Low-energy photo- and electroproduction for physical pions. II. Photoproduction phenomenology and extraction of the quark mass

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The background resonance $\Delta(1230)$, $N^*(1520)$, $N^*(1470)$, and $N^*(1535)$ pion and axial-vector amplitudes are first calculated in the soft-pion and on-shell configurations, respectively. Then a comparison is made with the usual soft-pion theorems and on-shell low-energy expansions of current algebra as worked out in the previous paper. The agreement is good, and we also deduce a nucleon dipole form-factor axial-vector mass of $m_A \approx 1.23$ GeV. Finally, an approximate value for the nonstrange current quark mass of $\hat{m} = 0.64 \pm 1.11\ \mu$ is extracted from the data.

I. INTRODUCTION

Given the theoretical analysis of the previous paper,[1] we are prepared to make a comparison with the data. To do this we must make use of the pion and radiative couplings of the baryon resonances which dominate the photoproduction backgrounds: the $\Delta(1230)$, $N^*(1520)$, $N^*(1470)$, and $N^*(1535)$. The required covariant resonance couplings are worked out in Appendix A. In Sec. II we approximate the soft-*pion* background amplitudes by these isobar resonance poles and find excellent agreement with the standard soft-pion theorems.[2,3]

Then we proceed to compare the on-shell photoproduction amplitudes with the data. We analyze threshold photoproduction data here with the idea of testing the on-shell low-energy theorems of I. Our threshold electroproduction current-algebra theorems will be analyzed elsewhere.[4] The background *axial-vector* amplitudes $B_j^{(+,-,0)}$ are obtained in Appendix B using both dispersion-theoretic and field-theoretic techniques with results which differ little in magnitude. In Sec. III these background amplitudes are decomposed into pole and nonpole parts for general values of the variables ν and t, but with $q_\pi^{\,2} = \mu^2$. Finally, in Sec. IV these background amplitudes are substituted into the low-energy current-algebra photoproduction amplitudes of (1) in Ref. 1. Good agreement with threshold data is found for $A_3^{(+,0)}$, $A_4^{(+,-,0)}$, and $A_1^{(-)}$. The threshold amplitude $A_3^{(-)}$ leads to a value for the axial-vector mass of $m_A = 1.31$ GeV assuming a dipole structure for the axial-vector-nucleon form factor, in good agreement with a recent neutrino scattering analysis[5] and with threshold electroproduction.[4]

This encourages us to make a detailed investigation of the nonstrange current-quark mass $\hat{m}$ occurring in the chiral-symmetry-breaking Σ terms in $A_1^{(+,0)}$. Looking at threshold multipole analyses for these two amplitudes, we find in Sec. IV that $\hat{m} \approx 0.64 \pm 1.11\ \mu$. While the quark mass is not well determined, the fact that it must be of $O(\mu)$ and *positive* on average is consistent with the almost exact agreement of the soft theorems of Sec. II when saturated by the isobar poles. In Sec. V we summarize the situation and note the need for more accurate low-energy photoproduction data.

II. THE SOFT-PION AMPLITUDES

We begin by testing the soft-pion theorems. We use the off-shell analog of Eq. (1a) in Ref. 1 for photoproduction ($k^2 = 0$):

$$\bar{A}_1^{(+,0)}(\nu, t, q^2) = -\frac{g_A(q^2)\kappa^{V,S}}{4mf_\pi} - \frac{q^2}{\mu^2}\,\frac{\Sigma_1^{(+,0)}}{f_\pi(m_V^{\,2}+\mu^2-q^2)} + B_1^{(+,0)}(\nu,t,q)\,, \tag{1}$$

where $\bar{A}_1 = A_1 - (A_1)_{\rm pole}$. We are interested in the soft-pion limit ($q \to 0$) corresponding to $\nu = t = 0$, so we recover the soft-pion result[2] from (1),

$$\bar{A}_1^{(+,0)}(0) = -\frac{g_A\kappa^{V,S}}{4mf_\pi} = -\frac{g(0)\kappa^{V,S}}{4m^2}\,, \tag{2}$$

where we have used the Goldberger-Treiman relation $mg_A = f_\pi g(0)$. Recall that $B_1^{(+,0)}$ comes from the axial-vector background term $q^\mu \bar{M}_{\mu\nu}^{(+,0)}$ which vanishes as $q \to 0$. Since K_ν^1 does not vanish as $q \to 0$, $B_1^{(+,0)}$ must do so. Then using the values $g(0) = 12.6$, $\kappa^V = 3.7$, $\kappa^S = -0.12$, we find

$$\bar{A}_1^{(+)}(0) = -\frac{11.6}{m^2} = -0.258\ \mu^{-2}\,, \tag{3a}$$

$$\bar{A}_1^{(0)}(0) = \frac{0.4}{m^2} = 0.008\ \mu^{-2}\,. \tag{3b}$$

We expect that $\overline{A}_1^{(+)}(0)$ will be dominated by the $\Delta(1230)$ intermediate state while there will be contributions to $\overline{A}_1^{(0)}(0)$ from the $N^*(1520)$, $N^*(1470)$, and $N^*(1535)$. Of course, the $\Delta(1230)$ will not contribute to $A_1^{(0)}$ since this amplitude represents $I=1$ pion production from an isoscalar photon which cannot couple to the $I=\frac{3}{2}$ $\Delta(1230)$. In the soft-pion-momentum limit these various isobar (narrow-width) contributions to $A_1^{(+,0)}$ are unique (see Appendix B) and give

$$\overline{A}_{1\,\mathrm{soft}}^{(+)}[\Delta(1230)] = \tfrac{2}{3}g^*\left[-\frac{m(M+m)}{3M^2}G_1 + \frac{M^2-m^2}{6M}G_2\right] = -\tfrac{1}{6}g^*G_M^*\,\frac{M+m}{mM}\,, \tag{4a}$$

$$\overline{A}_{1\,\mathrm{soft}}^{(+,0)}[N^*(1520)] = -\tfrac{1}{4}g_N^* \times\left[\frac{m(M-m)}{3M^2}H_1^{V,S} + \frac{M^2-m^2}{6M}H_2^{V,S}\right], \tag{4b}$$

$$\overline{A}_{1\,\mathrm{soft}}^{(+,0)}[N^*(1470)] = -\frac{g'\kappa'_{V,S}}{2M(M+m)}\,, \tag{4c}$$

$$\overline{A}_{1\,\mathrm{soft}}^{(+,0)}[N^*(1535)] = -\frac{g''\kappa''_{V,S}}{2M(M-m)}\,. \tag{4d}$$

The relative sign of $g^*G_M^*$ vs $g\kappa^V$ is positive as determined by SU_6, consistent with (2). We use data comparison (as given in Sec. IV) to choose the relative signs of the $N^*(1520)$ and the $N^*(1470)$ to $N^*(1535)$ contributions; all relative signs are also consistent with the multipole analysis of Adler and Gilman.[3] The coupling strength $g^*=14.24m^{-1}$ is narrow-width determined from the $\Delta^{++}\to p\pi^+$ decay. In Appendix A the remaining coupling constants in (4) are found to be $G_M^*=3.07$, $G_1=2.90\ \mathrm{GeV}^{-1}$, $G_2=-1.50\ \mathrm{GeV}^{-2}$, $g_N^*=-19.90m^{-1}$, $H_1^S=-1.95\ \mathrm{GeV}^{-1}$, $H_1^V=-3.24\ \mathrm{GeV}^{-1}$, $H_2^S=1.52\ \mathrm{GeV}^{-2}$, $H_2^V=1.33\ \mathrm{GeV}^{-2}$, $g'=-6.18$, $\kappa'_S=0.19$, $\kappa'_V=1.58$, $g''=0.52$, $\kappa''_S=-0.34$, $\kappa''_V=2.22$, where $m=m_N$ and M is the mass of the appropriate isobar. These coupling constants applied to (4) give

$$\begin{aligned}[\overline{A}_{1\,\mathrm{soft}}^{(+)}]_{\mathrm{isobar}} &= \overline{A}_{1\,\mathrm{soft}}^{(+)}[\Delta(1230)] + \overline{A}_{1\,\mathrm{soft}}^{(+)}[N^*(1520)] + \overline{A}_{1\,\mathrm{soft}}^{(+)}[N^*(1470)] + \overline{A}_{1\,\mathrm{soft}}^{(+)}[N^*(1535)]\\ &= (-0.283\pm0.012\ \mu^{-2}) + (-0.005\pm0.007\ \mu^{-2}) + (0.027\pm0.005\ \mu^{-2}) + (-0.012\pm0.002\ \mu^{-2})\\ &= -0.273\pm0.015\ \mu^{-2}\,,\end{aligned} \tag{5a}$$

$$\begin{aligned}[\overline{A}_{1\,\mathrm{soft}}^{(0)}]_{\mathrm{isobar}} &= \overline{A}_{1\,\mathrm{soft}}^{(0)}[N^*(1520)] + \overline{A}_{1\,\mathrm{soft}}^{(0)}[N^*(1470)] + \overline{A}_{1\,\mathrm{soft}}^{(0)}[N^*(1535)]\\ &= (0.009\pm0.007\ \mu^{-2}) + (0.004\pm0.005\ \mu^{-2}) + (0.002\pm0.002\ \mu^{-2})\\ &= 0.015\pm0.009\ \mu^{-2}\,.\end{aligned} \tag{5b}$$

The values of $\overline{A}_1^{(+,0)}$ given by Eqs. (5) are only slightly larger than the PCAC (partial conservation of axial-vector current) predictions (3). This is to be expected since we do not make narrow-width corrections for the various $\Delta N\pi$ and $N^*N\pi$ coupling constants as stressed by Adler and Gilman.[3] Indeed, if narrow-width corrections are made for the Δ *alone* in (5a) (not a consistent procedure), then $\overline{A}_{1\,\mathrm{soft}}^{(+)}$ becomes $-0.243\ \mu^{-2}$, which agrees very well with (3a) and Adler and Gilman, who obtain $\overline{A}_1^{(+)}(0)=-0.236\ \mu^{-2}$ $(V_1\to-2A_1)$.

