Critique on "Universal Theory of Primary Interactions and Nucleon-Nucleon Scattering"*

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We remark on a calculation of low-energy nucleon-nucleon scattering carried out by Chiang, Gleiser, Huq, and Saxena, who use Sudarshan's "universal vector and axial-vector interaction theory" in a one-boson-exchange potential approximation. We show that the test of high-mass meson exchange forces comes in fitting the lower partial waves ($l\leq 2$), not in fitting the higher partial waves ($l\geq 3$), since mere one-pion exchange accounts for most of the observed scattering in the higher partial waves. Thus Chiang *et al.*'s admittedly poor fit to some of the lower partial waves would seem to disprove Sudarshan's model. However, we point out an error in the treatment of the one-ρ-exchange potential, the correction of which will significantly improve the fit to the data, although the over-all agreement will still be poor. We also point out that Sudarshan's assumption, that the ω-nucleon coupling constant equals the ρ-nucleon coupling constant, is both arbitrary and in conflict with experiment, and that if g_ω^2 is altered to agree with experiment, the model will be in considerably better agreement with experiment. Finally, we remark that the model also lacks sufficient intermediate-range attraction, but that this is due in part to the neglect of the two-pion-exchange terms in Chiang *et al.*'s calculation. With the inclusion of 2π effects, the Sudarshan model should correctly predict the qualitative features of low-energy nucleon-nucleon scattering. However, a precise fit will likely require the exchange of the $\epsilon(715, T=0, J=0^+)$, and Sudarshan's model makes no provision for such a meson.

A UNIVERSAL vector and axial-vector interaction theory which links all strong, weak, and electromagnetic phenomena via vector and axial-vector currents has been proposed by Sudarshan.[1,2] This theory has interesting consequences for the nucleon-nucleon interaction because it proposes to fit all N-N data with a single adjustable constant. The theory is also interesting because it predicts short-ranged axial-vector meson exchange forces which do not conserve CP. If CPT holds, then these forces do not conserve time-reversal invariance (T) either. Since T has not been observed to be violated in nuclear physics, one might inquire why such a theory should be interesting. The reason is that the theory predicts only a very small CP violation for nucleon-nucleon scattering below 400 MeV incident laboratory energy; CP violation becomes significant only at higher energies. Thus a theory which is intrinsically CP violating is nevertheless reconcilable with present-day nuclear physics.

Sudarshan proposes the following Lagrangian for the nucleon-nucleon interaction:

$$\begin{aligned}\mathcal{L}^{\text{int}}=(4\pi)^{1/2}\bar{N}[&g_\rho\gamma_\mu\boldsymbol{\tau}\cdot\boldsymbol{\varrho}_\mu+(g_\rho'/m_\rho)\tfrac{1}{2}\sigma_{\mu\nu}\boldsymbol{\tau}\cdot\boldsymbol{\varrho}_{\mu\nu}\\&+g_\omega\gamma_\mu\omega_\mu+(g_\phi'/m_\phi)\tfrac{1}{2}\sigma_{\mu\nu}\phi_{\mu\nu}\\&+f_A\gamma_\mu\gamma_5\boldsymbol{\tau}\cdot\mathbf{A}_\mu+(f_A'/m_A)\tfrac{1}{2}\sigma_{\mu\nu}\gamma_5\boldsymbol{\tau}\cdot\mathbf{A}_{\mu\nu}\\&+f_Z\gamma_\mu\gamma_5Z_\mu+(f_Z'/m_Z)\tfrac{1}{2}\sigma_{\mu\nu}\gamma_5Z_\mu]N.\end{aligned}\quad(1)$$

In this expression $\boldsymbol{\varrho}$, ω, and ϕ represent fields associated with the ρ, ω, and ϕ quanta; $\mathbf{A}$ and Z stand for axial-vector fields whose associated axial-vector quanta are the A_1 meson and an isoscalar axial-vector meson, taken to be the $D(1285)$. The $\mathbf{A}$ and Z fields are assumed to satisfy the partially conserved axial-vector current (PCAC) condition, with the π thereby associated with the $\mathbf{A}$ field, and the η with the Z field. One defines $\boldsymbol{\varrho}_{\mu\nu}=\partial_\mu\boldsymbol{\varrho}_\nu-\partial_\nu\boldsymbol{\varrho}_\mu$, and $\phi_{\mu\nu}$, $\mathbf{A}_{\mu\nu}$, and $Z_{\mu\nu}$ are defined similarly.

Sudarshan links the N-N couplings of the $\boldsymbol{\varrho}$, ϕ, A, and Z fields through two $SU(4)$ symmetry groups. In the first set, the $\boldsymbol{\varrho}$ and ϕ act as the generators of an $SU(4)$ group, with $g_\rho'=(5/3)g_\rho$ and $g_\phi'=g_\rho$. In the second set, the $\boldsymbol{\varrho}$, $\mathbf{A}$, and Z fields act as the generators of the group; in this case the relevant coupling constants are not entirely fixed by the symmetry requirements, so Sudarshan chooses additionally to allow CP to be violated, and maximally. This yields $f_A=f_A'=(5/3)\times(\sqrt{\tfrac{1}{2}})g_\rho$ and $f_Z=f_Z'=(\sqrt{\tfrac{1}{2}})g_\rho$, where the mixed direct and derivative coupling of the A and Z fields brings about the CP violation.

Thus the coupling constants of the $\boldsymbol{\varrho}$, ϕ, $\mathbf{A}$, and Z fields are related to a single constant, taken to be the ρ-nucleon coupling constant g_ρ. Sudarshan remarks that the choice $f_A=(5/3)(\sqrt{\tfrac{1}{2}})g_\rho=(25/18)^{1/2}g_\rho$ yields the desired ratio of weak-interaction Fermi and Gamow-Teller couplings, given the assumption that the ρ and A_1 mesons mediate the nucleon-lepton weak interaction; also, $g_\rho'=(5/3)g_\rho$ yields the correct ratio of static charge to magnetic moment for the isovector component of the nucleon's electromagnetic form factors.

The field associated with the ω meson is in a different category from the fields thus far mentioned. The ω field is the only field appearing in the interaction Lagrangian [Eq. (1)] which is not linked through some underlying symmetry to the other particles. It is not included in the $SU(4)$ multiplet linking the ρ and ϕ, in place of the

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[1] E. C. G. Sudarshan, Proc. Roy. Soc. (London) **A305**, 319 (1968).

[2] T. Pradhan, E. C. G. Sudarshan, and R. P. Saxena, Phys. Rev. Letters **20**, 79 (1968).

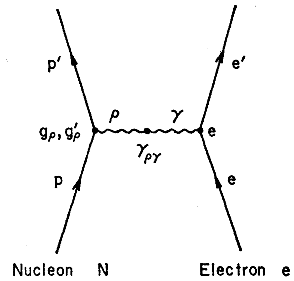

FIG. 1. Feynman diagram for ρ-dominated isovector electromagnetic form factor of the nucleon.

