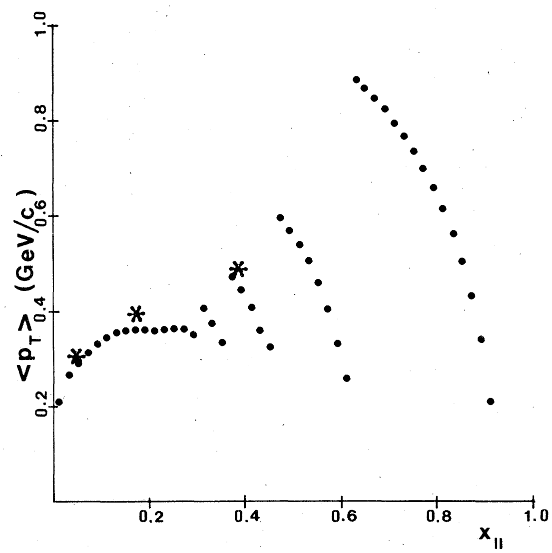

FIG. 5. The average of the component of the final-state hadron momentum (in GeV/c) transverse to the jet direction, plotted as a function of $x_\parallel = 2p_\parallel/E$. The data points indicated by asterisks are taken from Fig. 24 of Ref. 1 and the dots show the predictions of the model for a c.m. energy of 7.5 GeV. The oscillations in the model predictions at values of $x_\parallel$ above about 0.3 are the forward portions of the elliptical momentum decay spectra for low-multiplicity fireball decays. These fluctuations would be strongly damped in a model with a distribution of fireball invariant masses instead of the uniquely determined fireball mass used in this model.

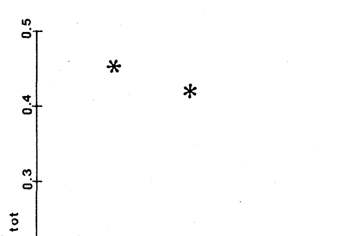

FIG. 6. Fractional charged-particle topological cross sections for $n_{ch} = 2$, 4, 6, and 8 at a c.m. energy of 3 GeV.

energy of 7.5 GeV, while the straight line indicates the approximate trend of the experimental data for c.m. energies between 7.0 and 7.8 GeV as presented in Fig. 11 of Ref. 1. The flat plateaus at large $x_{\parallel}$ are the contributions of the forward part of the momentum ellipse for low multiplicity fireball decays. In a more realistic model with a distribution of fireball masses, these plateaus would disappear. Bins with smaller values of $x_{\parallel}$ receive contributions from higher multiplicity fireballs, so no plateaus appear.

Figure 5 shows the average final-state hadron momentum transverse to the jet direction as a function of $x_{\parallel} = 2p_{\parallel}/E$. The data points indicated by asterisks are taken from Fig. 24 of Ref. 1 and the dots show the predictions of the model for a c.m. energy of 7.5 GeV. The oscillations in the model predictions at values of $x_{\parallel}$ above about 0.3 are the forward portions of the momentum ellipse for low multiplicity fireball decays. These fluctuations would be strongly damped in a model with a distribution of fireball invariant masses instead of the uniquely determined fireball mass used in this model.

Figures 6, 7, 8, and 9 show the fractional charged-prong topological cross sections for low-multiplicity events for four values of the total c.m. energy between 3 and 7.5 GeV.

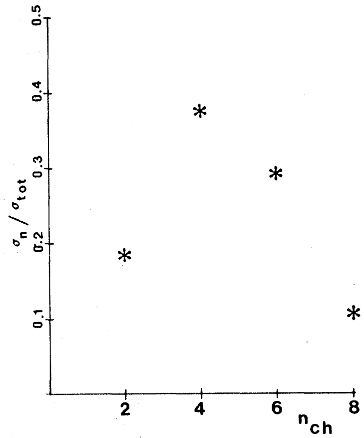

FIG. 8. Fractional charged-prong topological cross sections for $n_{ch} = 2$, 4, 6, and 8 at a c.m. energy of 6 GeV.

RELATIONSHIP TO OTHER MODELS

Earlier statistical models (Margolis *et al.*,[6] Carlson *et al.*[7]) and hydrodynamical models (Cooper *et al.*[8]) of electron-positron annihilation were reasonably successful in generating multiplicity distributions, but did not reproduce the jet structure which was subsequently found to be one of the most prominent features of e^+e^- annihilation processes.

The simple jet model presented in this paper suffers from the fact that it is not relativistically invariant. However, it seems likely that a relativistic model with similar features could be constructed along the lines of the "hadron ball" model of Eichmann, Elvekjaer, and Steiner.[9] Such a model would involve two "jet hadron balls" ($M_0 = 0$ in the notation of Ref. 9) decaying to hadrons with an invariant thermodynamic decay pattern. A jet mass distribution (dp/dM in the notation of Ref. 9) peaked around $M = s^{1/4}(p_Q^2 + m_Q^2)^{1/4}$ would lead to an $s^{1/4}$ growth in the average charged multiplicity such as that found in this paper and a total

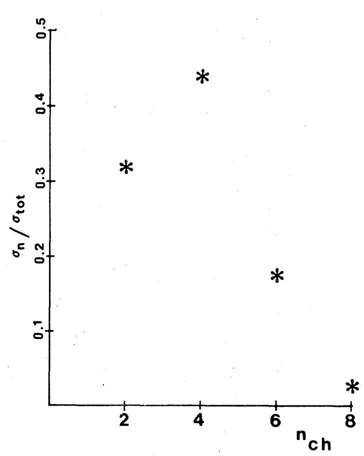

FIG. 7. Fractional charged-prong topological cross sections for $n_{ch} = 2$, 4, 6, and 8 at a c.m. energy of 4 GeV.

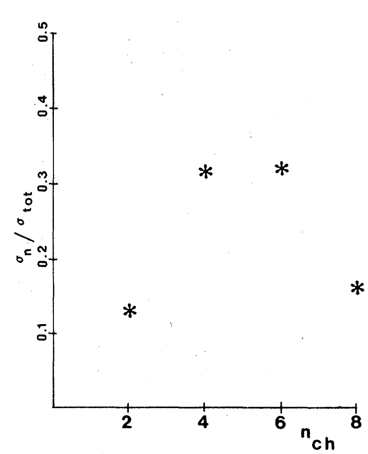

FIG. 9. Fractional charged-prong topological cross sections for $n_{ch} = 2$, 4, 6, and 8 at a c.m. energy of 7.5 GeV.

fireball momentum ($|Q|$ in the notation of Ref. 9) which is approximately $p_i - (p_Q{}^2 + m_Q{}^2)^{1/2}$, where p_i is the momentum of the initial quark and antiquark as found from Eq. (1).

A heuristic but relativistic formulation of the details of the evolution of quarks into hadrons has been given by Bjorken[10] and expanded upon by Brodsky and Weiss,[11] but it is not clear how their approach could be used to develop a model related to the one sketched in this paper.

CONCLUSION

A simple model has been developed for hadron production in electron-positron annihilation which roughly reproduces the experimental data. An extension of this model to incorporate additional quark flavors would require a model predicting the probability of producing higher-mass quarks. The corresponding extension of the model to the production of additional types of hadrons requires a model giving the percentage of different types of hadrons above the various production thresholds. However, in general, including other kinds of hadrons should lead to an increasing fraction of the c.m. energy going into neutral particles as the c.m. energy increases, as a result of kaon, meson-resonance, and baryon production.

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