Asymptotic $SU(3)$ Symmetry and Baryon Couplings

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(Received 10 April 1970)

General broken-$SU(3)$ sum rules for baryon transitions $B' \to (\frac{1}{2})^{\pm}+P$ (B' denotes a nonet or octet baryon with arbitrary spin and parity) are derived. The approach is based on the use of a chiral $SU(3)\otimes SU(3)$ charge algebra, the hypothesis of partially conserved axial-vector current (PCAC) and, in particular, the assumption of asymptotic $SU(3)$ symmetry formulated for the matrix elements of the vector charge V_K. The V_K is the $SU(3)$ raising or lowering operator. The sum rules thus obtained are always compatible with the Gell-Mann-Okubo mass splittings of hadrons. They exhibit a simple modification of exact-$SU(3)$ sum rules, but the effect is, in general, quite significant. As a specific application, the $Y_0^*(1405)$ transition is discussed in some detail in order to compare with the recent result of Gell-Mann, Oakes, and Renner (GOR), based on a different approximation for broken $SU(3)$ symmetry. It is shown that, in the absence of singlet-octet mixing, both approaches give the same result in this particular case, and that the GOR approximation can, in fact, be derived from our asymptotic $SU(3)$ symmetry. Our asymptotic $SU(3)$ symmetry, however, appears to be a more general and far-reaching prescription, useful in broken $SU(3)$ symmetry when combined with the use of equal-time commutation relations involving the charge V_K. A comment is also made about the hard-kaon and η-meson extrapolation resulting from the use of kaon and η-meson PCAC. The result is applied to the derivation of the values of the ΛNK and ΣNK couplings from the experimental information on the axial-vector semileptonic couplings of hyperons. The result is consistent with experiment.

I. OUTLINE OF APPROACH AND SUMMARY

VARIOUS attitudes have been taken toward the broken $SU(3)$ symmetry. The naive procedure most commonly utilized (especially in the experimental analysis) is to use exact $SU(3)$ with the usual modification due to particle mixing. If we seek higher accuracy, this is certainly unsatisfactory. In general, there is no guarantee that these exact-$SU(3)$ sum rules are compatible with the observed hadron mass splittings. This problem persists in a number of fundamental questions we would like to ask, such as: Are the vector and axial-vector Cabibbo angles equal? The original Cabibbo analysis[1] is based on exact-$SU(3)$ sum rules. We need to find broken-$SU(3)$ sum rules which are, at least, compatible with the Gell-Mann–Okubo (GO) mass splittings. It is also desirable that the prescription to derive such sum rules be simple, unique, and systematically applicable to any problem.

We have proposed[2] a prescription which seems to satisfy such criteria. The approach is based essentially on the following two basic ideas: (i) Instead of using the notion of exact $SU(3)$ symmetry, we use a set of equal-time commutation relations (such as the well-known chiral $SU(3)\otimes SU(3)$ charge algebra) involving the vector charge V_K which is an $SU(3)$ raising or lowering operator. As Gell-Mann stressed,[3] these commutators are valid even in broken $SU(3)$ symmetry. (ii) We use the idea of asymptotic $SU(3)$ symmetry formulated for the matrix elements of the vector charge V_K in the infinite-momentum limit. One can explicitly demonstrate[2] that this asymptotic assumption can be made in the presence of the GO mass splittings (including mixing). As a matter of fact [although the argument is slightly more dependent on the model of $SU(3)$ breaking] we are also led to conclude that if the asymptotic $SU(3)$ symmetry holds, there exist not only the GO mass splittings, but also simple *intermultiplet* mass relations among hadrons. It turns out that they include the $SU(6)$ mass formulas as special cases.[2,4]

These two ideas alone are already sufficient to derive some important sum rules such as the broken-$SU(3)$ Cabibbo sum rules for the hyperon axial-vector semileptonic decay couplings.[5] Also, for the vector meson $\to l+\bar{l}$ decay couplings we obtain, among others, the sum rules[6]

$$(G_\phi^2/m_\phi^2)+(G_\omega^2/m_\omega^2)=(G_{K^*}^2/m_{K^*}^2)=(G_\rho^2/m_\rho^2)\,,$$

which are also derived[7] by using spectral functions and imposing a different form of asymptotic condition from ours. Broken-$SU(3)$ sum rules for the radiative decays of hadrons are also derived.[8] For the problems involving the pseudoscalar mesons, it is convenient to add another

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[1] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

[2] S. Matsuda and S. Oneda, Phys. Rev. **158**, 1594 (1967); **174**, 1992 (1968); Nucl. Phys. **B9**, 55 (1969).

[3] M. Gell-Mann, Physics **1**, 63 (1964).

[4] For bosons, see S. Matsuda and S. Oneda, Phys. Rev. **179**, 1301 (1964); for baryons, see S. Matsuda and S. Oneda, Phys. Rev. D **1**, 944 (1970).

[5] S. Matsuda, S. Oneda, and P. Desai, Phys. Rev. **178**, 2129 (1969).

[6] S. Matsuda and S. Oneda, Phys. Rev. **171**, 1743 (1968).

[7] S. L. Glashow, H. J. Schnitzer, and S. Weinberg, Phys. Rev. Letters **19**, 137 (1967); T. Das, V. S. Mathur, and S. Okubo, *ibid.* **19**, 470 (1967).

[8] S. Matsuda and S. Oneda, Phys. Rev. **187**, 2107 (1969).

well-established hypothesis: (iii) partially conserved axial-vector current (PCAC).

The broken-$SU(3)$ sum rules thus obtained are different from exact-$SU(3)$ sum rules by the factors involving physical masses and mixing angles. [A notable exception is the Cabibbo sum rules mentioned before. In this case broken-$SU(3)$ sum rules take the same form as the hypothetical exact-$SU(3)$ sum rules.[5]] For example, for the strong decays $1^- \to 0^- + 0^-$ and $2^+ \to 1^- + 0^-$, we obtained,[2] among others,

$$(2g_{K^{*+}K^-\pi^0})/(g_{\rho^+\pi^-\pi^0}) \simeq (m_{K^*}/m_\rho)$$

and

$$(g_{A_2{}^+\rho^-\pi^0})/(2g_{K^{**+}K^{*-}\pi^0}) = (m_{A_2}{}^2 - m_\rho{}^2)/(m_{K^{**}}{}^2 - m_{K^*}{}^2)\,,$$

respectively. These ratios of coupling constants are unity in the $SU(3)$ limit. For the baryon transitions, $(\frac{1}{2})^+ \to (\frac{1}{2})^+ + \pi$, $(\frac{3}{2})^+ \to (\frac{1}{2})^+ + \pi$, and $(\frac{5}{2})^\pm \to (\frac{1}{2})^+ + \pi$, we have shown that exact-$SU(3)$ sum rules will be modified[4] (sometimes significantly) in broken $SU(3)$ symmetry.

Recently, Gell-Mann, Oakes, and Renner[9] (GOR) suggested a theoretical correction to the exact-$SU(3)$ sum rules for the transitions of $Y_0^*(1405)$, the $(\frac{1}{2})^-$ $SU(3)$ singlet. They also use PCAC for the pseudoscalar densities. They furthermore assume that at small four-momentum transfers the axial-vector currents retain their octet character even in the presence of symmetry breaking. They show that these prescriptions lead to a large violation of exact-$SU(3)$ sum rules.

In Sec. II we compare our approach with that of GOR in the same problem. We show that our asymptotic $SU(3)$ symmetry contains, as one of the results of its specific application, the GOR approximation. Therefore, our asymptotic $SU(3)$ symmetry appears to be a more general and far-reaching formulation in broken $SU(3)$ symmetry, and it is, in particular, explicitly consistent with the GO hadron mass splitting.

In Sec. III we write down the general form of broken-$SU(3)$ sum rules for the transition $B' \to (\frac{1}{2})^\pm + P$. Here B' denotes the nonet (or octet) baryon with arbitrary spin and parity, J^P. We obtain an interesting interplay of the physical masses in these sum rules. The effect of $SU(3)$ breaking is, in general, significant and depends on the relative spacings of the $SU(3)$ mass spectra of baryons involved. We use PCAC, $\partial_\mu A_\mu{}^\pi(x) = F_\pi m_\pi{}^2 \phi_\pi(x)$, $\partial_\mu A_\mu{}^K(x) = F_K m_K{}^2 \phi_K(x)$, etc., in the following way. We regard PCAC as an exact condition which provides a definition of a local pseudoscalar-meson field.[10] The values of F_π and F_K will then be fixed from the rates of $\pi \to \mu + \nu$ and $K \to \mu + \nu$ decays. A reasonable estimate of the value of (F_K/F_π) is around 1.22.[11] In our approach, where the use of infinite-momentum frame is always implied according to our asymptotic symmetry, the use of PCAC always involves a hard-pion or hard-kaon off-mass-shell extrapolation. As usual, we neglect the effect of hard-pion extrapolation. The hard-kaon extrapolation (which will be still much safer than the soft-kaon extrapolation) is certainly a considerable extrapolation.

In Sec. IV we gather some evidence which suggests that the effect of the neglect of hard-kaon extrapolation in our approach is of the order of $(F_K/F_\pi) - 1$, i.e., around 20%, in the amplitude. Furthermore, we suggest that the effect may be *effectively* taken into account by neglecting the extrapolation while at the same time replacing the value of F_K (which always accompanies the kaon PCAC) with that of F_π, i.e., by setting $F_K = F_\pi$.

In Sec. V we compute the $(\frac{1}{2})^+ \to (\frac{1}{2})^+ + K$ coupling using this prescription with the values of the axial-vector coupling constants of semileptonic hyperon decays determined from recent experiments. The values of the ΛNK and ΣNK couplings thus determined are consistent with the ones determined from recent experiments.