ϕ, because that would introduce a large Pauli coupling of the ω to the nucleon and thereby generate a large isoscalar magnetic moment,[1,3] assuming that the ω saturates the isoscalar form factor. This would contradict the experimental fact that this moment is nearly zero. Instead, Sudarshan introduces the ω *ad hoc*, adjusting the Dirac and Pauli couplings to the nucleon to yield the experimental static electromagnetic moments. Thus the Pauli coupling of the ω to the nucleon is set to zero to yield a zero isoscalar magnetic moment.

The Dirac coupling of the ω to the nucleon is determined through knowing the ω-photon interaction. In the model, Sudarshan assumes that the photon interacts with the nucleon only via the ρ or ω. Thus

$$\mathcal{L}_{\text{em}}{}^{\text{int}}=(\gamma_{\omega\gamma}m_\omega{}^2\omega_\mu+\gamma_{\rho\gamma}m_\rho{}^2\rho_\mu)\mathcal{A}_\mu. \qquad (2)$$

$\gamma_{\omega\gamma}$ is taken to be equal to $\gamma_{\rho\gamma}$. Fitting the static electric charges of the nucleon and proton, as depicted in Fig. 1 for ρ exchange, yields $g_\omega(-m_\omega{}^2)^{-1}\gamma_{\omega\gamma}m_\omega{}^2=\frac{1}{2}e$, and $g_\rho(-m_\rho{}^2)^{-1}\gamma_{\rho\gamma}m_\rho{}^2=\frac{1}{2}e$, so that

$$g_\omega=(\gamma_{\rho\gamma}/\gamma_{\omega\gamma})g_\rho. \qquad (3)$$

Since Sudarshan takes $\gamma_{\rho\gamma}=\gamma_{\omega\gamma}$, there results $g_\omega=g_\rho$.

This determination of the ω-nucleon interaction seems to be a weak point in Sudarshan's proposal. All the other meson-nucleon couplings result from $SU(4)$ symmetries. Furthermore, the assumption that the ρ and the ω couple equally strongly to the photon is in disagreement with experiment, as we shall describe farther on. Experiment dictates that if the ρ and ω saturate the form factors, g_ω must be much greater than g_ρ.

Let us defer for the moment the question of the appropriate coupling of the ω to the photon and to the nucleon, and consider the prediction that $\mathcal{L}^{\text{int}}$ of Eq. (1) makes for nucleon-nucleon scattering. Chiang, Gleiser, Huq, and Saxena (CGHS)[4] have carried out such calculations for N-N scattering up to 400 MeV laboratory scattering energy. They approximate the complete interaction by considering only one-boson-exchange terms plus terms resulting from iteration of the one-boson-exchange terms in the Lippmann-Schwinger equation. In short, they construct a one-boson-exchange potential (OBEP) from the meson Born terms and solve the Schrödinger equation for the phase shifts. CGHS also make another approximation in their calculations. They employ straight cutoff within a radius r_0; that is, they set $V(r)=V(r_0)$ for $0<r<r_0$. With these approximations, CGHS calculate the N-N phase shifts between 0 and 400 MeV laboratory scattering energy.[5] They look for a best fit to the data by varying the over-all coupling constant $g_\rho{}^2$ and the cutoff radius r_0. (The pion-nucleon coupling constant is presumably kept fixed near 15.) Data are represented in the form of the experimentally determined phase shifts found by MacGregor, Arndt, and Wright at several energies.[6] CGHS show fits to the data for two values of ρ-nucleon coupling,[7] $g_\rho{}^2=0.81$ and $g_\rho{}^2=1.02$. We have plotted their predictions in Fig. 2 for the case $g_\rho{}^2=0.81$, with $r_0=0.6$ F. Results are shown for phase shifts with orbital angular momentum $l=0$, 1, 2, 3, and 4. These are plotted as dashed curves, labeled "CGHS." We also plot the most recent phase shifts of MacGregor, Arndt, and Wright.[6,8]

Do the calculations of CGHS bear out Sudarshan's theory in the N-N domain, or do they disprove it? Although CGHS claim that they bear it out, it seems clear to us that their calculations, taken at face value, disprove Sudarshan's theory. Their calculated phase shift $\delta(^3P_2)$ disagrees with experiment and the calculated phase shifts $\delta(^3P_0)$ and $\delta(^3P_1)$ agree only qualitatively. Also the CGHS phase shift $\delta(^3F_4)$ fits experiment poorly, as do the $\delta(^1S_0)$ and $\delta(^1D_2)$ phase shifts. The $\delta(^3S_1)$, $\delta(^1P_1)$, and ϵ_1, and all the higher partial-wave parameters not previously cited more or less agree with the MacGregor-Arndt phase parameters, and it must be on account of these that CGHS claim to fit experiment. But correctly predicting the higher partial-wave parameters is no test of Sudarshan's vector and axial-vector theory; these phase shifts are primarily due just to the one-pion-exchange mechanism. That is, any model which includes one-pion exchange with a reasonable coupling constant is bound to fit all the

[3] E. C. G. Sudarshan (private communication).

[4] C. C. Chiang, R. J. Gleiser, M. Huq, and R. P. Saxena, Phys. Rev. **177**, 2167 (1969). These authors will be referred to as CGHS.

[5] CGHS also show predictions where they employ a hard core in certain of the states. We have not considered these predictions further because to include a hard core in some states and not others is really to include a new kind of force in addition to the Sudarshan dynamics, invalidating the test of the Sudarshan model. For instance, CGHS show a calculation where a hard core is used in the $^3P_0$ and $^3P_1$ states, but not in the $^3P_2$ state. The fit is greatly improved, but at the cost of introducing what amounts to a strong short-ranged spin-orbit potential plus a short-ranged central repulsion.

[6] M. H. MacGregor, R. A. Arndt, and R. M. Wright, Phys. Rev. **182**, 1714 (1969).

[7] There exists a discrepancy between the definitions of $g_\rho{}^2$ as given in Eq. (5) of Ref. 2 and Eq. (2.2) of Ref. 4, and as given in Eq. (3.9) of Ref. 4. We thank Dr. C. C. Chiang for informing us that values quoted for $g_\rho{}^2$ ought to be divided by a factor of 8 for use in Eq. (3.9) and Eqs. (3.10)–(3.14) of Ref. 4.

[8] Experimental phase shifts not shown at the lower energies for the higher partial-wave states have been fixed by MacGregor, Arndt, and Wright (Ref. 6) at the one-pion-exchange pole projection values.

