and

$$A_{I_s=2}\sim\beta\Gamma(1-\alpha(l))(s/s_0)^{\alpha(l)}.$$

Notice that the Regge form of $A_{I_s=2}$ is real, a result which, by duality, is necessary if the $I_s=2$ channel is to have no resonances. On the other hand, when we square the amplitudes to get the isospin cross sections, we will obviously get equal $I_s=1$ and $I_s=2$ cross sections. Comparing with our estimated cross sections, Fig. 4(a), we see that we must expect difficulties.

Neglecting this forewarning and continuing, we find, for the differential cross sections needed in (6.2),

$$\frac{d\sigma_{I_s=0}{}^{\rm el}}{d\tau}=\tfrac{1}{2}\pi\beta^2\Gamma^2(1-\alpha(\tau))\left(\frac{s}{s_0}\right)^{2\alpha(\tau)-2}\times\{3\sin^2[\tfrac{1}{2}\pi\alpha(\tau)]+1\},$$

$$\frac{d\sigma_{I_s=1}{}^{\rm el}}{d\tau}=\frac{d\sigma_{I_s=2}{}^{\rm el}}{d\tau}=\tfrac{1}{2}\pi\beta^2\Gamma^2(1-\alpha(\tau))\left(\frac{s}{s_0}\right)^{2\alpha(\tau)-2}.$$

To obtain the C^I's, Eq. (2.3a′), we cross to the t channel before doing the τ integration and find

$$\begin{pmatrix}C^0\\C^1\\C^2\end{pmatrix}=\tfrac{1}{2}\pi\beta^2\frac{\lambda(s,m_\pi{}^2,m_\pi{}^2)}{16\pi^3}\int d\tau\,\Gamma^2(1-\alpha(\tau))\left(\frac{s}{s_0}\right)^{2\alpha(\tau)-2}\times\begin{pmatrix}3+\sin^2[\frac{1}{2}\pi\alpha(\tau)]\\ \sin^2[\frac{1}{2}\pi\alpha(\tau)]\\ \sin^2[\frac{1}{2}\pi\alpha(\tau)]\end{pmatrix}.$$

Since $\alpha(0)\approx\frac{1}{2}$, the values of the integrands at $\tau=0$ (at the forward peak) are in the ratio 7:1:1 for $I=0$, 1, and 2, respectively. Taking the different widths of the forward peaks into account, we see that C^0, C^1, and C^2 are in the ratio r:1:1 with $r>7$. This implies that the low-energy contributions to the kernel—the parts with no singularity for $\lambda>0$—also stand in the ratio r:1:1. These ratios can be compared with those calculated directly from the low-energy resonances, where we found the R_I to stand in the ratio 0.79:0.31:0.01.

We see that the naive use of duality leads to the disastrous prediction that output $I=1$ and $I=2$ poles occur at the same value of λ. It also will probably lead to an excessively large separation between the output $I=0$ and $I=1$ poles.

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Induction of Quarklike Structure of Baryons*

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A set of self-consistency conditions for the interactions of a hypothetical set of even- and odd-parity mesons and baryons is studied. These conditions, previously derived, are written in a form that makes some of their implications transparent. Two types of $SU(n)$-invariant solutions are found; in only one of these do baryon exchanges participate in the bootstrapping of the baryons. The conditions imply that the meson-baryon-baryon interactions are proportional to matrix elements of the group generators in the space of a single quark, yet the identification of baryons with simple quarks does not lead to a solution. Thus, the baryons may be regarded as composites containing quarks. An approximate solution based on the $SU(6)$ group may correspond to experiment better than any exact solution.

I. INTRODUCTION

Two years ago, the author used fixed-angle dispersion relations and a simple dynamical assumption to derive bootstrap consistency conditions on the ratios of the trilinear coupling constants of a hypothetical set of mesons and baryons of both parities.[1,2] Recently, a slight modification of this set of consistency conditions was obtained from an idealized Veneziano model.[3] In this latter derivation, the "particles" are the lightest particles on Regge trajectories. In Ref. 3 (R3), an $SU(3)$-symmetric solution to these conditions was found, involving singlets, octets, and a decuplet.

