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Dissociation and Stripping of High-Energy Deuterons*

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The dissociation and stripping of high-energy deuterons are studied in a Glauber-model approach. In particular, our treatment of the dissociation problem includes both nuclear and Coulomb forecs. For heavy nuclei, the Coulomb dissociation cross section turns out to be much more important than the nuclear dissociation cross section and comparable in magnitude to the nuclear-stripping cross section. Comparison with experimental cross sections for 3.54-GeV/c deuterons shows satisfactory agreement.

INTRODUCTION

When high-energy deuterons are scattered by a nucleus, a variety of processes is possible. Of these, the stripping reactions constitute a particularly interesting class. In a stripping reaction, one of the nucleons of the deuteron traverses the nucleus without being scattered and reappears as a free particle after the collision. When this particle is the proton, we speak about a proton-stripping reaction and when it is the neutron we speak about a neutron-stripping reaction.

Experimentally, it is difficult to distinguish between a proper stripping reaction and a dissociation reaction. In a proper stripping reaction, one of the nucleons of the deuteron is scattered inelastically, i.e., causes a production of particles or a change of state of the target nucleus. This kind of process was first considered by Serber,[1] who used a black-sphere model for the nucleus. In a dissociation reaction, both the proton and the neutron reappear as free particles, and the state of the target nucleus does not change. This process was first considered by Glauber[2] and then by Feinberg[3] and by Akhieser and Sitenko.[4] These authors also treated the nucleus as a completely absorbing sphere.

In addition to the dissociation caused by the nuclear forces, there is a dissociation caused by the Coulomb field of the nucleus. This type of dissociation has been discussed by Dancoff[5] in the first Born approximation and for a pointlike nucleus. Akhieser and Sitenko[4] tried to extend these considerations to the black-sphere nucleus with a radius much larger than the deuteron radius. Both the dissociation and the stripping problem have also been discussed by Franco.[6]

Thus, since the first qualitative explanations of the stripping and dissociation processes, there have been a number of qualitative and semiquantitative discussions, but realistic estimates are still lacking. In particular, both the nucleus and the deuteron have been treated in an oversimplified manner, the Coulomb form factors have not been properly taken into account, and one has not been able to estimate the Coulomb nuclear interference term.

In this paper we report on a more realistic quantitative calculation of the dissociation and stripping cross sections. The nucleus is described by realistic nuclear density distributions. The basic formula for the stripping cross section is obtained from probability considerations.[6,7] As discussed in some detail, it is likely to be very accurate. The treatment of the dissociation problem is that of Glauber,[8,9] where the deuteron-nucleus amplitude is related to the corresponding proton-nucleus and neutron-nucleus amplitudes. The link is provided by the assumption of additive scattering phases, i.e., the deuteron scattering phase is the sum of the proton and neutron phases. A unified treatment of the Coulomb and nuclear dissociation reactions is obtained by extending this assumption to include also the Coulomb phase.

Up to now, only one relevant high-energy experiment has been performed,[10] and in this experiment the stripping cross section for 3.54-GeV/c deuterons was measured. However, the experiment did not distinguish between dissociation and pure stripping reactions. Our theoretical values for the sum of these reaction cross sections agree with the experimental values, within the rather large experimental errors, and thus set to rest some surmises of a severe disagreement between theory and experiment. The improved situation is essentially due to our more accurate treatment of the Coulomb forces.

I. NUCLEON-NUCLEUS AMPLITUDES

In the model which we shall employ, the deuteron-nucleus amplitude is constructed from the proton-nucleus and neutron-nucleus amplitudes. One is then neglecting the possibility that both proton and neutron can simultaneously interact with the same target

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nucleon. These particular interactions, which are the counterparts of the eclipse terms in proton-deuteron scattering, are naturally taken into account in the multiple scattering theory of Glauber, but as shown in Ref. 7, their contribution is usually quite small and will not be taken into account here.

We shall use a Gaussian form for the nucleon-nucleus amplitude. This approximation greatly simplifies the calculation of the nuclear dissociation cross section and the interference term between Coulomb and nuclear dissociation cross sections. However, in the stripping and pure Coulomb reactions, which make up the main part of the measured cross sections, this approximation is not made and the nucleus will there be described by realistic nucleon density distributions such as the Woods-Saxon distribution.

Neglecting spin-dependent terms, the impact parameter representation of the elastic nucleon-nucleus amplitude reads

$$F(\mathbf{q})=\frac{ik}{2\pi}\int d^2b\, e^{i\mathbf{q}\cdot\mathbf{b}}\Gamma(\mathbf{b})\,, \tag{1.1}$$

$$\Gamma(\mathbf{b})=1-e^{i\chi(\mathbf{b})}\,, \tag{1.2}$$

where $\chi(\mathbf{b})$ is twice the nucleon-nucleus asymptotic phase and $\Gamma(\mathbf{b})$ is the well-known profile function. In the Gaussian approximation we write

$$F(\mathbf{q})=F(0)e^{-a\mathbf{q}^2/2}\,, \tag{1.3}$$

$$\Gamma(\mathbf{b})=\gamma e^{-\mathbf{b}^2/2a}\,. \tag{1.4}$$

The dimensionless quantity γ and the slope parameter a can be determined directly from the measured nucleon-nucleus amplitudes. Because the necessary experimental information is available only at a few momenta, we shall use an alternative approach here and derive them from the better known nucleon-nucleon amplitudes.

In the diffraction scattering theory of Glauber,[9] the phase $\chi(\mathbf{b})$ of (1.2) is given by

$$\chi(\mathbf{b})=i\theta T(\mathbf{b})\,, \tag{1.5}$$

where $T(\mathbf{b})$ is the nuclear target thickness function obtained from the nuclear single-particle density function $\rho(\mathbf{r})$ through

$$T(\mathbf{b})=A\int_{-\infty}^{\infty}dz\,\rho(\mathbf{b}+\mathbf{k}z)\,, \tag{1.6}$$

$\mathbf{k}$ being the momentum of the incident nucleon. In order to define θ we introduce the nucleon-nucleon parameters

$$\theta_{pp}=\tfrac{1}{2}\sigma_{pp}(1-i\alpha_{pp})\,, \tag{1.7a}$$

$$\theta_{pn}=\tfrac{1}{2}\sigma_{pn}(1-i\alpha_{pn})\,, \tag{1.7b}$$

where α is the ratio between the real and imaginary parts of the forward elastic scattering amplitude. For proton-nucleus scattering we then have

$$\theta_p=(Z/A)\theta_{pp}+(N/A)\theta_{pn}\,, \tag{1.8a}$$

and for neutron-nucleus scattering

$$\theta_n=(N/A)\theta_{pp}+(Z/A)\theta_{pn}\,. \tag{1.8b}$$

In an ordinary nucleus, the number of protons is not very different from the number of neutrons, and it is consequently a good approximation to use the mean value

$$\theta=\tfrac{1}{2}(\theta_p+\theta_n)\,. \tag{1.9}$$

This will be done in the following.