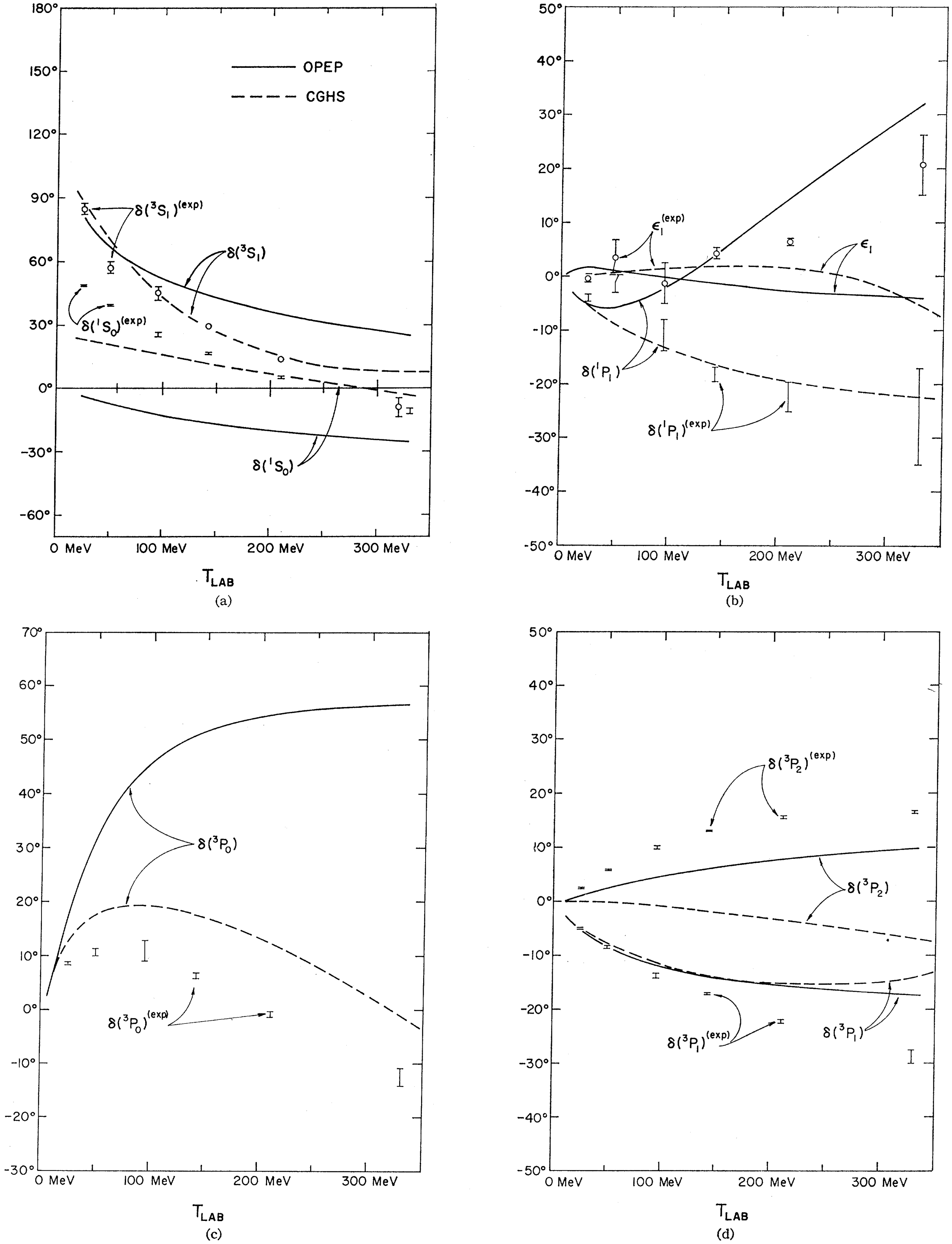

FIG. 2. Nuclear bar phase shifts for nucleon-nucleon scattering over the 0–350-MeV laboratory projectile energy range. The solid lines are predictions of the cutoff one-pion-exchange potential defined by Eqs. (4a) and (4b) of the text. The dashed curves depict calculations carried out by Chiang, Gleiser, Huq, and Saxena (Ref. 4), with $g_\rho{}^2=6.5/8=0.81$ and straight cutoff within 0.6 F. The error bars, which occur at 25, 50, 95, 142, 210, and 330 MeV, are experimental determinations of the phase shifts and mixing parameters found by Arndt, MacGregor, and Wright (Ref. 6). The dashed curves are suppressed where they overlap the solid curves.

