(3) We regard this theory as a tentative step in the right direction rather than a final result. In particular, it would be nice to introduce *CP* violation. It would also be nice not to have to require dynamical suppression of the non-octet parts of the nonleptonic interaction. Perhaps this could be achieved if strong interactions were taken into account at the outset.

(4) Since our interaction contains some more terms than the usual one, their presence may be tested with the help of other theoretical models or in several hard to observe reactions. We shall postpone detailed discussion of these points.

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Veneziano Amplitudes for $\pi\pi$, πK, and KK Scattering and Chiral Symmetry Breaking*

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$\pi\pi$, πK, and KK single- and multiple-term Veneziano amplitudes are studied as a coupled system. Adler and Adler-Weisberger conditions are imposed, and it is found that the single-term system cannot satisfy all of the PCAC (partial conservation of axial-vector current) and charge-algebra constraints. The multiple-term system, constructed to satisfy these constraints, results in much improved width predictions. These improved amplitudes are used to study chiral symmetry breaking by investigating the Σ terms. It is found that a single $(3,3^*)\oplus(3^*,3)$ representation is not sufficient to explain the symmetry breaking, whereas a mixture of $(3,3^*)\oplus(3^*,3)$ and $(1,8)\oplus(8,1)$ is sufficient (but not necessary). The admixture of $(1,8)\oplus(8,1)$ is considerable.

I. INTRODUCTION

CONSIDERABLE interest has been focused on the elegant amplitude construction of Veneziano.[1] Work has proceeded in many directions, including two in which we shall be most interested, namely, the comparison of Veneziano forms with (1) experimental data and (2) current-algebra off-mass-shell predictions.[2] For the latter, the Lovelace conjecture[3] has often been taken as a working hypothesis, that is, that the Veneziano amplitude with constant coefficients is the correct off-mass-shell extrapolator.

Much of this effort, however, has had somewhat of a patchwork quality with emphasis on a single amplitude at a time[4] (say, $\pi\pi$ elastic scattering), ignoring other systems (such as KK and $K\pi$ elastic scattering) which share common trajectories and are jointly constrained by factorization and current-algebra requirements. In this study we shall consider the Veneziano amplitudes for $\pi\pi$, πK, and KK[5] elastic scattering as a coupled system and attempt simultaneously and consistently to satisfy these constraints. (We have not included $\eta\eta$, $\eta\pi$, and ηK in our system because of the mixing problem.[6])

Initially, we investigate the single-term Veneziano forms (STV) constructed according to the duality diagram rules of Harari and Rosner.[7] These amplitudes have been constructed by Kawarabayashi, Kitakado, and Yabuki.[5] The $\pi\pi$ and πK system have been studied from the point of view of low-energy theorems and chiral symmetry breaking by several authors.[8] We find that we cannot consistently satisfy the Adler[9] and Alder-Weisberger[10] theorems with this single-term set of

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[1] G. Veneziano, Nuovo Cimento **57A**, 190 (1968).

[2] Review talks containing extensive lists of references on these and other aspects of the Veneziano model are M. Jacob, in Proceedings of the Lund Conference, 1969 (unpublished); C. Lovelace, in Proceedings of the Irvine Conference on Regge Poles, 1969 (unpublished).

[3] C. Lovelace, Phys. Letters **28B**, 265 (1968).

[4] For a variety of reasons satellite modifications to $\pi\pi$ and/or πK leading-term Veneziano amplitudes have been considered by Dennis Corrigan, Phys. Rev. **188**, 2465 (1969); Kashyap Vasavada, Phys. Rev. D **1**, 88 (1970); Kyungsik Kang, Brown University report (unpublished); N. G. Antoniou, A. Bartl, and F. Widder, Tubingen University report (unpublished).

[5] K. Kawarabayashi, S. Kitakado, and H. Yabuki, Phys. Letters **28B**, 432 (1969).

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[8] J. A. Cronin and K. Kang, Phys. Rev. Letters **23**, 1004 (1969); Hugh Osborn, Nucl. Phys. **B17**, 141 (1970); Riazuddin and Fayyazuddin, Phys. Rev. D **1**, 282 (1970).

[9] S. L. Adler, Phys. Rev. **139**, B1638 (1965).

[10] S. L. Adler, Phys. Rev. Letters **14**, 1051 (1965); W. I. Weisberger, *ibid.* **14**, 1047 (1965); S. L. Adler, Phys. Rev. **140**, B763 (1965); W. I. Weisberger, *ibid.* **143**, 1302 (1966).

Veneziano amplitudes.[11] Thus the Lovelace conjecture loses support in this extended STV system. We comment briefly on some other problems including a fundamental difficulty with the KK system in which the (ρ,ω) trajectory decouples in one of the channels.

We then consider a set of multiple-term Veneziano forms (MTV) which are constructed to satisfy consistently the current-algebra and PCAC constraints. As a by-product, certain width predictions improve. We then use this set as a model for off-mass-shell extrapolations to test a scheme of chiral symmetry breaking in which a smoothness approximation is combined with the assumption of a single $(3,3^*)\oplus(3^*,3)$ chiral $SU(3)$ representation, closely related to, although slightly more general than, the approximation scheme of Gell-Mann, Oakes, and Renner.[12] The results of this test are negative, but the ambiguities of the smoothness approximation preclude any clear-cut conclusions. In an attempt to avoid these ambiguities we apply a current-algebra theorem in which three of the four scattering particles are off mass shell and evaluate this amplitude using the MTV forms as the extrapolating function. We find that a single $(3,3^*)\oplus(3^*,3)$ representation will not suffice, in agreement with the result of Cronin and Kang.[8] If a $(1,8)\oplus(8,1)$ representation is included, the admixture is considerable.

In Secs. II and III the STV and MTV amplitudes are constructed and compared with experimental and current-algebra off-mass-shell results. In Sec. IV the MTV amplitudes are compared with models for chiral symmetry breaking. Results are discussed and summarized in Sec. V. An Appendix contains a derivation of the off-mass-shell theorem used in Sec. IV.

II. SINGLE-TERM VENEZIANO AMPLITUDES

In this section we wish to establish that the requirements of the commutation relation $[F_{4+i5}{}^5, F_{4-i5}{}^5] = Q^{\text{em}}+Y$ are not consistently met in the leading-term Veneziano model for the $\pi\pi$, πK, and KK scattering amplitudes.[11]

Let us first briefly review the construction of amplitudes in this model and list some of its prominent successes and failures. For our discussion, the $\pi^-\pi^+$, π^-K^+, K^-K^0, and K^-K^+ amplitudes will be sufficient. These may be written as[5]

$$T^{\pi^-\pi^+}(s,t,u)=\lambda_{\pi\pi}\frac{\Gamma(1-\alpha_\rho(s))\Gamma(1-\alpha_\rho(t))}{\Gamma(1-\alpha_\rho(s)-\alpha_\rho(t))}, \tag{1}$$

$$T^{\pi^-K^+}(s,t,u)=\lambda_{\pi K}\frac{\Gamma(1-\alpha^*(s))\Gamma(1-\alpha_\rho(t))}{\Gamma(1-\alpha^*(s)-\alpha_\rho(t))}, \tag{2}$$

$$T^{K^-K^0}(s,t,u)=\lambda_{KK}\frac{\Gamma(1-\alpha_\rho(s))\Gamma(1-\alpha_\phi(t))}{\Gamma(1-\alpha_\rho(s)-\alpha_\phi(t))}, \tag{3}$$

$$T^{K^-K^+}(s,t,u)=\lambda_{KK}\frac{\Gamma(1-\alpha_\rho(s))\Gamma(1-\alpha_\phi(t))}{\Gamma(1-\alpha_\rho(s)-\alpha_\phi(t))}+\lambda_{KK}\frac{\Gamma(1-\alpha_\rho(t))\Gamma(1-\alpha_\phi(s))}{\Gamma(1-\alpha_\rho(t)-\alpha_\phi(s))}, \tag{4}$$