From the elastic scattering amplitude (1.1) we can now easily derive the integrated elastic cross section and the total cross section. Introducing the effective nucleon number

$$N_0(\sigma;\alpha)=\frac{1}{\sigma}\int d^2b\left[1-e^{-\sigma(1-i\alpha)T(\mathbf{b})}\right], \tag{1.10}$$

we obtain

$$\sigma_{\mathrm{el}}=\sigma\,\mathrm{Re}N_0(\tfrac{1}{2}\sigma;\alpha)-\sigma N_0(\sigma;0)\,, \tag{1.11}$$

$$\sigma_{\mathrm{tot}}=\sigma\,\mathrm{Re}N_0(\tfrac{1}{2}\sigma;\alpha)\,. \tag{1.12}$$

Adjusting the Gaussian amplitude (1.3) to give these two results, we obtain

$$a=\sigma^2|N_0(\tfrac{1}{2}\sigma;\alpha)|^2/16\pi\sigma_{\mathrm{el}}\,, \tag{1.13}$$

$$\gamma=(4\sigma_{\mathrm{el}}/\sigma)[N_0(\tfrac{1}{2}\sigma;-\alpha)]^{-1}\,. \tag{1.14}$$

For the particular application we have in mind, deuteron dissociation and stripping at 3.54 GeV/c, we are fortunate enough to have available accurate determinations of the nucleon-nucleon amplitudes at 1.78 GeV/c. At this momentum[11]

$$\sigma_{pp}=47.49\text{ mb}\,,\quad \alpha_{pp}=-0.08\,, \tag{1.15a}$$

$$\sigma_{pn}=40.51\text{ mb}\,,\quad \alpha_{pn}=-0.41\,. \tag{1.15b}$$

This gives

$$\theta=-22.00(1+0.232i)\text{ mb}\,. \tag{1.16}$$

The parameters a and γ for this value of θ are given in Table I.

We have retained the imaginary part of γ, i.e., the real part of the nucleon-nucleus amplitude, in all our calculations. The influence of this term turns out to be insignificant. In future calculations, it can therefore be neglected from the outset.

TABLE I. Parameters for the Gaussian nucleon-nucleus amplitude (1.3) and (1.4) for nucleon momentum 1.78 GeV/c, using the mean value (1.16).

Nucleus	a (fm^2)	Reγ	Imγ
Pb	15.12	1.67	0.11
Cu	7.69	1.44	0.15
Al	4.87	1.19	0.16

[11] D. V. Bugg *et al.*, Phys. Rev. **146**, 980 (1966); A. A. Carter and D. V. Bugg, Phys. Letters **20**, 203 (1966).

II. DIFFRACTIVE DISSOCIATION

In a dissociation reaction the deuteron breaks up into a proton and a neutron, and the final deuteron state becomes a proton-neutron scattering state. In Ref. 7 we discussed this process in the framework of Glauber's high-energy diffraction theory. There, we also developed an approximation method which allowed us to calculate the dissociation cross section for a nucleus of arbitrary shape. Here we are working in a slightly different model and with Gaussian profile functions. For this particular case no approximations are necessary, and we shall now outline the calculations.

The matrix element for a transition from the initial deuteron state $|i\rangle = |\varphi_i(\mathbf{r})\rangle$ to a final deuteron state $|f\rangle = |\varphi_f(\mathbf{r})\rangle$ is

$$F_{fi}(\mathbf{q}) = \langle f|F(\mathbf{q},\mathbf{s})|i\rangle, \tag{2.1}$$

with

$$F(\mathbf{q},\mathbf{s}) = \frac{ik}{2\pi}\int d^2b\, e^{i\mathbf{q}\cdot\mathbf{b}}[1-e^{i\chi_d(\mathbf{b},\mathbf{s})}]. \tag{2.2}$$

Here $\mathbf{s}$ is the projection of $\mathbf{r}_p - \mathbf{r}_n$ onto the impact parameter plane and $\mathbf{b}$ is the impact parameter for the deuteron c.m. system. The additivity property of the phases says that χ_d is the sum of the proton-nucleus and neutron-nucleus phases. However, when the deuteron momentum is k, then the proton and the neutron each have a momentum $\frac{1}{2}k$. The additivity property thus reads

$$\chi_d(k;\mathbf{b},\mathbf{s}) = \chi_p(\tfrac{1}{2}k;\mathbf{b}_p) + \chi_n(\tfrac{1}{2}k;\mathbf{b}_n), \tag{2.3}$$

with the nucleon impact parameters

$$\mathbf{b}_p = \mathbf{b} + \tfrac{1}{2}\mathbf{s}, \tag{2.4}$$

$$\mathbf{b}_n = \mathbf{b} - \tfrac{1}{2}\mathbf{s}. \tag{2.5}$$

Defining the deuteron-nucleus profile function as

$$\Gamma_d(\mathbf{b},\mathbf{s}) = 1 - e^{i\chi_d(\mathbf{b},\mathbf{s})}, \tag{2.6}$$

we get from (2.3) the well-known composition law

$$\Gamma_d(\mathbf{b},\mathbf{s}) = \Gamma_p(\mathbf{b}_p) + \Gamma_n(\mathbf{b}_n) - \Gamma_p(\mathbf{b}_p)\Gamma_n(\mathbf{b}_n). \tag{2.7}$$

The indices p and n remind us to evaluate the nucleon-nucleus profile functions at momentum $\frac{1}{2}k$.

The dissociation cross section is obtained by summing over all final neutron-proton scattering states. Since we have only one bound neutron-proton state, this is equivalent to summing over all final neutron-proton states and subtracting the contribution from the bound state. Thus

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{diss}} = \sum_{|f\rangle\neq|i\rangle} |\langle f|F(\mathbf{q},\mathbf{s})|i\rangle^2$$
$$= \sum_{\text{all}|f\rangle} |\langle f|F(\mathbf{q},\mathbf{s})|i\rangle|^2 - |\langle i|F(\mathbf{q},\mathbf{s})|i\rangle|^2. \tag{2.8}$$

For small energy transfers to the deuteron c.m. system, we can, as a good approximation, invoke the closure relation

$$\sum_d \varphi_d^*(\mathbf{r}')\varphi_d(\mathbf{r}) = \delta(\mathbf{r}-\mathbf{r}') \tag{2.9}$$

to simplify this expression. We get

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{diss}} = \langle i|\,|F(\mathbf{q},\mathbf{s})|^2|i\rangle - |\langle i|F(\mathbf{q},\mathbf{s})|i\rangle|^2, \tag{2.10}$$

and from here the integrated cross section

$$\sigma_{\text{diss}} = \frac{1}{k^2}\int d^2q[\langle i|\,|F(\mathbf{q},\mathbf{s})|^2|i\rangle - |\langle i|F(\mathbf{q},\mathbf{s})|i\rangle|^2]. \tag{2.11}$$

Introducing the profile function (2.6) into (2.2) gives

$$\sigma_{\text{diss}} = \int d^2b\left[\int d^3r|\varphi(\mathbf{r})|^2|\Gamma(\mathbf{b},\mathbf{s})|^2 - \left|\int d^3r|\varphi(\mathbf{r})|^2\Gamma(\mathbf{b},\mathbf{s})\right|^2\right]$$
$$\equiv G_1^0 - G_2^0. \tag{2.12}$$

This expression exhibits the dissociation cross section as a difference between two rather large numbers. As we shall see in Sec. III, the Coulomb effects enter only into the second term and can therefore give appreciable corrections.