In the present paper, the bootstrap conditions of R3 are written in a more elegant form. This enables us to generalize the solution of R3 to the symmetry group $SU(n)$. The new form makes it clear (as shown in Sec. III) that any solution must have many of the features of the quark model. It may be that dynamical consistency conditions are the reason that the quark model works.

The most physical solution corresponds to the group $SU(6)$, and is discussed in Sec. IV. The baryon trajectories correspond to the $SU(6)$ multiplets and parities **56**$^+$, **70**$^+$, **70**$^-$, and **20**$^-$. An approximate solu-

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[1] R. H. Capps, Phys. Rev. **168**, 1731 (1968). This paper is referred to as R1.
[2] R. H. Capps, Phys. Rev. **171**, 1591 (1968). This paper is referred to as R2.
[3] R. H. Capps, Phys. Rev. D **1**, 2395 (1970). This paper is referred to as R3.

tion, which involves only the baryon multiplets **56**$^+$ and **70**$^-$, appears to correspond to experiment better than the exact solution.

II. CONSISTENCY CONDITIONS

A. Conditions from R1, R2, and R3

We consider a meson-baryon or meson-meson scattering amplitude, represented in the s, t, and u channels by

$$\begin{aligned} s:&\quad a+b\rightarrow c+d\,,\\ t:&\quad a+c\rightarrow \bar{b}+d\,,\\ u:&\quad c+b\rightarrow a+d\,. \end{aligned} \tag{1}$$

The symbols a and c denote meson states of even parity, while b and d denote either two baryon states of even parity or two meson states of even parity. The bar denotes an antiparticle. For convenience, we take the meson states a and c to be self-conjugate, i.e., $\bar{a}=a$, $\bar{c}=c$.

We consider first the case in which b and d denote baryons. There are three bootstrap equations, one for each pair of channels. The s-u condition is simplest, since virtual particles or Reggeons in both of these channels correspond to baryons. The s-u conditions derived in R1 and R3 are the same, and may be written

$$\sum_r (D_{cdr}D_{arb}-D_{adr}D_{crb}) -\sum_s (F_{cds}F_{asb}-F_{ads}F_{csb})=0\,, \tag{2}$$

where r refers to internal baryon states of even parity (or trajectories whose lowest states are of even parity), s refers to odd-parity baryon states or trajectories, and the D and F are constants of interaction of the external meson-baryon states with the even- and odd-parity internal baryon states or trajectories.[4] These constants satisfy the requirements

$$D_{ijk}=D_{ikj}^*\,,\quad F_{ijk}=F_{ikj}^*\,. \tag{3}$$

In R1, Eqs. (2) and (3) were obtained from a superconvergence assumption applied to $t=0$ dispersion relations; in this derivation, the particle labels refer both to internal quantum numbers and to spin components. In R3, the same equations were obtained from an idealized Veneziano model, in which all external particles were taken as spinless. At present, we are interested only in the properties of the equations, rather than in the nature of the quantum numbers specified by the labels. In Sec. II C, we will show that Eq. (2) can be derived from weaker assumptions than those used in either R1 ot R3.

If the parity of the baryon d is odd, while that of b is even, a different consistency condition results, i.e.,[2]

$$\sum_r (F_{cdr}D_{arb}+F_{adr}D_{crb}) -\sum_s (D_{cds}F_{asb}+D_{ads}F_{csb})=0\,. \tag{4}$$

We may complete the set of consistency requirements by writing equations analogous to Eqs. (2) and (4) that correspond to the s-t and u-t pairs of channels, applying the equations to every meson-meson and meson-baryon scattering amplitude, and assuming that the set of internal mesons and baryons is identical to the set of external mesons and baryons. (If only external particles of even parity are considered, this set is identical to the set of conditions given in R3.) We consider such a complete set of equations in Sec. II B.

This procedure has not been justified completely in previous references. If one uses the fixed-angle method of R1 and R2, Eq. (4) requires that the baryons of both parities be degenerate, and the extension to the s-t and u-t channel pairs requires that the meson mass equal the baryon mass. One of the advantages of the Veneziano-model derivation of R3 is that the degeneracy requirements are replaced by more realistic requirements, i.e., that the even- and odd-parity baryon trajectories are exchange degenerate and linear, and that the even- and odd-parity meson trajectories are exchange degenerate and parallel to the baryon trajectories. On the other hand, the Veneziano method has not been used to write the condition of Eq. (4), involving external particles of opposite parities. In the rest of the paper, we ignore these difficulties, and consider the set of algebraic conditions described above.