respectively. The constraints applied in determining these forms are summarized in the duality diagrams of Harari and Rosner,[7] each Veneziano term corresponding to an allowed diagram. The extremely restrictive nature of these rules is apparent in the K^-K^0 amplitude, where the (ρ,ω) trajectory is entirely absent in the t channel. The requirement that the pion and kaon PCAC consistency conditions be satisfied for these amplitudes, which is possible only if the arguments of the denominator gamma functions vanish,[3] yields with the exchange-degeneracy condition the following constraint on the leading trajectories[5]:

$$\alpha_\rho(s)=\alpha_\omega(s),\quad \alpha_\phi(s)-\alpha^*(s)=\alpha^*(s)-\alpha_\rho(s), \tag{5}$$

neglecting the pion mass-squared terms compared to $m_\rho{}^2$ and $m_K{}^2$. In terms of the vector-meson masses, this relation reads

$$m_\rho{}^2=m_\omega{}^2,\quad m_\phi{}^2-m_{K^*}{}^2=m_{K^*}{}^2-m_\rho{}^2, \tag{6}$$

which is the $SU(3)$ mass-squared relation with a mixing angle $\tan^2\theta=\frac{1}{2}$. The relations $(m_\rho{}^2-m_\pi{}^2)=(m_{K^*}{}^2-m_K{}^2)=1/2\alpha'$ also follow from the PCAC constraints. All of these relations are well satisfied experimentally. We are therefore encouraged by the consistency between the STV amplitude and PCAC for pions and kaons.

Applying pion PCAC twice, we can use the Adler-Weisberger (AW) theorems for the $\pi\pi$ and πK amplitudes. These relations, which utilize the commutator $[F_{\pi^+}{}^5,F_{\pi^-}{}^5]=2I_3$, yield[13]

$$-2F_\pi{}^2\frac{d}{ds}T^{\pi^-\pi^+}(s,t,u)\bigg|_{s=m_\pi{}^2;\,t=0}=+2F_\pi{}^2\alpha'\pi\lambda_{\pi\pi}=2 \tag{7}$$

and

$$-2F_\pi{}^2\frac{d}{ds}T^{\pi^-K^+}(s,t,u)\bigg|_{s=m_K{}^2;\,t=0}=+2F_\pi{}^2\alpha'\pi\lambda_{\pi K}=1, \tag{8}$$

where α' is the (universal) trajectory slope and F_π is the pion decay constant. By identifying the normalization constants with the residues at the ρ and K^* poles, we

[11] This difficulty is pointed out by H. Osborn (Ref. 8), who chooses to take the $\pi\pi$ and πK AW relations seriously and abandon the KK case. Our viewpoint is to consider *all* of the soft-K theorems to be bad (inconsistent) and to consider the consequence of demanding consistency.

[12] M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968).

[13] D. W. McKay and W. W. Wada, Phys. Rev. Letters **23**, 619 (1969).

have[14]

$$\lambda_{\pi\pi}=2f_{\rho\pi\pi}{}^2=2m_\rho{}^2/\pi F_\pi{}^2=2\lambda_{\pi K}=2f_{\rho KK}f_{\rho\pi\pi}=2f_{K^*K\pi}{}^2 \tag{9}$$

or

$$f_{\rho\pi\pi}=f_{\rho KK}=f_{K^*K\pi}=m_\rho/(\sqrt{\pi})F_\pi\,. \tag{10}$$

The following relations are obtained by normalizing the KK amplitudes to the ρ, ω, and ϕ poles:

$$\lambda_{KK}=f_{\rho KK}{}^2=f_{\omega KK}{}^2=\tfrac{1}{2}f_{\phi KK}{}^2\,. \tag{11}$$

[One also finds that forcing factorization at the ρ pole ensures that the σ pole (daughter of ρ) also factorizes in the coupled $\pi\pi$, πK, and KK STV amplitudes.] Taken together, these equations are the vector-meson nonet coupling relations with the mixing angle $\tan^2\theta=\frac{1}{2}$, consistent with the mass relations, Eq. (6). Similar relations hold for the tensor-meson nonet. The comparison of the widths determined by these couplings with the experimental widths is unsatisfactory, as indicated by Table I. Nevertheless, we can still pursue the question of the chiral symmetry content of the single-term Veneziano model.

The soft-kaon counterpart of Eq. (8), the πK AW relation, yields (in terms of the digamma function ψ) the equation

$$\begin{aligned}&-2F_K{}^2\frac{d}{ds}T^{\pi^-K^+}(s,t)\Big|_{s=m_\pi{}^2;\,t=0}\\&=+2F_K{}^2\alpha'\lambda_{\pi K}\Bigg\{\frac{\Gamma(1-\alpha_\rho(0))\Gamma(1-\alpha^*(m_\pi{}^2))}{\Gamma(2-\alpha_\rho(0)-\alpha^*(m_\pi{}^2))}\\&+\frac{\Gamma(1-\alpha_\rho(0))\Gamma(1-\alpha^*(m_\pi{}^2))}{\Gamma(1-\alpha_\rho(0)-\alpha^*(m_\pi{}^2))}\\&\times[\psi(1-\alpha^*(m_\pi{}^2))-\psi(2-\alpha^*(m_\pi{}^2)-\alpha_\rho(0))]\Bigg\}=1\end{aligned} \tag{12}$$

for the normalization constant $\lambda_{\pi K}$. {The charge commutator $[F_K{}^{+5},F_K{}^{-5}]=Q^{\rm em}+Y$ has been used in obtaining Eq. (12).} Combining Eqs. (8) and (12), we find[15] $(F_K/F_\pi)^2=1.55$ consistent with experimental estimates.[16]

Turning now to the KK amplitudes, a contradiction is evident when the two independent AW relations are evaluated.[11] These theorems read

$$-2F_K{}^2\frac{d}{ds}T^{K^-K^0}(s,t,u)\Big|_{s=m_K{}^2;\,t=0}=2\times(1.06)F_K{}^2\alpha' f_{\rho KK}{}^2=1 \tag{13}$$

and

$$-2F_K{}^2\frac{d}{ds}T^{K^-K^+}(s,t,u)\Big|_{s=m_K{}^2;\,t=0}=2\times(3.07)F_K{}^2\alpha' f_{\rho KK}{}^2=2\,, \tag{14}$$

which are inconsistent with each other and with the results of the AW relation for πK scattering with soft kaons [Eq. (12)].

Because no internal contradictions are encountered until the commutation rule $[F_K{}^{+5},F_K{}^{-5}]=Q^{\rm em}+Y$ is invoked, we believe that the single-term Veneziano model with constant coefficients is one in which the $SU(2)\otimes SU(2)$ algebra of charges is consistently satisfied, but the full $SU(3)\otimes SU(3)$ algebra is not. Therefore, the study of the *breaking* of the $SU(3)\otimes SU(3)$ symmetry generated by the vector and axial-vector charges has little meaning in the single-term Veneziano model for the $\pi\pi$, πK, and KK amplitudes.

Let us emphasize that, in the light of the many other difficulties of the Veneziano model, the charge algebra itself is not challenged but only the application of the leading-term Veneziano model to the study of the breaking of the chiral *symmetry* when the model is already inconsistent with the complete *algebra*. One way of studying chiral symmetry breaking in the framework of the Veneziano representation is to add secondary terms to the amplitudes, still keeping the basic rules of the duality diagrams and adjusting coefficients so that *all* of the low-energy theorems implied by $SU(3)\otimes SU(3)$ charge algebra can be consistently satisfied. We take up this problem in Sec. III.

