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New class of f-g fields relevant to quark confinement

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We propose an ansatz for the strong-gravity f field which is motivated by the quark-confinement problem and construct exact solutions of the vacuum f-g field equations.

Recently Salam and Strathdee[1] proposed a mechanism for quark confinement within the framework of the f-g field theory.[2] Their discussion is based on a de Sitter-type solution of the f-g field equations[3] which was also discovered by Isham and Storey.[4] The suggestion of Salam and Strathdee is to start with a de Sitter space-time and scale it down so that we end up with a micro-universe about the size of a hadron. This micro-universe is then regarded as being embedded in ordinary space-time and represents a kind of bag. In this note we shall point out a new class of f-g fields which is relevant for the quark-confinement problem. This class can be characterized by an ansatz for the strong-gravity field $f^{\mu\nu}$,

$$f^{\mu\nu}=\phi g^{\mu\nu}+H^{\mu\nu}. \tag{1}$$

where ϕ is in general a scalar field and $H^{\mu\nu}$ is a tensor field which we shall subsequently specialize. In addition $g^{\mu\nu}$ is the weak-gravity field which will be a solution of Einstein's equations. Salam and Strathdee chose $g^{\mu\nu}$ as an asymptotically de Sitter space-time. We shall later point out an alternative possibility for confinement, using a much larger class of solutions.

To simplify the field equations resulting from (1) we let the tensor field $H^{\mu\nu}$ satisfy[5]

$$H^{\mu\alpha}H_{\nu\alpha}=aH^{\mu}{}_{\nu}+b\delta^{\mu}{}_{\nu}, \tag{2}$$

which can always be arranged provided

$$(H^2-3aH+3a^2-4h)(H-2a)=0, \tag{3}$$

where a and b are scalar fields with

$$h=\tfrac{1}{4}H^{\alpha\beta}H_{\alpha\beta},\quad H=H^{\mu}{}_{\mu}.$$

The inverse of $f^{\mu\nu}$ is then given by

$$f_{\mu\nu}=(-b+\phi^2+a\phi)^{-1}[(\phi+a)g_{\mu\nu}-H_{\mu\nu}]. \tag{4}$$

For this choice of $H^{\mu\nu}$ the f-g field equations with the Isham and Storey[4] mixing term reduce to

$$G^{f}_{\mu\nu}=k_f{}^2(Xf_{\mu\nu}+YH_{\mu\nu}), \tag{5}$$

$$G^{g}_{\mu\nu}=k_g{}^2(Wg_{\mu\nu}+ZH_{\mu\nu}), \tag{6}$$

where

$$X=\alpha^2(g/f)^u(\phi+a)^{-1}\times[vI(\phi+a)+2(3\phi-3+H)(\phi^2+a\phi-b)], \tag{7a}$$

$$Y=2\alpha^2(g/f)^u(\phi+a)^{-1}(2\phi-a+H-3), \tag{7b}$$

$$W=\alpha^2(f/g)^v[uI-2\phi(3\phi-3+H)+2b], \tag{7c}$$

$$Z=-2\alpha^2(f/g)^v(2\phi-a+H-3), \tag{7d}$$

with

$$I=-12(\phi-1)^2-6(\phi-1)H-H^2+4h,$$

$$g/f=(b-\phi^2-a\phi)(-\phi^2+h+\tfrac{5}{4}aH+a\phi-\tfrac{1}{2}H^2-a^2-\phi H),$$

$$\alpha=M/2k_f.$$

This system of equations can be further simplified if we consider special forms of $H_{\mu\nu}$ satisfying (2) and (3). We shall find that for these cases field equations can be solved readily.

(a) $H_{\mu\nu}=\psi u_\mu u_\nu$, where ψ is a constant and u_μ is a timelike four-vector. It is remarkable that $Y=Z=0$ for the choice $\phi=\frac{3}{2}$ in which both fields $f_{\mu\nu}$ and $g_{\mu\nu}$ represent the solution of Einstein's equations with cosmological constants

$$\lambda_f=k_f{}^2X,\quad \lambda_g=k_g{}^2W, \tag{8}$$

respectively. This property was observed by Isham and Storey[4] in spherically symmetric space-times. Their solution falls into this class when $g_{\mu\nu}$ is taken as the usual de Sitter metric and $H=a=\psi$, $h=\frac{1}{4}\psi^2$, and $b=0$ with

$$\psi=-\tfrac{3}{2}+\tfrac{2}{3}\Delta^{-1},$$

$$u_\mu dx^\mu=(1-\tfrac{9}{4}\Delta)^{-1/2}[A^{1/2}(1-\tfrac{9}{4}B\Delta)^{1/2}dt+\tfrac{3}{2}A^{-1/2}\Delta^{1/2}(1-B)^{1/2}dr],$$

where

$$A=1-\frac{2\mu_g}{r}-\frac{\lambda_g r^2}{3},$$

$$B=\left(1-\frac{2\mu_f}{r}-\frac{2\lambda_f r^2}{9}\right)\Big/A,$$

and Δ is an integration constant.

(b) $H_{\mu\nu}=\phi k_\mu k_\nu$, where k_μ is a null vector. The scalars a, b, and H vanish. The field $f_{\mu\nu}$ and its determinant follow as

$$f_{\mu\nu}=\phi^{-1}(g_{\mu\nu}-k_\mu k_\nu), \tag{9}$$

$$g/f=\phi^4. \tag{10}$$

Here the form of $f_{\mu\nu}$ looks like the conformally scaled generalized Kerr-Schild metric. The Kerr-Schild[6-9] metric and its generalization[10-12] have been studied by several authors in general relativity. Without the conformal factor ϕ^{-1}, the Kerr-Schild metric has also been used by Aichelburg *et al.*[13] in f-g field equations. Mansouri and Urbantke[14] showed that there exists a unique solution of this type which is the well-known *pp*-wave metric. With the conformal factor we observe that there exists a large class of solutions. Among these solutions the cases where both metrics are scaled cosmological solutions with $\phi=\frac{3}{2}$ are particularly interesting.