B. Commutator Form of Baryon Conditions

The consistency equations were applied to meson-meson scattering in R2. It was shown that the mesons of each parity must correspond to the singlet and regular representations of a Lie group, that the interaction ratios are all fixed, and that the groups $SU(n)$, but not many other (possibly not any other) groups are satisfactory.

We assume the results of R2, and take the group to be $SU(n)$. We list below the meson-meson-meson interaction constants of R2. The quantum numbers of the meson state a, of either parity, may be expressed in terms of quarks Q and antiquarks $\bar{Q}$, i.e.,

$$a=\sum_{ij} A_{ij}\bar{Q}_iQ_j\,, \tag{5}$$

where $\sum_{ij}|A_{ij}|^2=1$. The matrix A_{ij} representing the meson singlet is a multiplet of the unit matrix. We denote the meson-meson-meson interactions corresponding to the cases for which the products of the three intrinsic parities are even and odd by the lower case letters d and f, respectively. If all meson states are taken to be self-conjugate, the interaction constants are proportional to traces of products of the matrices of

[4] The D and F given here are identical to those of R3. In R1 and R2 the parity-flip interaction matrices are anti-Hermitian. One can obtain the s-u channel condition of R1 from our Eq. (2) by making the substitutions $D_{abc}\rightarrow\gamma_{abc}{}^e$ and $(iF_{abc})\rightarrow\gamma_{abc}{}^o$. The D and F of R2 are identical to the γ^e and γ^o of R1.

Eq. (5), i.e.,

$$d_{abr}=\mathcal{C}\,\mathrm{Tr}[(AB+BA)R],\tag{6}$$

$$f_{abr}=\mathcal{C}\,\mathrm{Tr}[(AB-BA)R],\tag{7}$$

where $\mathcal{C}$ is a real constant.[5] The f_{ijk} are imaginary and proportional to the structure constants of the group.

Since all baryon-baryon states are exotic (free of resonances), the duality principle for the meson-exchange contributions to baryon-baryon amplitudes implies that the odd-parity and even-parity mesons of each internal quantum number couple identically to all baryon pairs. We assume that the baryon couplings satisfy this requirement, despite the fact that treatment of the baryon-antibaryon system with the duality principle leads to the prediction of some unwanted exotic baryon-antibaryon resonances.[6]

Because of this proportionality of the baryon couplings, we may limit attention to meson-baryon scattering amplitudes for which the external mesons are both of even parity. Furthermore, we can continue using the notation of Eqs. (2) and (4); the D are the couplings of any meson with two baryons of like parity, and the F are couplings with baryons of unlike parity.

It is convenient to consider D_{ijk} and F_{ijk} to be elements of matrices D_i and F_i in a vector space that is the direct sum of the even-parity and odd-parity baryon states. These matrices are Hermitian, as shown in Eq. (3). We may rewrite Eqs. (2) and (4) in terms of commutators, i.e.,

$$[D_c-F_c,\,D_a+F_a]=0.\tag{8}$$

Since the D connect only states of the same parity, and the F connect only states of opposite parity, the db matrix element of Eq. (8) is equivalent to both Eq. (2) and Eq. (4).

The consistency equations are Eq. (8) and the analogous equations that correspond to the s-t and u-t pairs of channels. It is convenient to write the difference and sum of the s-t and u-t equations; these are computed easily, since exchanging the mesons a and c is equivalent to exchanging the s and u channels. This procedure separates the conditions involving odd- and even-parity virtual mesons in the t channel, since the f and d meson-meson-meson interaction constants of Eqs. (7) and (6) are totally antisymmetric and totally symmetric, respectively. This procedure is used in R3 to derive the $a\rightleftarrows c$ antisymmetric condition for the case of external baryons of the same parity. We list below the antisymmetric and symmetric conditions, in terms of the D and F matrices:

$$[D_c+F_c,\,D_a+F_a]=\kappa\sum_m f_{acm}(D_m+F_m),\tag{9}$$

$$\{D_c+F_c,\,D_a+F_a\}_+=\kappa'\sum_m d_{acm}(D_m+F_m).\tag{10}$$

[5] See R2, Sec. III A. The F_{abc} of R2 is i times our f_{abc}.