III. MODIFIED VENEZIANO AMPLITUDES

There are two obvious troubles with STV amplitudes which we have already pointed out. One is the absence of (ρ,ω) in the t channel of K^-K^0 scattering, and the other is the conflict among the three independent soft-kaon AW relations. (Within the leading-term Veneziano framework, the only way to resolve both of these difficulties at once is to set $m_\pi=m_K$, which implies that the $I=0$, KK amplitude vanishes identically, a problem in itself, and that $\alpha_\rho=\alpha_\phi=\alpha_\omega=\alpha_{K^*}$.) We shall remove the conflict among the three AW relations by taking the leading-term Veneziano amplitudes with physical masses for π and K as the first approximation to the scattering amplitude and adding satellite terms as corrections. We maintain the PCAC consistency condition and demand consistency among all of the soft-pion and -kaon AW relations. We do not pursue the question of possible overrestrictiveness of the Harari-Rosner duality diagram prescription.

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[14] Our definitions of couplings relevant to normalization and factorization are as follows:

$$\begin{aligned}\mathcal{H}_I=&f_{\rho\pi\pi}\tilde\rho_\mu\cdot\tilde\pi\times\partial_\mu\tilde\pi+if_{\rho KK}\tilde\rho_\mu\cdot K^\dagger\tfrac{1}{2}\tilde\tau\overleftrightarrow{\partial}_\mu K+i\tfrac{1}{2}f_{\omega KK}\omega_\mu K^\dagger\overleftrightarrow{\partial}_\mu K\\&+i\tfrac{1}{2}f_{\phi KK}\phi_\mu K^\dagger\overleftrightarrow{\partial}_\mu K+if_{K^*K\pi}[K^*{}_\mu{}^\dagger\tfrac{1}{2}\tilde\tau K\overleftrightarrow{\partial}_\mu\cdot\tilde\pi-K^\dagger\overleftrightarrow{\partial}_\mu\tilde\pi\cdot\tfrac{1}{2}\tilde\tau K^*{}_\mu]\\&+g_{\sigma\pi\pi}\sigma\tilde\pi\cdot\tilde\pi+g_{\sigma KK}\sigma K^\dagger K+g_{\delta KK}\tilde\delta\cdot K^\dagger\tilde\tau K,\end{aligned}$$

where δ is the isovector scalar meson (daughter of ω) and tildes denote isovectors.

[15] Y. Oyanagi and N. Tokuda, Progr. Theoret. Phys. (Kyoto) **42**, 430 (1969); H. Osborn, Ref. 8.

[16] N. Brene, M. Roos, and A. Sirlin, Nucl. Phys. **B6**, 255 (1968).

TABLE I. Width predictions of the single-term Veneziano (STV) and our multiple-term Veneziano (MTV) models and the corresponding experimental widths [as compiled by the Particle Data Group, Rev. Mod. Phys. 42, 87 (1970)]. The last column gives the relative sizes of the coefficients of nonleading terms to the leading terms in MTV (Sec. III). There are no free parameters, and all widths are determined in terms of F_π^2 and m_ρ, taken as $F_\pi^2=0.47m_\pi^2$ and 765 MeV, respectively.

Amplitude	Width	STV width prediction (MeV)	MTV width prediction (MeV)	Experimental width (MeV)	Ratio of satellite coefficients to leading term
$T^{\pi^-\pi^+}$	$\Gamma(\rho\to\pi\pi)$	83	110	125 ± 20	$V_2^{12}/V_1^{11}=0.16$
	$\Gamma(f\to\pi\pi)$	69	165	151 ± 25	$V_2^{22}/V_1^{11}=-0.64$
	$\Gamma(\sigma\to\pi\pi)$	450	300	$\gg100$	
$T^{\pi^-K^+}$	$\Gamma(K^*(890)\to K\pi)$	25	33	50	$V_2^{12}/V_1^{11}=0.13$
	$\Gamma(K^*(1420)\to K\pi)$	26	42	47_{-5}^{+7}	$V_2^{21}/V_1^{11}=0.06$
					$V_2^{22}/V_1^{11}=-0.37$
$T^{K^-K^0}$ and $T^{K^-K^+}$	$\Gamma(\phi\to K\bar{K})$	1.9	2.67	3.1 ± 0.5	$V_2^{12}/V_1^{11}=-0.075$
	$\Gamma(f'\to K\bar{K})$	21	29	53_{-23}^{+28}	$V_2^{21}/V_1^{11}=0.175$

Let us first discuss the modifications of the KK scattering amplitudes in some detail, and then turn more briefly to the πK and $\pi\pi$ cases. For s-channel KK scattering, the isospin-one and -zero amplitudes are t-u symmetric and antisymmetric, respectively. In addition to the leading terms, we shall add two additional terms to $T^{(1)}$ to ensure that the soft-kaon Adler consistency condition is satisfied, and corresponding terms to $T^{(0)}$ to guarantee that the ϕ trajectory and its daughter are pure $I=0$. This will also maintain the restriction of the duality diagrams. Including all of the secondary terms which contain the ρ, ω, or ϕ poles is just sufficient for these purposes, so we write

$$\begin{aligned}T^{(1)}(s,t,u)&=\lambda_{KK}^{(1)}[V_1^{11}(\alpha_\rho(t),\alpha_\phi(u))+V_1^{11}(\alpha_\phi(t),\alpha_\rho(u))]\\&+\lambda_{KK}^{(2)}[V_2^{12}(\alpha_\rho(t),\alpha_\phi(u))+V_2^{12}(\alpha_\rho(u),\alpha_\phi(t))]\\&+\lambda_{KK}^{(3)}[V_2^{21}(\alpha_\rho(t),\alpha_\phi(u))+V_2^{21}(\alpha_\rho(u),\alpha_\phi(t))]\end{aligned}\quad(15)$$

and

$$\begin{aligned}T^{(0)}(s,t,u)&=\lambda_{KK}^{(1)}[V_1^{11}(\alpha_\rho(u),\alpha_\phi(t))-V_1^{11}((\alpha_\rho(t),\alpha_\phi(u))]\\&+\lambda_{KK}^{(2)}[V_2^{12}(\alpha_\rho(u),\alpha_\phi(t))-V_2^{12}(\alpha_\rho(t),\alpha_\phi(u))]\\&+\lambda_{KK}^{(3)}[V_2^{21}(\alpha_\rho(u),\alpha_\phi(t))-V_2^{21}(\alpha_\rho(t),\alpha_\phi(u))].\end{aligned}\quad(16)$$

In these expressions, the shorthand

$$V_k^{ij}(\alpha_a(x),\alpha_b(y))=\frac{\Gamma(i-\alpha_a(x))\Gamma(j-\alpha_b(y))}{\Gamma(k-\alpha_a(x)-\alpha_b(y))}$$

has been used. Applying the Adler consistency condition to $T^{(1)}$, we obtain

$$\lambda_{KK}^{(2)}[1-\alpha_\phi(m_K^2)]+\lambda_{KK}^{(3)}[1-\alpha_\rho(m_K^2)]=0.\quad(17)$$

This condition reduces the number of parameters to two, chosen as $\lambda_{KK}^{(1)}$ and $\lambda_{KK}^{(2)}$. Crossing to the u channel, we have

$$\begin{aligned}T^{K^-K^0}(u,t,s)&=\tfrac{1}{2}[T^{(1)}(s,t,u)+T^{(0)}(s,t,u)]\\&=\lambda_{KK}^{(1)}V_1^{11}(\alpha_\rho(u),\alpha_\phi(t))\\&+\lambda_{KK}^{(2)}\Bigg[V_2^{12}(\alpha_\rho(u),\alpha_\phi(t))\\&-\frac{1-\alpha_\phi(m_K^2)}{1-\alpha_\rho(m_K^2)}V_2^{21}(\alpha_\rho(u),\alpha_\phi(t))\Bigg]\end{aligned}\quad(18)$$

and

$$T^{K^-K^+}(u,t,s)=T^{(1)}(s,t,u).\quad(19)$$

We note that the (ρ,ω) pole is absent in the $T^{K^-K^0}$ t channel, consistent with the duality diagram prescription. Finally, applying the AW theorem to $T^{K^-K^0}(u,t,s)$ and $T^{K^-K^+}(u,t,s)$, we find the following conditions:

$$\begin{aligned}1&=2F_K^2\alpha'(\lambda_{KK}^{(1)}-10.5\lambda_{KK}^{(2)})\\&=-2F_K^2\frac{d}{du}T^{K^-K^0}(u,t,s)\Big|_{u=m_K^2;\,t=0}\end{aligned}\quad(20)$$

and

$$\begin{aligned}2&=2F_K^2\alpha'(3.08\lambda_{KK}^{(1)}-6.45\lambda_{KK}^{(2)})\\&=-2F_K^2\frac{d}{du}T^{K^-K^+}(u,t,s)\Big|_{u=m_K^2;\,t=0}.\end{aligned}\quad(21)$$