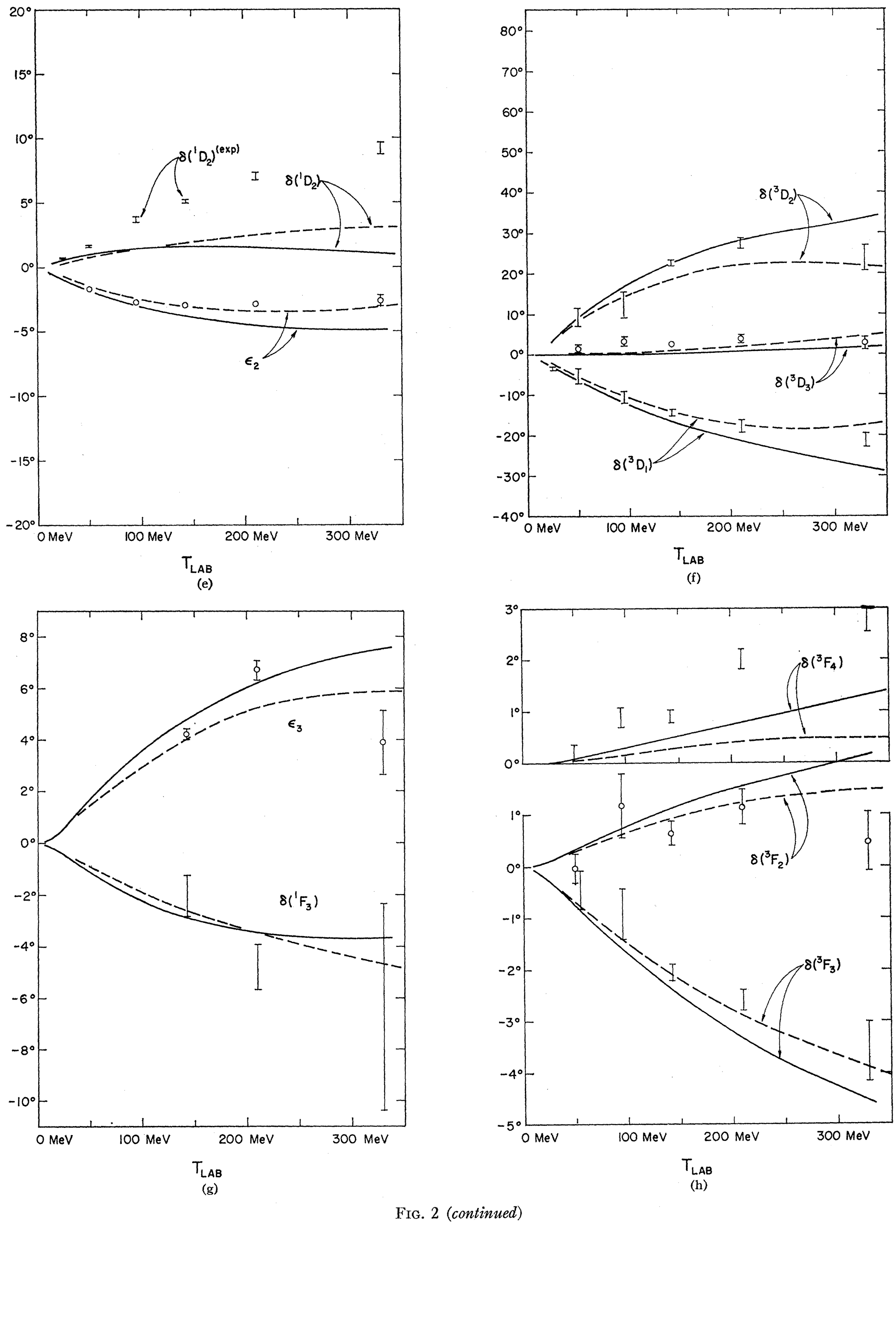

FIG. 2 *(continued)*

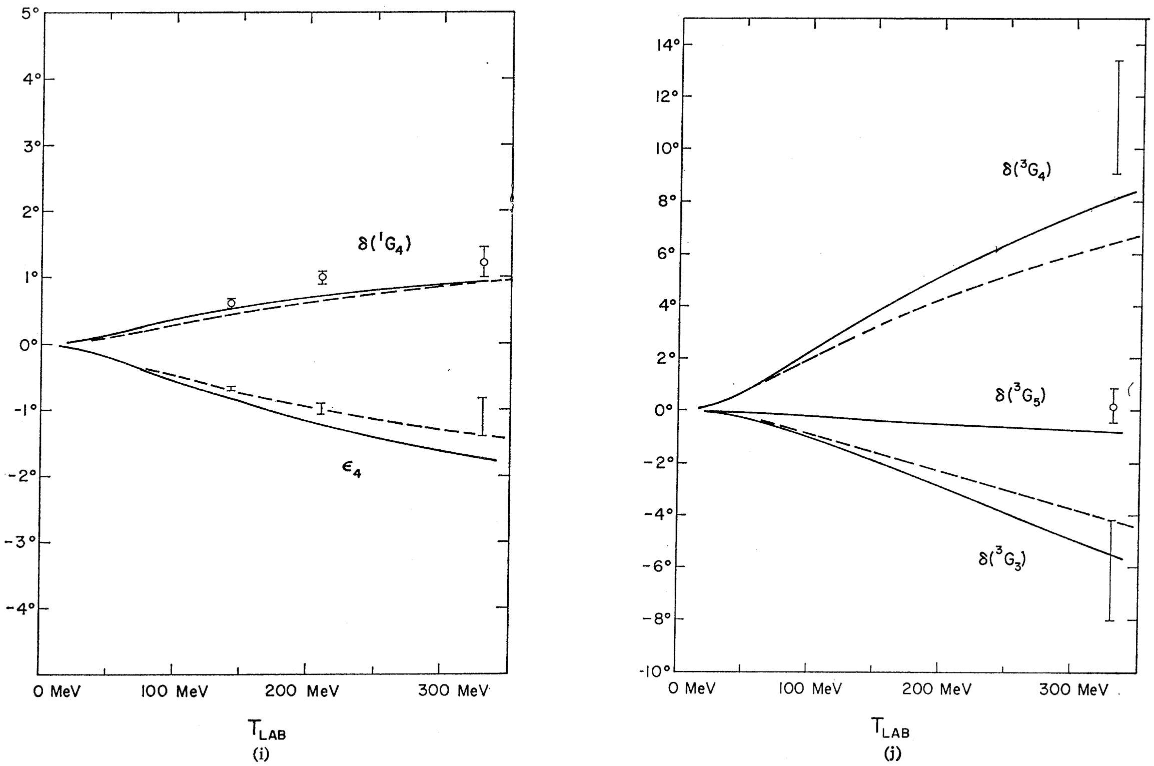

FIG. 2 (*continued*)

partial waves for $l\geq 2$ for scattering energies below 200 MeV, and for $l\geq 4$ below 400 MeV.

We have chosen to illustrate this by computing phase shifts for all partial waves using the one-pion-exchange potential $V^{(\pi)}$, modified by a short-range cutoff.[9] Since CGHS include $V^{(\pi)}$ plus a short-range cutoff of their own choosing, the success of Sudarshan's model may be judged in terms of how well CGHS succeed in fitting experiment beyond that which is accomplished through cutoff $V^{(\pi)}$ alone.

We have plotted curves for phase parameters predicted by $V^{(\pi)}$ plus cutoff as solid lines, labeled "OPEP," in Fig. 2. We take the well-known nonrelativistic approximation to OPEP, which is

$$V^{(\pi)}(m_\pi,r)=\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 g_\pi^2\left[\frac{m_\pi^2}{4M^2}\left(\frac{1}{3}+\frac{1}{m_\pi r}+\frac{1}{m_\pi^2 r^2}\right)\frac{e^{-m_\pi r}}{r}S_{12}+\left(\frac{m_\pi^2}{12M^2}\frac{e^{-m_\pi r}}{r}-\frac{1}{4M^2}\frac{4\pi}{3}\delta^{(3)}(\mathbf{r})\right)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right], \quad (4a)$$

where M is the nucleon mass, and subtract from it a cutoff factor $V^{(\pi)}(\Lambda,r)$, where Λ is the cutoff mass. Thus the cutoff OPEP, $V^{(\pi)}{}_{\rm cut}$, is

$$V^{(\pi)}{}_{\rm cut}=V^{(\pi)}(m_\pi,r)-V^{(\pi)}(\Lambda,r). \quad (4b)$$

The cutoff form corresponds to multiplying the OPEP momentum-space form, which is proportional to $(\boldsymbol{\sigma}_1\cdot\mathbf{q})(\boldsymbol{\sigma}_2\cdot\mathbf{q})/(\mathbf{q}^2+m_\pi^2)$, by the Feynman factor $\Lambda^2/(\Lambda^2+\mathbf{q}^2)$. For sufficiently large Λ, $V^{(\pi)}{}_{\rm cut}$ goes over to $V^{(\pi)}$. For lower values of Λ, $V^{(\pi)}{}_{\rm cut}$ resembles $V^{(\pi)}$ except that the $1/r^3$ and $\delta^{(3)}(\mathbf{r})$ singularities go over to $1/r$ singularities. The curves labeled OPEP are calculated for $g_\pi^2=13.5$, $m_\pi=135$ MeV, and $\Lambda=635$ MeV; $M=938.3$ MeV. (The pion-nucleon coupling constant is taken somewhat below 15 to compensate for the factor M^2/E^2 which is set equal to unity in the nonrelativistic form of OPEP. E is the center-of-mass energy of either nucleon.)