[6] See J. L. Rosner, Phys. Rev. Letters 21, 950 (1968); 21, 1468(E) (1968).

The κ and κ' are real proportionality constants, $\{\ \}_+$ denotes an anticommutator, and d and f are the $SU(n)$-invariant meson-meson-meson interaction constants given in Eqs. (6) and (7). The summations in the two equations are over the n^2 meson states of odd and even parity, respectively.

The baryon self-consistency conditions are the three equations (8)–(10). We must keep in mind the requirement that the D matrices connect only baryon states of the same parity, while the F connect only states of opposite parity. We refer to this as the parity requirement.

C. Self-Consistency Condition from General Duality Assumption

In this section we show that the basic condition of Eq. (2) does not depend on assumptions so detailed as those made in R1 and R3, but follows from a simple application of duality to backward scattering. A reader interested only in the solutions to the conditions of Sec. II B may skip to Sec. III.

We consider the s-channel meson-baryon amplitude of Eq. (1), in the intermediate energy range, and in the backward (small u) direction. In this region of s and u, the u channel is expected to dominate Regge exchange, so the duality assumption may be written[7]

$$\langle \mathrm{Im}T_{ui}\rangle=\langle \mathrm{Im}T_{si}\rangle,\tag{11}$$

where i denotes all the quantum numbers, T_{ui} is the amplitude written in terms of u-channel Regge exchanges, T_{si} is the same amplitude written in terms of s-channel resonances, and $\langle\ \rangle$ denotes some sort of local averaging in energy or angle or both. If the even- and odd-parity baryons occur in two exchange-degenerate sets, the left-hand side of Eq. (11) is proportional to $X_{ui}^{(+)}-X_{ui}^{(-)}$, where $X_{ui}^{(\pm)}$ is the sum of the residues of the trajectories of signature ±1.[8] (The signature is the parity of the physical states.) The contribution of a resonance to the right-hand side of Eq. (11) is proportional to the residue at the resonance energy of the s-channel trajectory through the resonance. We assume that the residues of the different trajectories are proportional, so that the proportionality constant relating the Regge-exchange and resonance regions is the same for all trajectories. If the right-hand side of Eq. (11) is dominated by resonances, this equation leads to the relation

$$X_{ui}^{(+)}-X_{ui}^{(-)}=\alpha^2X_{si}^{(+)}-\beta^2X_{si}^{(-)},\tag{12}$$

where α^2 and β^2 are positive constants that include the proportionality constant mentioned above. The negative sign occurs before the β^2 because an odd-parity

[7] A lucid discussion of the duality assumption is given by H. Harari, Phys. Rev. Letters 22, 562 (1969). This paper contains references to other discussions of duality.

[8] See C. B. Chiu and J. Finkelstein, Phys. Letters 27B, 510 (1968), Eq. (1). A minus sign is omitted from the exponent $(-i\pi A_2)$ in this equation.

resonance contributes negatively in the backward direction.

We consider an amplitude i that is exotic (no resonances) in the s channel, but for which $X_{ui}^{(+)}\neq 0$. For this amplitude, Eq. (12) implies that $X_{ui}^{(+)}-X_{ui}^{(-)}=0$. If we then consider the "crossed" amplitude (obtained from i by reversing the roles of the s and u channels), Eq. (12) implies $\beta^2=\alpha^2$. We next apply Eq. (12) to a process for which $X_{si}^{(+)}-X_{si}^{(-)}\neq 0$, and to its crossed amplitude. This leads to the result $\alpha^2=1$, implying

$$Y_{ui}=Y_{si}, \tag{13}$$

where $Y_{\alpha i}=X_{\alpha i}^{(+)}-X_{\alpha i}^{(-)}$. If Y_{si} vanishes for all i, the above argument fails, but Eq. (13) is clearly valid. If some $Y_{si}\neq 0$, Eq. (13) states that Y_{si} is the i component of an eigenvector of the s-u crossing matrix, with the eigenvalue 1.[9]

The X_s for the amplitude of Eq. (1) may be written in terms of coupling constants, i.e.,

$$X_{si}^{(+)}=\sum_r D_{cdr}D_{abr}^*\,,\quad X_{si}^{(-)}=\sum_s F_{cds}F_{abs}^*\,.$$

The D and F are normalized by these equations. The $X_{ui}^{(\pm)}$ may be obtained from the above expressions by interchanging the labels a and c. If these expressions are substituted into Eq. (13), and the Hermiticity properties of Eq. (3) are used, the consistency condition of Eq. (2) results. This completes the demonstration.