For the actual evaluation of (2.12) we shall use the Gaussian profile function (1.4) and also a Gaussian deuteron wave function. Because the dissociation cross section depends sensitively on the rms radius of the deuteron, it is essential to use a wave function which gives the correct rms radius rather than the correct binding energy. We shall therefore take

$$\varphi(\mathbf{r}) = (1/\pi R_d^2)^{3/4}e^{-\mathbf{r}^2/2R_d^2}, \tag{2.13a}$$

$$R_d = 3.27\text{ fm}. \tag{2.13b}$$

This value of R_d is obtained by adjusting the rms radius to the value given by the Gartenhaus wave function (see Appendix A).

With these approximations, we get

$$G_1^0 = 2\pi a|\gamma|^2\left[1 + \frac{1}{1+R_d^2/4a} - \tfrac{4}{3}\,\text{Re}\gamma\frac{1}{1+R_d^2/3a} + \tfrac{1}{4}|\gamma|^2\frac{1}{1+R_d^2/2a}\right], \tag{2.14}$$

$$G_2^0 = 4\pi a|\gamma|^2\left[\frac{1}{1+R_d^2/8a} - \tfrac{2}{3}\,\text{Re}\gamma\frac{1}{1+R_d^2/4a}\times\frac{1}{1+R_d^2/12a} + \tfrac{1}{8}|\gamma|^2\frac{1}{(1+R_d^2/4a)^2}\right]. \tag{2.15}$$

With the parameters of Sec. I, we obtain the following values for the dissociation cross section for 3.54-GeV/c deuterons:

$$\sigma_{\text{diss}}(\text{Al})=21.1\text{ mb},\quad \sigma_{\text{diss}}(\text{Cu})=23.9\text{ mb},\quad \sigma_{\text{diss}}(\text{Pb})=23.5\text{ mb}.$$

The model used to obtain these numbers is expected to work very well for heavy nuclei. For light nuclei, small discrepancies may be expected but as the nuclear dissociation cross section is much smaller than the stripping cross section they will not concern us here.

III. COULOMB CONTRIBUTIONS TO DISSOCIATION CROSS SECTION

We now want to discuss the effects of the Coulomb field. The Coulomb dissociation cross section, being essentially proportional to Z^2, will have a mass dependence quite different from the nuclear dissociation cross section. In fact the Coulomb dissociation cross section turns out to be negligible for light nuclei but quite considerable for heavy nuclei. For Pb it is found to be much larger than the nuclear dissociation cross section and comparable in magnitude to the nuclear-stripping cross section.

The inclusion of the Coulomb interaction is, in principle, quite straightforward. The additivity assumption for the asymptotic phases means that we replace (2.3) by

$$\chi_d(k;\mathbf{b},\mathbf{s})=\chi_C(\tfrac{1}{2}k;\mathbf{b}_p)+\chi_p(\tfrac{1}{2}k;\mathbf{b}_p)+\chi_n(\tfrac{1}{2}k;\mathbf{b}_n),\quad (3.1)$$

where χ_C is the phase produced by the Coulomb field of the nucleus. We choose to rewrite the scattering operator (2.2) in the form

$$F(\mathbf{q},\mathbf{s})=\frac{ik}{2\pi}\int d^2b\, e^{i\mathbf{q}\cdot\mathbf{b}}[1-e^{i\chi_C(\mathbf{b}_p)}] +\frac{ik}{2\pi}\int d^2b\, e^{i\mathbf{q}\cdot\mathbf{b}}e^{i\chi_C(\mathbf{b}_p)}[1-e^{i\chi_p(\mathbf{b}_p)+i\chi_n(\mathbf{b}_n)}];\quad (3.2)$$

i.e., we consider the Coulomb field as producing an independent scattering amplitude and in addition a distortion of the nuclear amplitude proper.

When discussing the Coulomb nuclear interference problem for proton-nuclear scattering, the Coulomb phase in the second term of (3.2) is usually replaced by $\chi_C(0)$ and taken outside the integral. The motivation for this approximation is that the partial waves important for nuclear scattering have almost the same Coulomb phase. As the Coulomb phase varies slowly with impact parameter, it is replaced by $\chi_C(0)$. For the scattering problem, this is a very good approximation for light nuclei, and for heavy nuclei the induced error is only a few percent. Nevertheless, for the dissociation problem this difference is important, and such a simplification is not possible.

We now introduce the proton-nucleus Coulomb amplitude

$$f_C(k;\mathbf{q})=\frac{ik}{2\pi}\int d^2b\, e^{i\mathbf{q}\cdot\mathbf{b}}[1-e^{i\chi_C(k;\mathbf{b})}]\quad (3.3)$$

and the nuclear amplitude function

$$F_N(\mathbf{q},\mathbf{s})=\frac{ik}{2\pi}\int d^2b\, e^{i\mathbf{q}\cdot\mathbf{b}}e^{i\chi_C(k/2;\mathbf{b}_p)} \times(1-e^{i\chi_p(k/2;\mathbf{b}_p)+i\chi_n(k/2;\mathbf{b}_n)}).\quad (3.4)$$

We can then rewrite (3.2) as

$$F(\mathbf{q},\mathbf{s})=2f_C(\tfrac{1}{2}k;\mathbf{q})e^{-i\mathbf{q}\cdot\mathbf{s}/2}+F_N(\mathbf{q},\mathbf{s}).\quad (3.5)$$

The dissociation cross section is still given by (2.11)

$$\sigma_{\text{diss}}=\frac{1}{k^2}\int d^2q[\langle i|\,|F(\mathbf{q},\mathbf{s})|^2|i\rangle-|\langle i|F(\mathbf{q},\mathbf{s})|i\rangle|^2] \equiv\sigma_C+\sigma_{CN}+\sigma_N,\quad (3.6)$$

with an obvious notation for the partial cross sections. Introducing the deuteron form factor

$$S(\mathbf{q})=\int d^3r|\varphi(\mathbf{r})|^2e^{i\mathbf{q}\cdot\mathbf{s}},\quad (3.7)$$

we arrive at the following expressions:

$$\sigma_C=\frac{4}{k^2}\int d^2q[1-S^2(\tfrac{1}{2}q)]|f_C(\tfrac{1}{2}k;\mathbf{q})|^2,\quad (3.8)$$

$$\sigma_{CN}=\frac{4}{k^2}\,\mathrm{Re}\int d^2q\, f_C(\tfrac{1}{2}k;\mathbf{q})[\langle i|F_N^*(\mathbf{q},\mathbf{s})e^{-i\mathbf{q}\cdot\mathbf{s}/2}|i\rangle -S(\tfrac{1}{2}q)\langle i|F_N^*(\mathbf{q},\mathbf{s})|i\rangle],\quad (3.9)$$

$$\sigma_N=\frac{1}{k^2}\int d^2q[\langle i|\,|F_N(\mathbf{q},\mathbf{s})|^2|i\rangle -|\langle i|F_N(\mathbf{q},\mathbf{s})|i\rangle|^2].\quad (3.10)$$

This is the general form of the dissociation cross section when both Coulomb and nuclear forces are taken into account. For simple nuclear models, such as the Gaussian and uniform models, a direct numerical evaluation is possible but intricate. When more accurate and systematic data become available, such a calculation will become necessary. At present, the experimental data are scanty and the experimental errors are large. We therefore feel justified in making some approximations in order to facilitate the numerical calculations.

