Possible Contribution of Regge Cut to High-Energy Cross Sections of pp, $\bar{p}p$, K^+p, and K^-p Reactions

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In this paper, we first discuss the elastic scattering cross-section data on $K^{\pm}p$ and pp reactions and investigate the possibility of explaining the data by considering the contribution of the Pomeranchukon pole. In the second half of the paper, we consider whether the contribution due to a cut in the complex angular momentum plane instead of that due to the Pomeranchukon can explain the sum of the total cross sections of the pp and $\bar{p}p$ reactions, and that of the K^+p and K^-p reactions. The calculated values have been compared with the older data along with the recently available Serpukhov machine data up to 55 BeV/c. We conclude that both sets of reactions may be largely explained by a cut. Finally, by considering the differences between the particle-particle and particle-antiparticle cross sections, we predict that at high energies, the pp and K^+p reactions should show a small decrease in their total cross sections.

I. INTRODUCTION

A PARTICLE corresponding to the Pomeranchukon trajectory has not been discovered so far. This may lead one to doubt the existence of the P trajectory and the role played by the P pole itself. One may ask: Cannot all the things said to be the contributions of a P pole be simulated by something else, as, for example, by a cut in the complex angular momentum plane?

In order to discuss this problem, we shall consider various experimental data available to us. In Sec. II, we shall discuss the elastic scattering cross sections of pp, K^+p, and K^-p reactions and find out whether a P pole can explain the data. In Sec. III, we shall consider the sum of the total cross sections $\sigma_{pp}+\sigma_{\bar{p}p}$ (sometimes referred to in the text as the NN case) and $\sigma_{K^+p}+\sigma_{K^-p}$ (referred to as the KN case), using the theory developed by Pursey and Sertorio,[1] and discuss the contribution to total cross sections due to a cut in the complex l plane. Finally, in Sec. IV we find an asymptotic form for the differences in the total cross sections of $\bar{p}p$, pp and K^-p, K^+p reactions.

II. ELASTIC SCATTERING CROSS SECTIONS OF pp, K^+p, AND K^-p REACTIONS AND THE POMERANCHUKON

In this section we propose to present a study of the pp, K^+p, and K^-p interactions in the elastic region in a high-energy interval. For inelastic reactions it has been experimentally proved[2] that the cross sections vary inversely as the nth power (n being a constant) of S, the square of the total c.m. energy. In the high-energy limit, the incident laboratory momentum $P_{\rm lab}$ is proportional to S; hence one can write this relation as $\sigma\propto(P_{\rm lab})^{-n}$. Morrison[2] analyzed the experimental data for different inelastic reactions in terms of Regge poles and assigned different numerical values for n. Depending upon the pole or poles exchanged, the values of n range from 0 to 4. For elastic scattering, a value of $n=0$ is expected if the Pomeranchukon is exchanged (since no quantum numbers are exchanged in the elastic reactions). In order to test this relation, we have studied the elastic scattering cross sections of the pp, K^+p, and K^-p reactions, starting from some energy at which the diffraction scattering predominates. We present on a log-log plot elastic scattering cross sections against incident laboratory momenta $P_{\rm lab}$, for pp scattering in the momentum range 1.66 to 5.5 BeV/c, using the experimental data from Refs. 3–7, Fig. 1(a); for K^+p scattering, range 1–5 BeV/c using the data from Refs. 8–12, Fig. 1(b); and for K^-p scattering, range 3–5

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BeV/c, from Refs. 13–17, Fig. 1(c). For the energy regions under consideration for each of the reactions, a straight-line visual fit has been presented. We find that the values of the slopes n of these fits are (i) pp, $n\approx 0.65$, (ii) K^+p, $n\approx 0.8$, and (iii) K^-p, $n\approx 0.35$. (A value of this order for K^-p can also be deduced from Ref. 17.)