Consistency between these two relations requires $\lambda_{KK}^{(1)}=-13.4\lambda_{KK}^{(2)}$. We can now write $T^{K^-K^0}$ in terms of one parameter $\lambda_{KK}^{(1)}$, and we have

$$\begin{aligned}T^{K^-K^0}(s,t,u)&=\lambda_{KK}^{(1)}\Bigg[V_1^{11}(\alpha_\rho(s),\alpha_\phi(t))\\&-0.075\left(V_2^{12}-\frac{1-\alpha_\phi(m_K^2)}{1-\alpha_\rho(m_K^2)}V_2^{21}\right)\Bigg],\end{aligned}\quad(22)$$

and similarly for $T^{K^-K^+}$. By identifying the residues at the ρ, ϕ, and ω poles [ignoring the absence of (ρ,ω) in the t channel for $T^{K^+K^0}$], we find the relations

$$\lambda_{KK}^{(1)}/1.08=f_{\rho KK}^2=f_{\omega KK}^2=(1/2.54)f_{\phi KK}^2 \quad (23)$$

for the vector-meson–kaon couplings. The σKK coupling is found to be

$$g_{\sigma KK}^2=0.33f_{\rho KK}^2m_\rho^2, \quad (24)$$

a relation which will be useful later when factorization of the σ pole is demanded in $\pi\pi$, πK, and KK scattering. We notice that if we use the relations $f_{\rho\pi\pi}^2=m_\rho^2/\pi F_\pi^2$ and $f_{\rho KK}^2=f_{\rho\pi\pi}^2$, obtained from the soft-pion AW theorems applied to the single-term Veneziano forms for $\pi\pi$ and πK, then Eq. (20), the AW relation for KK, yields $(F_K/F_\pi)^2=1.61$, compared with $(F_K/F_\pi)^2=1.55$ from the leading term of the πK amplitude.

The near consistency between KK and πK AW relations is encouraging, and one is tempted to stop here, retaining the STV forms for $\pi\pi$ and πK; however, by modifying T^{KK} we have destroyed the factorization at the σ pole. In addition the ρ and K^* widths determined by the AW relations and leading Veneziano amplitudes for $\pi\pi$ and πK are not very satisfactory. For these reasons, we must consider modifications to the πK and $\pi\pi$ amplitudes as well, similar to the modifications just discussed for the KK case. It is sufficient to discuss the AW relations in terms of the π^-K^+ and $\pi^-\pi^+$ elastic scattering amplitudes, so we shall restrict ourselves to these. (There is only one independent t-channel antisymmetric amplitude for each of these processes.)

In the case of πK, the soft-π and soft-K PCAC consistency conditions require that three secondary terms be added, or one more than is available from just those terms with a ρ or K^* pole.[17] We choose to add the term $V_2^{22}(\alpha_\rho(t),\alpha^*(s))$ in addition to those with the leading poles.[18] Enforcing the PCAC consistency condition for soft π and K, we have

$$\begin{aligned}T^{\pi^-K^+}(s,t,u)&=\lambda_{\pi K}^{(1)}V_1^{11}(\alpha^*(s),\alpha_\rho(t))\\&+\lambda_{\pi K}^{(2)}\bigg[V_2^{12}(\alpha^*(s),\alpha_\rho(t))+\frac{1-\alpha_\rho(m_K^2)}{1-\alpha^*(m_\pi^2)}\\&\times V_2^{21}(\alpha^*(s),\alpha_\rho(t))-\frac{2}{1-\alpha^*(m_\pi^2)}V_2^{22}(\alpha^*(s),\alpha_\rho(t))\bigg].\end{aligned} \quad (25)$$

The Adler-Weisberger relations for soft π's and K's read

$$\begin{aligned}1&=2F_\pi^2\alpha'\pi(\lambda_{\pi K}^{(1)}-\lambda_{\pi K}^{(2)})\\&=-2F_\pi^2\frac{d}{ds}T^{\pi^-K^+}(s,t,u)\bigg|_{t=0;\,s=m_K^2}\end{aligned} \quad (26)$$

and

$$\begin{aligned}1&=2F_K^2\alpha'(\lambda_{\pi K}^{(1)}-\lambda_{\pi K}^{(2)})\\&\times\{V_2^{11}(\alpha^*(m_\pi^2),\alpha_\rho(0))+V_1^{11}(\alpha^*(m_\pi^2),\alpha_\rho(0))\\&\times[\psi(1-\alpha^*(m_\pi^2))-\psi(2-\alpha^*(m_\pi^2)-\alpha_\rho(0))]\}\\&=-2F_K^2\frac{d}{ds}T^{\pi^-K^+}(s,t,u)\bigg|_{t=0;\,s=m_\pi^2},\end{aligned} \quad (27)$$

respectively. The V's are defined as before and the ψ is the digamma function. Equations (26) and (27) differ from the corresponding STV expressions (8) and (12) only in the replacement $\lambda_{\pi K}\to\lambda_{\pi K}^{(1)}-\lambda_{\pi K}^{(2)}$. The ratio $(F_K/F_\pi)^2$ determined from (26) and (27) is therefore the same as the STV model. In fact, any combination of the first five $V_k^{ij}(\alpha^*(s),\alpha_\rho(t))$, $k=1,2$ and $i,j\leq k$, which satisfies the soft-pion PCAC consistency condition *alone*, has the feature that the STV value of $(F_K/F_\pi)^2$ is reproduced.[19]

The normalization constants $\lambda_{\pi K}$,[1,2] determined in terms of $f_{K^*K\pi}^2$ and $f_{\rho\pi\pi}f_{\rho KK}$, are

$$\begin{aligned}f_{\rho\pi\pi}f_{\rho KK}&=\lambda_{\pi K}^{(1)}+\frac{1-\alpha_\rho(m_K^2)}{1-\alpha^*(m_\pi^2)}\lambda_{\pi K}^{(2)},\\f_{K^*K\pi}^2&=\lambda_{\pi K}^{(1)}+\lambda_{\pi K}^{(2)}.\end{aligned} \quad (28)$$

In addition, $g_{\sigma KK}g_{\sigma\pi\pi}$ satisfies the equation

$$\frac{1-\alpha_\rho(m_K)^2}{1-\alpha^*(m_\pi^2)}\lambda_{\pi K}^{(2)}=\frac{f_{\rho KK}f_{\rho\pi\pi}}{2}-\frac{g_{\sigma\pi\pi}g_{\sigma KK}}{m_\rho^2}. \quad (29)$$

Finally, let us consider the corrections to the $\pi^-\pi^+$ scattering amplitude which will permit us to simultaneously satisfy ρ and σ factorization in the coupled πK system. To achieve this, we need to introduce one additional parameter for the $\pi^-\pi^+$ amplitude. We choose the form[18]

$$\begin{aligned}T^{\pi^-\pi^+}(s,t,u)&=\lambda_{\pi\pi}^{(1)}V_1^{11}(\alpha_\rho(s),\alpha_\rho(t))\\&+\lambda_{\pi\pi}^{(2)}[V_2^{12}(\alpha_\rho(s),\alpha_\rho(t))+V_2^{21}(\alpha_\rho(s),\alpha_\rho(t))\\&-4V_2^{22}(\alpha_\rho(s),\alpha_\rho(t))],\end{aligned} \quad (30)$$

where PCAC consistency has been enforced. The AW relation imposes the condition

$$\begin{aligned}2&=2F_\pi^2\alpha'(\lambda_{\pi\pi}^{(1)}-\lambda_{\pi\pi}^{(2)})\pi\\&=-2F_\pi^2\frac{d}{ds}T^{\pi^-\pi^+}(s,t,u)\bigg|_{t=0;\,s=m_\pi^2}.\end{aligned} \quad (31)$$

The identifications of the residues at the ρ and σ poles yield

$$\lambda_{\pi\pi}^{(1)}+\lambda_{\pi\pi}^{(2)}=2f_{\rho\pi\pi}^2$$

[17] D. Corrigan, Ref. 4.