The nuclear dissociation term (3.10), which now depends on the Coulomb phase, will be evaluated without rufther approximations.

The Coulomb dissociation term (3.8) will be evaluated in the Born approximation, i.e., the general expression

(3.3) will be replaced by the expression given by the first Born approximation.

The interference term (3.9) is the most difficult one to evaluate. There we shall use the Born approximation for the absolute value of the Coulomb amplitude f_C, and the phase will be taken as the phase for the corresponding pointlike Coulomb amplitude, which is well known from elementary quantum mechanics. In this way f_C will have the correct phase for small momentum transfers which dominate the contribution to the dissociation cross section. We thus put

$$f_C(\tfrac{1}{2}k;q) = -\frac{Z\alpha}{v_p}k\frac{1}{q^2}\mathfrak{F}(q)\exp\left(2i\eta_0 - 2i\frac{Z\alpha}{v_p}\ln\frac{q}{k}\right), \quad (3.11)$$

where v_p is the velocity of the proton, $\mathfrak{F}(q)$ is the Coulomb nuclear form factor, and

$$e^{2i\eta_0} = \frac{\Gamma(1+iZ\alpha/v_p)}{\Gamma(1-iZ\alpha/v_p)} = \exp\left[2i\arg\Gamma\left(1+i\frac{Z\alpha}{v_p}\right)\right]. \quad (3.12)$$

The Coulomb form factor $\mathfrak{F}(q)$ is discussed in Appendix B. From (3.11), we immediately conclude that the integral (3.8) will diverge at $q=0$. This is due to our neglect of the deuteron binding which introduces a minimum momentum transfer λ_0. It can be determined by studying the dissociation in the first Born approximation which is certainly a very good approximation for small momentum transfers to the deuteron c.m. system. This has been done by several authors,[4–6] and we just quote their result

$$\lambda_0 = \epsilon/v_p, \quad (3.13)$$

where $\epsilon = 2.226$ MeV is the binding energy of the deuteron. The pure Coulomb dissociation cross section now becomes

$$\sigma_C = 8\pi\frac{Z^2\alpha^2}{v_p^2}\int_{\lambda_0}^{\infty} dq\,\frac{1}{q^3}\mathfrak{F}^2(q)[1-S^2(\tfrac{1}{2}q)]. \quad (3.14)$$

For small momentum transfers q we have

$$S(q) \approx 1 - \tfrac{1}{6}\langle \mathbf{r}^2\rangle q^2, \quad (3.15)$$

and because the main contribution to σ_C will come from small q values it is important to use a wave function which gives the correct rms radius for the deuteron. This is achieved by our choice (2.13) for the Gaussian wave functions. Of course, the integral (3.14) is easily evaluated numerically for any deuteron wave function and any Coulomb form factor. The numerical results for various wave functions and form factors are discussed in Sec. V.

We now turn to the nuclear dissociation term σ_N. Making the same division as in (2.12), we get

$$\sigma_N = G_1 - G_2, \quad (3.16)$$

and using (3.4),

$$G_1 = \frac{1}{k^2}\int d^2q\langle i|\,|F_N(\mathbf{q},\mathbf{s})|^2|i\rangle = \int d^3r\,|\varphi(\mathbf{r})|^2\int d^2b\,|\Gamma_d(\mathbf{b},\mathbf{s})|^2, \quad (3.17)$$

$$G_2 = \frac{1}{k^2}\int d^2q\,|\langle i|F_N(\mathbf{q},\mathbf{s})|i\rangle|^2 = \int d^2b\left|\int d^3r\,|\varphi(\mathbf{r})|^2 e^{i\chi_C(k/2;\mathbf{b}_p)}\Gamma_d(\mathbf{b},\mathbf{s})\right|^2, \quad (3.18)$$

where Γ_d is the profile function (2.7) for the nuclear part alone. The important conclusion to be drawn is that G_1 of (3.17) is not affected by the Coulomb interaction. It is exactly the same as G_1^0 of (2.12). However, G_2 does change. Because of the delicate cancellation in (3.16), small changes in G_2 can also produce appreciable changes in σ_N. Because $\Gamma_d(\mathbf{b},\mathbf{s})$ is mainly real, the effect of the Coulomb phase will be to decrease G_2 and thus increase the nuclear dissociation cross section. For Pb, where the effect is most pronounced, the increase in σ_N turns out to be twice as large as σ_N itself. A more detailed numerical discussion is given in Sec. V.

Finally, a few remarks about the Coulomb nuclear interference term σ_{CN}. We note that we need not bother about the lower integration limit in q. In fact since $S(0)=1$, the integrand vanishes for $q=0$ and we can integrate the whole way down to $q=0$, the contribution from q values smaller than λ_0 being extremely small. Furthermore, we found the contribution from the imaginary part of γ, i.e., the real part of the nucleon-nucleus amplitude, to be negligible. The Coulomb nuclear interference term itself is negative and about 25% of the pure Coulomb dissociation cross section σ_C.

IV. COULOMB PHASE

The high-energy diffraction theory can be used to describe the Coulomb scattering in a way very similar to that for the nuclear scattering. The long-range nature of the Coulomb field gives rise to new difficulties, but they can be handled by the usual screening procedure. The details have been worked out by Glauber both for the pointlike source[9] and for the general charge distribution.[12] We shall here recall these results and apply them to a few simple nuclear models.

The Coulomb phase is given by the general expression

$$\chi_C(k_p;\mathbf{b}) = -\frac{Z\alpha}{v_p}\int_{-\infty}^{\infty} dz\,\varphi((\mathbf{b}^2+z^2)^{1/2}), \quad (4.1)$$

$Z\alpha\varphi(\mathbf{r})$ being the Coulomb potential from the nucleus.

[12] R. J. Glauber, in *High-Energy Physics and Nuclear Structure*, edited by S. Devons (Plenum, New York, 1970).

The integral is divergent but this difficulty is circumvented by working with a screened Coulomb field, i.e., introducing

$$\varphi_f(r)=\varphi(r)f(r)\,, \tag{4.2}$$

$$f(r)=1\,,\quad r<a$$
$$=0\,,\quad r>a\,. \tag{4.3}$$

The screening is found to introduce an additional phase independent of **b**, which becomes infinite when a becomes infinite. As shown by Glauber, this additional phase just enters the Coulomb amplitude as a multiplicative phase factor and this will still be the case when the nuclear phase is added to it. The remaining finite part of χ_C, which is independent of a, then gives the desired phase. For a pointlike charge distribution, one obtains

$$\chi_{\mathrm{p.c.}}(k_p;\mathbf{b})=2(Z\alpha/v_p)\ln(k_pb)\,, \tag{4.4}$$

and this agrees with the result obtained from the angular momentum expansion of the Coulomb amplitude.