III. SOLUTIONS TO CONDITIONS

In this section we show that all solutions to Eqs. (8)–(10) have some simple quarklike properties. We find two classes of solutions, one of which includes the generalization to $SU(n)$ of the $SU(3)$-invariant solution of R3.

The second equation, Eq. (9), states that the D_i+F_i are a representation of the group generators, since the f are proportional to the structure constants. Together, Eqs. (8) and (9) imply that the even-parity baryon states correspond to a representation of the group, and the odd-parity states also correspond to a representation.

We next turn to Eq. (10), which is not listed in previous references. We consider first the terms for which the meson indices a, c, and m correspond to the regular representation. These terms state that the D_i+F_i anticommute like the group generators in the fundamental (one-quark) representation; the anticommutation relations are different for all other representations.[10] Thus, the baryon interactions correspond to one-quark operators.

[9] If all the $X_{si}^{(-)}$ and $X_{ui}^{(-)}$ are zero, Eq. (13) is a well-known bootstrap requirement of static meson-baryon models. Such a requirement was used first by G. F. Chew, Phys. Rev. Letters **9**, 233 (1962).

[10] See S. Gasiorowicz, *Elementary Particle Physics* (Wiley, New York, 1966), Chap. 17, Eq. (17.18). This result is given here for $SU(3)$ only, but applies to $SU(n)$ if the d_{ijk} are defined as in our Eq. (6).

Before finding a solution to all the equations, we show that the above conclusion remains valid if singlet meson states are included. If the initial and final baryon states (b and d) are taken as basis vectors of the quark representation, the elements of the n^2 matrices (D_i+F_i) are given by

$$\langle d|\,(D_a+F_a)\,|\,b\rangle=\lambda A_{db}\,, \tag{14}$$

where A_{db} is the coefficient in Eq. (5) and λ is a proportionality constant. It is seen from Eqs. (6) and (14) that the db element of the right-hand side of Eq. (10) is proportional to the quantity

$$\sum_M \mathrm{Tr}[(AC+CA)M]M_{db}. \tag{15}$$

The n^2 Hermitian matrices M satisfy the closure condition $\sum_M M_{ij}M_{kl}^*=\delta_{ik}\delta_{jl}$. Therefore, the expression of Eq. (15) is equal to $(AC+CA)_{db}$. This is proportional to the db element of the left-hand side of Eq. (10), completing the demonstration.

In the simplest conceivable solution, the baryons would correspond to a single fundamental representation, and so would all have the same parity. However, Eqs. (8) and (9) are inconsistent if all the F vanish, so baryons of both parities must exist. Thus, the baryons cannot be simple quarks, but must involve quarks, in order that the interaction constants be proportional to one-quark operators.

We have found two types of modified quark solutions, one without a static limit and one with a static limit. (We define this limit exactly later in this section.) In the simplest case of the first type, the baryons of each parity correspond to the quark representation. The D and F matrices may be written

$$D_a=\begin{pmatrix} xA & 0\\ 0 & yA\end{pmatrix},\quad F_a=\begin{pmatrix} 0 & zA\\ zA & 0\end{pmatrix}, \tag{16}$$

where the first row and column correspond to the even-parity states, and the second to the odd-parity states. The A are the matrices of Eq. (5), and x, y, and z are real constants. These matrices satisfy all the conditions only if $x=y$ and $z^2=x^2$.

One may construct more complicated solutions of this type by letting the baryons of each parity correspond to a quark plus something else, and by keeping Eq. (16) for the D and F matrices, where A operates only in the space of the one quark. In this class of solutions, all the baryons occur in exchange-degenerate pairs of opposite parity. Each of the two terms in the commutator of Eq. (8) vanishes separately. If we consider amplitudes for which the parities of the two external baryons are the same, the vector $X_s^{(+)}-X_s^{(-)}$ of Eq. (13) vanishes identically. We call this not having a static limit, as there is no analog to the static bootstrap condition of Ref. 9. The baryons are bootstrapped entirely by meson exchange forces. We discard this type of solution because it does not correspond to reality.