II. $Y_0^*(1405)$ TRANSITIONS AND COMPARISON WITH GOR APPROXIMATION

Consider the following weak matrix elements of the axial-vector currents (for notation, see Appendix A):

$$\langle Y'(\mathbf{q}')|A_\mu{}^{\pi^-}(0)|\Sigma^+(\mathbf{q})\rangle = i[\Sigma/E(\Sigma)]^{1/2}[Y'/E(Y')]^{1/2} \times g_{Y'\Sigma^+}(t)\bar{u}_Y(\mathbf{q}')\gamma_\mu u_\Sigma(\mathbf{q}) + \cdots\,,$$

and

$$\langle Y'(\mathbf{q}')|A_\mu{}^{K^-}(0)|p(\mathbf{q})\rangle = i[p/E(p)]^{1/2}[Y'/E(Y')]^{1/2} \times g_{Y'p}(t)\bar{u}_Y(\mathbf{q}')\gamma_\mu u_p(\mathbf{q}) + \cdots\,.$$

Here $t = -(q'-q)^2$ and $Y' \equiv Y_0^*(1405)$. (Here and hereafter, we use the particle symbol to denote the mass of the particle unless confusion arises. For example, $\Sigma \equiv M_\Sigma$.) The essence of the GOR approximation[9] for broken $SU(3)$ symmetry is $g_{Y'\Sigma^+}(0) \simeq g_{Y'p}(0)$ [in exact $SU(3)$, $g_{Y'\Sigma^+}(t) = g_{Y'p}(t)$]. Using the generalized Goldberger-Treiman (GT) relation, they obtain $g_{Y'\Sigma^+}(0) \times (Y'-\Sigma) \simeq F_\pi g_{Y'\Sigma^+\pi}$ and $g_{Y'p}(0)(Y'-p) \simeq F_K g_{Y'pK}$. Therefore, they obtain for the ratio of the coupling constants of the Y' decay,

$$R \equiv \frac{g_{Y'pK}}{g_{Y'\Sigma^+\pi}} \simeq \left(\frac{Y'-p}{Y'-\Sigma}\right)\left(\frac{F_\pi}{F_K}\right), \qquad (1)$$

instead of the $SU(3)$ value, unity.

We now study the same problem from our approach including the possible singlet-octet mixing which was not considered in deriving Eq. (1). Our asymptotic $SU(3)$ symmetry[2] states that for particles with extremely high momenta the matrix elements of the V_K behave

[9] M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968); see also C. Weil, *ibid.* **161**, 1682 (1967).

[10] For example, K. Nishijima, Phys. Rev. **133**, B204 (1964), and papers cited there.

[11] We assume one Cabibbo angle and use a value $\sin\theta \simeq 0.22$.

as if the V_K were exact-$SU(3)$ generators.[12] In other words, our asymptotic symmetry implies that the "in" and "out" states of a field transform according to a definite irreducible representation of $SU(3)$ even in broken symmetry, but only in the *infinite* momentum limit.[13] If the particle mixing takes place, the proper in and out states must be introduced by diagonalization.

This asymptotic symmetry is always compatible with the GO mass splittings. To see this, let us denote the usual $(\frac{1}{2})^+$ baryon octet as (N,Λ,Σ,Ξ) and the $(\frac{1}{2})^-$ baryon nonet as $(N',\Lambda',\Sigma',\Xi')$ and $Y'\equiv Y_0^*$. Here $N=(p,n)$, $N'=(p',n')$, etc. If we assume that the $SU(3)$-breaking interaction $H'(x)$ transforms like an $I=Y=0$ member of an $SU(3)$ octet, the commutator $[\dot{V}_{K^0},V_{K^0}]=0$ *always* holds. $\dot{V}_{K^0}$ is the time derivative of V_{K^0}. Consider the equations

$$\lim_{|\mathbf{q}|\to\infty}\langle\Xi^0(\mathbf{q})|[\dot{V}_{K^0},V_{K^0}]|n(\mathbf{q}')\rangle=0$$

and

$$\lim_{|\mathbf{q}|\to\infty}\langle\Xi^0(\mathbf{q})|[\dot{V}_{K^0},V_{K^0}]|n'(\mathbf{q}')\rangle=0.$$

By using our asymptotic $SU(3)$ symmetry for the V_{K^0}, we obtain the GO mass formulas from these equations,[14] namely,

$$\Sigma^2+3\Lambda^2=2(n^2+\Xi^2)$$

and

$$\Sigma'^2+3\Lambda'^2-2(n'^2+\Xi'^2)+3(Y'^2-\Lambda'^2)\sin^2\theta=0.$$

(For a general argument, see Appendix B.) In the above sum rules, θ denotes the Y'-Λ' mixing angle defined only in the infinite-momentum frame ($|\mathbf{q}|=\infty$) by

$$|\Lambda'(\mathbf{q})\rangle=\cos\theta|\Lambda_8'(\mathbf{q})\rangle+\sin\theta|Y_1'(\mathbf{q})\rangle$$

and (2)

$$|Y'(\mathbf{q})\rangle=-\sin\theta|\Lambda_8'(\mathbf{q})\rangle+\cos\theta|Y_1'(\mathbf{q})\rangle,$$

where $|\Lambda'\rangle\to|\Lambda_8'\rangle$ and $|Y'\rangle\to|Y_1'\rangle$ in the $SU(3)$ limit. Note that we always define the mixing angle only in the frame $|\mathbf{q}|\to\infty$, where the actual masses of the particles are not playing an important role.

Now we consider, for example, the following chiral $SU(3)\otimes SU(3)$ charge algebra:

$$[V_{\bar{K}^0},A_{\pi^-}]=A_{K^-}.$$

Here A_{π^-}, for example, denotes the isovector axial-vector charge. With our asymptotic $SU(3)$ symmetry, we obtain in the limit $|\mathbf{q}|=\infty$

$$\langle Y'(\mathbf{q})|A_{K^-}|p\rangle=\langle Y'(\mathbf{q})|V_{\bar{K}^0}|n'\rangle\langle n'|A_{\pi^-}|p\rangle-\langle Y'(\mathbf{q})|A_{\pi^-}|\Sigma^+\rangle\langle\Sigma^+|V_{\bar{K}^0}|p\rangle.$$

Our asymptotic $SU(3)$ symmetry implies (only in the limit $|\mathbf{q}|\to\infty$)

$$\langle\Lambda_8(\mathbf{q})|V_{\bar{K}^0}|n'(\mathbf{q})\rangle=-(\surd\tfrac{3}{2}),\quad\langle Y_1(\mathbf{q})|V_{\bar{K}^0}|n'(\mathbf{q})\rangle=0,$$

and

$$\langle\Sigma^+(\mathbf{q})|V_{\bar{K}^0}|p(\mathbf{q})\rangle=-1.$$

Therefore, the above equation becomes, with $|\mathbf{q}|=\infty$,

$$\langle Y'(\mathbf{q})|A_{K^-}|p\rangle=\langle Y'(\mathbf{q})|A_{\pi^-}|\Sigma^+\rangle+(\surd\tfrac{3}{2})\sin\theta\langle n'(\mathbf{q})|A_{\pi^-}|p\rangle.\quad(3)$$

By using PCAC for the A_{π^-}, we can relate, for example, the quantity

$$\lim_{|\mathbf{q}|\to\infty}\langle Y'(\mathbf{q})|A_{\pi^-}|\Sigma^+(\mathbf{q})\rangle$$

to the off-mass-shell coupling

$$g_{Y'\Sigma^+\pi}(Y'^2,\Sigma^2,\pi^2=0)\equiv g_{Y'\Sigma^+\pi}(\pi^2=0)$$

as follows (see Appendix C for normalization):

$$\lim_{|\mathbf{q}|\to\infty}\langle Y'(\mathbf{q})|A_{\pi^-}|\Sigma^+(\mathbf{q})\rangle=\frac{F_\pi g_{Y'\Sigma^+\pi}(\pi^2=0)}{Y'-\Sigma}.\quad(4)$$

This is, in fact, an example of the generalized GT relations. Thus by using kaon PCAC as well as pion PCAC, Eq. (3) now reads

$$\frac{F_K g_{Y'pK}(K^2=0)}{Y'-p}=\frac{F_\pi g_{Y'\Sigma^+\pi}(\pi^2=0)}{Y'-\Sigma}+(\surd\tfrac{3}{2})\sin\theta\frac{F_\pi g_{n'p\pi}(\pi^2=0)}{n'-p}.\quad(5)$$

Let us for the moment set $\theta=0$, i.e., the $Y_0^*(1405)$ is predominantly an $SU(3)$ singlet. Then we obtain

$$R'\equiv\frac{g_{Y'pK}(K^2=0)}{g_{Y'\Sigma^+\pi}(\pi^2=0)}=\left(\frac{Y'-p}{Y'-\Sigma}\right)\left(\frac{F_\pi}{F_K}\right).\quad(6)$$

This is essentially the result of GOR given by Eq. (1). (Our hard-kaon and hard-pion extrapolation corresponds to the neglect of terms other than the kaon or pion poles in deriving the GT relations.) However, in the above derivation we did not use the approximation $g_{Y'\Sigma\pi}(0)\simeq g_{Y'p}(0)$ utilized in the approach of GOR. However, we easily see that the above GOR approximation can, in fact, be derived from our asymptotic $SU(3)$ symmetry.