Isobar saturation of $\overline{A}_1^{(+,0)}$ was first considered by Fubini *et al.*,[6] but the data are better now and the consistency with *both* PCAC theorems is worth emphasizing. From our point of view for what is to follow, the soft theorems provide a test of the relative order of magnitudes and especially the *signs* of the various isobar coupling constants. The good agreement between (3) and (5) gives us further confidence to use the same type of isobar saturation to test the on-shell photoproduction theorems of Ref. 1 in the low-energy region.

III. THE ON-SHELL AXIAL-VECTOR BACKGROUND AMPLITUDE

We develop the background contribution to the pion photoproduction amplitude in terms of the quantities B_j^i defined by the expression

$$if_\pi^{-1}q^\mu\overline{M}_{\mu\nu}^i = \sum_{j=1}^4 B_j^i(\nu,t,k^2=0)K_\nu^j\,, \tag{6}$$

where the K_ν^j are the Chew-Goldberger-Low-Nambu (CGLN) photoproduction covariants. Our plan is to saturate these *axial-vector* background amplitudes B_j for an on-shell kinematical configuration of ν and t and $q^2\equiv\mu^2$ using the same type of isobar poles which were so successful in saturating the soft-*pion* background amplitudes $\overline{A}_1^{(+,0)}$ in Sec. II.

The details of the calculation of the resonant contributions to the photoproduction background amplitudes are given in Appendix B. We concentrate on the subtle spin-$\frac{3}{2}$ $\Delta(1230)$ contribution since from it the spin-$\frac{3}{2}$

$N^*(1520)$ contribution is easily obtainable. The spin-$\frac{1}{2}$ $N^*(1470)$ and $N^*(1535)$ isobar contributions are found in a straightforward manner.

There are two methods which can be used to calculate the spin-$\frac{3}{2}$ $\Delta(1230)$ contribution. One method is a dispersion theory approach which allows the use of the on-shell spin-$\frac{3}{2}$ propagator; alternatively, a field-theory approach necessitates the use of the ambiguous off-shell spin-$\frac{3}{2}$ propagator. Actually, what is usually done in the field-theory approach is to redefine the couplings so that the ambiguous part of the propagator does not contribute. This technique introduces two new parameters (Y at the photon vertex and Z at the pion vertex) which are determined by fitting to data.[7] The pole terms one obtains from these two methods are the same; the two methods differ only in the nonpole terms, as is discussed in Appendix B. The discrepancy between dispersion theory and field theory is only important in $B_4^{(+)}$ and $B_3^{(-)}$; it is of $O(\mu^2)$ in $B_1^{(+)}$. We now list the pole parts of the $\Delta(1230)$ contribution to the background amplitudes as derived in Appendix B.

$$B_{1P}^{(+)}[\Delta(1230)] = \frac{2}{3}\,\frac{g^*\nu_\Delta}{\nu_\Delta^2-\nu^2}\left[\left(-\frac{m(M+m)}{6M^2}(M^2-m^2)-\frac{3M^2+2mM+m^2}{6M^2}\mu^2+k\cdot q\right)G_1 + \left(\frac{(M^2-m^2)^2}{12M}-\frac{(M-m)^2}{12M}\mu^2-mk\cdot q\right)G_2\right], \quad (7a)$$

$$B_{2P}^{(+)}[\Delta(1230)] = \frac{2}{3}\,\frac{g^*\nu_\Delta}{\nu_\Delta^2-\nu^2}\left[G_1+\tfrac{1}{2}(M-m)G_2\right], \quad (7b)$$

$$B_{3P}^{(+)}[\Delta(1230)] = \frac{2}{3}\,\frac{g^*\nu}{\nu_\Delta^2-\nu^2}\left[\left(\frac{3M^2-2mM+m^2}{6M^2}(M+m)-m\frac{\mu^2}{6M^2}\right)G_1 + \left(\frac{5M-m}{12M}(M^2-m^2)+\frac{M-m}{12M}\mu^2-\tfrac{1}{2}k\cdot q\right)G_2\right], \quad (7c)$$

$$B_{4P}^{(+)}[\Delta(1230)] = \frac{2}{3}\,\frac{g^*\nu_\Delta}{\nu_\Delta^2-\nu^2}\left[\left(-\frac{3M^2+2mM-m^2}{6M^2}(M+m)-m\frac{\mu^2}{6M^2}\right)G_1 + \left(-\frac{M+m}{12M}(M^2-m^2)+\frac{M-m}{12M}\mu^2-\tfrac{1}{2}k\cdot q\right)G_2\right], \quad (7d)$$

$$B_{jP}^{(-)}[\Delta(1230)] = -\frac{\nu}{2\nu_\Delta}B_{jP}^{(+)}[\Delta(1230)], \quad j\neq 3 \quad (7e)$$

$$B_{3P}^{(-)}[\Delta(1230)] = -\frac{\nu_\Delta}{2\nu}B_{3P}^{(+)}[\Delta(1230)], \quad (7f)$$

where $M=M_\Delta=1230$ MeV, $m=m_N=939$ MeV, $\nu=\frac{1}{4}(s-u)$, and $\nu_\Delta=\frac{1}{2}(M^2-m^2-k\cdot q)$.

Note that $B_{1P}^{(+)}[\Delta(1230)]$ evaluated at the physical c.m. threshold

$$B_{1P}^{(+)}[\Delta(1230)]\,|_{\text{th}} = -0.335\ \mu^{-2} \quad (8)$$

is $O(\mu/m)$ different from the soft value obtained in Sec. II. This is reasonable since the discrepancy between the isobar denominators for the two cases is of $O(\mu/m)$.

We also note at this time the necessity of introducing the previously discussed nonpole terms. The background amplitude (i.e., $q^\mu \overline{M}_{\mu\nu}^t$) must vanish as $q\to 0$. Since the CGLN covariants K_ν^2, K_ν^3, and K_ν^4 vanish as $q\to 0$, this soft limit places no constraints on B_2, B_3, or B_4. However, K_ν^1 does not vanish as $q\to 0$ and it is therefore necessary for B_1 to vanish in this limit. $B_{1P}^{(+)}$ as given by (7a) does not satisfy this condition, underlying the importance of the extra nonpole term of $B_1^{(+)}$. As is derived in Appendix B, the nonpole term is given for dispersion theory by

$$B_{1NP}^{(+)}[\Delta(1230)] = -\tfrac{2}{3}g^*\left(\frac{1}{3M^2}[-m(M+m)+k\cdot q]G_1 + \frac{1}{6M}(M^2-m^2+k\cdot q)G_2\right), \quad (9a)$$

$$B_{jNP}^{i}[\Delta(1230)] = 0, \quad \begin{cases} i=(+), & j=2,3,4 \text{ or} \\ i\neq(+), & j=1,2,3,4. \end{cases} \quad (9b)$$

The nonpole terms for field theory are given in Ref. 6 as (in our notation)

$$B_{1NP}^{(+)}[\Delta(1230)] = -\tfrac{2}{3}g^*G_1\,\frac{1}{3M^2} \times\left[\tfrac{1}{4}\alpha\beta(t-\mu^2)+\tfrac{1}{4}\beta(5\mu^2-t)-m(M+m)\right] \quad (10a)$$

$$B_{4NP}^{(+)}[\Delta(1230)] = -\tfrac{2}{3}g^*G_1\,\frac{1}{3M^2}\left[\alpha\beta M+\tfrac{1}{2}m(\alpha\beta+\alpha+\beta)\right], \quad (10b)$$

$$B_{1\,\mathrm{NP}}^{(-)}[\Delta(1230)] = \tfrac{1}{3} g^* G_1 \frac{1}{3M^2}\beta(1-\alpha)\nu\,, \tag{10c}$$

$$B_{3\,\mathrm{NP}}^{(-)}[\Delta(1230)] = -\tfrac{1}{2} B_{4\,\mathrm{NP}}^{(+)}[\Delta(1230)]\,, \tag{10d}$$

where $\alpha = 1+4Z$, $\beta = 1+4Y$, and all other $B_{j\,\mathrm{NP}}^{i}$ are zero. Note that Eqs. (10) do not include the G_2 coupling which is included in Eqs. (9); both couplings are necessary to produce the observed $M1$ or G_M^*, Δ dominance. It is easy to see that $B_1^{(+)} = B_{1\mathrm{P}}^{(+)} + B_{1\,\mathrm{NP}}^{(+)}$ vanishes in the soft-pion limit if either (9a) or (10a) is used for $B_{1\,\mathrm{NP}}^{(+)}$. We shall henceforth consider only the dispersive approach.

As is shown in Appendix B, the $N^*(1520)$ contribution is obtained from that of the $\Delta(1230)$ by letting $M \to -M$ (i.e., $M_\Delta \to -M_{N^*}$), $\frac{2}{3} g^* \to \frac{1}{2} g_{N^*}$, $G_1 \to -\frac{1}{2} H_1^{V,S}$, $G_2 \to \frac{1}{2} H_2^{V,S}$, where the H_i^V contribute to the (+) amplitude and the H_i^S to the (0) amplitude. To obtain the contribution to the (−) amplitude the following relations should be used:

$$B_{j\mathrm{P}}^{(-)}[N^*(1520)] = \frac{\nu}{\nu_{N^*}} B_{j\mathrm{P}}^{(+)}[N^*(1520)]\,, \quad j \neq 3 \tag{11a}$$

$$B_{3\mathrm{P}}^{(-)}[N^*(1520)] = \frac{\nu_{N^*}}{\nu} B_{3\mathrm{P}}^{(+)}[N^*(1520)]\,, \tag{11b}$$

where $\nu_{N^*} = \frac{1}{2}(M^2 - m^2 - k\cdot q)$.