In examining the $V^{(\pi)}{}_{\rm cut}$ predictions in Fig. 2 one sees that $\delta(^1S_0)^{(\pi)}$ and $\delta(^3S_1)^{(\pi)}$ bear a qualitative resemblance to experiment, but of course are not expected to agree.[10] $\delta(^1P_1)^{(\pi)}$ does not agree with experiment above 25 MeV, but this is due to the short-range deep attraction which $V^{(\pi)}(\Lambda,r)$ did not succeed in removing.[11]

$\epsilon_1^{(\pi)}$ agrees with $\epsilon_1^{(\rm exp)}$ at low energies. The $T=1$ phase shifts $\delta(^3P_0)$, $\delta(^3P_1)$, and $\delta(^3P_2)$ agree with experiment at very low energies, but $\delta(^3P_0)^{(\pi)}$ moves sharply away from the experimental values as the energy increases, as do $\delta(^3P_1)^{(\pi)}$ and $\delta(^3P_2)^{(\pi)}$ on a lesser scale. This disparity is largely rectified by adding

[9] Some sort of cutoff must be introduced to render the Schrödinger equation solvable when this potential [Eq. (4) below] is used.

[10] We will refer to phase shifts calculated from $V^{(\pi)}{}_{\rm cut}$ as $\delta^{(\pi)}$.

[11] The more drastic straight cutoff employed by CGHS does remove it so that $\delta(^1P_1)^{(\pi)}$ agrees with experiment.

a spin-orbit force and constitutes the experimental evidence *for* the *LS* force.

The triplet *D* waves $\delta(^3D_1)$, $\delta(^3D_2)$, and $\delta(^3D_3)$ predicted by $V^{(\pi)}{}_{\rm cut}$ agree with experiment fairly well. This is partly due to the fact that the *D*-wave impact parameter penetrates only slightly the inner region dominated by Sudarshan's vector and axial-vector meson exchanges, even at $T_{\rm lab}=350$ MeV. Success of $V^{(\pi)}{}_{\rm cut}$ alone is also due to the fact that $V^{(\pi)}$ is very strong in $T=0$ states, such as these triplet *D* states, because the $\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 g_\pi{}^2$ factor is $(-3)(15)=-45$.

The $T=1$ *D*-state parameter $\delta(^1D_2)$ is less well predicted by $V^{(\pi)}{}_{\rm cut}$, since $V^{(\pi)}$ is weaker in $T=1$ states, but the remaining $T=1$ *D*-state parameter $\epsilon_2{}^{(\pi)}$ agrees fairly well with experiment. The $T=1$ *F*-state parameters $\delta(^3F_2)^{(\pi)}$ and $\delta(^3F_3)^{(\pi)}$ agree rather well with experiment. $\delta(^3F_4)^{(\pi)}$ agrees somewhat less well, and for the same reason that $\delta(^3P_2)^{(\pi)}$ fails to agree with experiment: lack of attractive *LS* force. (The $^3F_4$ state has matrix elements similar to those of the $^3P_2$ state and reflects the *N*-*N* potential similarly, except that it is only sensitive to the more distant regions of the potential.)

The remaining $l=3$ OPEP predicted phase parameters and all of the $l=4$ OPEP phase parameters are consistent with experiment, particularly when one takes into account the uncertainties in the experimental values. Thus phase parameters for $l\geq 4$ are given essentially by one-pion exchange, and correspond to impact parameters largely beyond the range of 2π- and other meson-exchange forces.

To judge the success of Sudarshan's model, one need compare only the lower partial-wave phase shifts that the model predicts with experiment. The higher partial-wave phase shifts will necessarily agree with experiment if one-pion exchange is properly included. Thus, the phase shifts which serve to test the model include only the *S*- and *P*-wave parameters [perhaps not even $\delta(^1P_1)$], and the $T=1$ *D*-wave parameters [$\delta(^1D_2)$ and ϵ_2]. The $T=1$ *F*-wave parameters need not be included since they merely reflect the *P*-wave fits and are largely given by OPEP anyway. All other high-l phase parameters contribute even less to a test of non-OPE processes. We are therefore not surprised to see that the CGHS phase shifts for $T=0$, $l=2$ states, and all $l=3$ and $l=4$ states, agree rather well with experiment and with the one-pion-exchange predictions [with the possible exception of $\delta(^3F_4)$]. Remaining differences between the CGHS and the OPEP phase parameters may be due to a somewhat different value of $g_\pi{}^2$ used in the two cases. (CGHS do not give the value of $g_\pi{}^2$ used in their calculations.)

The test of Sudarshan's model thus reduces to a comparison of the predicted $l=0$, 1, and the $T=1$, $l=2$ waves with experiment, taking into account how well $V^{(\pi)}{}_{\rm cut}$ alone fits these same data. On this basis one will have to conclude from the computations of CGHS that the theory accounts for the *N*-*N*, $T=0$ states satisfactorily but fails to account for $T=1$ scattering any better than does one-pion exchange. Specifically, for $l=1$ states, the CGHS $\delta(^3P_0)$ is closer to experiment than $\delta(^3P_0)^{(\pi)}$, their $\delta(^3P_1)$ is about equal to $\delta(^3P_1)^{(\pi)}$, and their $\delta(^3P_2)$ is farther from experiment than $\delta(^3P_2)^{(\pi)}$. For $l=2$, the CGHS $\delta(^1D_2)$ is farther from experiment than $\delta(^1D_2)^{(\pi)}$, while their ϵ_2 is closer to experiment than $\epsilon_2{}^{(\pi)}$. The remaining significant $T=1$ phase shift $\delta(^1S_0)$ is better predicted by CGHS than by $V^{(\pi)}{}_{\rm cut}$, but neither potential predicts quite the right slope, and it is the slope which tests the intermediate-range force [the core just adjusts the scattering length and scales $\delta(^1S_0)$ up or down]. An effective-interaction model ought to predict correctly the middle-range force (0.7 F$<r<$1.5 F) whether or not it predicts the core region correctly.