[18] If one adds V_2^{11} instead of V_2^{22}, the amplitude reduces to the STV form after soft-pion and -kaon PCAC constraints are applied. The MTV amplitude could have equivalently been written $T^{\pi^-\pi^+}=\lambda^1V_1^{11}+\lambda^2V_2^{11}+\lambda^3V_2^{22}$ for $\pi\pi$.

[19] The degree of generality of the insensitivity of the value of F_K/F_π to the addition of nonleading terms has not been determined.

and (32)

$$\lambda_{\pi\pi}^{(2)}=\tfrac{1}{2}f_{\rho\pi\pi}^2-2g_{\sigma\pi\pi}^2/m_\rho^2\,,$$

respectively.

We note that there are five normalization constants and five independent equations, three AW relations, and the factorization conditions for ρ and σ. The soft-kaon AW relation determines $(F_K/F_\pi)^2$, so that all normalizations can be expressed in terms of m_ρ^2 and F_π^2. There are two solutions to this system of equations, one of which yields a negative K^* width and will not be considered further. The other solution leads to the width predictions given in Table I. A consequence of this solution is the modified KSRF[20] relation

$$f_{\rho\pi\pi}^2=2m_\rho^2/1.52\pi F_\pi^2\,. \tag{33}$$

The ρ, ϕ, f, f', and $K^*(1420)$ widths are much improved compared to the predictions of STV and are now in agreement within experimental limits. The width of the σ has decreased substantially from the STV value of 450 MeV. The K^* width is still too small (about $\frac{2}{3}$ the experimental value).[21]

In general, we find that imposing the AW relations for soft K and enforcing factorization for ρ and σ has led to an improved representation of the low-energy region of the $\pi\pi$, πK, and KK systems. Having enforced the consequences of charge algebra for the coupled system, we are in a position to look into the question of symmetry breaking, and we take up this problem in Sec. IV.

IV. CHIRAL SYMMETRY BREAKING AND MODIFIED VENEZIANO FORMS

Our modified amplitudes [Eqs. (22), (25), and (30)] now satisfy the Adler consistency condition (that is, the restriction of PCAC) and the Adler-Weisberger relations (that is, the restriction of charge algebra). This has been achieved at the expense of adding satellite terms. We now investigate the chiral-symmetry-breaking content of these modified amplitudes.

Consider the forward elastic scattering of soft pseudoscalar mesons P_i on the pseudoscalar meson target P_j, where i and j are $SU(3)$ octet indices. By standard PCAC and reduction methods, we obtain[22]

$$\lim_{p'\to 0}\langle P_i(p')P_j(p)|T|P_i(p')P_j(p)\rangle =(i/F_i^2)\langle P_j(p)|[F_i^5,\partial\mathfrak{F}_i^5]|P_j(p)\rangle\,. \tag{34}$$

The Σ-term commutator can be evaluated in theories which specify the chiral symmetry breaking. In the scheme of Gell-Mann, Oakes, and Renner[12] we have

$$\frac{\partial}{\partial x_\mu}\mathfrak{F}_{\mu,i}^5(x)\equiv\partial\mathfrak{F}_i^5(x)=i[\mathcal{H}'(x),F_i^5(x_0)]\,, \tag{35}$$

where $\mathcal{H}'(x)$, the piece of Hamiltonian density which is not chiral symmetric, is given by $\mathcal{H}'(x)=-u_0(x)-cu_8(x)$, where the scalar densities u_0 and u_8 belong to a (single) $(3,3^*)\oplus(3^*,3)$ representation and hence transform like

$$[F_i^5,u_j]=-id_{ijk}v_k\,,\quad [F_i^5,v_j]=id_{ijk}u_k\,. \tag{36}$$

In terms of these scalar densities, we have

$$\begin{aligned}\lim_{p'\to 0}&\langle P_i(p')P_j(p)|T|P_i(p')P_j(p)\rangle\\ &=(1/F_i^2)\langle P_j(p)|[F_i^5,[F_i^5,u_0+cu_8]]|P_j(p)\rangle\\ &=(1/F_i^2)\sum_l d_{iil}[(\sqrt{\tfrac{2}{3}})+cd_{8ii}]\langle P_j(p)|u_l|P_j(p)\rangle\,.\end{aligned} \tag{37}$$

Thus for soft pions we have

$$\langle\pi_i(0)P_j(p)|T|\pi_i(0)P_j(p)\rangle =\frac{\sqrt{2}+c}{3F_\pi^2}\langle P_j(p)|\sqrt{2}u_0+u_8|P_j(p)\rangle \tag{38}$$

and for soft kaons

$$\langle K_i(0)P_j(p)|T|K_i(0)P_j(p)\rangle =\frac{\sqrt{2}-\frac{1}{2}c}{3F_K^2}\langle P_j(p)|\sqrt{2}u_0+\tfrac{1}{2}\sqrt{3}u_3-\tfrac{1}{2}u_8|P_j\rangle\,. \tag{39}$$

As a first approximation, let us evaluate the right-hand side of Eq. (37) using the smoothness assumption suggested in Ref. 12:

$$\begin{aligned}\langle P_i(p)|u_j|P_k(p)\rangle&\approx\langle P_i(0)|u_j|P_k(p)\rangle\\ &=d_{ijl}\frac{\langle 0|V_l|P_k(p)\rangle}{F_i}\approx d_{ijl}\frac{\langle 0|V_l|P_k(0)\rangle}{F_i}\\ &=d_{ijl}d_{lkm}\frac{\langle 0|u_m|0\rangle}{F_iF_k}\,,\end{aligned} \tag{40}$$

where we have used PCAC twice. In this strictest of all forms of momentum independence, at most a vacuum expectation value of u_0 is allowed, since the same assumptions imply

$$\begin{aligned}\langle P_i(p)|u_j|P_k(p)\rangle&\approx\langle P_i(p)|u_j|P_k(0)\rangle\\ &=d_{jkl}\frac{\langle P_i(p)|v_l|0\rangle}{F_k}\approx d_{jkl}\frac{\langle P_i(0)|v_l|0\rangle}{F_k}\\ &=d_{jkl}d_{ilm}\frac{\langle 0|u_m|0\rangle}{F_iF_k}\,.\end{aligned} \tag{41}$$

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[20] K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters **16**, 255 (1966); Riazuddin and Fayyazuddin, Phys. Rev. **147**, 1071 (1966).

[21] The relationship between the $K^*K\pi$ and $\rho\pi\pi$ couplings is still nearly that predicted by $SU(3)$.

[22] S. L. Adler, Phys. Rev. **140**, B736 (1965); **143**, 1144 (1966); W. I. Weisberger, *ibid.* **143**, 1302 (1966); K. Kawarabayashi and W. W. Wada, *ibid.* **146**, 1209 (1966).