Finally, we write down the $N^*(1470)$ contributions:

$$B_{1\mathrm{P}}^{(+,0)}[N^*(1470)] = -g'\kappa'_{V,S} \frac{M-m}{4M} \frac{\nu_{N^*}}{\nu_{N^*}{}^2 - \nu^2}\,, \tag{12a}$$

$$B_{2\mathrm{P}}^{(+,0)}[N^*(1470)] = 0\,, \tag{12b}$$

$$B_{3\mathrm{P}}^{(+,0)}[N^*(1470)] = \frac{g'\kappa'_{V,S}\nu}{4M(\nu_{N^*}{}^2 - \nu^2)}\,, \tag{12c}$$

$$B_{4\mathrm{P}}^{(+,0)}[N^*(1470)] = \frac{g'\kappa'_{V,S}\nu_{N^*}}{4M(\nu_{N^*}{}^2 - \nu^2)}\,, \tag{12d}$$

$$B_{j\mathrm{P}}^{(-)}[N^*(1470)] = \frac{\nu}{\nu_{N^*}} B_{j\mathrm{P}}^{(+)}[N^*(1470)]\,, \quad j\neq 3\,, \tag{12e}$$

$$B_{3\mathrm{P}}^{(-)}[N^*(1470)] = \frac{\nu_{N^*}}{\nu} B_{3\mathrm{P}}^{(+)}[N^*(1470)]\,, \tag{12f}$$

$$B_{1\,\mathrm{NP}}^{(+,0)}[N^*(1470)] = \frac{g'\kappa'_{V,S}}{2M(M+m)}\,, \tag{12g}$$

$$B_{j\,\mathrm{NP}}^{i}[N^*(1470)] = 0 \begin{cases} i = (+,0)\,, & j = 2,3,4 \quad \text{or} \\ i = (-)\,, & j = 1,2,3,4 \end{cases} \tag{12h}$$

where $g' = -5.23$ is obtained from the on-mass-shell decay rate for $N^* \to N\pi$ and $\kappa'_S = 0.19$ and $\kappa'_V = 1.58$ are obtained in Appendix A. The $N^*(1535)$ contribution is obtained from that of the $N^*(1470)$ by letting $g' \to g''$, $\kappa' \to \kappa''$, and $M \to -M$ in (12).

IV. DATA COMPARISON

We are now in a position to compare theory with data. McClaskey, Jacob, and Hite[8] (MJH) have obtained values for the CGLN photoproduction amplitudes at threshold through the use of interior dispersion relations and we begin by comparing our predictions with their results. We shall at first exclude the $j=1$, $i=(+)$, (0) cases since these contain the chiral-breaking Σ terms which we wish to extract from the data and will also exclude the $j=2$ cases since results for A_2^i are not given by MJH. The first thing we do is evaluate the background contributions at the physical threshold in the c.m. frame for the four isobars that we consider. These results are given in Table I. The background contributions are to be combined with the other terms contained in Eqs. (1) of Ref. 1 in order to obtain predictions of the photoproduction invariants that can be compared to the phenomenological values found by MJH. Note that MJH differ from us in their definitions of the A_j^i, namely,

$$(A_j^i)_{\mathrm{us}} = -\frac{1}{e}(A_j^i)_{\mathrm{MJH}}\,, \quad e = (4\pi\alpha)^{1/2}\,. \tag{13}$$

Also, the numbers quoted in MJH are in units of GeV^{-n} ($n=2$ for A_1, $n=3$ for $A_{3,4}$) so we must multiply by a numerical factor μ^n to get numbers in units of inverse pion masses. The tabulated results are given in Table II. As can be seen, the data and theory agree fairly well with the exception of $A_4^{(0)}$. This discrepancy is not serious since $A_4^{(0)}$ itself is very small.

The low-energy theorem for $A_3^{(-)}$ contains the form factor

$$G(t) = \frac{g_A(t) - g_A(\mu^2)}{t-\mu^2}\,. \tag{14}$$

To estimate $G(t)$ we have used a dipole form for $g_A(t)$

$$g_A(t) = \frac{g_A(0)}{(1-t/m_A{}^2)^2}\,, \tag{15a}$$

which gives at threshold for $g_A(0) \approx 1.25$,

$$G(t) \simeq G(-\mu^2) = \frac{2g_A(0)}{m_A{}^2}\,. \tag{15b}$$

If we demand the low-energy theorem for $A_3^{(-)}$ in Ref. 1 to be exactly satisfied for the reconstructed experimental value at threshold[8] $A_3^{(-)} = -0.074 \pm 0.001\,\mu^{-3}$, then we can solve for (15b) using $B_3^{(-)}$ from Table I as

$$\begin{aligned}\frac{G(-\mu^2)}{f_\pi} &= A_3^{(-)} + \frac{g\kappa^V}{8m^3(1+\mu/2m)} - B_3^{(-)}\Big|_{\mathrm{th}} \\ &= -0.074\,\mu^{-3} + 0.019\,\mu^{-3} + 0.098\,\mu^{-3} \\ &= 0.043\,\mu^{-3}\,,\end{aligned} \tag{16a}$$

$$m_A \simeq 1.31\ \mathrm{GeV}. \tag{16b}$$

This value of m_A is in good agreement with the re-

TABLE I. Isobar background contributions.

	Δ(1230)	N*(1520)	N*(1470)	N*(1535)	Total
$B_{1P}^{(+)}$ (μ^{-2})	−0.335 ±0.011	−0.009 ±0.008	+0.028 ±0.005	−0.013 ±0.002	−0.329 ±0.015
$B_{1NP}^{(+)}$ (μ^{-2})	+0.282 ±0.012	+0.005 ±0.007	−0.027 ±0.005	+0.012 ±0.002	+0.272 ±0.015
$B_1^{(+)}$ (μ^{-2})	−0.053 ±0.017	−0.003 ±0.010	+0.001 ±0.007	−0.001 ±0.004	−0.056 ±0.021
$B_2^{(+)}$ (μ^{-4})	+0.042 ±0.002	−0.005 ±0.002	0	0	+0.037 ±0.003
$B_3^{(+)}$ (μ^{-3})	+0.089 ±0.006	+0.003 ±0.001	−0.0015±0.0002	−0.000 13±0.000 03	+0.090 ±0.006
$B_4^{(+)}$ (μ^{-3})	−0.446 ±0.016	−0.008 ±0.002	−0.007 ±0.001	−0.0008 ±0.0002	−0.462 ±0.016
$B_{1P}^{(0)}$ (μ^{-2})	0	+0.009 ±0.008	+0.004 ±0.005	+0.002 ±0.002	+0.015 ±0.010
$B_{1NP}^{(0)}$ (μ^{-2})	0	−0.009 ±0.007	−0.004 ±0.005	−0.002 ±0.002	−0.015 ±0.010
$B_1^{(0)}$ (μ^{-2})	0	0.000 ±0.010	0.000 ±0.007	0.000 ±0.004	0.000 ±0.013
$B_2^{(0)}$ (μ^{-4})	0	0.000 ±0.002	0	0	0.000 ±0.002
$B_3^{(0)}$ (μ^{-3})	0	−0.001 ±0.001	−0.0002±0.0002	+0.000 02±0.000 03	−0.001 ±0.001
$B_4^{(0)}$ (μ^{-3})	0	−0.003 ±0.002	−0.001 ±0.001	+0.0001 ±0.0002	−0.004 ±0.002
$B_1^{(-)}$ (μ^{-2})	+0.072 ±0.003	−0.002 ±0.002	+0.006 ±0.001	−0.002 ±0.001	+0.074 ±0.004
$B_2^{(-)}$ (μ^{-4})	−0.0091±0.0003	−0.0009±0.0003	0	0	−0.0100±0.0004
$B_3^{(-)}$ (μ^{-3})	−0.102 ±0.007	+0.012 ±0.007	−0.007 ±0.001	−0.0008 ±0.0002	−0.098 ±0.010
$B_4^{(-)}$ (μ^{-3})	+0.096 ±0.004	−0.0015±0.0003	−0.0015±0.0002	−0.000 13±0.000 03	0.093 ±0.004

cent determination based upon neutrino scattering[5] and also threshold electroproduction.[4]

Since the data and the theory appear to be in good agreement, we proceed to a determination of the nonstrange quark mass $\hat{m}$ which is contained in the $\Sigma_1^{(+,0)}$ terms of Eq. (1a) of Ref. 1. There are two difficulties which prevent an accurate determination of $\hat{m}$. First, the values for the $E_{0+}^{(+,0)}$ multipoles at threshold given in the literature vary quite a bit from author to author. Second, since $\hat{m}$ is derived from a small number which is the difference between two large numbers, even small fractional uncertainties in the data will yield large fractional uncertainties in $\hat{m}$.

There are two ways to obtain a value of $\hat{m}$. It can be evaluated by using the recent determinations of $A_1^{(+)}$ and $A_1^{(0)}$ made by MJH or from threshold values of $E_{0+}^{(+)}$ and $E_{0+}^{(0)}$ given in the literature.