The main conclusion drawn for the energy regions under consideration is that the values of the slopes, at least for the first two reactions, are far from zero. It is true that for energies still higher than those considered in this section, the values of the slopes for these reactions may be in the vicinity of zero. However, one may ask whether the slopes will be really equal to zero? For example, high-energy $\pi^{\pm}p$ data[18] show a small but persistent decrease in the total cross sections. Hence the situation demands accurate data for elastic scattering cross sections also, at least up to the available machine energies. The next question is whether the use of the Pomeranchukon pole is all right at the energies we have considered. It is clear from the above discussions that at least it cannot explain the energy behavior of the elastic scattering cross section. As mentioned before,[2] inelastic cross sections have been interpreted in terms of Regge poles, including the P pole. Similarly, elastic scattering, such as e.g., K^+p scattering[19] in the momentum range considered in this paper, has been very well explained using exchange of Regge poles. Hence if the Regge model is correct, then we expect it to explain the relation $\sigma\propto S^{-n}$.

One other viewpoint may be that all the successful explanations given by using Regge poles including P are fortuitous and such a theory should be applicable only in the asymptotic regions, with the definition of asymptotic region being that energy at which the Pomeranchukon theorem is applicable and where the P pole dominates. In that case, that energy seems to be far off as indicated by Lindenbaum[18] for the $\pi^{\pm}p$ reactions; this limit is 25 000 BeV. However, if a P pole and its trajectory do not exist, then the definition of the asymptotic region loses its meaning. Is it then possible to have a Regge model with all the poles but the P pole replaced by something else, say, a cut in the l plane? In that case the cut has to simulate all the things done by a pole in a pure-pole explanation of the experimental data. However, unless proved, this remains very much a conjecture.

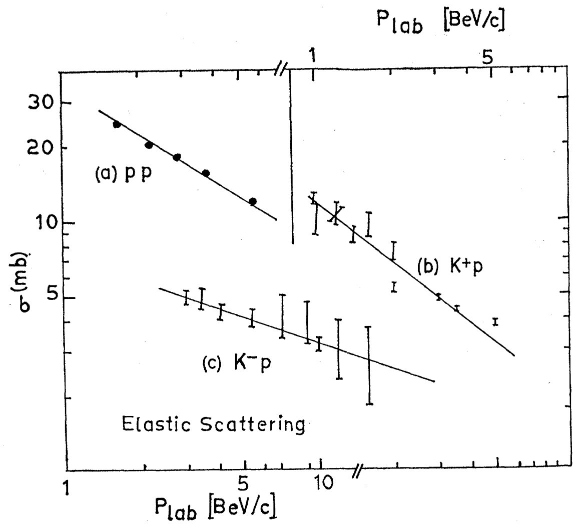

FIG. 1. Log-log plots of σ versus P_{lab}. (a) p-p elastic scattering (data used from Refs. 3–7); (b) K^+p elastic scattering (data used from Refs. 8–12); (c) K^-p elastic scattering (data used from Refs. 13–17).

III. SUM OF TOTAL CROSS SECTIONS AND REGGE CUT

Pursey and Sertorio[1] have already investigated the question of the existence of a Pomeranchukon and whether it can be replaced by a Regge cut using $\pi^{\pm}p$ total cross sections. They used an asymptotic form of the total cross section following Igi[20] and imposed a sum rule derived from the forward dispersion relations. We shall assume that the same theory is applicable in the NN and KN cases and do our calculations for the sum of the total cross sections $(\sigma_{pp}+\sigma_{\bar{p}p})$ and $(\sigma_{K^+p}+\sigma_{K^-p})$. The expressions for total cross sections for each of these reactions can be written in terms of different Regge poles P, P', A, ω, and ρ and it can be shown,[21] considering the leading poles only, that the sums of cross sections as written above depend only on P, P', and A_2 poles.