For the general charge distribution, one starts from the potential

$$\varphi(\mathbf{r})=\int d^2r\frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} \tag{4.5}$$

and arrives at the result

$$\chi_\rho(k_p;b)=4\pi\frac{Z\alpha}{Av_p}\left[\ln(k_pb)\int_0^b T(b')b'db' + \int_b^\infty T(b')\ln(k_pb')\,b'db'\right], \tag{4.6}$$

where the target thickness $T(\mathbf{b})$ is defined as in (1.6). The phase resulting from the screening turns out to be the same as for the pointlike charge distribution. We also note that there are other representations equivalent to (4.6), but we have found the form (4.6) to be the most convenient one.

An alternative way to derive the result (4.6) is to note that the phase $\chi_\rho(\mathbf{b})$ satisfies a two-dimensional Laplace equation. From (4.1), we obtain

$$\nabla_b{}^2\chi_\rho(\mathbf{b})=(Z\alpha/Av_p)T(\mathbf{b})\,, \tag{4.7}$$

which has a well-known solution.

We shall now use the representation (4.6) to calculate the phase for two simple nuclear models, the uniform and the Gaussian nuclear density distribution models. For these models, the phase can be expressed in elementary functions and this is very convenient in the numerical treatment. For details of the models we refer to Appendix B.

In the uniform nuclear model we have a target thickness function

$$T(\mathbf{b})=A(6/4\pi R_u{}^3)(R_u{}^2-b^2)^{1/2}\theta(R_u-b)\,, \tag{4.8}$$

and from (4.6) we obtain the phase

$$\chi_u(k_p;b)=2(Z\alpha/v_p)\{\theta(b-R_u)\ln(k_pb)+\theta(R_u-b)\times[\ln(k_pR_u)+\ln(1+(1-b^2/R_u{}^2)^{1/2}) -(1-b^2/R_u{}^2)^{1/2}-\tfrac{1}{3}(1-b^2/R_u{}^2)^{3/2}]\}\,. \tag{4.9}$$

This expression is finite for $b=0$, contrary to the phase obtained from a point charge (4.4).

In the Gaussian nuclear model we have a target thickness function

$$T(\mathbf{b})=(A/\pi R_G{}^2)e^{-\mathbf{b}^2/R_G{}^2}\,, \tag{4.10}$$

and this yields the Coulomb phase

$$\chi_G(k_p;b)=2(Z\alpha/v_p)[\ln(k_pb)+\tfrac{1}{2}E_1(b^2/R_G{}^2)]\,, \tag{4.11}$$

where the exponential integral $E_1(x)$ is defined by

$$E_1(x)=\int_x^\infty\frac{e^{-t}}{t}dt. \tag{4.12}$$

Also, χ_G is finite for $b=0$ since for small arguments the exponential integral has the expansion

$$E_1(x)=-\ln x-C+ax+\cdots\,, \tag{4.13}$$

C being Euler's constant.

For more general charge distributions, i.e., the Woods-Saxon distribution, the particular form (4.6) is less convenient since then already the target thickness $T(b)$ involves an integration. But starting from (4.6), or any equivalent expression, one easily derives

$$\chi_\rho(k_p;b)=2\frac{Z\alpha}{v_p}\left\{\ln(k_pb)+4\pi\int_b^\infty\rho(r)\left[-\left(1-\frac{b^2}{r^2}\right)^{1/2}+\ln\left(\frac{r}{b}+\left(\frac{r^2}{b^2}-1\right)^{1/2}\right)\right]r^2dr\right\}. \tag{4.14}$$

V. NUMERICAL RESULTS FOR DISSOCIATION CROSS SECTION

We shall first discuss our results for the pure Coulomb dissociation cross section. We then observe that for the Gaussian deuteron wave function (2.13) and the Gaussian form factor (B9) a closed expression for the Coulomb dissociation cross section can be obtained:

$$\sigma_C=R_d{}^2\frac{\pi}{2}\frac{Z^2\alpha^2}{v_p{}^2}\left\{\frac{8}{R_d{}^2\lambda_0{}^2}(1-e^{-\lambda_0{}^2R_d{}^2/8})e^{-\lambda_0{}^2R_G{}^2/2} -4\frac{R_G{}^2}{R_d{}^2}E_1\left(\frac{\lambda_0{}^2R_G{}^2}{2}\right)+\left(1+\frac{4R_G{}^2}{R_d{}^2}\right)\times E_1\left[\lambda_0{}^2\left(\frac{R_d{}^2}{2}+\frac{R_G{}^2}{8}\right)\right]\right\}. \tag{5.1}$$

The exponential integral $E_1(x)$ is defined as in (4.12).

TABLE II. The Coulomb dissociation cross section σ_C (mb) for Woods-Saxon form factor and various deuteron wave functions.

Wave function	Gaussian	Hulthén	Gartenhaus
Al	12.9	12.7	12.8
Cu	60.6	59.9	60.1
Pb	437.6	433.9	435.4

TABLE III. The Coulomb dissociation cross section σ_C (mb) for Gaussian deuteron wave function and various nuclear form factors.

Form factor	None	Gaussian	Uniform	Woods-Saxon
Al	17.2	13.2	13.1	12.9
Cu	85.3	61.9	61.4	60.6
Pb	682.3	447.4	442.4	437.6

The result for no form factor, i.e., $\mathfrak{F}(q)=1$, is

$$\sigma_C=R_d^2\frac{\pi}{2}\frac{Z^2\alpha^2}{v_p^2}\left[\frac{8}{R_d^2\lambda_0^2}(1-e^{-\lambda_0^2R_d^2/8})+E_1\left(\frac{\lambda_0^2R_d^2}{8}\right)\right]. \tag{5.2}$$

As can be seen from Table III (below), it is quite important to include the Coulomb form factor. Formula (5.2), which neglects the form factor, gives quite a misleading result.

In order to see in detail the effects of different wave functions and different form factors we give two tables. In Table II we give the dissociation cross section for the Woods-Saxon form factor and various deuteron wave functions. The conclusion drawn from these numbers is that once we have adjusted the Gaussian deuteron wave function to give the correct rms radius it also gives a satisfactory result for the Coulomb dissociation cross section. In Table III we give the results for the Gaussian deuteron wave function and various nuclear form factors. Now we conclude that with the present experimental accuracy the choice of the nuclear form factor does not matter very much.