(i) Kerr–de Sitter solution.[15,16] The nonvanishing parameters are found as

$$\frac{\Lambda_g}{\Lambda_f}=\tfrac{2}{3},$$

$$u=-\frac{3+\mu^2}{3+2\mu^2},\quad v=\frac{9+8\mu^2}{6+4\mu^2},$$

where $\mu^2=k_g^2/k_f^2\sim 10^{-38}$. The forms of $g_{\mu\nu}$ and k_μ are given in Ref. 16.

(ii) Accelerated de Sitter solution.[8] The solution is

$$g_{\mu\nu}=\eta_{\mu\nu}-\tfrac{1}{3}\lambda_1 R^2\partial_\mu\tau\partial_\nu\tau,$$

$$f_{\mu\nu}=\tfrac{2}{3}[\eta_{\mu\nu}-\tfrac{1}{3}R^2(\lambda_1+\lambda_2)\partial_\mu\tau\partial_\nu\tau],$$

where $\eta_{\mu\nu}$ is the Minkowski metric, R and τ are the retarded distance and affine parameter on a geodesic, respectively.[9] Cosmological constants λ_f and λ_g are found as

$$\lambda_g=\lambda_1,$$

$$\lambda_f=\tfrac{3}{2}(\lambda_1+\lambda_2).$$

It is interesting to note that the classes given above are also the solutions of the f-g field equations corresponding to the mixing term in Ref. 1 [Eq. (3.1c)], provided $x+(1-x)\phi=0$. In this case the field equations decouple as they do in general when the determinant of $H_{\mu\nu}$ vanishes.

We shall remark on an interesting feature of our ansatz which may be relevant to Salam and Strathdee's confinement program. When the weak part is subtracted from $f^{\mu\nu}$ we get

$$f^{\mu\nu}=g^{\mu\nu}+k_f^2 T^{\mu\nu}, \tag{11}$$

where

$$T^{\mu\nu}=\frac{1}{k_f^2}[(\phi-1)g^{\mu\nu}+H^{\mu\nu}]. \tag{12}$$

This tensor formally looks like an energy-momentum tensor. In particular we can recognize a perfect fluid (null or non-null) energy-momentum tensor with constant pressure and constant energy density for the above solutions. If processes inside the hadronic matter are governed by the strong-gravity field $f^{\mu\nu}$, then this tensor $T^{\mu\nu}$, especially the $(\phi-1)/k_f^2$ term, should play an important role in the problem of quark confinement. We should therefore search for solutions where ϕ is not unity.

Exact solutions with constant ϕ such as those we have presented above may be relevant to the confinement problem along the lines suggested by Salam and Strathdee. However, their choice for the size of the de Sitter micro-universe appears arbitrary, and it would be desirable if the scaling that they have suggested could be obtained in a dynamical way. In this regard we believe that conformal mapping, particularly Penrose's[17] conformal treatment of infinity, can play an important role, and this would provide further motivation of our ansatz (1). A dynamical conformal factor ϕ could also serve as the definition of the boundaries of the bag.

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[1]A. Salam and J. Strathdee, Phys. Rev. D 18, 4596 (1978).

[2]C. J. Isham, A. Salam, and J. Strathdee, Phys. Rev. D 3, 367 (1971).

[3]A. Salam and J. Strathdee, Phys. Rev. D 16, 2668 (1977).

[4]C. J. Isham and D. Storey, Phys. Rev. D 18, 1047 (1978).

[5]Our convention and notation is the same as that of Ref. 4. Contracting and raising and lowering of indices are

done with the metric $g_{\mu\nu}$.
[6]G. C. Debney, R. P. Kerr, and A. Schild, J. Math. Phys. 10, 1842 (1969); R. P. Kerr and A. Schild, in *Proceedings of Symposium in Applied Mathematics*, edited by H. Grad (American Mathematical Society, Providence, R.I., 1966), Vol. XVII, p. 199.
[7]M. M. Schiffer, R. J. Adler, J. Mark, and C. Sheffield, J. Math. Phys. 14, 52 (1973).
[8]M. Gürses and F. Gürsey, J. Math. Phys. 16, 2385 (1975).
[9]W. B. Bonnor and P. C. Vaidya, in *General Relativity*, papers in honor of J. L. Synge, edited by L. O'Raifeartaigh (Dublin Institute for Advanced Studies, Dublin, 1972), p. 119.
[10]I. Robinson and J. R. Robinson, Int. J. Theor. Phys. 2, 231 (1964).
[11]C. J. Talbot, Commun. Math. Phys. 13, 45 (1969).
[12]B. C. Xanthopoulos, J. Math. Phys. 19, 1607 (1978).
[13]P. C. Aichelburg, R. Mansouri, and H. K. Urbantke, Phys. Rev. Lett. 27, 1533 (1971).
[14]R. Mansouri and H. K. Urbantke, Commun. Math. Phys. 26, 301 (1972).
[15]B. Carter, Phys. Lett. 26A, 399 (1968).
[16]B. Carter, in *Black Holes*, 1972 Les Houches Lectures, edited by C. M. DeWitt and B. S. DeWitt (Gordon and Breach, New York, 1973).
[17]R. Penrose, in *Relativity, Groups and Topology*, 1963 Les Houches Lectures, edited by C. M. DeWitt and B. S. DeWitt (Gordon and Breach, New York, 1964).