We shall not go into the derivation presented by Pursey and Sertorio,[1] but shall mention only the im-

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portant formulas used. Some of the important assumptions about the cut made by these authors are (i) that the discontinuity across the cut is reasonable and has a factor α, and (ii) that the cut contribution depends on two adjustable parameters as it does in case of Regge poles. The formula for $\sigma(E)$, which stands for the sum of the total cross sections, is

$$\sigma(E)=\sigma_+(E)+\sigma_-(E)$$
$$=\sum_{j=1}^{n} b_j\left(\frac{E}{\bar{E}_j}\right)^{\alpha_j-1}+\left[B-\sum_{j=1}^{n}\frac{(1-\alpha_j)}{\alpha_j}b_j\right]\times\left[\frac{\ln(\bar{E}|u)}{\ln(E|u)}\right]^{\gamma+2}\left[\frac{\ln(E|u)}{\gamma+1}-1\right], \quad (1)$$

where the summation is over all the contributing poles; b_j is the residue term for the jth pole. E represents the total energy in laboratory frame, and $\bar{E}$ is the energy at which the asymptotic region begins. μ is the pion mass. The parameter γ is the cut parameter, and α is the usual trajectory parameter in the Regge model. The value of B is found[1] from the equation

$$EB=\int_0^{P_{\text{lab}}}(\sigma_++\sigma_-)dk-4\pi^2\left[D_1(\mu)+\frac{f^2}{M}\right]-\bar{E}[\sigma_+(\bar{E})+\sigma_-(\bar{E})], \quad (2)$$

where $D(\mu)$ is the real part of the scattering amplitude, f is the coupling constant, k is the laboratory momentum $\bar{P}_{\text{lab}}$ is the asymptotic limit of the laboratory momentum, and M is the nucleon mass. Using a numerical value of B and experimental cross-section values of $\pi^{\pm}p$ reactions, limits of the cut parameter were found,[1] which are $0.562\leqslant 1+\gamma\leqslant 0.609$. Using these values, Pursey and Sertorio[1] could prove, for the case of $\pi^{\pm}p$, that if the P pole and cut only are considered, the contribution to the cross section due to the cut dominates; and similar results were obtained using P'+cut. As suggested by these authors, it will be interesting to investigate whether the values of the cut parameters can explain other processes such as pp, $\bar{p}p$, K^+p, and K^-p. In using Pursey and Sertorio's theory, we shall use the same values of the cut parameter, and similarly use $\alpha=0.62$ for P'. Since we do not know the value of B, we shall treat it as a variable and use the experimental data[22,23] on pp, $\bar{p}p$, K^+p, and K^-p reactions available upto 55 BeV/c. The residue parameter b can be expressed in the following form, using the equations from Ref. 1:

$$b=\frac{\alpha}{(1-\alpha)-[\ln(\bar{E}|\mu)]/(\gamma+1)}\times\left[B\left(1-\frac{\ln(\bar{E}|\mu)}{\gamma+1}\right)-\bar{\sigma}\frac{\ln(\bar{E}|\mu)}{\gamma+1}\right], \quad (3)$$

where $\bar{\sigma}$ stands for the sum of the total cross sections at asymptotic energy $\bar{E}$. Using this formula, one can determine b in terms of B.

In this section, we shall present some of the calculations done for NN and KN reactions. The calculated values have been presented for the momenta 6 and from 20 to 1000 BeV/c. Since the Serpukhov machine data[23] in the region 20 to 55 BeV/c are available, it will be interesting to compare the calculated values with the experiments in this high-energy region. Calculations are presented for only one value of the cut parameter, namely, $\gamma+1=0.562$, since for the other end, i.e., $\gamma+1=0.609$, the calculated values differ by about 2 mb only. In the momentum region 20–55 BeV/c, pp and K^+p experimental cross sections are not available; however, from the trend of the data on pp and K^+p up to 20 BeV/c, let us expect constant cross sections for these reactions in the higher momentum region. The values of $\sigma_{pp}+\sigma_{\bar{p}p}$ and $\sigma_{K^+p}+\sigma_{K^-p}$ have been obtained assuming this. The asymptotic values of the cross sections used are $\sigma_{pp}=38.7\pm0.6$ mb and $\sigma_{K^+p}=17.5\pm0.1$ mb. Though we considered many values between 6 and 20 BeV/c as the asymptotic momenta, the calculations are presented only for two values of $\bar{P}_{\text{lab}}$ equal to 6 and 20 BeV/c. Following Ref. 1, we calculated the sum of the cross sections approximating it as due to a cut and one of the poles. However, we present the calculations here as if the sum is obtained by one asymptotic form alone. We shall discuss this point again.