Why does the model not fit the $T=1$ states any better than does one-pion exchange? The immediate cause for the misfit is easily seen: The model has too little spin-orbit potential; indeed, it may be of the wrong sign. As an example of how a properly attractive *LS* force can bring about agreement with the experimental phase shifts, we plot in Fig. 3 the $^3P_0$, $^3P_1$, and $^3P_2$ phase shifts due to $V^{(\pi)}{}_{\rm cut}$ (copied from Fig. 2 and plotted as solid lines) and phase shifts due to $V^{(\pi)}{}_{\rm cut}$ plus a negative spin-orbit potential (plotted as dashed curves). The specific potential used is

$$[V^{(\pi)}(m_\pi,r)-V^{(\pi)}(\Lambda,r)] \\ +[V_{LS}(m_V,r)-V_{LS}(\Lambda',r)]\mathbf{L}\cdot\mathbf{S}\,, \quad \text{(5a)}$$

where

$$V_{LS}(m,r)=\frac{3g^2}{2M^2}\frac{1}{r}\frac{d}{dr}\left(\frac{e^{-mr}}{r}\right), \quad \text{(5b)}$$

and $g^2=30$, $m_V=770$ MeV, and $\Lambda'=1500$ MeV. (The potential has the form of the *LS* part of one-vector meson exchange.) It may be seen from Fig. 3 that the *LS* force greatly lowers $\delta(^3P_0)$, lowers $\delta(^3P_1)$ somewhat, and raises $\delta(^3P_2)$. This is precisely what is needed in the case of the CGHS phase shifts, as one may see by examining Fig. 2.

It seems strange that Sudarshan's model should fail because it predicts the wrong spin-orbit potential. Other models with ρ, ω, and ϕ exchange mechanisms predict the experimental *LS* force very well. Examples are the one-boson-exchange models of Sawada *et al.*,[12] Bryan and Scott,[13] and Scotti and Wong.[14] In the case of the CGHS calculation the difficulty may be traced to the *LS* part of the ρ-meson-exchange potential, which we denote as $V_{LS}{}^{(\rho)}$. In Table I of the CGHS paper, we see that $V_{LS}{}^{(\rho)}$ is positive, in contrast to $V_{LS}{}^{(\omega)}$ and $V_{LS}{}^{(\phi)}$ of that table, which are negative. The fact is that $V_{LS}{}^{(\rho)}$ should be negative also, thus yielding an

[12] S. Sawada, T. Ueda, W. Watari, and M. Yonezawa, Progr. Theoret. Phys. (Kyoto) **32**, 380 (1964).
[13] R. A. Bryan and B. L. Scott, Phys. Rev. **177**, 1435 (1969).
[14] A. Scotti and D. Y. Wong, Phys. Rev. **138**, B145 (1965).

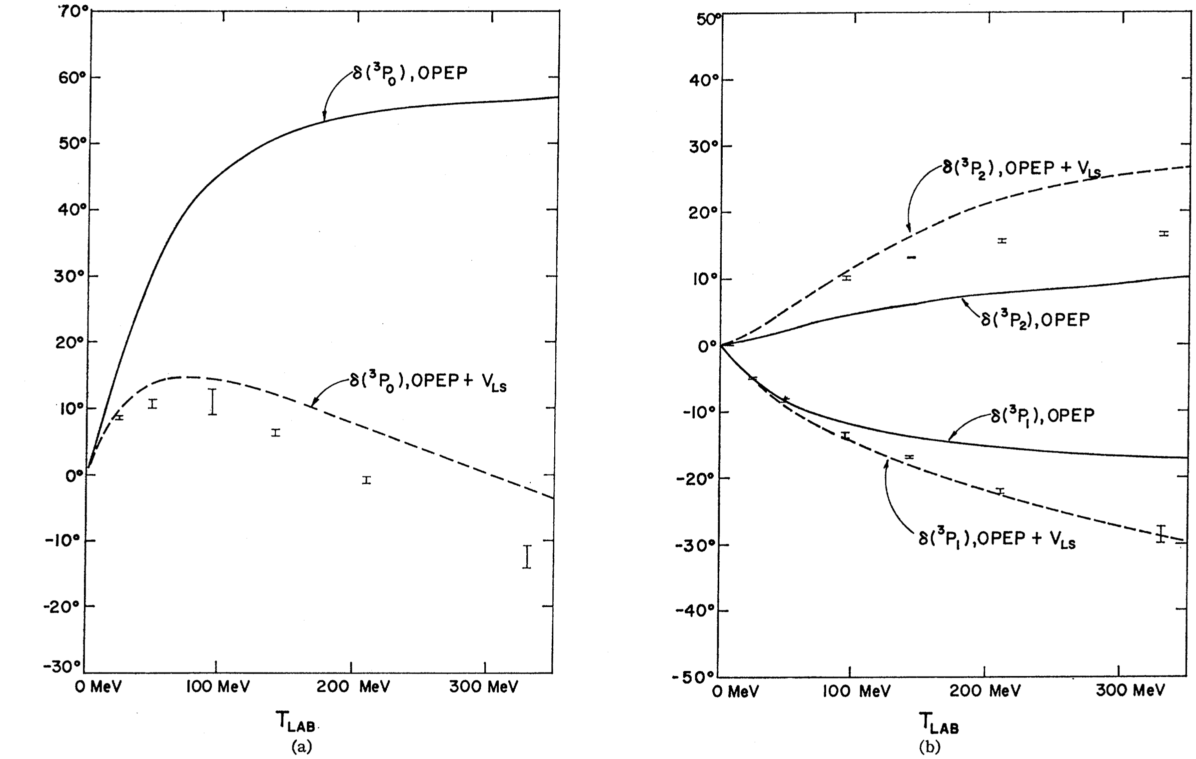

FIG. 3. Theoretical predictions for the $^3P_0$, $^3P_1$, and $^3P_2$ phase shifts. The solid curves are predictions of the cutoff one-pion-exchange potential, copied from Fig. 2. The dashed curves are predictions of the same cutoff one-pion-exchange potential, plus an attractive spin-orbit potential described in the text [Eqs. (5a) and (5b)].

over-all negative spin-orbit potential, but CGHS have unaccountably used the wrong sign for the ratio of Pauli to Dirac coupling of the ρ to the nucleon.[15]

The one-ρ-exchange Born term due to Sudarshan's Lagrangian [Eq. (1)] is

$$\{\bar{u}(p')[g\gamma_\mu{}^{(a)}+(g'/2m_\rho)(q^{(a)}\gamma_\mu{}^{(a)}-\gamma_\mu{}^{(a)}q^{(a)})]u(p)\}\times(-1/2\pi)(q^2-m_\rho{}^2)^{-1}\{\bar{u}(n')[g\gamma_\mu{}^{(b)}-(g'/2m_\rho)\times(q^{(b)}\gamma_\mu{}^{(b)}-\gamma_\mu{}^{(b)}q^{(b)})]u(n)\},$$

where p and p' are the initial and final four-momenta of nucleon a, and n and n' are the initial and final four-momenta of nucleon b; $q^2=(p'-p)^2=(p_0'-p_0)^2-|\mathbf{p}'-\mathbf{p}|^2$, $q^{(a)}=(p_0'-p_0)\gamma_0{}^{(a)}-(\mathbf{p}'-\mathbf{p})\cdot\boldsymbol{\gamma}^{(a)}$, and $q^{(b)}=(n_0'-n_0)\gamma_0{}^{(b)}-(\mathbf{n}'-\mathbf{n})\cdot\boldsymbol{\gamma}^{(b)}$, where $\gamma_0=\beta$ and $\gamma_j=\beta\alpha_j$, as in the paper by Feynman.[16] This matrix element is depicted in Fig. 4. The corresponding coordinate-space potentials are correctly given in Ref. 13 as well as in many other articles and, for $g_\rho'/g=5/3$, yield a negative $V_{LS}{}^{(\rho)}$, not positive.[17]