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[12] By this we mean that the V_K has nonvanishing matrix elements only between the states belonging to the same $SU(3)$ multiplet (in the absence of mixing) and the values of the matrix elements take the $SU(3)$ values. However, this is assumed only in the asymptotic limit. We also emphasize that the mixing angles are defined only in this asymptotic limit. At finite momentum, the mixing will not, in general, take such simple forms. We think that the usual mixing angle introduced in the GO mass formulas find a correct interpretation only in our formulation of mixing problems. For more details, see S. Oneda, H. Umezawa, and S. Matsuda, Phys. Rev. Letters **25**, 71 (1970).

[13] Intuitively speaking, we assume that the $SU(2)$ and $SU(3)$ symmetries are well realized among particles with extremely large momenta where masses are not important.

[14] For the earlier use of the commutator $[\dot{V}_{K^0},V_{K^0}]=0$ to derive the GO mass formulas see, for example, S. Fubini and G. Furlan, Ann. Phys. (N.Y.) **1**, 229 (1965); G. Furlan, F. Lannoy, C. Rossetti, and G. Segrè, Nuovo Cimento **40**, 597 (1965); K. Nishijima and J. Swank, Nucl. Phys. **B3**, 553 (1967). The concept of asymptotic $SU(3)$ symmetry was not introduced in this literature.

Furthermore, we think that *unless* our asymptotic $SU(3)$ symmetry is valid, the GOR approximation cannot be justified. To see this, let us now consider each term of Eq. (3) as the weak matrix elements of the axial-vector currents. Then Eq. (3) reads

$$g_{Y'p}(0)=g_{Y'\Sigma^+}(0)+(\sqrt{\tfrac{3}{2}})\sin\theta\, g_{n'p}(0)\,. \tag{7}$$

The essential step in deriving Eq. (7) is the use of asymptotic $SU(3)$ symmetry. If there is no Y'-Λ mixing, we reproduce the GOR approximation $g_{Y'p}(0)=g_{Y'\Sigma^+}(0)$. Of course, in the presence of mixing, Eq. (7) is the correct sum rule which is compatible with the GO mass formulas for the $(\frac{1}{2}^+)$ and $(\frac{1}{2}^-)$ baryons with mixing derived above. The asymptotic result $g_{Y'p}(0)=g_{Y'\Sigma^+}(0)$ coincides with the exact $SU(3)$ result. However, this is a rather *accidental* situation in our approach. If the Y', for example, has a higher spin, the asymptotic relation between the $g_{Y'p}(0)$ and $g_{Y'\Sigma^+}(0)$ involves the physical masses and does not coincide with the exact $SU(3)$ relation. Thus we have seen that the GOR approximation is justified (in this particular case) from our approach as one of the specific applications of our asymptotic $SU(3)$ symmetry formulated for the vector charge V_K.

As an approximation in broken $SU(3)$ symmetry, our asymptotic $SU(3)$ symmetry is certainly a more general one which can be utilized in a systematic way combined with the equal-time commutation relations involving the charge V_K. The sum rules obtained are guaranteed to be consistent with the GO hadron mass splittings including mixing.

As will be discussed in Sec. IV, the effect of the hard-kaon extrapolation may be effectively taken into account by neglecting the extrapolation and at the same time replacing F_K by F_π. Then we obtain from Eq. (5) (neglecting, of course, hard-pion extrapolation) for the physical couplings (with $\theta=0$)

$$R=\frac{g_{Y'pK}}{g_{Y'\Sigma^+\pi}}\simeq\frac{Y'-p}{Y'-\Sigma}\simeq 2.2\,.$$

Kim and Von Hippel[15] obtained $R\simeq 2.6\pm 0.2$ from the analysis of experiment. The rather impressive agreement seems to indicate that the GOR approximation or our asymptotic $SU(3)$ symmetry (and also our prescription for the hard-kaon extrapolation) are sensible approximations. Of course, one should test the more accurate formula, which includes mixing, obtained from Eq. (5),

$$\frac{g_{Y'pK}}{Y'-p}=\frac{g_{Y'\Sigma^+\pi}}{Y'-\Sigma}+(\sqrt{\tfrac{3}{2}})\sin\theta\,\frac{g_{n'p\pi}}{n'-p}. \tag{8}$$

A preliminary test of the broken-$SU(3)$ sum rules, such as Eq. (8), has been already carried out by Tripp and his co-workers.[16] Our impression is that the time is getting ripe for the test of broken-$SU(3)$ sum rules.

III. GENERAL BROKEN-SU(3) SUM RULES FOR TRANSITION: NONET (OR OCTET) BARYON $\rightarrow(\frac{1}{2})^{\pm}$ OCTET BARYON+P

We now generalize the argument presented in Sec. II. Let us denote the nonet baryon with an *arbitrary* J^P as B' (N', Λ', Σ', Ξ', and Y'), where Y' belongs to an $SU(3)$ singlet in the symmetry limit. Let B (N,Λ,Σ,Ξ) denote either the $(\frac{1}{2})^+$ or $(\frac{1}{2})^-$ octet baryons. Extension to the case where the B is a nonet is straightforward. We consider a general transition $B'\rightarrow B+P$. The transition involves only one partial wave with a definite parity.[17] The $\Lambda'-Y'$ mixing is taken care of by defining a mixing angle θ through the equations

$$\begin{aligned}\langle\Lambda'(\mathbf{q})| &=\cos\theta\langle\Lambda_8'(\mathbf{q})|+\sin\theta\langle Y_1'(\mathbf{q})|\,,\\ \langle Y'(\mathbf{q})| &=-\sin\theta\langle\Lambda_8'(\mathbf{q})|+\cos\theta\langle Y_1'(\mathbf{q})|\,,\end{aligned} \tag{9}$$

in the frame $|\mathbf{q}|=\infty$. Here $\langle\Lambda'(\mathbf{q})|\rightarrow\langle\Lambda_8'(\mathbf{q})|$ and $\langle Y'(\mathbf{q})|\rightarrow\langle Y_1'(\mathbf{q})|$ in the $SU(3)$ limit. As explained in a specific example in Sec. II, we make a full use of the usual chiral $SU(3)\otimes SU(3)$ charge algebra involving the charge V_K (which holds in broken symmetry) and apply our asymptotic $SU(3)$ symmetry. We then obtain a set of sum rules involving various matrix elements of the axial-vector charges, $\langle B'(\mathbf{q}')|A_i|B(\mathbf{q})\rangle$ with $|\mathbf{q}|=\infty$. Here A_i stands for A_π, A_K, and A_η. We list these sum rules by introducing the hypothetical couplings

$$G_{B'BP_i}\equiv\lim_{|\mathbf{q}|\rightarrow\infty}\langle B'(\mathbf{q})|A_i|B(\mathbf{q})\rangle\,. \tag{10}$$

For example,

$$G_{p'n\pi}\equiv\lim_{|\mathbf{q}|\rightarrow\infty}\langle p'(\mathbf{q})|A_{\pi^+}|n(\mathbf{q})\rangle,$$

$$G_{p'\Lambda K}\equiv\lim_{|\mathbf{q}|\rightarrow\infty}\langle p'(\mathbf{q})|A_{K^+}|\Lambda(\mathbf{q})\rangle,$$

and

$$G_{p'p\eta}\equiv\lim_{|\mathbf{q}|\rightarrow\infty}\langle p'(\mathbf{q})|A_\eta|p(\mathbf{q})\rangle,\quad\text{etc.}$$

In Appendix A we list our notation and conventions in more detail. We express the G's in terms of four inde-

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[15] J. K. Kim and F. Von Hippel, Phys. Rev. **184**, 1961 (1969). Other theoretical approaches will be found here. See also R. D. Tripp, R. O. Bangerter, A. Barbaro-Galtieri, and T. S. Mast, Phys. Rev. Letters **21**, 1721 (1968).

[16] R. D. Tripp, LRL Report No. UCRL-19361 (unpublished). The treatment of the $\frac{1}{2}^-\rightarrow\frac{1}{2}^++P$ transition discussed in this paper using the correction factor of $SU(3)$ breaking is justified from our asymptotic $SU(3)$ symmetry. The quantities x_1, x_2, x_3, y_1, y_2, and y_3 defined in this paper which denote the coupling strengths of the $\Lambda'\Sigma\pi$, $\Lambda'\Lambda\eta$, $\Lambda'NK$, $Y'\Sigma\pi$, $Y'\Lambda\eta$, and YNK decays, respectively, are related to our G's in the following way. If we parametrize the G's by taking $G_{Y'\Sigma^+\pi}=-\sin\theta(\sqrt{\frac{3}{5}})g_d+\cos\theta(\sqrt{\frac{3}{8}})g_1$, $G_{\Sigma'^+\Lambda\pi}=(\sqrt{\frac{3}{5}})g_d$, and $G_{\Sigma'^+\Sigma^0\pi}=-g_f$, then $x_1=-G_{\Lambda'\Sigma^+\pi}$, $y_1=G_{Y'\Sigma^+\pi}$, $x_2=(\sqrt{\frac{2}{3}})G_{\Lambda'\Lambda\eta}$, $y_2=-(\sqrt{\frac{2}{3}})G_{Y'\Lambda\eta}$, $x_3=-(\sqrt{\frac{2}{3}})G_{\Lambda'pK}$, and $y_3=(\sqrt{\frac{2}{3}})G_{Y'pK}$. However, the sign of our mixing angle θ is opposite to the one used by Tripp.