A. *NN* Case

(*i*) *Cut and P or P′ Pole Contribution*

In Table I, column 3, we present the calculated values for $\sigma_{pp}+\sigma_{p\bar{p}}$, considering the contribution from the cut alone. For the case $\bar{P}_{\text{lab}}=6$ BeV/c, some of the calculated values up to 50 BeV/c are lower and some higher by about 1 to 2 standard deviations from the experimental $\sigma_{pp}+\sigma_{\bar{p}p}$. For $\bar{P}_{\text{lab}}=20$ BeV/c, the calculated values are within experimental errors up to 25 BeV/c, after which they become consistently lower by 2–4 standard deviations. In case 6 BeV/c is the asymptotic momentum region, the value of B is about 43, whereas for $\bar{P}_{\text{lab}}=20$ BeV/c, B is about 20. When a value of B is known from Eq. (2) it will help us to decide about $\bar{P}_{\text{lab}}$. From the calculated values, we then conclude that a small amount of pole contribution may give a statistically good fit; however, we do not attempt this, as

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TABLE I. Calculated values are presented for $\sigma_{pp}+\sigma_{\bar{p}p}$ for different momenta for the case of cut plus P- or P'-pole contribution, assuming values for the limits of the asymptotic momenta 6 and 20 BeV/c. The value of the cut parameter is taken as $\gamma+1=0.562$.

		Calculated cross sections in mb			
	Experimental total	Cut contribution only		P'-pole contribution only	
P_{lab}	cross section in mb	$\bar{P}_{lab}=6$	20	6	20
(BeV/c)	$\sigma_{pp}+\sigma_{\bar{p}p}$	$B=43.3$, $b=0$	$B=19.734$, $b=0$	$B=0$, $b=99.9$	$B=0$, $b=87.7$
6	99.90±1.25	99.90	99.10	99.90	138.65
20	87.70±1.25	88.38	87.70	63.48	87.70
25	84.80±0.85	86.23	85.57	58.33	80.55
30	85.80±0.85	84.58	83.91	54.44	75.19
35	84.20±0.92	83.20	82.57	51.35	70.91
40	83.70±0.92	82.06	81.45	48.80	67.41
45	83.60±0.92	81.12	80.44	46.66	64.48
50	82.30±1.00	80.28	79.58	44.83	61.93
70		77.65	77.05	39.46	54.51
100		75.11	74.49	34.46	47.60
1000		62.66	62.20	14.36	19.84

our aim is only to show the possible importance of the cut contribution in NN reactions.

(ii) P' Pole Contribution

The calculated values for the P'-pole contribution alone are presented in column 4 of Table I, for the same range of the asymptotic momenta. We note that the cross sections decrease very fast for both $\bar{P}_{lab}$ values; the differences at 50 BeV/c from the experimental values are 37 and 21 mb, respectively. The values at higher momenta up to 1000 BeV/c will become even lower than the pp cross-section values. However, as can be seen from Table I, this does not happen in the case of the cut contribution where the cross sections fall gradually, and even at 1000 BeV/c the calculated values are lower by about 20 mb from the experimental values at 50 BeV/c.

Thus we conclude for the NN case that if a cut is to contribute to pp and $\bar{p}p$ reactions, its contribution completely dominates that of either the P or the P' pole. It is possible that the data may be explained by using a combination of poles; however, what we want to emphasize here is that a similar conclusion to that drawn in the $\pi^{\pm}p$ case[1] can be drawn in the NN case also.