To obtain $\hat{m}$ directly from $A_1^{(+,0)}$, we use Eq. (1a) of Ref. 1 evaluated at physical threshold in the c.m. frame $\nu \simeq m\mu$, $t \simeq -\mu^2$:

$$A_1^{(+,0)}\Big|_{\rm th} = \frac{g}{4m^2(1+\mu/2m)} - \frac{g_A(\mu^2)\kappa^{V,S}}{4mf_\pi} - \frac{\Sigma_1^{(+,0)}}{f_\pi m_V^2} + B_1^{(+,0)}\Big|_{\rm th}, \tag{17}$$

where

$$\frac{\Sigma_1^{(+)}}{f_\pi m_V^2} \rightarrow \frac{\hat{m}}{f_\pi m_\rho{}^2} = 0.049\ \hat{m}/\mu^3 \tag{18a}$$

and

$$\frac{\Sigma_1^{(0)}}{f_\pi m_V^2} \rightarrow \frac{5}{9}\ \frac{\hat{m}}{f_\pi m_8{}^2} = 0.019\ \hat{m}/\mu^3\,. \tag{18b}$$

To use $E_{0+}^{(+,0)}$ data for the determination of $\hat{m}$, we

TABLE II. Data comparison with MJH for amplitudes containing no current-algebra terms.

	Nucleon poles	Background	Total	MJH
$A_3^{(+)}$ (μ^{-3})	+0.275 ±0.014	+0.090±0.006	+0.365±0.015	+0.363 ±0.003
$A_4^{(+)}$ (μ^{-3})	−0.019 ±0.001	−0.462±0.016	−0.481±0.016	−0.431 ±0.007
$A_3^{(0)}$ (μ^{-3})	−0.0089 ±0.0004	−0.001±0.001	−0.010±0.001	−0.0097±0.0001
$A_4^{(0)}$ (μ^{-3})	+0.000 62±0.000 03	−0.004±0.002	−0.003±0.002	−0.0135±0.0001
$A_1^{(-)}$ (μ^{-2})	−0.998 ±0.050	+0.074±0.004	−0.924±0.050	−1.000 ±0.010
$A_4^{(-)}$ (μ^{-3})	+0.275 ±0.014	+0.093±0.004	+0.368±0.015	+0.357 ±0.003

need $E_{0+}^{(+,0)}$ evaluated at the c.m. threshold:

$$\tilde{E}_{0+}^{(+,0)}\Big|_{\rm th} \equiv \frac{8\pi(m+\mu)^2}{e\mu(2m+\mu)[m(m+\mu)]^{1/2}} E_{0+}^{(+,0)}\Big|_{\rm th}$$
$$= -A_1^{(+,0)} - \mu\, \frac{1+\mu/2m}{1+\mu/m} A_3^{(+,0)}$$
$$- \mu\, \frac{\mu/2m}{1+\mu/m} A_4^{(+,0)}\Big|_{\rm th}, \qquad (19)$$

where we have defined $\tilde{E}_{0+}^{(+,0)}$ to make the numerical analysis easier, with $\tilde{E}_{0+}=47.55\, E_{0+}/\mu$ at the c.m. threshold. Using Eqs. (1) of Ref. 1 to solve (19) for the chiral-breaking terms we find $[g_A(\mu^2) \simeq 1.28$ from (15)$]$

$$\frac{\Sigma_1^{(+,0)}}{f_\pi m_V^2} = \frac{g(1+\kappa^{V,S})}{4m^2(1+\mu/2m)} - \frac{g_A(\mu^2)\kappa^{V,S}}{4mf_\pi}$$
$$- \tilde{E}_{0+}^{(+,0)}\Big|_{\rm th} + \tilde{E}_{0+}\Big|_{\rm th}, \qquad (20)$$

where the background threshold multipoles are from Table I:

$$\tilde{E}_{0+}^{(+,0)}\Big|_{\rm th} = -B_1^{(+,0)}\Big|_{\rm th} - \mu\, \frac{1+\mu/2m}{1+\mu/m} B_3^{(+,0)}\Big|_{\rm th}$$
$$- \mu\, \frac{\mu/2m}{1+\mu/m} B_4^{(+,0)}\Big|_{\rm th}$$
$$= \begin{cases} 0.002 \pm 0.022\ \mu^{-2} & (+) \\ 0.001 \pm 0.013\ \mu^{-2} & (0). \end{cases} \qquad (21)$$

The values of $\hat{m}$ obtained independently from the isovector and isoscalar are tabulated in Table III. Narrow-width corrections to (21) can be expected to be negligible. Thus we may apply the various multipole analyses considered in the literature.[9-20]

As can be seen from Table III, there is a large variation in the values of the extracted $\hat{m}$ with reasonably large uncertainties, Nevertheless, the scale of the quark mass is small since it is obtained from the difference between much larger terms. Therefore, the few (unphysical) negative values of $\hat{m}$ in Table III are of no great concern. If we average over all the predictions of $\hat{m}$ in Ta-

TABLE III. Quark-mass values.

Method	$\hat{m}$ (π^+ masses)	E_{0+} ($10^{-2}\,\mu^{-1}$) or $A_{\rm MJH}$ (GeV^{-2})	$\tilde{E}_{0+}$ (μ^{-2}) or $A_1^{\rm us}$ (μ^{-2})	Source
A_1	3.65 ± 0.62	$A_1^{(+)} = 6.63 \pm 0.15$	$A_1^{(+)} = -0.429 \pm 0.010$	MJH (IDR) (Ref. 8)
A_1	1.81 ± 0.61	$A_1^{(+)} = 5.24 \pm 0.13$	$A_1^{(+)} = -0.339 \pm 0.008$	MJH (HDR) (Ref. 20)
A_1	-1.07 ± 0.83	$A_1^{(0)} = -1.51 \pm 0.02$	$A_1^{(0)} = 0.098 \pm 0.001$	MJH (IDR) (Ref. 8)
A_1	-0.96 ± 0.83	$A_1^{(0)} = -1.48 \pm 0.02$	$A_1^{(0)} = 0.096 \pm 0.001$	MJH (HDR) (Ref. 20)
E_{0+}	3.66 ± 1.96	$E_{0+}^{(+)} = 0.250 \pm 0.175$	$\tilde{E}_{0+}^{(+)} = 0.119 \pm 0.083$	MJH (IDR) (Ref. 8)
E_{0+}	1.82 ± 1.01	$E_{0+}^{(+)} = 0.060 \pm 0.024$	$\tilde{E}_{0+}^{(+)} = 0.029 \pm 0.011$	MJH (HDR) (Ref. 20)
E_{0+}	-0.04 ± 0.98	$E_{0+}^{(+)} = -0.130$	$\tilde{E}_{0+}^{(+)} = -0.062$	Nolle (Ref. 14)
E_{0+}	1.76 ± 0.98	$E_{0+}^{(+)} = 0.055$	$\tilde{E}_{0+}^{(+)} = 0.026$	Donnachie (Ref. 15)
E_{0+}	1.45 ± 0.98	$E_{0+}^{(+)} = 0.022$	$\tilde{E}_{0+}^{(+)} = 0.011$	De Baenst (Ref. 16)
E_{0+}	0.29 ± 0.98	$E_{0+}^{(+)} = -0.097$	$\tilde{E}_{0+}^{(+)} = -0.046$	Olsson (Refs. 7 and 17)
E_{0+}	2.37 ± 0.98	$E_{0+}^{(+)} = 0.117$	$\tilde{E}_{0+}^{(+)} = 0.056$	Berends (Ref. 18)
E_{0+}	0.64 ± 1.32	$E_{0+}^{(+)} = -0.06 \pm 0.09$	$\tilde{E}_{0+}^{(+)} = -0.029 \pm 0.043$	Adamovich (Ref. 13)
E_{0+}	1.80 ± 0.98	$E_{0+}^{(+)} = 0.059$	$\tilde{E}_{0+}^{(+)} = 0.028$	Von Gehlen (Ref. 12)
E_{0+}	-0.98 ± 1.73	$E_{0+}^{(0)} = -0.184 \pm 0.060$	$\tilde{E}_{0+}^{(0)} = -0.087 \pm 0.029$	MJH (IDR) (Ref. 8)
E_{0+}	-0.93 ± 0.89	$E_{0+}^{(0)} = -0.180 \pm 0.014$	$\tilde{E}_{0+}^{(0)} = -0.086 \pm 0.007$	MJH (HDR) (Ref. 20)
E_{0+}	2.07 ± 0.80	$E_{0+}^{(0)} = -0.060$	$\tilde{E}_{0+}^{(0)} = -0.029$	Nolle (Ref. 14)
E_{0+}	-0.03 ± 0.80	$E_{0+}^{(0)} = -0.145$	$\tilde{E}_{0+}^{(0)} = -0.069$	Donnachie (Ref. 15)
E_{0+}	-1.72 ± 0.80	$E_{0+}^{(0)} = -0.212$	$\tilde{E}_{0+}^{(0)} = -0.101$	De Baenst (Ref. 16)
E_{0+}	0.13 ± 0.80	$E_{0+}^{(0)} = -0.138$	$\tilde{E}_{0+}^{(0)} = -0.066$	Olsson (Refs. 7 and 17)
E_{0+}	0.39 ± 0.80	$E_{0+}^{(0)} = -0.127$	$\tilde{E}_{0+}^{(0)} = -0.061$	Berends (Ref. 18)
E_{0+}	0.07 ± 1.50	$E_{0+}^{(0)} = -0.14 \pm 0.05$	$\tilde{E}_{0+}^{(0)} = -0.067 \pm 0.024$	Adamovich (Ref. 13)
E_{0+}	0.34 ± 0.80	$E_{0+}^{(0)} = -0.13$	$\tilde{E}_{0+}^{(0)} = -0.062$	Von Gehlen (Ref. 12)
E_{0+}	0.39 ± 1.32	$E_{0+}^{(0)} = -0.127 \pm 0.041$	$\tilde{E}_{0+}^{(0)} = -0.061 \pm 0.020$	Shaw (Ref. 19)

ble III we obtain $\hat{m}=0.64\pm1.11\ \mu$. Clearly, better low-energy data are needed in order to obtain a more accurate value of the nonstrange current-quark mass.