[17] There is also no difficulty in extending our discussion to the cases where more partial waves are involved. The cases $(\frac{3}{2})^+\rightarrow(\frac{1}{2})^++0^-$, $(\frac{5}{2})^+\rightarrow(\frac{3}{2})^++0^-$, $(\frac{5}{2})^-\rightarrow(\frac{3}{2})^++0^-$, and $(\frac{3}{2})^+\rightarrow(\frac{1}{2})^-+0^-$ were treated by G. Fourez, thesis, University of Maryland, 1969 (unpublished); Nucl. Phys. **B18**, 189 (1970).

pendent quantities, i.e., $G_{\Sigma'^+\Lambda\pi}\equiv d$, $G_{\Sigma'^+\Sigma^0\pi}\equiv f$, $G_{Y'\Sigma^+\pi}\equiv s$, and the mixing angle θ. [Note that in exact $SU(3)$ we also have four parameters: D and F couplings, g_d and g_f, singlet coupling, g_s, and θ.] The sum rules thus obtained are

$$G_{p'n\pi}=(\sqrt{\tfrac{3}{2}})d-(\sqrt{\tfrac{1}{2}})f\,, \tag{11}$$

$$G_{p'\Lambda K}=-\tfrac{1}{2}d+\tfrac{1}{2}\sqrt{3}f\,, \tag{12}$$

$$G_{p'\Sigma^0K}=\tfrac{1}{2}\sqrt{3}d+\tfrac{1}{2}f\,, \tag{13}$$

$$G_{\Sigma'^+pK}=(\sqrt{\tfrac{3}{2}})d+(\sqrt{\tfrac{1}{2}})f\,, \tag{14}$$

$$G_{\Sigma'^+\Xi^0K}=(\sqrt{\tfrac{3}{2}})d-(\sqrt{\tfrac{1}{2}})f\,, \tag{15}$$

$$G_{\Lambda'\Sigma^+\pi}=(1/\cos\theta)d+\tan\theta\, s\,, \tag{16}$$

$$G_{\Lambda'pK}=(-\tfrac{3}{2}\cos\theta+1/\cos\theta)d+\tfrac{1}{2}\sqrt{3}\cos\theta\, f+\tan\theta\, s\,, \tag{17}$$

$$G_{\Lambda'\Xi^-K}=(-\tfrac{3}{2}\cos\theta+1/\cos\theta)d-\tfrac{1}{2}\sqrt{3}\cos\theta\, f+\tan\theta\, s\,, \tag{18}$$

$$G_{Y'pK}=\tfrac{3}{2}\sin\theta\, d-\tfrac{1}{2}\sqrt{3}\sin\theta\, f+s\,, \tag{19}$$

$$G_{Y'\Xi^-K}=\tfrac{3}{2}\sin\theta\, d+\tfrac{1}{2}\sqrt{3}\sin\theta\, f+s\,, \tag{20}$$

$$G_{\Xi'^-\Sigma^0K}=\tfrac{1}{2}\sqrt{3}d-\tfrac{1}{2}f\,, \tag{21}$$

$$G_{\Xi'^-\Lambda K}=-\tfrac{1}{2}d-\tfrac{1}{2}\sqrt{3}f\,, \tag{22}$$

$$G_{\Xi'^-\Xi^0\pi}=(\sqrt{\tfrac{3}{2}})d+(\sqrt{\tfrac{1}{2}})f\,, \tag{23}$$

$$G_{p'p\eta}=-\tfrac{1}{2}(\sqrt{\tfrac{1}{2}})d-\tfrac{1}{2}(\sqrt{\tfrac{3}{2}})f\,, \tag{24}$$

$$G_{\Sigma'^+\Sigma^+\eta}=(\sqrt{\tfrac{1}{2}})d\,, \tag{25}$$

$$G_{\Lambda'\Lambda\eta}=-(\sqrt{\tfrac{1}{2}})(2\cos\theta-1/\cos\theta)d+(\sqrt{\tfrac{1}{2}})\tan\theta\, s\,, \tag{26}$$

$$G_{Y'\Lambda\eta}=\sqrt{2}\sin\theta\, d+(\sqrt{\tfrac{1}{2}})s\,, \tag{27}$$

$$G_{\Xi'^0\Xi^0\eta}=-\tfrac{1}{2}(\sqrt{\tfrac{1}{2}})d+\tfrac{1}{2}(\sqrt{\tfrac{3}{2}})f\,. \tag{28}$$

For the $SU(2)$ couplings see Appendix A.

These sum rules hold irrespective of the spins and parities of the baryons involved and they are the broken-$SU(3)$ sum rules which are compatible with the GO mass splittings with $\Lambda'-Y'$ mixing. By using the commutator $[\dot{V}_{K^0},V_{K^0}]=0$, we can explicitly demonstrate this. {Furthermore, we can show that the GO mass splittings and our asymptotic $SU(3)$ symmetry are consistent with having commutators such as $[\dot{V}_{K^0},A_{K^0}]=0$. See Appendix B.} If we identify the $G_{B'BP}$ with the coupling for the transition $B'\rightarrow B+P$, these sum rules are, in fact, of the same form as the ones obtained by using exact $SU(3)$ and the usual prescription of mixing. However, the G's are not the physical coupling constants $g_{B'BP}$. They are defined [with $(\Box-\pi^2)\phi_\pi(x)=-J_\pi(x)$] by

$$\langle B'(\mathbf{q}')|J_\pi(0)|B(\mathbf{q})\rangle=ig_{B'BP}\left\{\frac{B'}{E(B')}\frac{B}{E(B)}\right\}^{1/2}\times\bar{u}_{\mu_1\cdots\mu_n}(\mathbf{q}')\,[1\text{ or }\gamma_5]\,u_B(\mathbf{q})p_{\mu_1}\cdots p_{\mu_n}.$$

Here $p=q'-q$, and $u_{\mu_1\cdots\mu_n}$ is the Rarita-Schwinger wave function ($\mu_i=1, 2, 3, 4$) of the B' which has spin $n+\frac{1}{2}$ ($n=0, 1, 2, \cdots$). u_B is the Dirac spinor for the B. The choice of 1 or γ_5 depends on the spins and parities of B' and B. Now by using PCAC for the charges A_i (A_π, A_K, A_η), we can relate the $G_{B'BP_i}$ to the off-mass-shell $B'\rightarrow B+P_i$ coupling $g_{B'BP_i}$ (B'^2, B^2, $P_i^2=0$). As in Sec. II, we identify these couplings with the physical coupling $g_{B'BP_i}$. We find (see Appendix C for derivation) the following general relation:

$$G_{B'BP_i}=\alpha_{B'BP_i}F_ig_{B'BP_i}. \tag{29}$$

Here, corresponding to $P_i=\pi$, K, and η, we have $F_i=F_\pi$, F_K, and F_η, respectively. According to our prescription for taking into account effectively the hard-P-meson extrapolation (see the discussion in Secs. IV and V), we set[18] $F_\pi=F_K=(1/\sqrt{2})F_\eta$. Then the F_i will disappear from the sum rules for the physical couplings $g_{B'BP_i}$. [In Eq. (29) we set $F_\pi=F_K=1$ and $F_\eta=\sqrt{2}$ so that the η coupling is multiplied by $\sqrt{2}$.] Then the correction due to $SU(3)$ breaking comes solely from the factors $\alpha_{B'BP_i}$. They are given (B' has spin $n+\frac{1}{2}$, where $n=0, 1, 2, \cdots$) by

$$\alpha_{B'BP_i}=\left(\frac{1}{2\sqrt{2}}\right)^n\left(\frac{B'^2-B^2}{B'}\right)^n\frac{1}{B'+B}, \tag{30}$$

when

$$B'=(\tfrac{1}{2})^+,(\tfrac{3}{2})^-,(\tfrac{5}{2})^+,\cdots\quad\text{and}\quad B=(\tfrac{1}{2}^+)$$

or when

$$B'=(\tfrac{1}{2})^-,(\tfrac{3}{2})^+,(\tfrac{5}{2})^-,\cdots\quad\text{and}\quad B=(\tfrac{1}{2})^-\,,$$

and

$$\alpha_{B'BP_i}=\left(\frac{1}{2\sqrt{2}}\right)^n\left(\frac{B'^2-B^2}{B'}\right)^n\frac{1}{B'-B}, \tag{31}$$

when

$$B'=(\tfrac{1}{2})^-,(\tfrac{3}{2})^+,(\tfrac{5}{2})^-,\cdots\quad\text{and}\quad B=(\tfrac{1}{2})^+$$

or when

$$B'=(\tfrac{1}{2})^+,(\tfrac{3}{2})^-,(\tfrac{5}{2})^+,\cdots\quad\text{and}\quad B=(\tfrac{1}{2})^-.$$

We consider some specific examples below. For the transitions $(\frac{1}{2})^+\rightarrow(\frac{1}{2})^++P_i$ (see Sec. V) and $(\frac{3}{2})^+\rightarrow(\frac{1}{2})^++P_i$, we obtain $\alpha=(B'+B)^{-1}$ and $\alpha\propto(B'+B)/B'$, respectively. Although the $(\frac{3}{2})^+$ baryons belong to a decuplet (Δ, Σ^*, Ξ^*, Ω), we can immediately read off the effect of $SU(3)$ violation[2,19] since there is only one independent coupling,

$$\frac{g_{\Xi^*\Xi\pi}}{g_{\Sigma^*\Sigma\pi}}=\frac{(\Sigma^*+\Sigma)}{(\Xi^*+\Xi)}\left(\frac{\Xi^*}{\Sigma^*}\right), \tag{32}$$

[18] See Appendix A for the definition of the A_η.