We may obtain another type of solution by letting the quantum numbers of the baryon states be those of a direct product of the $SU(n)$ representations α, β, γ, etc., where α is the fundamental (quark) representation. The conditions of Eqs. (9) and (10) will be satisfied if the matrices $(D+F)$ are operators in the α-quark space, i.e.,

$$D_a+F_a=\kappa A^{\alpha}, \tag{17}$$

where

$$\langle\alpha'\beta'\gamma'\cdots|A^{\alpha}|\alpha\beta\gamma\cdots\rangle=A_{\alpha'\alpha}\delta_{\beta'\beta}\delta_{\gamma'\gamma}\cdots,$$

$A_{\alpha'\alpha}$ is the matrix element of Eq. (5), and κ is a real constant of proportionality. The s-u channel condition, Eq. (8), will be satisfied if D_c-F_c is the appropriate operator κC^{β} in β space. In this case,

$$D_a=\tfrac{1}{2}\kappa(A^{\alpha}+A^{\beta}), \tag{18}$$

$$F_a=\tfrac{1}{2}\kappa(A^{\alpha}-A^{\beta}). \tag{19}$$

These expressions will satisfy all three of the consistency equations [Eqs. (8)–(10)], provided they may be made consistent with the parity requirement of Sec. II B. It must be possible to assign each irreducible representation a definite parity in such a way that the D connect only states of the same parity and the F connect only states of opposite parity. It is seen from Eqs. (18) and (19) that this may be accomplished if β, as well as α, is a quark representation, if A^{β} in the β-quark space is equal to A^{α} in α-quark space, and if states of opposite symmetry under the exchange $\alpha\rightleftarrows\beta$ correspond to baryons of opposite parity.

The simplest solution of this type is a two-quark solution, with the parities of the symmetric and antisymmetric states opposite. This solution does not correspond to reality. In the next simplest solution, the baryons are composites of three quarks, labeled α, β, and γ. The total number of baryon states is n^3, and parity corresponds to symmetry under the interchange $\alpha\rightleftarrows\beta$. If the symmetry group is $SU(3)$, this is the solution of R3; there are a decuplet and octet of one parity and a singlet and octet of the other. The corresponding $SU(6)$ solution is discussed in Sec. IV. We note that in this type of solution, the matrix elements of D between like-parity states and of F between unlike-parity states may be computed from Eq. (17) alone. Thus, all the interactions allowed by the parity requirement are proportional to operators in α-quark space.

This type of solution has a static limit, in the sense that there is a nonvanishing vector Y_{si} that satisfies Eq. (13). The two terms of the commutator of Eq. (8) do not vanish separately for all amplitudes.

Recently, Mandelstam has written a series of papers on the Veneziano representation for hadron-hadron scattering, in which the hadrons are regarded as having the quantum numbers of composites of quarks and antiquarks.[11] Spin is treated relativistically in this series. The Mandelstam amplitudes must satisfy self-consistency conditions that are analogous to ours, although they cannot be written in the simple form of Eqs. (8)–(10). In a paper on the baryon trajectories, Mandelstam discusses a particular solution.[11] This solution is of the type we have characterized as not having a static limit, as each baryon trajectory is parity doubled.

[11] S. Mandelstam, Phys. Rev. D 1, 1745 (1970).

IV. POSSIBLE PHYSICAL REALIZATION

It is well known that s-u channel crossing for meson-baryon amplitudes mixes states of opposite normality. [Normality $\pm$ states are states for which the parity is $\pm(j-\frac{1}{2})$.] Consequently, the consistency conditions of Sec. II B are realistic physically only if the baryon spin components are included in the particle labels. Thus, the physical group is $SU(6)$, not $SU(3)$. It has been pointed out in R1 that the three-hadron interactions must be treated by the $SU(6)_W$ form of this group.