We next discuss the nuclear cross section σ_N. In the presence of the Coulomb field we must use formulas (3.16)–(3.18). The integrals are now so complicated that we have to evaluate them for the Gaussian deuteron wave function. Furthermore, the nuclear dissociation cross section is very sensitive to the rms radius of the deuteron and it is necessary to use our parametrization (2.13). This was found to give excellent results for the Coulomb dissociation cross section and we think this is also the case for the nuclear dissociation cross section. We thus have $\sigma_N=G_1-G_2$, with G_2 given by

$$\begin{aligned}G_2=8\pi a|\gamma|^2\frac{1}{\zeta}\int_0^\infty sds\int_0^\infty s'ds'\\ \times\exp[i\chi_C(\tfrac{1}{2}b;s(2a)^{1/2})-i\chi_C(\tfrac{1}{2}k;s'(2a)^{1/2})]\\ \times\exp[-\zeta^{-1}(s^2+s'^2)]\left\{I_0\left(\frac{2ss'}{\zeta}\right)\right.\\ \times\exp[-(s^2+s'^2)]A+I_0\left(\frac{2ss'}{\zeta}\frac{(1+\zeta)^2}{1+2\zeta}\right)\frac{1}{1+2\zeta}\\ \left.\times\exp\left[-\frac{1+\zeta}{1+2\zeta}(s^2+s'^2)\right]B\right\},\end{aligned} \tag{5.3a}$$

$$\begin{aligned}A=1+2\frac{1}{1+\zeta}\exp\left(s^2-\frac{s'^2}{1+\zeta}\right)\\ -2\gamma^*\frac{1}{1+\zeta}\exp\left(-\frac{s'^2}{1+\zeta}\right),\end{aligned} \tag{5.3b}$$

$$B=1-2\gamma^*\exp(-s^2)+|\gamma|^2\exp[-(s^2+s'^2)], \tag{5.3c}$$

$$\zeta=R_d^2/4a. \tag{5.3d}$$

Here $I_0(Z)$ is the modified Bessel function of order zero. In Table IV we have compared the results obtained from this formula with the results for no Coulomb field. It is immediately seen that for heavy nuclei the Coulomb field produces a considerable enhancement of the nuclear dissociation cross section. For medium heavy nuclei the effect is much smaller and for light nuclei it is very small. This is in agreement with our previous findings that the Coulomb effects are most important for heavy nuclei.

We finally discuss the Coulomb-nuclear interference term σ_{CN} of (3.9). Also here we shall use the Gaussian deuteron wave function (2.13) to simplify the numerical integrations. After some algebraic simplifications we can write (3.9) as

$$\begin{aligned}\sigma_{CN}=8\pi(Z\alpha/v_p)\operatorname{Im}e^{-2i\eta_0}[\gamma X\\ +(1+R_d^2/2a)^{-1}\gamma Y],\end{aligned} \tag{5.4a}$$

$$\begin{aligned}X=\int_0^\infty bdb\, e^{i\chi_C(k/2;b)}e^{-b^2/2a}\\ \times\int_0^\infty\frac{dq}{q}J_0(qb)(1-e^{-q^2R_d^2/8})\\ \times\exp\left(i\frac{2Z\alpha}{v_p}\ln\frac{q}{k}\right)\mathfrak{F}(q),\end{aligned} \tag{5.4b}$$

$$\begin{aligned}Y=\int_0^\infty bdb\, e^{i\chi_C(k/2;b)}\exp\left(-\frac{b^2}{2a}\frac{1}{1+R_d^2/2a}\right)\\ \times(1-\gamma e^{-b^2/2a})\int_0^\infty\frac{dq}{q}\left[J_0(qb)-J_0\left(qb\frac{1+R_d^2/4a}{1+R_d^2/2a}\right)\right.\\ \left.\times\exp\left(-\frac{q^2R_d^2}{8}\frac{1+R_d^2/4a}{1+R_d^2/2a}\right)\right]\\ \times\exp\left(i\frac{2Z\alpha}{v_p}\ln\frac{q}{k}\right)\mathfrak{F}(q),\end{aligned} \tag{5.4c}$$

TABLE IV. The nuclear dissociation cross section σ_N (mb) for various Coulomb phase functions.

Coul. phase function	None	Uniform	Gaussian	Woods-Saxon
Al	21.1	21.9	22.0	21.8
Cu	23.9	29.6	30.6	29.3
Pb	23.5	77.6	86.7	74.9

TABLE V. The Coulomb-nuclear interference term σ_{CN} (mb) for various Coulomb phase functions.

Coul. phase function	Uniform	Gaussian	Woods-Saxon
Al	−2.8	−3.1	−2.6
Cu	−16.7	−18.2	−15.8
Pb	−109.5	−122.8	−104.6

where η_0 is defined as in (3.12). In Table V we have collected our results for the Coulomb phase functions under consideration. The general features are the same as before. We have a large interference term for heavy nuclei and small interference terms for medium heavy and light nuclei. Again the fluctuations from one phase function to another are quite small.

Let us now sum up the different contributions to the dissociation cross section. In order to have some consistency, we give the Coulomb dissociation cross section for the Gaussian deuteron wave function and for the partial contributions we take the number obtained with the Woods-Saxon nuclear density distribution. We get

$$\sigma_{\mathrm{diss}}(\mathrm{Pb})=407.9\text{ mb}\,,\quad \sigma_{\mathrm{diss}}(\mathrm{Cu})=74.1\text{ mb}\,,$$
$$\sigma_{\mathrm{diss}}(\mathrm{Al})=32.1\text{ mb}\,.$$

The results for the Gaussian and uniform density distributions are not very different.

VI. NUCLEAR-STRIPPING REACTIONS

In a proton-stripping reaction, the proton of the deuteron does not undergo any collision, whereas the neutron undergoes an inelastic collision. Also, the role of proton and neutron can be reversed and then we have a neutron-stripping reaction. The cross sections for these stripping reactions are most easily obtained from probability considerations as done in Refs. 6 and 7. For completeness we shall here repeat the argument and finally give two tables of generalized nucleon numbers so that the stripping cross section could be estimated also for other situations. We also remark that the stripping cross section can be defined without reference to probability arguments; this has been done by Franco and Glauber.[13]

The possibility that the proton should pass through the nucleus without being scattered is $e^{-\sigma_p T(\mathbf{b}_p)}$, $\mathbf{b}_p=\mathbf{b}+\frac{1}{2}\mathbf{s}$ being the proton impact parameter. In the same way the probability that the neutron should suffer a collision is $1-e^{-\sigma_n T(\mathbf{b}_n)}$, $\mathbf{b}_n=\mathbf{b}-\frac{1}{2}\mathbf{s}$ being the neutron impact parameter. Here σ_p and σ_n refer to the average proton-nucleon and neutron-nucleon cross sections. The probability for stripping is the product of these probabilities. Properly weighting this probability over the deuteron wave function gives the stripping cross section

$$\sigma_{p,\mathrm{strip}}=\int d^2b\int d^3r|\varphi(\mathbf{r})|^2e^{-\sigma_pT(\mathbf{b}_p)}(1-e^{-\sigma_nT(\mathbf{b}_n)})\,. \quad (6.1)$$

[13] R. J. Glauber (private communication).