Let us verify that g_ρ'/g_ρ of Eq. (1) should indeed be $+5/3$ if the ρ is to yield the correct isovector anomalous magnetic moment of the nucleon. From Eqs. (1) and (2), the electron-nucleon interaction in lowest order is

$$\{\bar{u}(p')[g\gamma_\mu{}^{(N)}+(g'/2m_\rho)(q\gamma_\mu{}^{(N)}-\gamma_\mu{}^{(N)}q)]u(p)\}\times(m_\rho{}^2-q^2)^{-1}[-m_\rho{}^2\gamma_{\rho\gamma}/2\pi^2q^2]\{\bar{u}(e')e\gamma_\mu{}^{(e)}u(e)\}, \quad (6)$$

where p and p' are, respectively, the initial and final four-momenta of the nucleon, and e and e' the corresponding momenta for the electron. This scattering process is illustrated in Fig. 4. If this interaction is to yield the observed isovector components of the nucleon electric charge and magnetic moment, then $g(\gamma_{\rho\gamma}/m_\rho{}^2)$ must equal $0.5e$ and $(g'/2m_\rho)(\gamma_{\rho\gamma}/m_\rho{}^2)$ must equal $1.85e/4M$, where M is the nucleon mass. Thus g'/g must equal $3.70(m_\rho/2M)=1.51$, which is about 5/3, not $-5/3$.

If CGHS were to redo their calculations correctly, the over-all LS potential would receive a large negative contribution from reversing of the sign of g_ρ'/g_ρ. This may be seen from the leading terms of $V_{LS}{}^{(\rho)}$, which go as

$$-\left(3g^2+\frac{8M}{m_\rho}gg'\right)\frac{1}{2M^2}\frac{1}{r}\frac{d}{dr}\left(\frac{e^{-m_\rho r}}{r}\right)\mathbf{L}\cdot\mathbf{S}.$$

[15] It is well known that the nucleon electromagnetic isovector form factors predict an attractive one-ρ-exchange spin-orbit potential. See, e.g., Ref. 12 and P. T. Matthews, in *Proceedings of the Aix-en-Provence Conference on Elementary Particles, 1961* (Centre d'Etudes Nucléaires de Saclay, Gif-sur-Yvette, Seine et Oise, Saclay, France, 1961), Vol. II, p. 87.

[16] R. P. Feynman, Phys. Rev. **76**, 749 (1949).

[17] Dr. C. C. Chiang kindly informs us that the signs of the $g_\rho g_\rho'$ cross terms in the potentials listed in Eqs. 3.10–3.12 and 3.14 of Ref. 4 were mistakenly changed to the wrong sign in the proofs. Only the LS potential, Eq. (3.13) of Ref. 4, has the correct sign for the cross term.

The predicted triplet-P phase shifts would then be in better agreement with experiment.

Unfortunately, the over-all agreement would still not be all that good. In fact, a much larger ω-nucleon coupling constant is required for a good fit to the data. We know this from studies of one-ω-exchange contributions in other one-boson-exchange models. To show this, consider the N-N potential in the form

$$V=V^{(0)}+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2V^{(1)}, \tag{7a}$$

where each

$$V^{(i)}=V_C^{(i)}(r)+V_{\sigma\sigma}^{(i)}(r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+V_T^{(i)}(r)S_{12}+V_{LS}^{(i)}(r)\mathbf{L}\cdot\mathbf{S},\quad i=0,1. \tag{7b}$$

Only OBEP of $T=0$ mesons contribute to $V^{(0)}$, and only OBEP of $T=1$ mesons contribute to $V^{(1)}$. Now, it takes a g_ω^2 of about 20 to fit the experimental $V_{LS}^{(0)}$ potential when there are no other important contributors besides the ω. This is shown in Fig. 2(b) of Ref. 18. In the CGHS model, the ω, ϕ, and D OBEP contribute to $V_{LS}^{(0)}$, but the ϕ and D contributions are less than $\frac{1}{3}$ as important as the ω contribution because of their shorter range. We estimate, therefore, that g_ω^2 ought to be ≈ 20 in the CGHS model for an optimum fit to the data.

In the Sudarshan model, $g_\omega^2=g_\rho^2$, and $g_\rho^2\approx 1$, so the fit is quite bad, but there is no reason for this to be so. Sudarshan's choice of setting $\gamma_{\omega\gamma}=\gamma_{\rho\gamma}$, and hence $g_\omega^2=g_\rho^2$, is quite arbitrary. Moreover, it disagrees with current experimental information. Ting, reporting on the results of electron-positron clashing beam experiments, wherein ω, ϕ, and ρ mesons are produced as intermediate states, states that $\gamma_{\rho\gamma}^2=(9.0\pm3.0)\gamma_{\omega\gamma}^2$, taking an average of world experiments.[19] This predicts that $g_\omega^2=(9.0\pm3.0)g_\rho^2$, assuming the ρ and ω saturate the electromagnetic form factors. Taking this figure will greatly improve the agreement with experiment of the CGHS calculations.

There remain two states which will not be well fitted, however: the $^1S_0$ and the $^1D_2$ states. In fact, these will be fitted worse. The CGHS model predicts too little intermediate-range attraction in the central potential to give $\delta(^1S_0)$ and $\delta(^1D_2)$ correctly. One knows that $V_C^{(0)}$ in Eq. (7) must be moderately negative in the range[20] 1 F$<r<$2 F to yield the right $\delta(^1D_2)$, and to yield the right slope to $\delta(^1S_0)$ above 25 MeV. However, there *is* no meson in the CGHS model which contributes an attraction to $V_C^{(0)}$. Only the $(T=0)$ ω, ϕ, η, and D mesons could contribute, but the η makes no contribution, the D makes no appreciable contribution, and the ω and ϕ provide only repulsion. This explains why the CGHS $\delta(^1D_2)$ is too small at 320 MeV, and why the CGHS $\delta(^1S_0)$ falls off too slowly with energy. [A stronger middle-range attraction calls for a stronger short-range repulsion. With the potentials in the two regions adjusted to give the correct $\delta(^1S_0)$ at 25 MeV, $\delta(^1S_0)$ will decrease more rapidly with increasing energy as its impact parameter approaches the strengthened repulsive core.]

