a right to doubt its existence. On the other hand, the existence of a cut in complex plane was never doubted. By considering the cut contribution to the total cross sections of π^+p and π^-p, Pursey and Sertorio[1] have already explained the $\pi^\pm p$ experimental data. Using their theory, we have drawn the same conclusions for NN and KN reactions. As regards the asymptotic limit of the energy, we find, in the case where a P pole is absent, that the asymptotic limit may very well lie in the energy region considered in this paper. In Sec. II, we have discussed the elastic scattering cross sections in about this energy region and found from the experimental results a relation $\sigma_{el} \propto S^{-n}$, where n is nonzero. We find that we cannot derive such a simple formula even in the case where no P pole is there and a cut replaces it. From the energy dependence of the cut, one can only say that there will be a dependence of the cross section on energy.

In case a cut has to replace a Pomeranchukon, many problems such as those mentioned in Ref. 1 will arise. At the moment, we can question the existence of a Pomeranchukon and say only that the contribution from a cut can explain some part of the available experimental data.

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Algebra of Pion Sources

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Assuming that the space integrals of source terms (sources) in the Klein-Gordon equation for the pion fields together with isospin generators form an $SU(2)\otimes SU(2)$ algebra which, in the soft-pion limit, is a good symmetry of the strong interactions, we calculate S-wave scattering lengths for the collision of pions with hadron targets as well as the pion-nucleon coupling constant. The results are in excellent agreement with experiment.

Many successful calculations[1] of pure strong-interaction processes have been carried out during recent years with the aid of an assumed $SU(2)\otimes SU(2)$ structure of vector and axial-vector currents of hadrons. In all these calculations, a vital role is played by the partially conserved axial-vector current (PCAC) hypothesis which provides the necessary link between the axial-vector current and strong interactions. This hypothesis, however, is not precisely defined[2] and may be the source of some of the failures[3] of this brand of algebraic approach to strong-interaction physics. It would therefore be worthwhile to look for alternative approaches where the PCAC hypothesis would be unnecessary for calculation of strong-interaction parameters. It may be noted in this context that the scattering matrix for pion-hadron collisions is usually written in terms of the retarded commutator of pion-source densities. It is, therefore, natural to ask whether these pion sources, i.e., the space integrals of these densities, together with isospins, which are known to be constants of motion for strong interactions, do form a closed algebra. In this paper we discuss the consequences of an $SU(2)\otimes SU(2)$ algebraic structure of the isospin generators and pion sources. We show that predictions for scattering lengths and coupling constants can be made in the soft-pion limit, provided we assume that for hadrons at rest this $SU(2)\otimes SU(2)$ is a good symmetry of the Hamiltonian for strong interactions in this limit. The results agree well with experiment for pion-nucleon scattering.

The invariant T matrix for forward scattering of charged pions on a hadron target at rest can be written

[1] Y. Tomozawa, Nuovo Cimento **46A**, 707 (1966); K. Raman and E. C. G. Sudarshan, Phys. Letters **21**, 450 (1966); A. P. Balachandran, M. Gundzik, and F. Nicodemi, Nuovo Cimento **44A**, 1257 (1966); S. Weinberg, Phys. Rev. Letters **17**, 616 (1966); L. N. Chang, Phys. Rev. **162**, 1497 (1967).

[2] No precise meaning can be given to the smooth-variation hypothesis of matrix elements of the pion source density with the square of momentum transfer required for the application of the PCAC equation. Further, there are ambiguities when matrix elements between multiparticle states are considered. See, for instance, S. L. Adler and R. F. Dashen, *Current Algebras and Applications to Particle Physics* (Benjamin, New York, 1968), Chap. 1. Besides, in the presence of electromagnetic and weak interactions, the PCAC equation itself gets altered. See, for instance, S. L. Adler, Phys. Rev. **177**, 2426 (1969); S. R. Choudhury, Y. Tomozawa, and Y. P. Yao, Nucl. Phys. **B17**, 430 (1970).