Although the spin components are taken into account in one of the derivations of Eq. (8) (that of R1 and R2), accounting for spins in a Veneziano-type model or any Reggeized model is not trivial. In such a model, the consistency conditions of Sec. II B are not exact, but apply only to the extent that the spin components transform like internal quantum numbers under crossing. We do not study this problem here, but rather study the baryon spectrum to see if the gross features of the three-quark, $SU(6)$ solution of Sec. III may correspond to experiment. In this solution, the baryon multiplets are $\mathbf{56}^+$, $\mathbf{70}^+$, $\mathbf{70}^-$, $\mathbf{20}^-$, where the superscript is the parity. Each of these multiplets should correspond to a Regge trajectory. The quantum numbers of the lowest states on the even-parity trajectories should be those of the **56** and **70** multiplets. These lowest even-parity states should be approximately degenerate, i.e., the trajectories through the $j^P=\frac{3}{2}^+$ states should lie a unit above the trajectories through the $j^P=\frac{1}{2}^+$ states. The spins of the lowest physical states on the odd-parity trajectories should be one unit higher than the $SU(6)$ spins, and even- and odd-parity trajectories of the same $SU(6)$ spins should be nearly degenerate. For example, the spin and $SU(3)$ quantum numbers of the 20-fold multiplet of $SU(6)$ are $(\frac{3}{2},1)$ and $(\frac{1}{2},8)$. The corresponding $\mathbf{20}^-$ trajectories should be degenerate with the $(\frac{3}{2},10)$ and $(\frac{1}{2},8)$ trajectories, respectively, of the $\mathbf{56}^+$, but the lowest states on the $\mathbf{20}^-$ trajectories should correspond to spin and $SU(3)$ multiplicities $(\frac{5}{2},1)$ and $(\frac{3}{2},8)$.[12]

Experimentally, there is strong evidence for $\mathbf{56}^+$ and $\mathbf{70}^-$ trajectories of this type, and practically no evidence for either a $\mathbf{70}^+$ or $\mathbf{20}^-$ trajectory. No exact solution of our equations involves only the $\mathbf{56}^+$ and $\mathbf{70}^-$. However, we may look for an approximate solution. We consider

[12] We assume that the bryon trajectories thaat should correspond to our solution are those on which the lightest even-parity baryons lie, and the odd-parity (odd-signature) trajectories that are approximately exchange-degenerate with them.

the six[3] three-quark baryon states of the exact solution, and consider the two independent 70-fold multiplets that are distinguished by their symmetry under exchange of the β and γ quarks, rather than the α and β quarks. The $\beta\gamma$ and $\alpha\beta$ symmetries are related by the equations

$$\mathbf{70}_{\beta\gamma^+} = -(\tfrac{1}{4})^{1/2}\mathbf{70}_{\alpha\beta^+} + (\tfrac{3}{4})^{1/2}\mathbf{70}_{\alpha\beta^-},$$
$$\mathbf{70}_{\beta\gamma^-} = -(\tfrac{3}{4})^{1/2}\mathbf{70}_{\alpha\beta^+} - (\tfrac{1}{4})^{1/2}\mathbf{70}_{\alpha\beta^-}.$$

Thus, the antisymmetric $\beta\gamma$ state is composed mainly of the symmetric $\alpha\beta$ state. We now modify the rule of Sec. III that gives the parity in terms of $\alpha\beta$ symmetry. We let the $\mathbf{70}_{\beta\gamma^-}$ (as well as the **56**) correspond to even parity, while the $\mathbf{70}_{\beta\gamma^+}$ (as well as the **20**) corresponds to odd parity. We continue to determine the D and F interactions from Eq. (17). Consequently, the two consistency conditions involving the t channel, Eqs. (9) and (10), will remain satisfied. Furthermore, since the **56** and **20** are completely symmetric and completely antisymmetric, respectively, and since operators in α-quark space cannot change the $\beta\gamma$ symmetry, the $\mathbf{56}^+$ and new $\mathbf{70}^-$ states will decouple from the new $\mathbf{70}^+$ and $\mathbf{20}^-$ states, and satisfy Eqs. (9) and (10) by themselves.

Unfortunately, the D and F operators determined by this procedure do not satisfy Eq. (8), the s-u consistency condition. Thus, this two-multiplet solution is only approximate. We consider this approximate solution for two reasons. First, as our knowledge of strong-interaction dynamics increases, we may learn that some self-consistency conditions are more accurate than others, so that an appropriate approximate solution may be physical. Second, some of the predictions concerning coupling-constant ratios of the two solutions are measurable experimentally.