[19] If we use the usual method of spin summation for the $(\frac{3}{2})^+$ particles we obtain, for example, $(g_{\Xi^*\Xi\pi})/(g_{\Sigma^*\Sigma\pi})=(\Sigma^*+\Sigma)(\Xi^*+\Xi)^{-1}(\Sigma^{*2}+\Xi^{*2}+\Sigma^*\Xi^*)(3\Sigma^{*2})^{-1}$ (see, for example, Ref. 17) compared with the one given in this paper, $(g_{\Xi^*\Xi\pi})/(g_{\Sigma^*\Sigma\pi})=(\Sigma^*+\Sigma)(\Xi^*+\Xi)^{-1}(\Xi^*/\Sigma^*)$, which is obtained by choosing a particular spin-wave function for the $(\frac{3}{2})^+$ particle. However, these two expressions are essentially equivalent, since $(\Xi^*/\Sigma^*)\simeq(\Sigma^{*2}+\Xi^{*2}+\Sigma^*\Xi^*)(3\Sigma^{*2})^{-1}$.

$$\frac{g_{\Xi^*\Xi\pi}}{g_{\Delta N\pi}}=\frac{(\Delta+N)}{(\Xi^*+\Xi)}\left(\frac{\Xi^*}{\Delta}\right), \tag{33}$$

$$\frac{g_{\Delta N\pi}}{g_{\Sigma^*\Lambda\pi}}=\frac{(\Sigma^*+\Lambda)}{(\Delta+N)}\left(\frac{\Delta}{\Sigma^*}\right). \tag{34}$$

These coupling ratios are normalized to unity in the $SU(3)$ limit.

Since the masses appear summed in the α's for the transitions $(\frac{1}{2})^+\to(\frac{1}{2})^+ + P_i$ and $(\frac{3}{2})^+\to(\frac{1}{2})^+ + P_i$, the $SU(3)$-breaking effects are not very significant. [If the predictions, Eqs. (32)–(34), are not consistent with experiment, we have to take into account the effect of mixing, presumably between the ground-state and higher-lying $(\frac{3}{2})^+$ baryons.[20]] However, in general, the α involves the nonzero power of the mass difference $(B'-B)$, and the effect could be very conspicuous in some cases, such as the Y_0^* transition discussed in Sec. II. For the $(\frac{3}{2})^-\to(\frac{1}{2})^+ + P_i$, we have $\alpha\propto(B'-B)/B'$. Corresponding to the sum rule, Eq. (8), for the Y_0^* transition, we obtain for the $Y'\equiv\Lambda(1520)$ [$I=0$, $(\frac{3}{2})^-$ baryon]

$$\left(\frac{Y'-p}{Y'}\right)g_{Y'pK}=\left(\frac{Y'-\Sigma}{Y'}\right)g_{Y'\Sigma^+\pi}+(\sqrt{\tfrac{3}{2}})\sin\theta\left(\frac{n'-p}{n'}\right)g_{n'p\pi}. \tag{35}$$

Here the primed particles belong to the $(\frac{3}{2})^-$ nonet.[16] θ is the $Y'-\Lambda'$ mixing angle.

Since the hard-pion extrapolation is least dangerous, the pion transition sum rules will be the most trustworthy ones. Equations (11), (16), and (23) read, for the $(\frac{1}{2})^-\to(\frac{1}{2})^+ + \pi$ transitions,

$$\frac{1}{p'-n}g_{p'n\pi}=(\sqrt{\tfrac{3}{2}})\frac{1}{\Sigma'-\Lambda}g_{\Sigma'^+\Lambda\pi}-(\sqrt{\tfrac{1}{2}})\frac{1}{\Sigma'-\Sigma}g_{\Sigma'^+\Sigma^0\pi}, \tag{36}$$

$$\frac{1}{\Lambda'-\Sigma}g_{\Lambda'\Sigma^+\pi}=\frac{1}{\cos\theta}\frac{1}{\Sigma'-\Lambda}g_{\Sigma'^+\Lambda\pi}+\tan\theta\frac{1}{Y'-\Sigma}g_{Y'\Sigma^+\pi}, \tag{37}$$

$$\frac{1}{\Xi'-\Xi}g_{\Xi'^-\Xi^0\pi}=(\sqrt{\tfrac{3}{2}})\frac{1}{\Sigma'-\Lambda}g_{\Sigma'^+\Lambda\pi}+(\sqrt{\tfrac{1}{2}})\frac{1}{\Sigma'-\Sigma}g_{\Sigma'^+\Sigma^0\pi}. \tag{38}$$

For the $(\frac{3}{2})^-\to(\frac{1}{2})^+ + \pi$ transitions, we have

$$\frac{p'-n}{p'}g_{p'n\pi}=(\sqrt{\tfrac{3}{2}})\frac{\Sigma'-\Lambda}{\Sigma'}g_{\Sigma'^+\Lambda\pi}-(\sqrt{\tfrac{1}{2}})\frac{\Sigma'-\Sigma}{\Sigma'}g_{\Sigma'^+\Sigma^0\pi}, \tag{39}$$

$$\frac{\Lambda'-\Sigma}{\Lambda'}g_{\Lambda'\Sigma^+\pi}=\frac{1}{\cos\theta}\frac{\Sigma'-\Lambda}{\Sigma'}g_{\Sigma'^+\Lambda\pi}+\tan\theta\frac{Y'-\Sigma}{Y'}g_{Y'\Sigma^+\pi}, \tag{40}$$

$$\frac{\Xi'-\Xi}{\Xi'}g_{\Xi'^-\Xi^0\pi}=(\sqrt{\tfrac{3}{2}})\frac{\Sigma'-\Lambda}{\Sigma'}g_{\Sigma'^+\Lambda\pi}+(\sqrt{\tfrac{1}{2}})\frac{\Sigma'-\Sigma}{\Sigma'}g_{\Sigma'^+\Sigma^0\pi}. \tag{41}$$

For the kaon coupling, we have to be aware of the hard-kaon extrapolation involved. However, our prescription (neglecting the extrapolation and at the same time setting $F_K=F_\pi$) may be reasonable as exemplified in Secs. IV and V. Therefore, it is inetresting to see how well the sum rules obtained [by setting $F_K=F_\pi=(1/\sqrt{2})F_\eta$ and neglecting extrapolation] are satisfied in reality. However, for the η coupling, we have an extra complication besides the hard-η-meson extrapolation. That is, in broken $SU(3)$ symmetry the η-meson PCAC should be written, in general, as

$$\partial_\mu A_\mu{}^\eta(x)=F_\eta m_\eta{}^2\phi_\eta(x)+F_{\eta'}m_{\eta'}{}^2\phi_{\eta'}(x)\,,$$

where η' is $\eta'(958)$. In this paper we set $F_{\eta'}=0$. The neglect of the η' contribution should be kept in mind. We expect that the sum rules involving the η-meson coupling may be less well satisfied compared with the ones involving only the pion and kaon couplings.

So far we have used only the algebraic information given by the chiral $SU(3)\otimes SU(3)$ charge algebra. Actually we think that we have a commutator involving $\dot{V}_K$, $[\dot{V}_{K^0},A_{K^0}]=0$. This commutator is valid under a rather general assumption of $SU(3)$ breaking and always gives rise to the GO mass formulas when combined with our asymptotic $SU(3)$ symmetry (see Appendix B). This commutator gives a constraint for the couplings. In the case when the B' is also an octet, this commutator fixes the ratio of the $G_{\Sigma'^+\Lambda\pi}\equiv d$ and $G_{\Sigma'^+\Sigma^0\pi}\equiv f$ couplings which corresponds to fixing the D/F ratio in exact $SU(3)$. This is discussed in Appendix D.

IV. COMMENT ON HARD-KAON MASS EXTRAPOLATION

We may reinterpret our previous calculations[2] in the following way. In our approach we can relate the off-mass-shell coupling, for example, $g_{K^*K\pi}(K^2=0)\equiv g_{K^*K\pi}(K^2,K^2=0,\pi^2)$, to the physical coupling $g_{K^*K\pi}$ as follows. Consider, for example, a commutator $\frac{1}{2}A_{K^-}=[V_{K^-},A_{\pi^0}]$ and insert it between the states $\langle\pi^0(\mathbf{q})|$ and $|K^{*+}(\mathbf{q})\rangle$ with $|\mathbf{q}|=\infty$. By using our asymptotic $SU(3)$, we obtain a sum rule

$$\tfrac{1}{2}\langle\pi^0(\mathbf{q})|A_{K^-}|K^{*+}(\mathbf{q})\rangle=-(\sqrt{\tfrac{1}{2}})\langle K^+(\mathbf{q})|A_{\pi^0}|K^{*+}(\mathbf{q})\rangle,\quad|\mathbf{q}|=\infty\,.$$

[20] For different approaches see, for example, R. Graham, S. Pakvasa, and K. Raman, Phys. Rev. **163**, 1774 (1967).

Then the use of kaon as well as pion PCAC for the A_K and A_π gives

$$\frac{g_{K^*K\pi}(K^2=0)}{g_{K^*K\pi}}=\frac{F_\pi}{F_K}\simeq 0.82. \qquad (42)$$

Here we set $g_{K^*K\pi}(\pi^2=0)\simeq g_{K^*K\pi}$. In the same way we also obtain for the $K^{**}(1420)$ ($I=\frac{1}{2}$, $J^P=2^+$ meson)

$$\frac{g_{K^{**}K\pi}(K^2=0)}{g_{K^{**}K\pi}}=\frac{K^{**2}-\pi^2}{K^{**2}-K^2}\left(\frac{F_\pi}{F_K}\right)\simeq 0.92, \qquad (43)$$

and for the κ ($I=\frac{1}{2}$, $J^P=0^+$ meson)

$$\frac{g_{\kappa K\pi}(K^2=0)}{g_{\kappa K\pi}}=\frac{\kappa^2-\pi^2}{\kappa^2-K^2}\left(\frac{F_\pi}{F_K}\right). \qquad (44)$$

For a mass of κ around 1100 MeV, the ratio is about 1.