[3] For instance, $\pi^0 \to 2\gamma$, $\eta \to 2\gamma$, and $\eta \to \pi^+\pi^-\pi^0$ are incorrectly obtained: D. G. Sutherland, Nucl. Phys. **B2**, 433 (1967); Phys. Letters **23**, 384 (1966).

as

$$T^{\mp}(k)=2iM\int d^4x\, e^{-ik\cdot x}\theta(x_0)\langle t|[j^{\pm}(x),j^{\mp}(0)]|t\rangle$$

$$-2iM\int d^4x\, e^{-ik\cdot x}\delta(x_0)\langle t|\{[j^{\pm}(x),\dot{\phi}^{\mp}(0)] +ik_0[j^{\pm}(x),\phi^{\mp}(0)]\}|t\rangle, \quad (1)$$

where $|t\rangle\equiv$hadron state, $-j^{\pm}(x)=(\Box-m^2)\phi^{\pm}(x)$, $\phi^{\pm}$ being pion field operators, M is the target mass, and k is the pion four-momentum. The superscripts $\pm$ represent the charge of the pion. At threshold, i.e., for $\mathbf{k}=0$, this equation becomes

$$T^{\mp}(m)=2iM\int_{-\infty}^{\infty} dx_0\, e^{imx_0}\theta(x_0)\langle t|[P^{\pm}(x_0),P^{\mp}(0)]|t\rangle$$

$$-2iM\langle t|[P^{\pm}(0),\dot{\Phi}^{\mp}(0)]|t\rangle +2Mm\langle t|[P^{\pm}(0),\Phi^{\mp}(0)]|t\rangle, \quad (2)$$

where

$$P^{\pm}(x_0)=\int d^3x\, j^{\pm}(x)$$

is the pion source operator and

$$\Phi^{\pm}(x_0)=\int d^3x\, \phi^{\pm}(x).$$

Using the Klein-Gordon equation

$$(\partial^2/\partial x_0{}^2+m^2)\Phi^{\pm}(x_0)=P^{\pm}(x_0),$$

and the Jacobi identity to convert $\langle t|[P^{\pm}(0),\dot{\Phi}^{\mp}(0)]|t\rangle$ and $\langle t|[P^{\pm}(0),\ddot{\Phi}^{\mp}(0)]|t\rangle$ into $\langle t|[\dot{P}^{\pm}(0),\Phi^{\mp}(0)]|t\rangle$ and $\langle t|[\ddot{P}^{\pm}(0),\Phi^{\mp}(0)]|t\rangle$, Eq. (2) can be brought to the form

$$T^{\mp}(m)=2iM\int_{-\infty}^{\infty} dx_0\, e^{imx_0}\theta(x_0)\langle t|[P^{\pm}(x_0),P^{\mp}(0)]|t\rangle$$

$$+2iM\langle t|[\dot{P}^{\pm}(0),\Phi^{\mp}(0)]|t\rangle -(2M/m)\langle t|[\ddot{P}^{\pm}(0),\Phi^{\mp}(0)]|t\rangle +(2M/m)\langle t|[P^{\pm}(0),P^{\mp}(0)]|t\rangle. \quad (3)$$

Further simplification of this result can be made if we make the following two assumptions:

(i) The operators $P^{\pm}$ and P^0, together with the isospin generators, form an $SU(2)\otimes SU(2)$ group, i.e.,

$$[I^{\alpha},I^{\beta}]=i\epsilon_{\alpha\beta\gamma}I^{\gamma}, \quad [I^{\alpha},P^{\beta}(0)]=i\epsilon_{\alpha\beta\gamma}P^{\gamma}(0),$$
$$[P^{\alpha}(0),P^{\beta}(0)]=i\epsilon_{\alpha\beta\gamma}I^{\gamma}. \quad (4)$$

The second commutation relation defines the transformation law of the pion sources under isospin rotations, while the last one fixes the scale of their matrix elements.

(ii) The $SU(2)\otimes SU(2)$ defined in Eq. (4) is a good symmetry of the strong-interaction Hamiltonian for hadrons in the zero-pion-mass limit in which case $\dot{P}^{\pm}(x_0)=0$, in addition to $\dot{I}^{\pm}=0$.

The validity of these assumptions can be tested by comparing with experiment the consequences that follow from them. From Eq. (3) we get

$$\lim_{m\to 0} mT^{\mp}(m)=\lim_{m\to 0}\langle t|\{2iMm[\dot{P}^{\pm}(0),\Phi^{\mp}(0)] -2M[\ddot{P}^{\pm}(0),\Phi^{\mp}(0)]+2M[P^{\pm}(0),P^{\mp}(0)]\}|t\rangle, \quad (5)$$

since the first term on the right-hand side of Eq. (3) does not have any singularity at $m=0$. If we now make use of our assumption (ii), Eq. (5) takes the simple form

$$\lim_{m\to 0} mT^{\mp}(m)=2M\langle t|[P^{\pm}(0),P^{\mp}(0)]|t\rangle =\pm 2M\langle t|2I_3|t\rangle, \quad (6)$$

from which it follows that

$$\lim_{m\to 0} mT_{\text{odd}}(m)=-2M, \quad \lim_{m\to 0} mT_{\text{even}}(m)=0, \quad (7)$$

where T_{odd} and T_{even} are defined by the equation

$$T^{\mp}(\omega)=T_{\text{even}}(\omega)\mp\langle t|2I_3|t\rangle T_{\text{odd}}(\omega). \quad (8)$$