V. CONCLUSION

We conclude that the low-energy photoproduction data are reasonably consistent with the soft-pion predictions of current algebra. Furthermore, the on-shell Ward identities derived in Ref. 1 appear to be in agreement with a recent extraction of the c.m. threshold invariant amplitudes.[8] Finally, various determinations of the chiral-symmetry-breaking quark mass roughly average to a value of $\hat{m}=0.64\pm1.11\ \mu$.

In order that future data analyses zero in on the quark mass, we suggest the data be analyzed in terms of the invariant amplitudes *below* threshold, as is now done for πN scattering (see, e.g., Nielsen and Oades[21]) rather than in terms of multipoles at threshold or slightly above threshold. In this connection it should prove useful to work with the amplitudes F, A_2, A_3, A_4 with

$$F\equiv A_1+\frac{\nu}{m}A_3+\frac{k\cdot q}{2m}A_4\,, \tag{22}$$

replacing A_1. This amplitude F is analogous to the πN amplitude $F=A+\nu B$ and both are useful because the nonpole parts (in dispersion theory) vanish identically in the single-soft-pion limit. That is, the large contact terms of g^2/m in πN scattering and $g\kappa/4m^2$ for photoproduction are automatically subtracted out of the background amplitudes $\overline{F}$. For photoproduction it is the sum $A_1+g\kappa/4m^2$ which is the quantity of interest in determining the chiral-breaking Σ terms. If we use Eqs. (1) of Ref. 1 to write an off- (the pion) mass-shell expression for $\overline{F}$ we find

$$\overline{F}^{(+,0)}=\frac{g(q^2)\kappa^{V,S}}{4m^2}\left(1-\frac{mg_A(q^2)}{f_\pi g(q^2)}\right)-\frac{q^2}{\mu^2}\;\frac{\Sigma_1^{(+,0)}}{f_\pi(m_V^2+q^2-\mu^2)}+\overline{F}_{\rm res}^{(+,0)}\,, \tag{23}$$

where

$$\overline{F}_{\rm res}^{(+,0)}=B_1^{(+,0)}+\frac{\nu}{m}\,B_3^{(+,0)}+\frac{k\cdot q}{2m}\,B_4^{(+,0)}\,.$$

From this expression it is clear that $\overline{F}\to0$ as $q\to0$ since the B amplitudes vanish in the soft-pion limit. Furthermore, we see that PCAC requires that $\overline{F}$ be small on the pion mass shell. Thus it is the natural amplitude to use to investigate chiral-breaking terms.

Finally, if our low-energy analysis is to be sharpened, the narrow-width connections to *all four* isobar coupling constants should be found by folding in the relevant phase-shift data near resonance. This has been considered for the $\Delta(1230)$ resonance in photoproduction[3] and in πN scattering,[22] but also must be done for the 1520, 1470, and 1535 photoproduction resonances.

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APPENDIX A: RESONANCE COUPLINGS

There are a total of eight couplings required to calculate the resonance background contributions to photoproduction. They are: $\gamma N\Delta$, $\pi N\Delta$, and γNN^* and πNN^* for the $N^*(1520)$, $N^*(1470)$, and $N^*(1535)$. We take these couplings to be given by the following:

$$\langle\Delta^k(K)|j_\pi^i|N(p)\rangle=-g^*\delta^{ik}\overline{\Delta}^\alpha(K)q_\alpha N(p)\,, \tag{A.1a}$$

$$\langle\Delta^j(K)|V_\nu^\gamma|N(p)\rangle=i\delta^{j3}\overline{\Delta}^\alpha(K)(G_1\mathcal{K}_{\alpha\nu}^1+G_2\mathcal{K}_{\alpha\nu}^2)N(p)\,, \tag{A.1b}$$

$$\langle N^*(K)|j_\pi^i|N(p)\rangle_{1520}=-g_{N^*}\overline{N}^{*\alpha}(K)\tau^i q_\alpha\gamma_5 N(p)\,, \tag{A.1c}$$

$$\langle N^*(K)|V_\nu^\gamma|N(p)\rangle_{1520}=\overline{N}^{*\alpha}(K)(H_1J_{\alpha\nu}^1+H_2J_{\alpha\nu}^2)N(p)\,, \tag{A.1d}$$

$$\langle N^*(K)|j_\pi^i|N(p)\rangle_{1470}=g'\overline{N}^*(K)\tau^i\gamma_5N(p)\,, \tag{A.1e}$$

$$\langle N^*(K)|V_\nu^\gamma|N(p)\rangle_{1470}=\overline{N}^*(K)\,\frac{\kappa'}{2M}\,i\sigma_{\nu\alpha}k^\alpha N(p)\,, \tag{A.1f}$$

$$\langle N^*(K)|j_\pi^i|N(p)\rangle_{1535}=g''\overline{N}^*(K)\tau^iN(p)\,, \tag{A.1g}$$

$$\langle N^*(K)|V_\nu^\gamma|N(p)\rangle_{1535}=\overline{N}^*(K)\,\frac{\kappa''}{2m}\,i\sigma_{\nu\alpha}k^\alpha\gamma_5N(p)\,, \tag{A.1h}$$

with $Q=\frac{1}{2}(K+p)$ and where

$$\mathcal{K}_{\alpha\nu}^1=(k_\alpha\gamma_\nu-\gamma\cdot kg_{\alpha\nu})\gamma_5\,,$$
$$\mathcal{K}_{\alpha\nu}^2=(k_\alpha Q_\nu-k\cdot Qg_{\alpha\nu})\gamma_5\,,$$
$$J_{\alpha\nu}^i=-\mathcal{K}_{\alpha\nu}^i\gamma_5\,,$$
$$H_i=\tfrac{1}{2}(H_i^S+H_i^V\tau^3)\,,$$
$$\kappa'=\tfrac{1}{2}(\kappa_s'+\kappa_v'\tau_3)\,,$$
$$\kappa''=\tfrac{1}{2}(\kappa_S''+\kappa_V''\tau_3)\,.$$

The values of g^*, g_{N^*}, g', and g'' are obtained from the decay rates for resonance $\to \pi N$ and are given by $g^* = 14.24 m^{-1}$, $g_{N^*} = -19.90 m^{-1}$, $g' = -6.18$, and $g'' = 0.52$. All the isobar couplings that we use are obtained from information contained in Ref. 23 (BCP).

The manner in which the photon coupling constants are obtained deserves comment. The coupling-constant data of BCP are given in terms of helicity amplitudes whereas our coupling constants do not correspond to definite-helicity states. It is easy to relate our coupling constants to helicity coupling constants by expanding the relevant spinors into helicity states. When this is done the following results are obtained (also see Ref. 24):

$$\sqrt{3}\, G_{1/2} = \frac{M^2 - m^2}{2M} \left[2mG_1 - M(M-m) G_2\right], \tag{A.2a}$$

$$G_{3/2} = (M^2 - m^2)\left[G_1 + \tfrac{1}{2}(M-m) G_2\right], \tag{A.2b}$$

$$\sqrt{3}\, H_{1/2} = -\frac{M^2 - m^2}{2M} \left[2mH_1 + M(M+m) H_2\right], \tag{A.2c}$$

$$H_{3/2} = -(M^2 - m^2)\left[H_1 + \tfrac{1}{2}(M+m) H_2\right], \tag{A.2d}$$

$$\kappa'_{1/2} = -\sqrt{2}\,(M^2 - m^2)\,\frac{\kappa'}{2M}, \tag{A.2e}$$

$$\kappa''_{1/2} = \sqrt{2}\,(M^2 - m^2)\,\frac{\kappa''}{2M}. \tag{A.2f}$$

These coupling constants are still not those of the BCP tables. In order to determine the relationship between our constants and the $\tilde{A}_{3/2,1/2}$ of the BCP tables we compare rate expressions. We obtain the following:

$$\Gamma^w_{\Delta} = \tfrac{2}{3} e^2 \,\frac{(M^2 - m^2)}{32\pi M^3}\,(G_{3/2}^2 + G_{1/2}^2), \tag{A.3a}$$

$$\Gamma^w_{N^*(1520)} = e^2 \,\frac{(M^2 - m^2)}{32\pi M^3}\,(H_{3/2}^2 + H_{1/2}^2), \tag{A.3b}$$

$$\Gamma^w_{N^*(1470)} = e^2 \,\frac{(M^2 - m^2)}{16\pi M^3}\,(\kappa'_{1/2})^2, \tag{A.3c}$$

$$\Gamma^w_{N^*(1535)} = e^2 \,\frac{(M^2 - m^2)}{16\pi M^3}\,(\kappa''_{1/2})^2, \tag{A.3d}$$

while the Particle Data Group tables[25] give

$$\Gamma^w = m\,\frac{(M^2 - m^2)^2}{4M^3\pi}\,\frac{2}{2j+1}\,(\tilde{A}_{3/2}^2 + \tilde{A}_{1/2}^2), \tag{A.4}$$

where j = spin of the decaying resonance.