B. KN Case

(i) Cut and P or P' Pole Contribution

In Table II, column 3, we have presented calculations for $\sigma_{K^+p}+\sigma_{K^-p}$ considering only the cut contribution. For $\bar{P}_{lab}=6$ BeV/c, the calculated values at 20 BeV/c are less by about 2 mb than the experimental values, and this difference becomes about 6 mb at 55 BeV/c. At $\bar{P}_{lab}=20$ BeV/c, the calculated values are less by about 1.5 mb at 30 BeV/c and this difference becomes about 4 mb at 55 BeV/c. Thus it seems that a small amount of pole contribution may make up this difference. A statistically good fit, however, will not be attempted, for the same reasons mentioned in the NN case.

(ii) P' Pole Contribution

The calculated values are given in column 4 of Table II. For both the $\bar{P}_{lab}$ values, the disagreement with the data increases with momenta. At 55 BeV/c, the calcu-

TABLE II. Calculated values are presented for $\sigma_{K^+p}+\sigma_{K^-p}$ for different momenta for the case of cut plus P- or P'-pole contribution, assuming values for the limits of the asymptotic momenta 6 and 20 BeV/c. The value of the cut parameter is taken as $\gamma+1=0.562$.

		Calculated cross sections in mb			
	Experimental total	Cut contribution only		P'-pole contribution only	
P_{lab}	cross section in mb	$\bar{P}_{lab}=6$	20	6	20
(BeV/c)	$\sigma_{K^+p}+\sigma_{K^-p}$	$B=11.875$, $b=0$	$B=6.92$, $b=0$	$B=0$, $b=41$	$B=0$, $b=38.7$
6	41.00±0.31	41.00	44.08	41.00	61.11
20	38.70±0.61	35.98	38.70	25.99	38.70
25	38.20±0.41	35.16	37.82	23.87	35.55
30	38.80±0.31	34.53	37.13	22.27	33.17
35	38.30±0.31	34.02	36.61	21.00	31.28
40	38.40±0.31	33.59	36.14	19.96	29.74
45	38.10±0.31	33.22	35.74	19.09	28.44
50	38.50±0.41	32.91	35.38	18.34	27.32
55	39.00±0.61	32.63	35.08	17.69	26.35
65		32.02	34.56	16.14	24.74
100		30.95	33.30	14.10	21.00
1000		26.20	28.17	5.88	8.75

lated values are lower than the experimental values by about 15–20 mb. The calculations at still higher momenta show a very rapidly decreasing total cross section. Compare this with the cut contribution case, where the values are gradually falling; even at 1000 BeV/c the calculated values are only about 10–13 mb less than the experimental cross section at 55 BeV/c.

Thus we also conclude for the KN case that though a small contribution from a pole cannot be ruled out, the cut contribution may largely explain the available experimental values in the high-energy region.

IV. ASYMPTOTIC FORMS FOR DIFFERENCES IN CROSS SECTIONS AND COMMENTS

In this section we discuss the differences in the cross sections $\sigma_- - \sigma_+$ for all the old and newer data available. In the last section we discussed the sum of the cross sections $\sigma_- + \sigma_+$, and the way we have presented the calculations in Tables I and II makes it appear that the sum is approximated to an asymptotic form. Actually, as remarked there, some small amount of pole contribution is necessary to get a better fit to the experimental data. The question we now take up is whether the available data for the difference $\sigma_- - \sigma_+$ can be approximated by an asymptotic form. A useful parametrization is $(\sigma_- - \sigma_+) \propto E_{\text{lab}}^{-n}$, where E_{lab} is the total energy in the laboratory system and n is a constant. Using simple Regge-pole theory, one can write $(\sigma_- - \sigma_+) \propto E_{\text{lab}}^{1-\alpha(0)}$. For the NN and KN cases, considering only leading poles, $\alpha(0)$ is the value of trajectory parameter for ω at $t=0$. We use the above parametrization and present in Fig. 2 a plot on a log-log scale of $(\sigma_- - \sigma_+)$ against E_{lab} for values of the momenta from 6 to 55 BeV/c. For the