On using the definition,

$$T(m)=8\pi(1+M/m)a,$$

for the dimensionless s-wave scattering length a, we obtain from Eq. (7)

$$a_{\text{odd}}=-1/4\pi, \quad a_{\text{even}}=0. \quad (9)$$

The results for nucleon, pion, and kaon targets are given in Table I, along with the corresponding results obtained from vector–axial-vector algebra and the experimental results. It will be noticed that our scattering lengths are different from those of the V-A current algebra, which are expressed in terms of weak-interaction parameters. Our results do not contain such parameters because we work in terms of strong-interaction operators alone. For π-N collisions, our scattering lengths are the experimental value, whereas the V-A current-algebra scattering lengths are higher.

A further application of our scheme can be made in the determination of the pion-nucleon coupling constant. This is done by combining Eq. (7) with the unsubtracted dispersion relation

$$\text{Re}T_{\text{odd}}(\omega)=-\frac{4M\omega f^2(1-m^2/4M^2)}{\omega^2-(m^2/2M)^2} +\frac{2M\omega}{\pi}\int_m^{\infty} d\omega'\frac{(\omega'^2-m^2)^{1/2}}{\omega'^2-\omega^2} \times[\sigma_{\text{tot}}{}^{p\pi^+}(\omega')-\sigma_{\text{tot}}{}^{p\pi^-}(\omega')]. \quad (10)$$

TABLE I. S-wave scattering lengths for collision of pions with N, K, and π.

Target	Scattering lengths from algebra of meson sources	Scattering lengths from algebra of V and A currents	Experimental values
Nucleon	$a_{1/2}=\frac{1}{2\pi}=0.16$	$a_{1/2}=\frac{1}{2\pi}\left(\frac{M_N}{M_N+m}\right)\left(\frac{2g_V^2m^2}{f_\pi^2}\right)=0.20$	$a_{1/2}=0.17$
	$a_{3/2}=-\frac{1}{4\pi}=-0.08$	$a_{3/2}=-\frac{1}{4\pi}\left(\frac{M_N}{M_N+m}\right)\left(\frac{2g_V^2m^2}{f_\pi^2}\right)=-0.10$	$a_{3/2}=-0.09$
Kaon	$a_{1/2}=\frac{1}{2\pi}=0.16$	$a_{1/2}=\frac{1}{2\pi}\left(\frac{M_K}{M_K+m}\right)\left(\frac{2g_V^2m^2}{f_\pi^2}\right)=0.18$	...
	$a_{3/2}=-\frac{1}{4\pi}=-0.08$	$a_{3/2}=-\frac{1}{4\pi}\left(\frac{M_K}{M_K+m}\right)\left(\frac{2g_V^2m^2}{f_\pi^2}\right)=-0.09$	...
Pion	$a_0=\frac{7}{16\pi}=0.14$	$a_0=\frac{7}{16\pi}\left(\frac{2g_V^2m^2}{f_\pi^2}\right)=0.20$	...
	$a_2=-\frac{1}{8\pi}=-0.04$	$a_2=-\frac{1}{8\pi}\left(\frac{2g_V^2m^2}{f_\pi^2}\right)=-0.06$	...

This leads to the result

$$2f^2=1\,, \tag{11}$$

since only the pole term of Eq. (10) survives in the limit considered in Eq. (7). However, in this zero-pion-mass limit, the nucleon pole and the threshold branch point of the T matrix coincide. The pole would thus lie on the contour employed in obtaining the dispersion relation. For this reason, the pole term should be multiplied by a factor of $\frac{1}{2}$ in this limit. Therefore, relation (11) should be replaced by

$$f^2=1 \quad \text{or} \quad f^2/4\pi=1/4\pi=0.079\,, \tag{12}$$

which agrees very well with the result $f^2/4\pi=0.081 \pm 0.002$ obtained by comparing forward dispersion relations with experiment.[4]

The good agreement of our results with experiment implies that the scale fixed by the last commutation relation of Eq. (4) for the matrix element of the pion source operators is correct and that the $SU(2)\times SU(2)$ algebra defined by this equation is a good symmetry of the strong interactions in the soft-pion limit.

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[4] J. Hamilton and W. S. Woolcock, Rev. Mod. Phys. **35**, 737 (1963).