This argument also gives the double stripping cross section, where both proton and neutron collide inelastically. We get

$$\sigma_{pn,\mathrm{strip}}=\int d^2b\int d^3r|\varphi(\mathbf{r})|^2(1-e^{-\sigma_nT(\mathbf{b}_n)})\times(1-e^{-\sigma_pT(\mathbf{b}_p)})\,. \quad (6.2)$$

Adding up the stripping cross sections, we get the total reaction cross section

$$\sigma_{p,\mathrm{strip}}+\sigma_{n,\mathrm{strip}}+\sigma_{pn,\mathrm{strip}}=\int d^2b\int d^3r|\varphi(\mathbf{r})|^2(1-e^{-\sigma_pT(\mathbf{b}_p)-\sigma_nT(\mathbf{b}_n)})\,. \quad (6.3)$$

In expressions (6.1)–(6.3) we use the total nucleon-nucleon cross section rather than the elastic one. Therefore, one might be inclined to believe that (6.1) contains part of the dissociation cross section. That this is not the case can be understood in the following way.

The sum of the elastic and dissociation cross sections is given by the first term of the Eq. (2.11). For purely imaginary nucleon-nucleon amplitudes, this term can be rewritten to give

$$\sigma_{\mathrm{diss}}+\sigma_{\mathrm{el}}=\int d^2b\int d^3r|\varphi(\mathbf{r})|^2\times(1-e^{-\frac{1}{2}\sigma_pT(\mathbf{b}_p)-\frac{1}{2}\sigma_nT(\mathbf{b}_n)})^2\,. \quad (6.4)$$

The total cross section can be calculated via the optical theorem; Eq. (2.1), specialized to elastic scattering, yields

$$\sigma_{\mathrm{tot}}=2\int d^2b\int d^3r|\varphi(\mathbf{r})|^2(1-e^{-\frac{1}{2}\sigma_pT(\mathbf{b}_p)-\frac{1}{2}\sigma_nT(\mathbf{b}_n)})\,. \quad (6.5)$$

We also have the obvious relation

$$\sigma_{\mathrm{tot}}=\sigma_{\mathrm{el}}+\sigma_{\mathrm{diss}}+\sigma_{p,\mathrm{strip}}+\sigma_{n,\mathrm{strip}}+\sigma_{pn,\mathrm{strip}}\,. \quad (6.6)$$

We now remark that Eqs. (6.4) and (6.5) are completely general relations and do not rely on probability considerations. But subtracting (6.4) from (6.5) we obtain (6.3). This supports the correctness of our probability arguments and the correctness of our assumption that the stripping cross section as defined through (6.1) does not, in fact, contain any part of the dissociation cross section.

For the numerical evaluation of (6.1), we shall use the Gaussian deuteron wave function (2.13). Although a direct evaluation is straightforward, we would first like to rewrite the result in a form which is more closely related to the generalized nucleon numbers introduced in Ref. 7. Adding and subtracting suitable terms, we can write

$$\sigma_{p,\mathrm{strip}}=\sigma_n N_0(\sigma_n)-(\sigma_p+\sigma_n)\delta N_0(\sigma_p,\sigma_n)\,, \quad (6.7)$$

where the nucleon numbers N_0 and δN_0 are defined by

$$N_0(\sigma)=\frac{1}{\sigma}\int d^2b[1-e^{-\sigma T(\mathbf{b})}]$$

$$\delta N_0(\sigma_p,\sigma_n)=\frac{1}{\sigma_p+\sigma_n}\int d^2b\int d^2s|\varphi(\mathbf{s})|^2\{1-e^{-\sigma_p T(\mathbf{b}_p)}\}\times\{1-e^{-\sigma_n T(\mathbf{b}_n)}\}$$

$$=\frac{4\pi}{(\sigma_p+\sigma_n)R_d^2}\int_0^\infty b_+db_+\int_0^\infty b_-db_-\times I_0\left(\frac{2b_+b_-}{R_d^2}\right)\exp\left(-\frac{b_+^2+b_-^2}{R_d^2}\right)\times(1-e^{-\sigma_p T(b+)})(1-e^{-\sigma_n T(b-)})\,. \quad (6.8)$$

Only δN_0 depends on the particular choice of deuteron wave function. For neutron-stripping reactions, we obtain

$$\sigma_{n,\mathrm{strip}}=\sigma_p N_0(\sigma_p)-(\sigma_p+\sigma_n)\delta N_0(\sigma_p,\sigma_n)\,. \quad (6.9)$$

At high energies the cross sections σ_p and σ_n are close to their mean value σ. Put $\sigma_p=\sigma+\delta\sigma$ and $\sigma_n=\sigma-\delta\sigma$. Expanding the integrand of (6.8) in powers of $\delta\sigma$, we conclude that the difference between $\delta N_0(\sigma_p,\sigma_n)$ and $\delta N_0(\sigma,\sigma)$ is so small that it is an unnecessary luxury to keep the dependence on two variables in δN_0. With the aid of Tables VI and VII we can therefore get a quite accurate estimate of the stripping cross section for any desired situation.

For our particular case, deuteron stripping at 3.54 GeV/c, only the proton stripping cross section has been observed. We have evaluated (6.7), keeping the two-parameter dependence and using cross sections σ_p and σ_n calculated from (1.9) and (1.7) and the Woods-Saxon form (B10) for the nuclear density distribution.

TABLE VI. Effective nucleon number $N_0(\sigma)$.

σ (mb)	30	35	40	45	50	55
^{16}O	9.71	9.09	8.54	8.06	7.63	7.24
^{27}Al	14.25	13.16	12.23	11.42	10.72	10.10
^{40}Ca	18.82	17.22	15.87	14.72	13.73	12.88
^{64}Cu	26.00	23.54	21.51	19.81	18.37	17.14
^{108}Ag	36.92	33.08	29.98	27.44	25.31	23.50
^{140}Ce	43.81	39.08	35.30	32.22	29.65	27.48
^{208}Pb	56.74	50.32	45.24	41.13	37.74	34.89

TABLE VII. Effective nucleon number $\delta N_0(\sigma)=\delta N_0(\sigma,\sigma)$.

σ (mb)	30	35	40	45	50	55
^{16}O	1.71	1.72	1.71	1.68	1.65	1.61
^{27}Al	3.13	3.05	2.96	2.85	2.74	2.63
^{40}Ca	4.74	4.52	4.30	4.08	3.88	3.69
^{64}Cu	7.45	6.96	6.50	6.09	5.72	5.39
^{108}Ag	11.85	10.84	9.97	9.22	8.58	8.02
^{140}Ce	14.72	13.35	12.20	11.23	10.41	9.70
^{208}Pb	20.23	18.16	16.47	15.07	13.89	12.90

The results we obtain are

$$\sigma_{p,\mathrm{strip}}(\mathrm{Pb})=500.5\text{ mb}\,,\quad \sigma_{p,\mathrm{strip}}(\mathrm{Cu})=345.1\text{ mb}\,,$$
$$\sigma_{p,\mathrm{strip}}(\mathrm{Al})=257.5\text{ mb}\,.$$

Let us like to close with a few remarks about these numbers. First, the probability argument used to derive (6.1) neglects the imaginary parts of θ_n and θ_p, as defined through (1.9). The author thinks that there is strong evidence for believing that they only affect the first term of (6.1). The possible generalizations of this term suggest that their contributions to the stripping cross section are at most a few percent. Secondly, (6.1) negects the eclipse terms discussed in Ref. 7. These give a small negative contribution which is about 2%. Both corrections are thus far smaller than the present experimental errors.