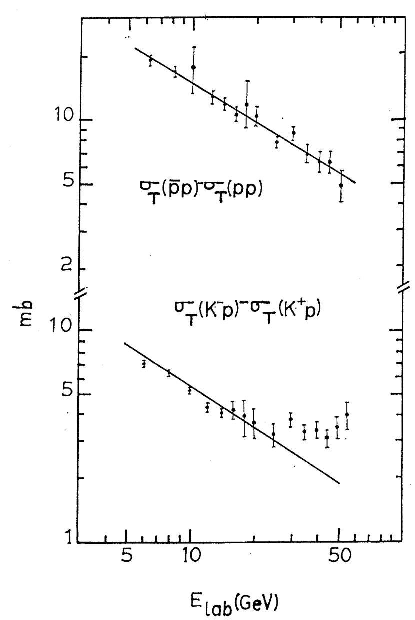

FIG. 2. Log-log plot of $(\sigma_- - \sigma_+)$ versus E_{lab} for NN and KN reactions. Experimental data used are from Refs. 22 and 23.

NN case a straight-line visual fit has been plotted. The calculated slope of this line is about 0.6, making the value of $\alpha_\omega(0)=0.4$, which is in good agreement with the value $\alpha_\omega(0)=0.38\pm0.4$ given in Ref. 24. For the KN case, however, all the points do not seem to lie on a straight line. At most, one may think of a fit with the sum of two asymptotic terms, e.g.,

$$\sigma_{K^-p} - \sigma_{K^+p} = C_1 E_{\text{lab}}^{-n_1} + C_2 E_{\text{lab}}^{-n_2},$$

where C_1 and C_2 are constants such that $C_1 > C_2$. n_1 and n_2 are also constants with $n_1 > n_2$. If n_2 is very near zero, the second term will dominate at higher energies. We have presented such a fit for the KN case in Fig. 2. The fit for the experimental points up to about 20 BeV gives a straight line of slope about 0.6, again showing an agreement with the value of $\alpha_\omega(0)$ as in the NN case. This kind of parametrization then suggests that one has to consider another pole in addition to ω in the KN case. Such a pole contributing to the $(\sigma_- - \sigma_+)$, however, will not be there in the sum of the cross sections.[21]

The reason we have tried the asymptotic parametrization for the differences in the cross-section values is to point out that if the sum is also approximated by an asymptotic form having a different energy dependence, we are going to face the problem of keeping the pp and K^+p total cross sections constant for all higher energies, as we have assumed in calculating the values in Tables I and II. We shall not, however, attempt here to find a balance between the two asymptotic forms, but shall discuss what the theory of Ref. 1 says if it is true. Let us consider the predicted sum of the cross sections along with our Fig. 2. Figure 2 tell us that σ_- is always greater than σ_+. Hence, although the σ_- values corresponding to $\bar{p}p$ and K^-p show a trend towards crossing the σ_+ values at some higher energies, we expect that σ_- will eventually level off and stay above σ_+. On the other hand, if the predicted cross sections in Tables I and II only are correct, and if σ_+ is going to remain constant, σ_- will have to become less than σ_+ and continue to decrease further at higher energies.

If both the predicted values for the sum and Fig. 2 are correct, then the σ_+ values cannot remain constant; they have to go on decreasing in such a way that they are always less than the σ_- values. Note that experimental values for the total cross sections of pp and K^+p are available up to about 20 BeV/c. Secondly, as mentioned earlier the $\pi^\pm p$ total cross sections appear to be decreasing slowly at high energies.[18] However, if the σ_+ values for pp and K^+p are experimentally found to be constant for all higher energies, then the theory of Ref. 1 followed here will be in trouble.