These results indicate that for the $K'K\pi$ coupling, $g_{K'K\pi}(K^2=0)/g_{K'K\pi}\simeq(F_\pi/F_K)$, if the mass of K' meson with an arbitrary J^P is sufficiently larger than the kaon mass. The effect of the neglect of the hard-kaon extrapolation in these cases is *less* than $F_K/F_\pi-1\simeq 0.2$ in the amplitude. Furthermore, these examples indicate an interesting tendency for the hard-kaon extrapolation effect to be compensated if we replace the F_K (which always accompanies the use of kaon PCAC) by F_π. This prescription is not terribly good in the case of the κ meson, since the κ mass (around 1100 MeV) is relatively small. For the η meson, we obtain a similar result and we use a similar prescription. If we use this prescription, we predict for the $\phi^0\to K^++K^-$ coupling[2] (θ is the ω-ϕ mixing angle)

$$\frac{g_{\phi^0K^+K^-}}{g_{K^{*+}K^+\pi^0}}=-\sqrt{3}\cos\theta\left(\frac{K^{*2}+\phi^2}{2K^{*2}}\right)\left(\frac{F_\pi}{F_K}\right)\simeq-\sqrt{3}\cos\theta\left(\frac{\phi}{K^*}\right).$$

For the 2^+-decay $A_2{}^0\to K^++K^-$, we also predict[2] (see Ref. 19)

$$\frac{g_{A_2K^+K^-}}{g_{K^{*0}K^+\pi^-}}=\left(\frac{K^{**2}-\pi^2}{A_2{}^2-K^2}\right)\left(\frac{K^{**4}+A_2{}^4+4K^{**2}A_2{}^2}{6K^{**4}}\right)\left(\frac{F_\pi}{F_K}\right)$$

$$\simeq\left(\frac{K^{**2}-\pi^2}{A_2{}^2-K^2}\right)\left(\frac{A_2}{K^{**}}\right)^2.$$

It is interesting to see whether experiment is consistent with the $SU(3)$-breaking factors obtained above by using the asymptotic $SU(3)$ and PCAC. For the case of baryon couplings it is not possible to make a similar study and we simply determine whether the prescription described above will work here. The sum rule obtained in Sec. II, Eq. (8) (which was obtained by setting $F_K=F_\pi$) will also provide us one such test. In Sec. V we test the above prescription for the ΛNK and ΣNK couplings.

TABLE I. Determination of the $\frac{1}{2}^+\to\frac{1}{2}^++P$ couplings.

Coupling constant	Asymptotic $SU(3)$+PCAC[a]	Experiment	Exact $SU(3)$[b]
$g_{\Lambda pK}{}^2/4\pi$	17.5	16.0±2.5[c]; 13±3[d]	16.1
$g_{\Sigma^0 pK}{}^2/4\pi$	0.678	0.3±0.5[c]; 0±1[d]	0.59
$g_{\Xi^-\Sigma^0K}/4\pi$	26.0		14.5
$g_{\Xi^-\Lambda K}{}^2/4\pi$	2.92		2.24
$g_{\Sigma^+\Lambda\pi}{}^2/4\pi$	10.2		6.54
$g_{\Sigma^0\Sigma^-\pi}{}^2/4\pi$	15.9		10.3
$g_{\Xi^0\Xi^-\pi}{}^2/4\pi$	1.82		0.744

[a] $g_{pn}(0)=1.27$, $g_{p\Lambda}(0)=-0.945$, $g_{\Sigma^+\Lambda}(0)=0.618$, and $(g_{pp\pi}{}^2/4\pi)=14.6$ are input (see Ref. 5).
[b] $f\equiv F/(D+F)=0.42$ and $(g_{pp\pi}{}^2/4\pi)\cong 14.6$ are input.
[c] Reference 21.
[d] Reference 22.

V. KAON COUPLING IN $(\frac{1}{2})^+\to(\frac{1}{2})^++K$ INTERACTION

Here we determine the $(\frac{1}{2})^+\to(\frac{1}{2})^++K$ couplings using the generalized GT relation for the charge A_K and the prescription for the hard-kaon extrapolation discussed in Sec. IV. The $(\frac{1}{2})^+\to(\frac{1}{2})^++\pi$ couplings were discussed previously.[4]

We first consider the ΛpK coupling. From Eq. (29), kaon PCAC gives

$$\lim_{|\mathbf{q}|\to\infty}\langle p(\mathbf{q})|A_{K^+}|\Lambda(\mathbf{q})\rangle\propto F_K(p+\Lambda)^{-1}g_{p\Lambda K}(K^2=0).$$

However, we also obtain

$$\lim_{|\mathbf{q}|\to\infty}\langle p(\mathbf{q})|A_{K^+}|\Lambda(\mathbf{q})\rangle\propto g_{p\Lambda}(0).$$

$g_{p\Lambda}(0)$ is the weak axial-vector coupling (at zero four-momentum transfer) for the $\Lambda^0\to p+e^-+\bar{\nu}$ decay. Thus $g_{p\Lambda}(0)=F_K(p+\Lambda)^{-1}g_{p\Lambda K}(K^2=0)$. This is the generalized GT relation. The well-known GT relation for the $n\to p+e^-+\bar{\nu}$ decay is $g_{np}(0)=F_\pi(p+n)^{-1}\times g_{pn\pi}(\pi^2=0)$. Thus we obtain

$$(g_{p\Lambda K}/g_{pn\pi})=(p+n)(p+\Lambda)^{-1}[g_{p\Lambda}(0)/g_{pn}(0)].$$

Here we have used the prescription $[g_{p\Lambda K}(K^2=0)=g_{p\Lambda K}$ and $F_K=F_\pi]$ discussed in Sec. IV. Previously we have determined the values of the weak g's by using the broken-$SU(3)$ sum rules for the g's [obtained by using asymptotic $SU(3)$ symmetry] and some experiments on hyperon leptonic decays. Using the values thus determined,[5] i.e., $g_{pn}(0)\simeq 1.27$ and $g_{p\Lambda}(0)\simeq -0.945$, and $(g_{pp\pi}{}^2/4\pi)\simeq 14.6$, we then obtain from the above equation $(g_{p\Lambda K}{}^2/4\pi)\simeq 17.5$. Kim[21] deduced a value $(g_{p\Lambda K}{}^2/4\pi)\simeq 16.0\pm 2.5$ from experiment (see also Chan and Meiere[22] and Šoln[23]). In broken $SU(3)$ symmetry

[21] J. K. Kim, Phys. Rev. Letters **19**, 1074 (1967); **19**, 1079 (1967).
[22] C. H. Chan and F. T. Meiere, Phys. Rev. Letters **20**, 568 (1968). For earlier determination see, for example, M. Lusignoli, M. Restignoli, G. A. Snow, and G. Violini, Phys. Letters **21**, 229 (1966); N. Zovko, *ibid.* **23**, 143 (1966), and the papers cited there.
[23] For the approach using a chiral $U(3)\otimes U(3)$ phenomenological Lagrangian see, for example, J. Šoln, University of Wisconsin-Milwaukee report (unpublished), and the papers cited there.

the $(\frac{1}{2})^+ \to (\frac{1}{2})^+ + P_i$ couplings do not satisfy the exact $SU(3)$ sum rules, although the axial-vector semileptonic hyperon couplings do.[5] If we nevertheless use (for comparison) exact $SU(3)$ with the F/D ratio determined from the weak semileptonic decays[5] [$\alpha = D/(D+F) \simeq 0.58$ or $f = F/(D+F) \simeq 0.42$], we obtain $(g_{p\Lambda K}{}^2/4\pi) \simeq 16.1$ from $(g_{pp\pi}{}^2/4\pi) \simeq 14.6$.

In a similar way, from the generalized GT relation (with our prescription) we obtain $(g_{\Sigma^0 pK}{}^2/4\pi) \simeq 0.678$, where we have used[5] $g_{n\Sigma^-}(0) \simeq 0.244$. From experiment, Kim[21] deduced a value (0.3 ± 0.5) (see also Refs. 22 and 23), whereas the hypothetical exact $SU(3)$ with $f \simeq 0.42$ gives 0.59.

In Table I we summarize the results. Our result is based on these values of the g's: $g_{pn}(0) \simeq 1.27$, $g_{p\Lambda}(0) \simeq -0.945$, and $g_{\Sigma\Lambda}(0) \simeq 0.618$ ($f \simeq 0.42$). They are subject to the errors (which are still rather large) in the present experiments on the semileptonic hyperon decays. The difference between the broken-$SU(3)$ sum rules and the exact-$SU(3)$ ones is not very large, since the effect of $SU(3)$ violation enters as the sum of the baryon masses.

ACKNOWLEDGMENTS

One of us (S.O.) wishes to express his thanks to Professor R. Jaggard, Professor H. Umezawa, and Professor Y. Chow, and other members of Department of Physics, University of Wisconsin-Milwaukee for their hospitality. He is especially indebted to Professor H. Umezawa for his useful discussions and enlightening remarks. Conversation with Professor N. Papastamatiou, Professor D. Welling, and Professor J. Šoln have been useful. We also thank L. Bessler for his careful reading of the manuscript.