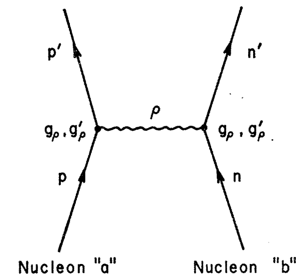

FIG. 4. Feynman diagram for the exchange of a ρ meson between two nucleons.

Some of this attraction in $V_C^{(0)}$ can come from the 2π-exchange terms neglected by CGHS. Binstock reports that 2π-exchange terms with both N and N^* intermediate baryons exhibit some central attraction.[21] Also, Partovi and Lomon report that 2π-exchange terms yield considerable central attraction in calculations where they treat the Bethe-Salpeter equation in a Blankenbecler-Sugar approximation generalized to spin-$\frac{1}{2}$ particles.[22]

Additional attraction beyond this may be required, particularly since an increased ω-nucleon coupling increases the "hard-core" repulsion characteristic of vector-meson exchange. A possible source of additional attraction in $V_C^{(0)}$ is ϵ exchange. The ϵ, with quantum numbers $T=0$, $J^{PG}=0^{++}$, has a mass of 715 MeV and a width possibly as great as 400 MeV.[23] If it is indeed this wide, the low-mass components will yield a Yukawa attraction of about the required range, corresponding to that of a zero-width meson of mass 550–600 MeV.[24,25] However, Sudarshan's model makes no provision for scalar mesons, so it is not clear how meaningful it would be to add the ϵ contribution to the model.

Even though the vector and axial-vector interaction theory may not be general enough to describe nucleon-nucleon scattering, it does point out the interesting possibility of violation of time-reversal invariance in

[18] R. A. Bryan and B. L. Scott, Phys. Rev. **135**, B434 (1964).
[19] S. C. C. Ting, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968), p. 43.
[20] This may be seen from examining $V_C^{(0)}$ of the Yale and Hamada-Johnston potentials plotted in Fig. 2(a) of Ref. 18.

[21] J. Binstock (private communication); and unpublished.
[22] M. H. Partovi and E. L. Lomon, Phys. Rev. Letters **22**, 438 (1969).
[23] A. Barbaro-Galtieri, S. E. Derenzo, L. R. Price, A. Rittenberg, A. H. Rosenfeld, N. Barash-Schmidt, C. Bricman, M. Roos, P. Söding, and C. G. Wohl, Rev. Mod. Phys. **42**, 87 (1970).
[24] This is about the mass predicted for a meson with the ϵ's quantum numbers by the OBEP models, see, e.g., Ref. 13; N. Hoshizaki, S. Otsuki, W. Watari, and M. Yonezawa, Progr. Theoret. Phys. (Kyoto) **27**, 1199 (1962); V. V. Babikov, *ibid.* **29**, 712 (1963).
[25] Since the present article was first written, the author has received an article by Chiang, Gleiser, and Huq [Phys. Rev. D **1**, 2184 (1970)], wherein the CGHS calculation was repeated with the addition of an ϵ-exchange potential. The predicted scattering phase shifts agree better with experiment than previously, but the Pauli coupling of the ρ meson was still treated incorrectly.

nuclear physics. CGHS estimate that the maximal CP violation (brought about by setting $f_A'=f_A$ and $f_Z'=f_Z$) causes only a 3% discrepancy between the proton-proton polarization and asymmetry at 400 MeV, and only a 5% discrepancy between these quantities at 600 MeV. Should experimental measurements eventually show that time reversal is not violated, the vector and axial-vector model could still accommodate this by our taking $f_A'=f_Z'=0$. Incidentally, this reminds one that Sudarshan's model has several adjustable constants and not just one, as stated in Ref. 4. Variables are g_ρ^2, g_ω^2, f_ω/g_ω, g_π^2, and g_η^2.

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Possible Tests of Anomalously Large Leptonic Weak Interactions in High-Energy e^+e^- Collisions*

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Some consequences of two possible mechanisms of a breakdown in $V-A$ theory leading to anomalously large weak vertices in hitherto untested leptonic reactions are discussed. We consider the processes $e^+e^-\to\mu^+\mu^-$ and $e^+e^-\to\nu_e\bar{\nu}_e\gamma$ with total c.m. energies around 10 GeV. A purely leptonic neutral current can cause a measurable forward-backward muon asymmetry and a muon polarization in the first reaction. Expressions are computed for these effects in terms of the most general nonderivative coupling. On the other hand, some intrinsic anomaly in diagonal weak vertices can make the cross section for the second process large and measurable; in that case, study of the γ spectrum can reveal information about the structure of such an anomalous vertex. The angular distribution and the energy spectrum of the photon are computed for the $V-A$ and $S-P$ theories up to terms that are first order in (total c.m. energy)^2/2(intermediate-boson mass)^2.

I. INTRODUCTION

At the present time, all observed weak processes can be described by the interaction Hamiltonian density

$$H_{\rm int}(x)=(G/\sqrt{2})J^\mu(x)J_\mu^\dagger(x)\,. \quad (1)$$

An interaction of the current×current type was first proposed by Fermi[1] in 1934, following the neutrino hypothesis of Pauli. However, it was only after the discovery of parity nonconservation that Sudarshan and Marshak[2] and Feynman and Gell-Mann[2] gave the present form (1) for the weak Hamiltonian density $H_{\rm int}(x)$. In Eq. (1), G is the Fermi constant:

$$G=(1.01\pm0.01)\times10^{-5}M_p^{-2},$$

M_p being the mass of the proton. This value of G is obtained from the muon lifetime. The current $J_\mu(x)$ is the sum of the leptonic current $J_\mu^{(l)}$ and the hadronic current $J_\mu^{(h)}$:

$$J_\mu(x)=J_\mu^{(l)}(x)+J_\mu^{(h)}(x)\,, \quad (2)$$

where[3]

$$J_\mu^{(l)}=\bar{\nu}_e\gamma_\mu(1-\gamma_5)e+\bar{\nu}_\mu\gamma_\mu(1-\gamma_5)\mu \quad (3)$$

and e, ν_e, μ, ν_μ are the electron, electron-neutrino, muon, and muon-neutrino fields, respectively. The hadronic current $J_\mu^{(h)}(x)$ also has a $V-A$ form and was successfully described by Cabibbo[4] in 1963 in terms of

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[1] E. Fermi, Z. Physik 88, 161 (1934); Nuovo Cimento 11, 1 (1934).
[2] E. C. G. Sudarshan and R. E. Marshak, Phys. Rev. 109, 1860 (1958); R. P. Feynman and M. Gell-Mann, *ibid.* 109, 193 (1958).
[3] Our metric and γ-matrix conventions are those used in J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1954), see summary in Appendix A, p. 281.
[4] N. Cabibbo, Phys. Rev. Letters 10, 531 (1963). See also M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1960).