Equating (40) and (41), we can easily show that $\langle 0|u_i|0\rangle=0$ for $i\neq 0$. Thus we have

$$\langle P_i(p)|u_i|P_j(p)\rangle \approx d_{ijk}(\sqrt{\tfrac{2}{3}})\langle 0|u_0|0\rangle/F_iF_j, \quad (42)$$

consistent with results of Ref. 12. [We have not, however, made the assumption of $SU(3)$ vertex symmetry and are not forced to take $F_K=F_\pi$.] For the calculation of the meson-meson terms we need only diagonal elements of $\langle P_i(p)|u_j|P_k(p)\rangle$, so we need not assume all of the smoothness inherent in the equating of (40) and (41). In particular, the off-diagonal elements in this equality, which imply $\langle 0|u_8|0\rangle=0$, need not be imposed; this is tantamount to allowing a κ meson to spoil the smoothness approximation by contributing momentum dependence to matrix elements such as $\langle K|u_0|\pi\rangle$. This allows a vacuum expectation value for u_8, $\langle 0|u_8|0\rangle\neq 0$, which in fact is necessary to support κ domination of the strangeness-changing (PCVC) vector current; that is,

$$\begin{aligned} m_\kappa^2F_\kappa &= \langle 0|\partial\mathfrak{F}_4|\kappa_4\rangle = i\langle 0|i[\mathcal{H}',F_4]|\kappa_4\rangle \\ &= ic\langle 0|[u_8,F_4]|\kappa_4\rangle = cf_{485}\langle 0|u_5|\kappa_4\rangle \\ &\approx \tfrac{3}{4}c\langle 0|u_8|0\rangle/F_\kappa, \quad (43)\end{aligned}$$

where we have used partial conservation of vector current (PCVC) and made a smoothness approximation in the last step.[23]

Thus our approximation for the diagonal scalar density-matrix elements is

$$\langle P_i(p)|u_j|P_i(p)\rangle \approx (1/F_i^2) \times[(\sqrt{\tfrac{2}{3}})d_{iji}\langle 0|u_0|0\rangle + d_{iji}d_{ii8}\langle 0|u_8|0\rangle]. \quad (44)$$

In this approximation we are also constrained by the mass formula

$$\begin{aligned} \lim_{p\to 0}\langle P_i(p)|-u_0-cu_8|P_i(p')\rangle &= m_i^2 \approx \langle 0|[F_i^5,[F_i^5,-u_0-cu_8]]|0\rangle/F_i^2 \\ &= [(\sqrt{\tfrac{2}{3}})+cd_{ii8}] \\ &\quad\times[(\sqrt{\tfrac{2}{3}})\langle 0|u_0|0\rangle+\langle 0|u_8|0\rangle d_{ii8}]/F_i^2, \quad (45)\end{aligned}$$

where we have used PCAC, smoothness, and the evaluation (40).

Inserting (44) into (37), we have

$$\begin{aligned} \langle P_i(0)P_j(p)|T|P_i(0)P_j(p)\rangle &\approx \frac{[(\sqrt{\frac{2}{3}})+cd_{ii8}]}{F_i^2F_j^2}\sum_l d_{iil}[(\sqrt{\tfrac{2}{3}})d_{jlj}\langle 0|u_0|0\rangle + d_{jlj}d_{jj8}\langle 0|u_8|0\rangle] \\ &= \frac{1}{F_j^2}\sum_l (d_{iil}d_{jlj})\frac{[\frac{2}{3}+(\sqrt{\frac{2}{3}})cd_{ii8}]}{F_i^2}[\langle 0|u_0|0\rangle+(\sqrt{\tfrac{3}{2}})d_{jj8}\langle 0|u_8|0\rangle] \\ &= \frac{1}{F_j^2}\sum_l (d_{iil}d_{jjl})m_i^2\left[\frac{\langle 0|u_0|0\rangle+(\sqrt{\frac{3}{2}})d_{jj8}\langle 0|u_8|0\rangle}{\langle 0|u_0|0\rangle+(\sqrt{\frac{3}{2}})d_{ii8}\langle 0|u_8|0\rangle}\right], \quad (46)\end{aligned}$$

where in the last step we have used Eq. (45). Thus we have

$$\langle \pi_i(0)\pi_j(p)|T|\pi_i(0)\pi_j(p)\rangle \approx m_\pi^2/F_\pi, \quad (47)$$

$$\langle \pi_i(0)K_j(p)|T|\pi_i(0)K_j(p)\rangle \approx \frac{1}{2}\frac{m_\pi^2}{F_K^2}\frac{\langle 0|u_0|0\rangle-\langle 0|u_8|0\rangle/2\sqrt{2}}{\langle 0|u_0|0\rangle+\frac{1}{2}\langle 0|u_8|0\rangle}, \quad (48)$$

$$\langle K_i(0)\pi_j(p)|T|K_i(0)\pi_j(p)\rangle \approx \frac{1}{2}\frac{m_K^2}{F_\pi^2}\frac{\langle 0|u_0|0\rangle+\frac{1}{2}\langle 0|u_8|0\rangle}{\langle 0|u_0|0\rangle-\langle 0|u_8|0\rangle/2\sqrt{2}}, \quad (49)$$

$$\langle K_i(0)K_i(p)|T|K_i(0)K_i(p)\rangle \approx m_K^2/F_K^2, \quad (50)$$

$$\langle K_{4,5}(0)K_{6,7}(p)|T|K_{4,5}(0)K_{6,7}(p)\rangle \approx \tfrac{1}{2}m_K^2/F_K^2. \quad (51)$$

Note that in the limit $\langle 0|u_8|0\rangle=0$ the Σ terms are proportional to the projectile mass squared and the target decay constant and have no explicit dependence on c or $\langle 0|u_0|0\rangle$. Even with $\langle 0|u_8|0\rangle\neq 0$ they have no explicit dependence on the symmetry-breaking parameter, this dependence having been replaced, through the use of the mass formula, by the dependence on the projectile mass. Thus this crude approximation scheme does not allow us to add to the many determinations[24] of c (which is essentially fixed by the mass formula) but does enable us to test the consistency of $(3^*,3)\oplus(3,3^*)$ breaking and the smoothness assumptions with the Veneziano multiple term forms.

The results of this test are summarized in Table II. Single-term Veneziano forms are also tabulated, but one must be cautious in comparing them with the $(3^*,3)\oplus(3,3^*)$ evaluation since they do not all, as we have shown in the previous sections, consistently satisfy the AW relations. We have written the MTV results in terms of F_π and F_K, but the ratio $(F_K/F_\pi)^2$ is fixed by the analysis of the previous section.

It is interesting to note that the STV amplitude for $\pi\pi$ scattering agrees to within terms of order m_π^2/m_ρ^2 with

[23] This is the same as the result of the pole saturation version of PCVC, as in P. R. Auvil and N. G. Deshpande, Phys. Rev. **183**, 1463 (1969).

[24] A list of references is contained in B. Renner and A. Sudberry, Nucl. Phys. **B13**, 27 (1960). Two recent papers not included there are N. H. Fuchs and T. K. Kuo, Nuovo Cimento **64A**, 382 (1969); Y. Y. Lee, *ibid.* **64A**, 474 (1969).

TABLE II. Comparison of STV and MTV "Σ-term" amplitudes with evaluation of the corresponding matrix elements by $(3^*,3)\oplus(3,3^*)$ and the smoothness approximation of Eq. (40).