Comparing (A.4) and (A.3) to obtain the G's, H's, and F in terms of the $\tilde{A}$'s and then combining the results with the inverted form of (A.2), we find

$$G_1 = -(\tfrac{3}{2})^{1/2}\,\frac{M}{(M^2 - m^2)(M+m)}\left(\frac{m(M^2 - m^2)}{\pi\alpha}\right)^{1/2} \times (\tilde{A}^{\Delta}_{3/2} + \sqrt{3}\,\tilde{A}^{\Delta}_{1/2}), \tag{A.5a}$$

$$G_2 = -(\tfrac{3}{2})^{1/2}\,\frac{2}{(M^2 - m^2)^2}\left(\frac{m(M^2 - m^2)}{\pi\alpha}\right)^{1/2} \times (m\tilde{A}^{\Delta}_{3/2} - \sqrt{3}\,M\tilde{A}^{\Delta}_{1/2}), \tag{A.5b}$$

$$H_1 = -\frac{M}{(M^2 - m^2)(M-m)}\left(\frac{m(M^2 - m^2)}{\pi\alpha}\right)^{1/2} \times (\tilde{A}^{N^*}_{3/2} - \sqrt{3}\,\tilde{A}^{N^*}_{1/2}), \tag{A.5c}$$

$$H_2 = \frac{2}{(M^2 - m^2)^2}\left(\frac{m(M^2 - m^2)}{\pi\alpha}\right)^{1/2} \times (m\tilde{A}^{N^*}_{3/2} - \sqrt{3}M\tilde{A}^{N^*}_{1/2}), \tag{A.5d}$$

$$-\kappa', \kappa'' = \frac{\sqrt{2}M}{(M^2 - m^2)}\left(\frac{m(M^2 - m^2)}{\pi\alpha}\right)^{1/2} \tilde{A}^{N^*}_{1/2}, \tag{A.5e}$$

where $\alpha = e^2/4\pi$.

These expressions imply from the BCP helicity couplings[23]

$$G_1 = 2.90 \pm 0.09\ \text{GeV}^{-1}, \tag{A.6a}$$

$$G_2 = -1.50 \pm 0.55\ \text{GeV}^{-2}, \tag{A.6b}$$

$$H_1^p = -2.60 \pm 0.22\ \text{GeV}^{-1}, \tag{A.6c}$$

$$H_1^n = 0.64 \pm 0.40\ \text{GeV}^{-1}, \tag{A.6d}$$

$$H_2^p = 1.43 \pm 0.17\ \text{GeV}^{-2}, \tag{A.6e}$$

$$H_2^n = 0.09 \pm 0.30\ \text{GeV}^{-2}, \tag{A.6f}$$

$$\kappa'_p = 0.88 \pm 0.18, \tag{A.6g}$$

$$\kappa'_n = -0.69 \pm 0.19, \tag{A.6h}$$

$$\kappa''_p = 0.94 \pm 0.22, \tag{A.6i}$$

$$\kappa''_n = -1.28 \pm 0.39. \tag{A.6j}$$

We need the isovector and isoscalar coupling constants $H^V = H^p - H^n$, $H^S = H^p + H^n$, etc. These are

$$H_1^V = -3.24 \pm 0.46\ \text{GeV}^{-1}, \tag{A.7a}$$

$$H_2^V = 1.33 \pm 0.34\ \text{GeV}^{-2}, \tag{A.7b}$$

$$H_1^S = -1.95 \pm 0.46\ \text{GeV}^{-1}, \tag{A.7c}$$

$$H_2^S = 1.52 \pm 0.34\ \text{GeV}^{-2}, \tag{A.7d}$$

$$\kappa'_V = 1.58 \pm 0.26, \tag{A.7e}$$

$$\kappa'_S = 0.19 \pm 0.26, \tag{A.7f}$$

$$\kappa''_V = 2.22 \pm 0.45, \tag{A.7g}$$

$$\kappa''_S = -0.34 \pm 0.45. \tag{A.7h}$$

APPENDIX B: CALCULATION OF THE AXIAL-VECTOR BACKGROUND

As mentioned in the text, we choose to evaluate the background contribution of spin-$\frac{3}{2}$ isobars through the use of dispersion theory. Since dispersion theory calculations keep the intermediate states on the mass shell, it is possible to use the unambiguous on-shell spin-$\frac{3}{2}$ propagator. Ambiguities would be present if subtraction constants were needed, but the high-energy behavior of the CGLN invariant amplitudes guarantees that no subtractions are necessary because each amplitude falls off at least as fast as $\nu^{\alpha-1}$.

To calculate the background we must actually calculate a part of the Δ contribution to the axial-vector photoproduction amplitude $(M^i_{\mu\nu})^\Delta$. We do not need the full amplitude since many of the double-index covariants required for the description of axial-vector photoproduction have zero contraction with q^μ. There are 16 double-index covariants needed:

$$K^1_{\mu\nu} = (k\cdot q g_{\mu\nu} - k_\mu q_\nu)\gamma_5\,, \tag{B.1a}$$

$$K^2_{\mu\nu} = (k\cdot q g_{\mu\nu} - k_\mu q_\nu) k\cdot\gamma\gamma_5\,, \tag{B.1b}$$

$$K^3_{\mu\nu} = (k\cdot q P_\mu - \nu k_\mu) K^1_\nu\,, \tag{B.1c}$$

$$K^4_{\mu\nu} = P_\mu K^2_\nu - 2\nu(k_\mu P_\nu - \nu g_{\mu\nu})\gamma_5\,, \tag{B.1d}$$

$$K^5_{\mu\nu} = P_\mu K^3_\nu - \nu(k_\mu\gamma_\nu - g_{\mu\nu} k\cdot\gamma)\gamma_5\,, \tag{B.1e}$$

$$K^6_{\mu\nu} = P_\mu K^4_\nu - \nu\epsilon_{\mu\nu}(\gamma k)\,, \tag{B.1f}$$

$$K^7_{\mu\nu} = \tfrac{1}{2}\{\gamma_\mu, K^3_\nu\} - k_\mu K^1_\nu - 2(k_\mu P_\nu - \nu g_{\mu\nu})\gamma_5\,, \tag{B.1g}$$

$$K^8_{\mu\nu} = \tfrac{1}{2}\{\gamma_\mu, K^4_\nu\} - 2P_\mu K^1_\nu + 2m(k_\mu\gamma_\nu - g_{\mu\nu} k\cdot\gamma)\gamma_5 = \epsilon_{\mu\nu}(kq)\,, \tag{B.1h}$$

$$K^9_{\mu\nu} = (k\cdot q q_\mu - q^2 k_\mu) K^1_\nu\,, \tag{B.1i}$$

$$K^{10}_{\mu\nu} = q_\mu K^2_\nu - 2q^2(k_\mu P_\nu - \nu g_{\mu\nu})\gamma_5\,, \tag{B.1j}$$

$$K^{11}_{\mu\nu} = q_\mu K^3_\nu - q^2(k_\mu\gamma_\nu - g_{\mu\nu} k\cdot\gamma)\gamma_5\,, \tag{B.1k}$$

$$K^{12}_{\mu\nu} = q_\mu K^4_\nu - q^2\epsilon_{\mu\nu}(\gamma k)\,, \tag{B.1l}$$

$$K^{13}_{\mu\nu} = k_\mu K^1_\nu\,, \tag{B.1m}$$

$$K^{14}_{\mu\nu} = (k_\mu P_\nu - \nu g_{\mu\nu})\gamma_5\,, \tag{B.1n}$$

$$K^{15}_{\mu\nu} = (k_\mu\gamma_\nu - g_{\mu\nu} k\cdot\gamma)\gamma_5 = \tfrac{1}{2}\{\gamma_\mu, K^1_\nu\}\,, \tag{B.1o}$$

$$K^{16}_{\mu\nu} = \epsilon_{\mu\nu}(\gamma k) = \tfrac{1}{2}[\gamma_\mu, K^1_\nu]\,. \tag{B1.p}$$

Here the single index K's are the standard CGLN covariants. Note that all of these covariants are electromagnetic-current conserving while only the first 12 conserve the axial-vector current. Along with these 16 covariants there are two "equivalence theorems" (see, e.g., Refs. 26):

$$m[k_\mu K^4_\nu - k\cdot q K^{16}_{\mu\nu}] = K^1_{\mu\nu} - \tfrac{1}{2}k\cdot q K^7_{\mu\nu} + \nu K^8_{\mu\nu} + \tfrac{1}{2}K^9_{\mu\nu}\,, \tag{B.2a}$$

$$m[\gamma_\mu K^2_\nu + k_\mu(K^4_\mu + 2mK^1_\nu) - 2\nu K^{15}_{\mu\nu} + 4mK^{14}_{\mu\nu}] = 2\nu K^1_{\mu\nu} + 2mK^5_{\mu\nu} - 2(m^2 + \tfrac{1}{2}k\cdot q)K^7_{\mu\nu} + 2\nu K^8_{\mu\nu} + K^9_{\mu\nu} - mK^{12}_{\mu\nu}\,. \tag{B.2b}$$