CONCLUSIONS

In the experiment[10] which measured the stripping cross section for 3.54-GeV/c deuterons, no distinction was made between pure stripping reactions and dissociation reactions. Before comparing with the experimental yields, we must therefore add these reaction cross sections together. This gives

$$\sigma_{\mathrm{theor}}(\mathrm{Pb})=910\text{ mb}\,,\quad \sigma_{\mathrm{expt}}(\mathrm{Pb})=(950\pm240)\text{ mb}\,,$$
$$\sigma_{\mathrm{theor}}(\mathrm{Cu})=420\text{ mb}\,,\quad \sigma_{\mathrm{expt}}(\mathrm{Cu})=(550\pm110)\text{ mb}\,,$$
$$\sigma_{\mathrm{theor}}(\mathrm{Al})=310\text{ mb}\,,\quad \sigma_{\mathrm{expt}}(\mathrm{Al})=(290\pm70)\text{ mb}\,.$$

As can be seen, the agreement between theory and experiment is satisfactory. Of course, because of the large experimental errors this is not a very detailed check on the theory. In particular, we might expect small changes in the theoretical cross sections when the microscopic approach[7] to the deuteron-nucleus collision is used instead of the one used here. However, it is clear that the proper inclusion of the Coulomb forces removes the previous drastic disagreement between theory and experiment. In view of this improved situation, it would be interesting to have more accurate measurements and, if possible, also separate determinations of the pure stripping and dissociation cross sections.

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APPENDIX A: DEUTERON FORM FACTORS

We give the explicit expressions for the form factor

$$S(\mathbf{q})=\int d^3x\, e^{i\mathbf{q}\cdot\mathbf{x}}|\varphi_d(\mathbf{x})|^2 \tag{A1}$$

for the wave functions considered in the text.

For the *Gaussian* deuteron wave function:

$$\varphi(\mathbf{x})=(\pi R_d^2)^{-3/4}e^{-\mathbf{x}^2/2R_d^2}, \tag{A2}$$

$$\langle \mathbf{r}^2\rangle=\tfrac{3}{2}R_d^2, \tag{A3}$$

$$S(\mathbf{q})=e^{-\mathbf{q}^2R_d^2/4}. \tag{A4}$$

For the *Hulthén* deuteron wave function:

$$\varphi(\mathbf{r})=(N/r)(e^{-\alpha r}-e^{-\beta r}), \tag{A5}$$

$$N=\frac{\alpha+\beta}{\alpha-\beta}\left[\frac{\alpha\beta}{2\pi(\alpha+\beta)}\right]^{1/2}, \tag{A6}$$

with $\alpha=0.232$ fm^{-1} and $\beta=1.202$ fm^{-1};

$$\langle \mathbf{r}^2\rangle=15.99\ \mathrm{fm}^2, \tag{A7}$$

$$S(q)=\frac{1}{qX}\arctan\frac{qX}{1+q^2Y+q^4Z}, \tag{A8a}$$

$$X=(\alpha-\beta)^2/2\alpha\beta(\alpha+\beta), \tag{A8b}$$

$$Y=[4\alpha\beta(\alpha+\beta)^2]^{-1}[2(\alpha+\beta)^2+(\alpha-\beta)^2], \tag{A8c}$$

$$Z=[4\alpha\beta(\alpha+\beta)^2]^{-1}. \tag{A8d}$$

For the *Gartenhaus* deuteron wave function:

$$\varphi(\mathbf{x})=\frac{N}{r}\sum_{i=1}^{4}(e^{-\alpha_i r}-e^{-\beta_i r}), \tag{A9}$$

$$\alpha_1=0.232,\quad \alpha_2=3.49,\quad \alpha_3=4.322,\quad \alpha_4=4.40,$$
$$\beta_1=1.822,\quad \beta_2=1.90,\quad \beta_3=2.732,\quad \beta_4=5.99,$$

$$\langle \mathbf{r}^2\rangle=16.04\ \mathrm{fm}^2, \tag{A10}$$

$$S(q)=\frac{C}{q}\sum_{i,j=1}^{4}\arctan\frac{qX_{ij}}{1+q^2Y_{ij}+q^4Z_{ij}}, \tag{A11a}$$

$$X_{ij}=X_{ji}=\frac{1}{\alpha_i+\alpha_j}+\frac{1}{\beta_i+\beta_j}-\frac{1}{\alpha_i+\beta_j}-\frac{1}{\alpha_j+\beta_i}, \tag{A11b}$$

$$Z_{ij}=Z_{ji}=[(\alpha_i+\alpha_j)(\alpha_i+\beta_j)(\alpha_j+\beta_i)(\beta_i+\beta_j)]^{-1}, \tag{A11c}$$

$$Y_{ij}=Y_{ji}=Z_{ij}\tfrac{1}{2}[(\alpha_i+\alpha_j)^2+(\beta_i+\beta_j)^2+(\alpha_j+\beta_i)^2+(\alpha_i+\beta_j)^2], \tag{A11d}$$

$$\frac{1}{C}=\sum_{i,j=1}^{4}X_{ij}. \tag{A11e}$$

APPENDIX B: NUCLEAR MODELS AND FORM FACTORS

The Coulomb nuclear form factor is the Fourier transform of the nucleon density distribution

$$\mathfrak{F}(r)=\frac{4\pi}{q}\int_0^\infty rdr\,\rho(r)\sin(qr), \tag{B1}$$

$$\mathfrak{F}(0)=1. \tag{B2}$$

(1) In the *uniform* model the nuclear density distribution is given by

$$\rho_u(r)=(3/4\pi R_u^3)\theta(R_u-r), \tag{B3}$$

where the radius R_u is chosen so that the correct rms radius is obtained. Therefore,

$$R_u=(5/3)^{1/2}R_{\mathrm{rms}}. \tag{B4}$$

From electron scattering data, one finds

$$R_u=1.120A^{1/3}+2.009A^{-1/3}-1.513A^{-1}\ (\mathrm{fm}). \tag{B5}$$

The form factor is given by

$$\mathfrak{F}_u(q)=[3/(qR_u)^3][\sin(qR_u)-qR_u\cos(qR_u)]. \tag{B6}$$

(2) In the *Gaussian* nuclear density distribution,

$$\rho_G(r)=(\pi^{3/2}R_G^3)^{-1}e^{-r^2/R_G^2}, \tag{B7}$$

which gives the correct rms radius when

$$R_G=(\sqrt{\tfrac{2}{3}})R_{\mathrm{rms}}. \tag{B8}$$

Also, the form factor has a Gaussian shape:

$$\mathfrak{F}_G(q)=e^{-\mathbf{q}^2R_G^2/4}. \tag{B9}$$

(3) The *Woods-Saxon* density distribution is given by

$$\rho(r)=\rho_0/\{1+\exp[(r-c)/d]\}. \tag{B10}$$

We adopt the parameters

$$c=1.14A^{1/3}\ \mathrm{fm},\quad d=0.545\ \mathrm{fm}. \tag{B11}$$

In this model, the form factor (B1) cannot be simplified any further.