"Σ-term" amplitude	$(3,3^*)\oplus(3^*,3)$ with smoothness approximation (Eq. 40)	STV (single-term Veneziano); $(F_K/F_\pi)^2=1.55$	MTV (multiple-term Veneziano); $(F_K/F_\pi)^2=1.55$
$\langle\pi^-\pi^+\vert T\vert\pi^-\pi^+\rangle$	$\frac{m_\pi^2}{F_\pi^2}=\frac{0.019}{F_\pi^2}$	$\frac{m_\pi^2}{F_\pi^2}+O\left(\frac{m_\pi^2}{m_\rho^2}\right)=\frac{0.019}{F_\pi^2}$	$\frac{0.016}{F_\pi^2}$
$\langle\pi^-K^+\vert T\vert\pi^-K^+\rangle$, π soft	$\frac{m_\pi^2}{2F_K^2}\left[\frac{\langle 0\vert u_0\vert 0\rangle-\langle 0\vert u_8\vert 0\rangle/2\sqrt{2}}{\langle 0\vert u_0\vert 0\rangle+\frac{1}{2}\langle 0\vert u_8\vert 0\rangle}\right]$	$\frac{m_\pi^2}{2F_\pi^2}+O\left(\frac{m_\pi^2}{m_\rho^2}\right)=\frac{0.01}{F_\pi^2}$	$\frac{0.0107}{F_\pi^2}$
$\langle\pi^-K^+\vert T\vert\pi^-K^+\rangle$, K soft	$\frac{m_K^2}{2F_\pi^2}\left[\frac{\langle 0\vert u_0\vert 0\rangle+\frac{1}{2}\langle 0\vert u_8\vert 0\rangle}{\langle 0\vert u_0\vert 0\rangle-\langle 0\vert u_8\vert 0\rangle/2\sqrt{2}}\right]$	$\frac{0.154}{F_K^2}$	$\frac{0.159}{F_K^2}$
$\langle K^-K^+\vert T\vert K^-K^+\rangle$	$\frac{m_K^2}{F_K^2}=\frac{0.24}{F_K^2}$	$\frac{0.378}{F_K^2}$	$\frac{0.46}{F_K^2}$
$\langle K^-K^0\vert T\vert K^-K^0\rangle$	$\frac{m_K^2}{2F_K^2}=\frac{0.12}{F_K^2}$	$\frac{0.215}{F_K^2}$	$\frac{0.234}{F_K^2}$

results of the $(3,3^*)\oplus(3^*,3)$ and smoothness approximation model.[25] [This amplitude, however, is not sensitive to an admixture of $(1,8)\oplus(8,1)$.] The MTV amplitude for $\pi^+\pi^-$ scattering is somewhat lower than the single-term form and the agreement is somewhat poorer (~20%).

The MTV amplitude for πK scattering with soft π can be fitted with the $(3,3^*)\oplus(3^*,3)$ form, but since $F_\pi\neq F_K$ one must invoke considerable admixture of $\langle 0|u_8|0\rangle$:

$$\frac{\langle 0|u_0|0\rangle-\langle 0|u_8|0\rangle/2\sqrt{2}}{\langle 0|u_0|0\rangle+\frac{1}{2}\langle 0|u_8|0\rangle}\approx 1.1\left(\frac{F_K}{F_\pi}\right)^2\approx 1.7, \quad (52)$$

which implies $\langle 0|u_8|0\rangle/\langle 0|u_0|0\rangle\approx -0.6$. With this admixture of $\langle 0|u_8|0\rangle$ we are able to calculate the smoothness $(3,3^*)\oplus(3^*,3)$ model prediction for π^-K^+ scattering with K soft, with the result

$$\langle\pi^-K^+|T|\pi^-K^+\rangle_{K\text{ soft}}=\frac{1}{2}\frac{m_K^2}{F_\pi^2}\frac{1}{1.7}=\frac{0.109}{F_K^2}, \quad (53)$$

which differs from the MTV amplitude by 60%. This disagreement becomes progressively worse and is especially acute in the KK amplitudes, which are independent of the $\langle 0|u_8|0\rangle$ admixture, and which differ from the $(3,3^*)\oplus(3^*,3)$ smoothness model by 100%. [The K^-K^+ amplitude is, in fact, in this model independent of $(1,8)\oplus(8,1)$ admixture, that is, still given by m_K^2/F_K^2 even when $(1,8)\oplus(8,1)$ is taken into account because the mass formula changes in a compensating way.]

Thus the results of this test of the consistency between the MTV amplitude Σ terms and the $(3,3^*)\oplus(3^*,3)$ and smoothness approximation scheme of Eq. (44) are negative. We can as yet, however, draw no clear-cut conclusion because of all the smoothness assumptions which went into Eq. (44).

In order to avoid these ambiguities, let us restrict ourselves to the πK amplitude and sidestep the smoothness approximation by invoking a divergence-charge-algebra theorem in which three mesons are off mass shell (but only two are soft). This theorem is derived in the Appendix. (We are forced to restrict oursleves to the πK amplitude because it is necessary that target and projectile mass be distinct in order that the theorem go through.) The results of this theorem are

$$\langle\pi^-(-m_K^2)K^+(0)|T|\pi^-(0)K^+(-m_K^2)\rangle F_\pi^2F_Km_\pi^2/(m_\pi^2-m_K^2)$$
$$\equiv T^{\pi^-K^+}(-m_K^2,0,0,-m_K^2;m_K^2,m_K^2,0)F_\pi^2F_Km_\pi^2/(m_\pi^2-m_K^2)=\langle 0|[F_{\pi^+}{}^5(0),[F_{K^+}{}^5(0),\partial\mathfrak{F}_{\pi^-}{}^5(0)]]|K^+\rangle \quad (54)$$

and

$$\langle\pi^-(0)K^+(-m_\pi^2)|T|\pi^-(-m_\pi^2)K^+(0)\rangle F_\pi F_K^2m_K^2/(-m_\pi^2+m_K^2)$$
$$\equiv T^{\pi^-K^+}(0,-m_\pi^2,-m_\pi^2,0;m_\pi^2,m_\pi^2,0)F_\pi F_K^2m_K^2/(-m_\pi^2+m_K^2)=\langle 0|[F_{K^-}{}^5[F_{\pi^-}{}^5,\partial\mathfrak{F}_{K^+}{}^5]]|\pi^-\rangle, \quad (55)$$

where Σ commutators are assumed to carry no exotic quantum numbers and we have used the notation $T^{\pi^-K^+}(P_{\pi^-}{}^2,P_{K^+}{}^2,P'_{\pi^-}{}^2,P'_{K^+}{}^2;s,t,u)$.

Assuming a single $(3,3^*)\oplus(3^*,3)$ symmetry-breaking term, we can evaluate the double commutators:

$$T^{\pi^-K^+}(-m_K^2,0,0,-m_K^2;m_K^2,m_K^2,0)F_\pi^2F_K\times\frac{m_\pi^2}{m_\pi^2-m_K^2}=\frac{\sqrt{2}+c}{2\sqrt{3}}\frac{\langle 0|v_4-iv_5|K^+\rangle}{\sqrt{2}}, \quad (56)$$

[25] H. Osborn, Ref. 8.

$$T^{\pi^-K^+}(0,\,-m_\pi^2,\,-m_\pi^2,\,0;\,m_\pi^2,\,m_\pi^2,\,0)F_\pi F_K^2 \times\frac{m_K^2}{-m_\pi^2+m_K^2}=\frac{\sqrt{2}-\frac{1}{2}c}{2\sqrt{3}}\frac{\langle 0|v_1+iv_2|\pi^-\rangle}{\sqrt{2}}. \quad (57)$$

From the definitions of the PCAC constants and the form of the divergences in terms of the pseudoscalar densities we have

$$\frac{\langle 0|\partial\mathfrak{F}_{1+i2}{}^5|\pi^-\rangle}{\sqrt{2}}=m_\pi^2F_\pi=\frac{\sqrt{2}+c}{\sqrt{3}}\frac{\langle 0|v_1+iv_2|\pi^-\rangle}{\sqrt{2}}, \quad (58)$$

$$\frac{\langle 0|\partial\mathfrak{F}_{4-i5}{}^5|K^+\rangle}{\sqrt{2}}=m_K^2F_K=\frac{\sqrt{2}-\frac{1}{2}c}{\sqrt{3}}\frac{\langle 0|v_4-iv_5|K^+\rangle}{\sqrt{2}}. \quad (59)$$