V. CONCLUSION

A particle corresponding to the Pomeranchukon trajectory is yet to be discovered. Until that time, one has

[24] V. Barger, Topical Conference on High-Energy Collisions of Hadrons, CERN Report No. 68-7, 1968 (unpublished).

a right to doubt its existence. On the other hand, the existence of a cut in complex plane was never doubted. By considering the cut contribution to the total cross sections of π^+p and π^-p, Pursey and Sertorio[1] have already explained the $\pi^\pm p$ experimental data. Using their theory, we have drawn the same conclusions for NN and KN reactions. As regards the asymptotic limit of the energy, we find, in the case where a P pole is absent, that the asymptotic limit may very well lie in the energy region considered in this paper. In Sec. II, we have discussed the elastic scattering cross sections in about this energy region and found from the experimental results a relation $\sigma_{el} \propto S^{-n}$, where n is nonzero. We find that we cannot derive such a simple formula even in the case where no P pole is there and a cut replaces it. From the energy dependence of the cut, one can only say that there will be a dependence of the cross section on energy.

In case a cut has to replace a Pomeranchukon, many problems such as those mentioned in Ref. 1 will arise. At the moment, we can question the existence of a Pomeranchukon and say only that the contribution from a cut can explain some part of the available experimental data.

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Algebra of Pion Sources

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Assuming that the space integrals of source terms (sources) in the Klein-Gordon equation for the pion fields together with isospin generators form an $SU(2) \otimes SU(2)$ algebra which, in the soft-pion limit, is a good symmetry of the strong interactions, we calculate S-wave scattering lengths for the collision of pions with hadron targets as well as the pion-nucleon coupling constant. The results are in excellent agreement with experiment.

Many successful calculations[1] of pure strong-interaction processes have been carried out during recent years with the aid of an assumed $SU(2) \otimes SU(2)$ structure of vector and axial-vector currents of hadrons. In all these calculations, a vital role is played by the partially conserved axial-vector current (PCAC) hypothesis which provides the necessary link between the axial-vector current and strong interactions. This hypothesis, however, is not precisely defined[2] and may be the source of some of the failures[3] of this brand of algebraic approach to strong-interaction physics. It would therefore be worthwhile to look for alternative approaches where the PCAC hypothesis would be unnecessary for calculation of strong-interaction parameters. It may be noted in this context that the scattering matrix for pion-hadron collisions is usually written in terms of the retarded commutator of pion-source densities. It is, therefore, natural to ask whether these pion sources, i.e., the space integrals of these densities, together with isospins, which are known to be constants of motion for strong interactions, do form a closed algebra. In this paper we discuss the consequences of an $SU(2) \otimes SU(2)$ algebraic structure of the isospin generators and pion sources. We show that predictions for scattering lengths and coupling constants can be made in the soft-pion limit, provided we assume that for hadrons at rest this $SU(2) \otimes SU(2)$ is a good symmetry of the Hamiltonian for strong interactions in this limit. The results agree well with experiment for pion-nucleon scattering.

The invariant T matrix for forward scattering of charged pions on a hadron target at rest can be written

[1] Y. Tomozawa, Nuovo Cimento **46A**, 707 (1966); K. Raman and E. C. G. Sudarshan, Phys. Letters **21**, 450 (1966); A. P. Balachandran, M. Gundzik, and F. Nicodemi, Nuovo Cimento **44A**, 1257 (1966); S. Weinberg, Phys. Rev. Letters **17**, 616 (1966); L. N. Chang, Phys. Rev. **162**, 1497 (1967).

[2] No precise meaning can be given to the smooth-variation hypothesis of matrix elements of the pion source density with the square of momentum transfer required for the application of the PCAC equation. Further, there are ambiguities when matrix elements between multiparticle states are considered. See, for instance, S. L. Adler and R. F. Dashen, *Current Algebras and Applications to Particle Physics* (Benjamin, New York, 1968), Chap. 1. Besides, in the presence of electromagnetic and weak interactions, the PCAC equation itself gets altered. See, for instance, S. L. Adler, Phys. Rev. **177**, 2426 (1969); S. R. Choudhury, Y. Tomozawa, and Y. P. Yao, Nucl. Phys. **B17**, 430 (1970).

[3] For instance, $\pi^0 \to 2\gamma$, $\eta \to 2\gamma$, and $\eta \to \pi^+\pi^-\pi^0$ are incorrectly obtained: D. G. Sutherland, Nucl. Phys. **B2**, 433 (1967); Phys. Letters **23**, 384 (1966).