While it is true that these 16 covariants form a complete set, using them as such necessitates the introduction of kinematic singularities into the invariant amplitudes multiplying them in processes involving an axial-vector current.[27] While this practice presents no problem in any dynamical calculation, it will lead to erroneous results in dispersion theory. We therefore need to introduce two more covariants, the use of which will eliminate the need for the introduction of kinematic singularities:

$$K^{17}_{\mu\nu} = P_\mu K^1_\nu\,, \tag{B.3a}$$

$$K^{18}_{\mu\nu} = q_\mu K^1_\nu\,. \tag{B.3b}$$

We need only consider contributions to our background amplitude from the covariants $K^{13}_{\mu\nu}$ through $K^{18}_{\mu\nu}$ since

$$q^\mu K^j_{\mu\nu} = 0\,, \quad j = 1-12\,, \tag{B.4a}$$

$$q^\mu K^{13}_{\mu\nu} = k\cdot q K^1_\nu\,, \tag{B.4b}$$

$$q^\mu K^{14}_{\mu\nu} = \tfrac{1}{2}K^2_\nu\,, \tag{B.4c}$$

$$q^\mu K^{15}_{\mu\nu} = K^3_\nu\,, \tag{B.4d}$$

$$q^\mu K^{16}_{\mu\nu} = K^4_\nu\,, \tag{B.4e}$$

$$q^\mu K^{17}_{\mu\nu} = \nu K^1_\nu\,, \tag{B.4f}$$

$$q^\mu K^{18}_{\mu\nu} = q^2 K^1_\nu\,. \tag{B.4g}$$

Next we discuss the coupling at the ΔNA ($A \Rightarrow$ axial-vector) vertex, defined as in the πN scattering case (see, e.g., Ref. 27)

$$\langle\Delta^k|A^i_\mu|N\rangle = -\delta^{ik} f_\pi \bar{\Delta}^\alpha(K)\left[g^*(q^2)\left(g_{\alpha\mu} - \frac{q_\alpha q_\mu}{q^2-\mu^2}\right) + (\text{three other covariants})\right]N(p)\,. \tag{B.5a}$$

The three other covariants are divergenceless and would only contribute to the $K^1_{\mu\nu}$ through $K^{12}_{\mu\nu}$ covariants, contributions in which we are not interested since they do not correspond to pion amplitudes. The term proportional to $q_\alpha q_\mu$ has a pole for physical pions and cannot contribute to the on-shell pion amplitude. It must therefore give the Δ contribution to the first term in $R^i_{\mu\nu}$ of Eq. (42) of Ref. 1. To avoid double counting we ignore it here.

We are now ready to evaluate the background amplitudes $B(\Delta)$. We use Eq. (B.1b) for the photon vertex coupling and

$$\langle\Delta^k|A^i_\mu|N\rangle = -\delta^{ik} g^* f_\pi \bar{\Delta}^\alpha(K) g_{\alpha\mu} N(p) \tag{B.6}$$

at the axial-vector vertex. Using these couplings we find that

$$if_\pi^{-1}(\bar{M}^i_{\mu\nu})^\Delta_s = -\tfrac{2}{3}(I^i_+ - \tfrac{1}{2}I^i_-)\frac{g^*}{M^2-s} g_{\mu\alpha} \times \mathcal{P}^{\alpha\beta}(K)(G_1\mathcal{K}^1_{\beta\nu} + G_2\mathcal{K}^2_{\beta\nu}) \tag{B.7}$$

for the s channel, where $\mathcal{P}^{\alpha\beta}(K)$ is the on-shell spin-$\frac{3}{2}$ projection operator [the $\delta_{ij}-\frac{1}{3}\tau_i\tau_j$ isospin part of (B.8) yields for the isospin couplings the structure $I_+-\frac{1}{2}I_-$ in (B.7)]:

$$\mathcal{P}^{\alpha\beta}_{ij}(K)=-\left(g^{\alpha\beta}-\tfrac{1}{3}\gamma^\alpha\gamma^\beta-\frac{1}{3M}(K^\alpha\gamma^\beta-\gamma^\alpha K^\beta)-\frac{2}{3M^2}K^\alpha K^\beta\right)(\gamma\cdot K+M)(\delta_{ij}-\tfrac{1}{3}\tau_i\tau_j). \tag{B.8}$$

We now work out (B.7), ignoring contributions from $K^1_{\mu\nu}$ through $K^{12}_{\mu\nu}$ which will not contribute to our final result. We obtain

$$\begin{aligned}
if_\pi^{-1}(\bar{M}^i_{\mu\nu})^\Delta_s=\tfrac{2}{3}(I^i_+-\tfrac{1}{2}I^i_-)\frac{g^*}{M^2-s}\Bigg\{&K^{13}_{\mu\nu}\left[G_1\left(1-\frac{1}{6M^2}(\mu^2+k\cdot q)+\frac{(M+m)(M-2m)}{6M^2}-\frac{2\nu}{3M^2}\right)\right.\\
&\left.+G_2\left(-m-\frac{1}{12M}(\mu^2+k\cdot q)+\frac{M^2-m^2}{6M}-\frac{\nu}{3M}\right)\right]+K^{14}_{\mu\nu}[2G_1+(M-m)G_2]\\
&+K^{15}_{\mu\nu}\left[G_1\left(-\frac{\mu^2}{6M}+\tfrac{1}{3}(M+m)+\frac{M-m}{6M^2}(\mu^2+k\cdot q)+\frac{M-m}{3M^2}\nu\right)\right.\\
&\left.+G_2\left(\tfrac{1}{6}\mu^2-\frac{M+m}{12M}(\mu^2+k\cdot q)+\frac{5M-m}{6M}\nu\right)\right]\\
&+K^{16}_{\mu\nu}\left[G_1\left(-\frac{\mu^2}{6M}-\tfrac{2}{3}(M+m)+\frac{M-m}{6M^2}(\mu^2+k\cdot q)+\frac{M-m}{3M^2}\nu\right)\right.\\
&\left.+G_2\left(\tfrac{1}{6}\mu^2-\frac{M+m}{12M}(\mu^2+k\cdot q)-\tfrac{1}{2}k\cdot q-\frac{M+m}{6M}\nu\right)\right]\\
&+K^{17}_{\mu\nu}\left[G_1\left(\frac{(M+m)(M-2m)}{3M^2}-\frac{2\nu}{3M^2}\right)+G_2\left(\frac{M^2-m^2}{3M}-\frac{\nu}{3M}\right)\right]\\
&+K^{18}_{\mu\nu}\left[G_1\left(-\frac{M^2+mM+m^2}{3M^2}-\frac{\nu}{3M^2}\right)+G_2\left(\frac{m(M-m)}{6M}-\frac{\nu}{6M}\right)\right]\Bigg\}.
\end{aligned} \tag{B.9}$$

The u-channel result follows from this expression by letting $s\to u$, $\nu\to-\nu$, $K^i_{\mu\nu}\to K^i_{\mu\nu}$ $i=13, 14, 16, 18$, $K^i_{\mu\nu}\to -K^i_{\mu\nu}$ $i=15, 17$, and $I^i_-\to -I^i_-$. After adding the s- and u-channel contributions together we find the contribution to $M_{\mu\nu}$ which will not vanish in $q^\mu M_{\mu\nu}$:

$$\begin{aligned}
if_\pi^{-1}(\bar{M}^i_{\mu\nu})^\Delta=\tfrac{2}{3}I^i_+\frac{g^*}{\nu_\Delta^2-\nu^2}\Bigg\{&K^{13}_{\mu\nu}\left[G_1\left[\left(1-\frac{1}{6M^2}(\mu^2+k\cdot q)+\frac{(M+m)(M-2m)}{6M^2}\right)\nu_\Delta-\frac{2\nu^2}{3M^2}\right]\right.\\
&\left.+G_2\left[\left(-m-\frac{1}{12M}(\mu^2+k\cdot q)+\frac{M^2-m^2}{6M}\right)\nu_\Delta-\frac{\nu^2}{3M}\right]\right]+\nu_\Delta K^{14}_{\mu\nu}[2G_1+(M-m)G_2]\\
&+\nu K^{15}_{\mu\nu}\left[G_1\left(-\frac{\mu^2}{6M}+\tfrac{1}{3}(M+m)+\frac{M-m}{6M^2}(\mu^2+k\cdot q)+\frac{M-m}{3M^2}\nu_\Delta\right)\right.\\
&\left.+G_2\left(\tfrac{1}{6}\mu^2-\frac{M+m}{12M}(\mu^2+k\cdot q)+\frac{5M-m}{6M}\nu_\Delta\right)\right]\\
&+K^{16}_{\mu\nu}\left[G_1\left[\left(-\frac{\mu^2}{6M}-\tfrac{2}{3}(M+m)+\frac{M-m}{6M^2}(\mu^2+k\cdot q)\right)\nu_\Delta+\frac{M-m}{3M^2}\nu^2\right]\right.\\
&\left.+G_2\left[\left(\tfrac{1}{6}\mu^2-\frac{M+m}{12M}(\mu^2+k\cdot q)-\tfrac{1}{2}k\cdot q\right)\nu_\Delta-\frac{M+m}{6M}\nu^2\right]\right]\\
&+\nu K^{17}_{\mu\nu}\left[G_1\left(\frac{(M+m)(M-2m)}{3M^2}-\frac{2\nu_\Delta}{3M^2}\right)+G_2\left(\frac{M^2-m^2}{3M}-\frac{\nu_\Delta}{3M}\right)\right]\\
&+K^{18}_{\mu\nu}\left[G_1\left(-\frac{M^2+mM+m^2}{3M^2}\nu_\Delta-\frac{\nu^2}{3M^2}\right)+G_2\left(\frac{m(M-m)}{6M}\nu_\Delta-\frac{\nu^2}{6M}\right)\right]\Bigg\}\\
-\tfrac{1}{3}I^i_-\frac{g^*}{\nu_\Delta^2-\nu^2}\Bigg\{&\nu K^{13}_{\mu\nu}\left[G_1\left(1-\frac{1}{6M}(\mu^2+k\cdot q)+\frac{(M+m)(M-2m)}{6M^2}-\frac{2\nu_\Delta}{3M^2}\right)\right.\\
&\left.+G_2\left(-m-\frac{1}{12M}(\mu^2+k\cdot q)+\frac{M^2-m^2}{6M}-\frac{\nu_\Delta}{3M}\right)\right]+\nu K^{14}_{\mu\nu}[2G_1+(M-m)G_2]
\end{aligned}$$