APPENDIX A: NOTATIONS AND CONVENTIONS

In a quark model, the vector and axial-vector currents are defined by

$$V_\mu{}^i(x) = i\bar{q}(x)\gamma_\mu(\tfrac{1}{2}\lambda_i)q(x)$$

and

$$A_\mu{}^i(x) = i\bar{q}(x)\gamma_5\gamma_\mu(\tfrac{1}{2}\lambda_i)q(x)\,,$$

respectively. $i = \pi^+, \pi^0, \pi^-, K^+, K^0, \bar{K}^0, K^-$, and η correspond to $\lambda_1 + i\lambda_2$, λ_3, $\lambda_1 - i\lambda_2$, $\lambda_4 + i\lambda_5$, $\lambda_6 + i\lambda_7$, $\lambda_6 - i\lambda_7$, $\lambda_4 - i\lambda_5$, and λ_8, respectively. The charges are defined, for example, as

$$V_{\pi^+} = -i\int d^3x\; V_4{}^{\pi^+}(\mathbf{x},0)$$

and

$$A_{\pi^+} = -i\int d^3x\; A_4{}^{\pi^+}(\mathbf{x},0)\,.$$

Instead of using exact $SU(3)$, we make a full use of the chiral $SU(3) \otimes SU(3)$ algebra involving the charges V_K. To derive the sum rules involving only the pion couplings, the use of the commutator such as $[V_{K^0}, A_{\pi^-}] = 0$ provides a short-cut way. F_π, F_K, and F_η are defined by

$$\partial_\mu A_\mu{}^{\pi^+}(x) = F_\pi m_\pi{}^2 \phi_{\pi^+}(x)\,,$$
$$\partial_\mu A_\mu{}^{K^+}(x) = F_K m_K{}^2 \phi_{K^+}(x)\,,$$

and

$$\partial_\mu A_\mu{}^{\eta}(x) = F_\eta m_\eta{}^2 \phi_\eta(x)\,.$$

Thus $\partial_\mu A_\mu{}^{\pi^0}(x) = 1/\sqrt{2} F_\pi m_\pi{}^2 \phi_{\pi^0}(x)$, and in the $SU(3)$ limit we have $F_\pi = F_K = \sqrt{2} F_\eta$.

Owing to the conventions used for the A_i and V_i, the $SU(2)$ relations between the G's are as follows:

$$G_{\Sigma'^+\Sigma^+\pi} = -(1/\sqrt{2})G_{\Sigma'^+\Sigma^0\pi} = -(1/\sqrt{2})G_{\Sigma'^0\Sigma^+\pi} = -G_{\Sigma'^-\Sigma^-\pi} = (1/\sqrt{2})G_{\Sigma'^-\Sigma^0\pi} = (1/\sqrt{2})G_{\Sigma'^0\Sigma^-\pi}\,,$$
$$G_{\Sigma'^-\Lambda^0\pi} = G_{\Sigma'^+\Lambda^0\pi} = \sqrt{2}G_{\Sigma'^0\Lambda^0\pi}\,,$$
$$G_{\Sigma'^0 pK} = (1/\sqrt{2})G_{\Sigma'^+ pK} = (1/\sqrt{2})G_{\Sigma'^- nK} = -G_{\Sigma'^0 nK}\,,$$
$$G_{\Xi'^-\Sigma^0 K} = (1/\sqrt{2})G_{\Xi'^-\Sigma^- K} = (1/\sqrt{2})G_{\Xi'^0\Sigma^+ K} = -G_{\Xi'^0\Sigma^0 K}\,,$$
$$G_{p'p\pi} = \tfrac{1}{2}G_{p'n\pi} = \tfrac{1}{2}G_{n'p\pi} = -G_{n'n\pi}\,,$$
$$G_{\Xi'^-\Xi^-\pi} = \tfrac{1}{2}G_{\Xi'^-\Xi^0\pi} = \tfrac{1}{2}G_{\Xi'^0\Xi^-\pi} = -G_{\Xi'^0\Xi^0\pi}\,,$$
$$G_{\Lambda'^0 nK} = G_{\Lambda' pK}, \quad G_{\Xi'^0\Lambda^0 K} = G_{\Xi'^0\Lambda^0 K}\,,$$
$$G_{Y' nK} = G_{Y' pK}, \quad \text{etc.}$$

APPENDIX B: DERIVATION OF GO MASS FORMULAS FROM COMMUTATORS $[\dot{V}_{K^0}, V_{K^0}] = 0$ AND $[\dot{V}_{K^0}, A_{K^0}] = 0$

If the $SU(3)$-breaking interaction transforms like an $I = Y = 0$ member of the $SU(3)$ octet, the commutator $[\dot{V}_{K^0}, V_{K^0}] = 0$ holds. Our asymptotic $SU(3)$ symmetry is always compatible with the GO mass splitting. For example, consider the equation

$$\lim_{|\mathbf{q}|\to\infty} \langle n'(\mathbf{q}) | [\dot{V}_{K^0}, V_{K^0}] | \Xi'^0 \rangle = 0\,.$$

Among the complete set of intermediate states inserted between the two charges $\dot{V}_{K^0}$ and V_{K^0}, we need to retain only the states Σ'^0, Λ', and Y' from our asymptotic $SU(3)$ and the values of the matrix elements of the V_K take the $SU(3)$ values. Then this equation gives the GO mass formulas for the B' nonet which are quadratic, i.e.,

$$2(n'^2 + \Xi'^2) - (\Sigma'^2 + 3\Lambda'^2) - 3(Y'^2 - \Lambda'^2)\sin^2\theta = 0\,. \quad \text{(B1)}$$

Suppose now that the commutator $[\dot{V}_{K^0}, A_{K^0}] = 0$ is also valid, and consider the equation

$$\lim_{|\mathbf{q}|\to\infty} \langle n'(\mathbf{q}) | [\dot{V}_{K^0}, A_{K^0}] | \Xi'^0 \rangle = 0\,.$$

By using the asymptotic $SU(3)$ symmetry, we obtain in the limit $|\mathbf{q}|\to\infty$

$$(\sqrt{\tfrac{1}{2}})(n'^2-\Sigma'^2)G_{\Sigma'^0\Xi'^0K}-(\sqrt{\tfrac{3}{2}})\cos\theta(n'^2-\Lambda'^2)G_{\Lambda'\Xi'^0K}+(\sqrt{\tfrac{3}{2}})\sin\theta(n'^2-Y'^2)G_{Y'\Xi'^0K}+(\sqrt{\tfrac{1}{2}})(\Sigma'^2-\Xi'^2)\times G_{n'\Sigma'^0K}-(\sqrt{\tfrac{3}{2}})\cos\theta(\Lambda'^2-\Xi'^2)G_{n'\Lambda'K}+(\sqrt{\tfrac{3}{2}})\sin\theta(Y'^2-\Xi'^2)G_{n'Y'K}=0.$$

As we did in Sec. IV (but now both the baryons are nonets), we express all the G's in the above equation in terms of the three independent ones, $G_{\Lambda'\Sigma'^+\pi}$, $G_{\Sigma'^0\Sigma'^+\pi}$, and $G_{Y'\Sigma'^+\pi}$. The coefficients of the $G_{\Lambda'\Sigma'^+\pi}$ and $G_{Y'\Sigma'^+\pi}$ turn out to be zero, whereas the coefficient of the $G_{\Sigma'^0\Sigma'^+\pi}$ turns out to be

$$2(n'^2+\Xi'^2)-(\Sigma'^2+3\Lambda'^2)-3(Y'^2-\Lambda'^2)\sin^2\theta.$$

Therefore, unless $G_{\Sigma'^0\Sigma'^+\pi}=0$ (which implies that $g_{\Sigma'^0\Sigma'^+\pi}=0$) we obtain again the GO mass formulas, Eq. (B1). Consistent with this observation, the commutator is, in fact, valid under a rather general assumption of $SU(3)$ breaking. Namely, for example, in a quark model we can admit the following general $SU(3)$ breaking:

$$H'=\alpha S_8(x)+\beta d_{8ij}J_\mu{}^i(x)J_\mu{}^j(x).$$

Here

$$S_8(x)=\bar{q}(x)\lambda_8 q(x),$$

d_{ijk} is the Gell-Mann d symbol, and

$$J_\mu{}^i(x)J_\mu{}^j(x)=V_\mu{}^i(x)V_\mu{}^j(x)+\gamma A_\mu{}^i(x)A_\mu{}^j(x).$$

The coefficients α, β, and γ are arbitrary. We therefore believe that the commutator $[\dot{V}_{K^0},A_{K^0}]=0$ provides an important constraint for the theory. Indeed, we have derived[4] simple *intermultiplet* mass relations from this commutator combined with the use of our asymptotic $SU(3)$ symmetry. These intermultiplet mass formulas comprise the $SU(6)$ mass formulas as special cases. The commutator also gives a constraint for the couplings. This will be discussed in Appendix D.

APPENDIX C: EVALUATION OF $G_{B'BP_i}$

By using PCAC, we obtain

$$G_{B'BP_i}\equiv\lim_{|\mathbf{q}|\to\infty}\langle B'(\mathbf{q}')|A_i|B(\mathbf{q})\rangle \propto\lim_{|\mathbf{q}|\to\infty}(2\pi)^3\delta^3(\mathbf{q}-\mathbf{q}')\frac{F_i}{(q_0'-q_0)}\left(\frac{B'B}{q_0'q_0}\right)^{1/2}\times\bar{u}_{\mu_1\cdots\mu_n}(\mathbf{q}')\ (1\text{ or }\gamma_5)\times u_B(\mathbf{q})p_{\mu_1}\cdots p_{\mu_n}g_{B'BP_i}(P_i{}^2=0).$$

The factor $F_i(q_0'-q_0)^{-1}$ comes from the use of PCAC and $p_\mu=q_\mu'-q_\mu$. We do not bother about the irrelevant factors such as i. We obtain, in the limit,

$$G_{B'BP_i}\propto\lim_{|\mathbf{q}|\to\infty}\left(\frac{2q_0F_i}{B'^2-B^2}\right)\frac{(B'B)^{1/2}}{q_0}\bar{u}_{0\cdots0}(\mathbf{q}')\times[1\text{ or }\gamma_5]u_B(\mathbf{q})(p_0)^n g_{B'BP_i}.$$

Here $(p_0)^n=(B'^2-B^2)^n(2q_0)^{-n}$ in the limit $|\mathbf{q}|\to\infty$. Therefore, the problem reduces to the evaluation of $\lim_{|\mathbf{q}|\to\infty}\bar{u}_{0\cdots0}(\mathbf{q}')[1\text{ or }\gamma_5]u_B(\mathbf{q})$.