In both the exact and approximate solutions, the $\mathbf{56}^+$-$\mathbf{70}^-$-M interaction ratios (where M is a meson) are given by $SU(6)_W$ symmetry, and thus measurements of these interaction constants test the symmetry prediction of the model, but do not distinguish between the exact and approximate solutions. The $SU(6)_W$ symmetry implies, for example, that the $SU(3)$ F/D ratios of the decays of the $j^P=\frac{5}{2}^-$ and $\frac{3}{2}^-$ octets into pseudoscalar mesons and baryons are $-1/3$ and $5/3$, respectively. The experimental numbers are about -0.14 and 1.2, respectively.[13]

On the other hand, the ratios of $\mathbf{70}^-$-$\mathbf{70}^-$-M interaction constants depend on an arbitrary parameter, analogous to the F/D parameter of $SU(3)$ interactions. This parameter is different for the exact and approximate ($\mathbf{56}^+$,$\mathbf{70}^-$) solutions. This $SU(6)$ parameter is not very familiar, so we have listed in Table I some $SU(3)$ F/D values that are predicted by the exact and approximate solutions. The interactions are those of the

TABLE I. Predicted $\mathbf{70}^-$-$\mathbf{70}^-$-meson F/D ratios corresponding to the exact (4-multiplet) and approximate ($\mathbf{56}^+$, $\mathbf{70}^-$) solutions.

	Baryon octets		
	$\frac{5}{2}^-$-$\frac{5}{2}^-$	$\frac{3}{2}^-$-$\frac{3}{2}^-$	$\frac{5}{2}^-$-$\frac{3}{2}^-$
Exact solution	-1	1	0
Approximate solution	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$

pseudoscalar meson octet with the $j=\frac{5}{2}$ and $j=\frac{3}{2}$ octets of the $\mathbf{70}^-$ trajectory. [The j is one larger than the $SU(6)$ spin, as explained earlier in this section.] If one assumes that F/D values are the same for Regge recurrences, the numbers in the table are measurable, in principle. For example, the predicted $(-\frac{1}{3})$ in column 1 implies that the $\Lambda_{9/2^-}$ [recurrence of the $\Lambda_{5/2^-}$ (1830)] will not decay into a $\bar{K}$-$N^*_{5/2^-}$ (1680). Admittedly, this kind of measurement is difficult.

V. CONCLUSIONS

One of the mysteries of hadron physics is that the quark model is so successful, despite the fact that no actual quarks have been observed to exist. This paper is the fourth of a series of papers that suggests a solution to this apparent paradox, namely, that hadron interactions must satisfy a set of self-consistency conditions that requires quarklike properties.

The set of self-consistency conditions that we propose is given in Sec. II B. We summarize here the most striking implications of this set, listing first results of earlier papers of the series. It was shown in R2 that mesons of both parities must correspond to the identity and regular representations of a Lie group. No satisfactory group other than $SU(n)$ was found. This group must apply to particle spins in the $SU(n)_W$ form. The baryons of both parities must also correspond to representations of the group, as shown in R3.

One of the principal new results of this paper is Eq. (10), obtained from a simple linear combination of the baryon consistency conditions. This equation shows that the meson-baryon-baryon interaction constants are proportional to the matrix elements of group generators in the one-quark representation. However, the other conditions, Eqs. (8) and (9), imply that the baryons cannot correspond simply to one fundamental representation. In the simplest solutions that we have found of those that have static limits (i.e., baryon exchange forces help bootstrap the baryons), the baryons behave like either two-quark or three-quark composites. The requirement of half-odd-integral spin eliminates the two-quark possibility. These results may help explain the success of the duality-diagram assumption, which requires that interactions behave as one-quark operators.[7]

It is shown in Sec. IV that an approximate solution, with the baryon trajectories corresponding to the $SU(6)$ multiplets and parities $\mathbf{56}^+$ and $\mathbf{70}^-$, may be more physical than an exact solution.

[13] The experimental F/D values are taken from the compilation of R. D. Tripp, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968), pp. 173–191. The α parameter of this reference is related to F/D by the formula $\alpha=D/(F+D)$.