$$
\begin{aligned}
&+K^{15}_{\mu\nu}\left[G_1\left[\left(-\frac{\mu^2}{6M}+\tfrac{1}{3}(M+m)+\frac{M-m}{6M^2}(\mu^2+k\cdot q)\right)\nu_\Delta+\frac{M-m}{3M^2}\nu^2\right]\right.\\
&\qquad\left.+G_2\left[\left(\tfrac{1}{6}\mu^2-\frac{M+m}{12M}(\mu^2+k\cdot q)\right)\nu_\Delta+\frac{5M-m}{6M}\nu^2\right]\right]\\
&+\nu K^{16}_{\mu\nu}\left[G_1\left(-\frac{\mu^2}{6M}-\tfrac{2}{3}(M+m)+\frac{M-m}{6M^2}(\mu^2+k\cdot q)-\frac{M-m}{3M^2}\nu_\Delta\right)\right.\\
&\qquad\left.+G_2\left(\tfrac{1}{6}\mu^2-\frac{M+m}{12M}(\mu^2+k\cdot q)-\tfrac{1}{2}k\cdot q-\frac{M+m}{6M}\nu_\Delta\right)\right]\\
&+K^{17}_{\mu\nu}\left[G_1\left(\frac{(M+m)(M-2m)}{3M^2}\nu_\Delta-\frac{2\nu^2}{3M^2}\right)+G_2\left(\frac{M^2-m^2}{3M}\nu_\Delta-\frac{\nu^2}{3M}\right)\right]\\
&+\nu K^{18}_{\mu\nu}\left[G_1\left(-\frac{M^2+mM+m^2}{3M^2}-\frac{\nu_\Delta}{3M^2}\right)+G_2\left(\frac{m(M-m)}{6M}-\frac{\nu_\Delta}{6M}\right)\right]\Bigg\}\,. \qquad \text{(B.10)}
\end{aligned}
$$

To obtain the dispersion-theory expression from (B.10) we let $\nu^2\to\nu_\Delta{}^2$ in the coefficients of the crossing-even quantities, that is, we let $\nu^2\to\nu_\Delta{}^2$ inside the square brackets. After making this substitution, we contract with q^μ to obtain the contribution to the pion photoproduction amplitude. Note that when q^μ is contracted with the $K^{17}_{\mu\nu}$ term of the I^i_+ part of the amplitude, a term proportional to ν^2 is introduced. To obtain the pole part of $B_1^{(+)}$, (9a), this ν^2 should be replaced by $\nu_\Delta{}^2$, while the nonpole term is obtained by replacing this ν^2 by $\nu^2-\nu_\Delta{}^2$. That is, if one wishes to write $B_1^{(+)}$ in the natural form $B_1^{(+)}=B_{1\mathrm{P}}^{(+)}+B_{1\,\mathrm{NP}}^{(+)}$, one should replace ν^2 with $\nu^2=\nu_\Delta{}^2+(\nu^2-\nu_\Delta{}^2)$.

As was mentioned in the body of the paper, the contribution from the $N^*(1520)$ is easily obtainable from that of the Δ. This can be seen from the expression for $(\overline{M}^i_{\mu\nu})^{N^*}_{s\ \mathrm{channel}}$ and using $\gamma_5^2=-1$, $\gamma_5\gamma_\mu\gamma_5=\gamma_\mu$:

$$
\begin{aligned}
if_\pi{}^{-1}(\overline{M}^i_{\mu\nu})^{N^*}_{s\ \mathrm{channel}}&=-\frac{\tau^i}{2}\,\frac{g^*_N}{M^2-s}\,g_{\mu\alpha}\gamma_5\mathcal{P}^{\alpha\beta}(K)[-H_1\mathcal{K}^1_{\beta\nu}\gamma_5-H_2\mathcal{K}^2_{\beta\nu}\gamma_5]\\
&=-\frac{\tau^i}{2}\,\frac{g^*_N}{M^2-s}\,g_{\mu\alpha}\mathcal{P}^{\alpha\beta}(K)\Big|_{M\to-M}[H_1\mathcal{K}^1_{\beta\nu}-H_2\mathcal{K}^2_{\beta\nu}]\\
&=\frac{1}{4}\,\frac{g^*_N}{M^2-s}\,g_{\mu\alpha}\mathcal{P}^{\alpha\beta}(K)\Big|_{M\to-M}[-(H_1^SI_0^i+H_1^VI_+^i+H_1^VI_-^i)\mathcal{K}^1_{\beta\nu}\\
&\qquad+(H_2^SI_0^i+H_2^VI_+^i+H_2^VI_-^i)\mathcal{K}^2_{\beta\nu}]\,, \qquad \text{(B.11)}
\end{aligned}
$$

where we have used in analogy with (B.6)

$$
\langle N^*|A^i_\mu|N\rangle=-g_{N^*}f_\pi\bar{N}^{*\alpha}(K)\,\tfrac{1}{2}\tau^i g_{\alpha\mu}\gamma_5 N(p)\,. \qquad \text{(B.12)}
$$

Comparing (B.7) and (B.11) we can see $\Delta\to N^*$ implies that we should let $\frac{2}{3}g^*\to\frac{1}{2}g^*_N$, $G_1\to-\frac{1}{2}H_1^{V,S}$, and $G_2\to\frac{1}{2}H_2^{V,S}$ for $(+,0)$ and $M\to-M$. The $(-)$ amplitude is then obtained from the expressions

$$
B_j^{(-)N^*}=\frac{\nu}{\nu_{N^*}}\,B_{j\mathrm{P}}^{(+)N^*}\,,\quad j\neq 3 \qquad \text{(B.13a)}
$$

$$
B_3^{(-)N^*}=\frac{\nu_{N^*}}{\nu}B_3^{(+)N^*}\,. \qquad \text{(B.13b)}
$$

For completeness the alternate field-theory background as obtained in Ref. 6 is given by Eqs. (7) and (10).

Finally, we outline the derivation of the soft-isobar expressions for the pion amplitude, Eqs. (4a) and (4b). We first note that contracting (B.6) with q^μ and dividing by f_π yields the coupling for $\Delta N\pi$. This means that if we first contract (B.10) with q^μ and then let $\nu^2\to\nu_\Delta{}^2$ (rather than performing these operations in the opposite order as was done above) we will obtain the on-shell expression for the pion amplitude. Letting $q\to 0$ in the coefficient of $I^i_+K^1_\nu$ will yield Eq. (4a) and the use of the $\Delta\to N^*$ prescription given after Eq. (B.11) will then yield Eq. (4b).

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[1]J. T. MacMullen and M. D. Scadron, preceding paper, Phys. Rev. D 20, 1069 (1979), referred to as I.

[2]S. Fubini, G. Furlan, and C. Rossetti, Nuovo Cimento 40, 1171 (1965).

[3]S. L. Adler and F. J. Gilman, Phys. Rev. 152, 1460 (1966).

[4]F. D. Gault and M. D. Scadron, Nucl. Phys. B (to be published).
[5]J. Bell *et al.*, Phys. Rev. Lett. 41, 1012 (1978).
[6]S. Fubini, G. Furlan, and C. Rossetti, Nuovo Cimento 43, 161 (1966).
[7]M. G. Olsson and E. T. Osypowski, Nucl. Phys. B87, 399 (1975).
[8]R. McClaskey, R. J. Jacob, and G. E. Hite, Arizona State University report (unpublished); Phys. Rev. D 18, 660 (1978).
[9]R. G. Moorhouse, H. Oberlack, and A. H. Rosenfeld, Phys. Rev. D 9, 1 (1974).
[10]F. A. Berends and A. Donnachie, Nucl. Phys. B84, 342 (1975).
[11]W. Pfeil and D. Schwela, Nucl. Phys. B45, 379 (1972).
[12]G. von Gehlen, Bonn University Report No. PI 2-80, 1970 (unpublished).
[13]M. I. Adamovich *et al.*, Yad. Fiz. 11, 657 (1970) [Sov. J. Nucl. Phys. 11, 369 (1970)].
[14]P. Nolle, Diploma thesis, Bonn, 1972 (unpublished).
[15]A. Donnachie and G. Shaw, Ann. Phys. (N. Y.) 37, 333 (1966).
[16]P. de Baenst, Nucl. Phys. B24, 633 (1970).
[17]M. G. Olsson and E. T. Osypowski, Phys. Rev. D 17, 174 (1978).
[18]F. A. Berends, A. Donnachie, and D. L. Weaver, Nucl. Phys. B4, 54 (1974).
[19]G. Shaw and A. Donnachie, Nucl. Phys. 87, 556 (1967).
[20]The values for the A_1^i and E_{0+}^i were obtained using hyperbolic boundary conditions and were given to us by MJH in a private communication.
[21]H. Nielsen and G. C. Oades, Nucl. Phys. B72, 310 (1974).
[22]G. Höhler, H. P. Jacob, and R. Strauss, Nucl. Phys. B39, 237 (1972).
[23]I. M. Barbour, R. L. Crawford, and N. H. Parsons, Nucl. Phys. B141, 253 (1978).
[24]H. F. Jones and M. D. Scadron, Ann. Phys. (N. Y.) 81, 1 (1973).
[25]Particle Data Group, Rev. Mod. Phys. 48, S1 (1978).
[26]H. F. Jones and M. D. Scadron, Phys. Rev. 173, 1734 (1968); Nucl. Phys. B10, 17 (1969); M. D. Scadron, Univ. of Arizona report, 1970 (unpublished).
[27]M. D. Scadron and L. R. Thebaud, Phys. Rev. D 9, 1544 (1974).