We use the technique developed by Fourez[17] to derive a conveinent explicit solution of the Rarita-Schwinger equation. The Rarita-Schwinger wave function satisfies

$$q_{\mu_i}'u_{\mu_i\cdots\mu_i\cdots\mu_n}(\mathbf{q}')=0\quad\text{and}\quad\gamma_{\mu_i}u_{\mu_i\cdots\mu_i\cdots\mu_n}(\mathbf{q}')=0.$$

We now seek a solution of the form

$$u_{\mu_1\cdots\mu_n}(\mathbf{q}')=e_{\mu_1}(\mathbf{q}')\cdots e_{\mu_n}(\mathbf{q}')u_{B'}(\mathbf{q}').$$

Here $u_{B'}(\mathbf{q}')$ is a Dirac spinor with momentum $\mathbf{q}'$. $e_{\mu_i}(\mathbf{q}')$ $(i=1,2,\cdots,n)$ are the identical four-vectors which satisfy $q_\mu'e_\mu(\mathbf{q}')=0$ and $\gamma_\mu e_\mu(\mathbf{q}')\ u_{B'}(\mathbf{q}')=0$. We take a coordinate system $q_\mu'=(|\mathbf{q}|,0,0;iq_0')$, and choose for the normalized spinor $u_{B'}(\mathbf{q}')$ a solution

$$u_{B'}{}^*(\mathbf{q}')=(q_0'+B')^{1/2}(2B')^{-1/2}(1\ 0\ 0\ |\mathbf{q}|(q_0'+B')^{-1}).$$

Then the normalized vector $e_\mu(\mathbf{q}')$ is given by

$$e_\mu(\mathbf{q}')=(\sqrt{\tfrac{1}{2}})(q_0'/B',\ i,\ 0,\ i|\mathbf{q}|/B').$$

Thus

$$\bar{u}_{0\cdots0}(\mathbf{q}')\propto(\sqrt{\tfrac{1}{2}})^n(|\mathbf{q}|/B')^n\bar{u}_{B'}(\mathbf{q}').$$

By choosing

$$u_B{}^*(\mathbf{q})=(q_0+B)^{1/2}(2B)^{-1/2}(1\ 0\ 0\ |\mathbf{q}|(q_0+B)^{-1})$$

we obtain

$$\lim_{|\mathbf{q}|\to\infty}\bar{u}_{B'}(\mathbf{q}')u_B(\mathbf{q})=(2B)^{-1/2}(2B')^{-1/2}(B'+B).$$

By choosing

$$u_B{}^*(\mathbf{q})=(q_0+B)^{1/2}(2B)^{-1/2}(0\ 1\ |\mathbf{q}|(q_0+B)^{-1}\ 0)$$

we get

$$\lim_{|\mathbf{q}|\to\infty}\bar{u}_{B'}(\mathbf{q}')\gamma_5u_B(\mathbf{q})=(2B)^{-1/2}(2B')^{-1/2}(B'-B).$$

Combining these results, we arrive at the expression for the $G_{B'BP_i}$ given in the text.

APPENDIX D: CONTRAINT FOR $B'\to B+P$ COUPLINGS FROM COMMUTATOR $[\dot{V}_{K^0},A_{K^0}]=0$

Suppose B' and B are the nonet and octet baryons, respectively. By considering the equation

$$\lim_{|\mathbf{q}|\to\infty}\langle n'(\mathbf{q})|[\dot{V}_{K^0},A_{K^0}]|\Xi(\mathbf{q})\rangle=0$$

and using asymptotic $SU(3)$ symmetry for the V_K, we obtain a relation

$$(\sqrt{\tfrac{1}{2}})(n'^2-\Sigma'^2)G_{\Sigma'^0\Xi^0K}-(\sqrt{\tfrac{3}{2}})\cos\theta(n'^2-\Lambda'^2)G_{\Lambda'\Xi^0K}+(\sqrt{\tfrac{3}{2}})\sin\theta(n'^2-Y'^2)G_{Y'\Xi^0K}+(\sqrt{\tfrac{1}{2}})(\Sigma^2-\Xi^2)G_{n'\Sigma^0K}-(\sqrt{\tfrac{3}{2}})(\Lambda^2-\Xi^2)G_{n'\Lambda^0K}=0.$$

We express the G's in the above equation in terms of the three independent parameters d, f, and s, defined in Sec. IV. Then we obtain a constraint for the d, f, and s couplings. By using the GO mass formulas which can also be derived from the same commutator $[\dot{V}_{K^0}, A_{K^0}]=0$, we obtain

$$\sqrt{3}d[2(\Sigma'^2+\Lambda'^2-n'^2-\Xi'^2)-(\Sigma^2-\Lambda^2)] +2f[(n'^2-\Xi'^2)-(n^2-\Xi^2)] +2\sqrt{3}\sin\theta s(\Lambda'^2-Y'^2)=0. \quad \text{(D1)}$$

The equation

$$\lim_{|\mathbf{q}|\to\infty} \langle n(\mathbf{q})|[\dot{V}_{K^0}, A_{K^0}]|\Xi'(\mathbf{q})\rangle=0,$$

also leads to the same constraint. If both the B and B' are octets, Eq. (D1) takes a form

$$\sqrt{3}d[(\Sigma'^2-\Lambda'^2)-(\Sigma^2-\Lambda^2)] +2f[(n'^2-\Xi'^2)-(n^2-\Xi^2)]=0. \quad \text{(D2)}$$

As an example consider the case $B'=(\frac{1}{2})^-$ and $B=(\frac{1}{2})^+$; Eq. (D1) gives a constraint for the physical couplings:

$$\sqrt{3}(\Sigma'-\Lambda)^{-1}[2(\Sigma'^2+\Lambda'^2-n'^2-\Xi'^2)-(\Sigma^2-\Lambda^2)]g_{\Sigma'^+\Lambda\pi} +2(\Sigma'-\Sigma)^{-1}[(n'^2-\Xi'^2)-(n^2-\Xi^2)]g_{\Sigma'^+\Sigma^0\pi} +2\sqrt{3}\sin\theta(Y-\Sigma)^{-1}(\Lambda'^2-Y'^2)g_{Y'\Sigma^-\pi}=0. \quad \text{(D3)}$$

Since this constraint is sensitive to the errors in the mass values (because it involves mass differences), it may not be very useful at present.

PHYSICAL REVIEW D VOLUME 2, NUMBER 5 1 SEPTEMBER 1970

Broken Chiral Symmetry and the η Meson*

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(Received 27 March 1970)

The scheme of broken chiral symmetry given by Gell-Mann, Oakes, and Renner is generalized to include the η meson with proper consideration of the octet-singlet mixing. It is shown that at least one of the η and the η' must violate the partially conserved axial-vector current (PCAC) condition (or Adler's condition). This is not simply because they are much heavier than the pion. The analysis of the decays $\eta\to 3\pi$ and $\eta\to 2\gamma$, which are known for large deviations from the simple SU_3 predictions, indicates that PCAC for the η is violated rather severely.

I. INTRODUCTION AND SUMMARY

An investigation has been made by Gell-Mann, Oakes, and Renner[1] of the consequence of the Hamiltonian

$$\mathcal{H}=\mathcal{H}_0+\mathcal{H}', \quad (1.1)$$

where $\mathcal{H}_0$ is invariant under $SU_3\times SU_3$, while $\mathcal{H}'$ takes a simple form

$$\mathcal{H}'=-u_0-cu_8 \quad (1.2)$$

with $c\approx-\sqrt{2}$. This Hamiltonian leads to a very satisfactory understanding of the mesons as far as pions and K mesons are concerned. It turns out, however, that one needs something extra beyond the straightforward extension of the scheme if one tries to include the η meson in the consideration. The argument is given briefly in Sec. II, together with a proposal of a possible way out of the difficulties. Although the same scheme has also been proposed recently by Lee,[2] we exploit its consequence with a different emphasis. A natural and interesting conclusion is that there is not as strong a reason for assuming partial conservation of axial-vector current (PCAC) for the η meson as that for the pion and K meson. By PCAC we mean here that the amplitude in the soft-meson limit can be calculated on the basis of the equal-time commutator involving the corresponding axial-vector charge. In this sense, its violation also leads to the violation of Adler's condition.[2a]

One may argue that this is not surprising simply because the η is much heavier than the pion. This argument is, however, not always correct. Note, for example, that there is so far no reason to prevent PCAC from holding for the K meson which is as heavy as the η.[3] It is also interesting to note that Adler's condition can be satisfied rather naturally for the pion as well as for the K meson in the Veneziano-type meson-meson scattering amplitudes, while the same is not true for the η,[4–7] indicating that PCAC does not hold for the η. The last two sections are devoted to

* Supported in part by the U. S. Atomic Energy Commission.
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