Thus we can write

$$\frac{F_\pi^2m_\pi^2}{-m_K^2+m_\pi^2}T^{\pi^-K^+}(-m_K^2,\,0,\,0,\,-m_K^2;\,m_K^2,\,m_K^2,\,0) =\tfrac{1}{2}m_K^2\frac{\sqrt{2}+c}{\sqrt{2}-\frac{1}{2}c}, \quad (60)$$

$$\frac{F_K^2m_K^2}{m_K^2-m_\pi^2}T^{\pi^-K^+}(0,\,-m_\pi^2,\,-m_\pi^2,\,0;\,m_\pi^2,\,m_\pi^2,\,0) =\tfrac{1}{2}m_\pi^2\frac{\sqrt{2}-\frac{1}{2}c}{\sqrt{2}+c}. \quad (61)$$

We eliminate c by taking the product of these two equations:

$$T^{\pi^-K^+}(m_\pi^2,m_\pi^2,0)T^{\pi^-K^+}(m_K^2,m_K^2,0) =-(m_K^2-m_\pi^2)^2/4F_\pi^2F_K^2. \quad (62)$$

Evaluating the off-mass-shell amplitudes with the MTV forms we have

$$T^{\pi^-K^+}(m_K^2,m_K^2,0)T^{\pi^-K^+}(m_\pi^2,m_\pi^2,0) =(-0.4)\left(\frac{m_\rho^2}{0.9\pi}\right)^2\left(\frac{1}{F_\pi^2}\right)^2. \quad (63)$$

Equating (62) and (63) we have $F_K^2/F_\pi^2=0.78$, which is inconsistent with our previous determination (from the AW relations) of $F_K^2/F_\pi^2=1.55$. This result is similar to that of Cronin and Kang,[8,26] although it is derived from a different off-mass-shell theorem, as discussed in the Appendix.

Thus it is clear that the single $(3,3^*)\oplus(3^*,3)$ representation model of chiral symmetry breaking is inconsistent with the MTV $K\pi$ amplitude (assuming, of course, that the Veneziano form is the correct off-mass-shell extrapolator). This conclusion is stronger than our previous (also negative) results because it is independent of any smoothness assumption for the matrix elements of u_0 and u_8. [The single-term Veneziano amplitude for $K\pi$ scattering, we remark for completeness, fares no better in satisfying (62); using it as the extrapolating function yields $(F_K/F_\pi)^2=0.8$, which is nearly the same result as for MTV $K\pi$ and again inconsistent with determination from the AW relations.]

We can, however, satisfy Eqs. (60) and (61) if we assume either an additional, linearly independent, $(3,3^*)\oplus(3^*,3)$ representation or an admixture of $(1,8)\oplus(8,1)$ $(\mathcal{H}=\mathcal{H}_{\text{sym}}-u_0-cu_8-g_8)$. For example, if we assume the latter, Eqs. (60) and (61) reduce to two equations in the unknowns c, $\langle 0|(h_1+ih_2)/\sqrt{2}|\pi^-\rangle\equiv\eta_\pi$, and $\langle 0|(h_4-ih_5)/\sqrt{2}|K^+\rangle\equiv\eta_K$. If one makes the approximation $\eta_\pi\approx\eta_K$, then one finds $c=-0.88\sqrt{2}$ with

$$\langle 0|(h_1+ih_2)/\sqrt{2}|\pi^-\rangle/\langle 0|(v_1+iv_2)/\sqrt{2}|\pi^-\rangle \approx(\eta_\pi/m_\pi^2F_\pi)[(\sqrt{\tfrac{2}{3}})+c/\sqrt{3}]=0.75\,,$$

a measure of $(1,8)\oplus(8,1)$ admixture, which is considerable. The stability of c ($\approx-\sqrt{2}$) is perhaps not surprising considering the voluminous literature[24] devoted to showing $(c+\sqrt{2})\approx 0$.

V. SUMMARY AND CONCLUSIONS

We have considered the $\pi\pi$, πK, and KK Veneziano amplitudes as a coupled system. Imposing the Adler conditions on the STV amplitudes results, it is well known, in trajectory constraints which imply mass relations which are experimentally well satisfied. The soft-pion AW relations and the requirement that the ρ pole factorizes normalize the amplitudes and yield coupling-constant relations identical to $SU(3)$ predictions (which are not entirely satisfactory). The set of amplitudes considered as a whole, however, does not consistently satisfy the soft-kaon AW relations.

This set of leading-term amplitudes was then modified by adding secondary terms but leaving the trajectories unchanged (thus retaining the good mass relations) and enforcing all of the soft-pion and soft-kaon Adler and Adler-Weisberger relations. Coupling-constant relations improved, but the problem of the K^*/ρ width ratio remained. [A ratio close to the $SU(3)$ value is predicted by the MTV amplitudes, whereas the experimental value is too large by roughly the ratio (3:2) of the "old" width to the presently accepted width.] This is a funda-

[26] It has been pointed out by Osborn (Ref. 8) and by F. Csikor [Phys. Letters **31B**, 141 (1970)] that certain multiplicative off-shell extrapolation factors which contain poles at all of the 0^- daughters of the π or K (exchange degenerate) trajectories can be introduced to remove the contradiction, noted by Cronin and Kang (Ref. 8) between the πK Veneziano amplitude and the consequences of a single $(3,3^*)\oplus(3^*,3)$ representation breaking. Osborn remarks that the asymptotic behavior of the simplest such extrapolation factor, which is essentially a form factor as discussed by H. Suura [Phys. Rev. Letters **23**, 551 (1969)], would be "appalling." In fact, the well-behaved form suggested by Suura as reasonable for off-shell pion extrapolation, along with the corresponding kaon factor, does *not* remove the contradiction with $(3,3^*)\oplus(3^*,3)$ breaking. We have not explored possible multiplicative PCAC modifications of this kind because they would not remove the lack of consistency among the soft-kaon AW relations and because all of the low-energy theorems which we employ are of the fixed-point variety, where extrapolations of the order of pion or kaon masses are involved. For such applications Adler and Adler-Weisberger relations indicate that PCAC for pions and kaons is good to within 15% (see Ref. 10).

mental problem with possible roots in the Harari-Rosner duality diagram prescription [which may be too restrictive in that too much $SU(3)$ is implicitly imposed]. These troubles may also be related to the decoupling of the (ρ,ω) trajectory in the KK t channel, a direct consequence of the planar duality diagram prescription.

Finally, in possession of a set of amplitudes which satisfy the constraints of PCAC (Adler condition) and charge algebra (Adler-Weisberger condition) we investigated chiral symmetry breaking by using the Veneziano forms to go off mass shell to the Σ-term points. We found, in a series of tests, that a single $(3,3^*)\oplus(3^*,3)$ representation for the symmetry breaking is not sufficient in the context of the Veneziano amplitudes. This conclusion is most clearly demonstrated with the $K\pi$ amplitude alone, and the result follows from either the STV or MTV form.

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APPENDIX

We sketch here the derivation of a low-energy theorem of which all those used in the text are special cases.

Let us define an amplitude $M_{\mu\nu}{}^{ijkl}(p_i,p_j,p_k,p_l)$ as follows:

$$M_{\mu\nu}{}^{ijkl}(p_i\cdots p_l)=\int d^4x_i d^4x_j d^4x_k\, e^{-ip_ix_i-ip_jx_j+ip_kx_k} \times\frac{\langle 0|T(\mathfrak{F}_{\mu,j}{}^5(x_j)\mathfrak{F}_{\nu,k}{}^5(x_k)\partial\mathfrak{F}_i{}^5(x_i))|P_l\rangle}{m_i{}^2F_i}, \quad \text{(A1)}$$

where the $\mathfrak{F}_{\mu,i}{}^5$'s are the octet of axial-vector currents and PCAC is assumed in the form $(\partial/\partial x_\mu)\mathfrak{F}_{\mu,i}{}^5(x)\equiv\partial\mathfrak{F}_i{}^5(x)=m_i{}^2F_i\varphi_i(x)$. Forming the scalar $p_\mu{}^jp_\nu{}^kM_{\mu\nu}{}^{ijkl}$ and using the usual soft-meson techniques, we find that the limits $p_\mu{}^j\to0$ and $p_\nu{}^k\to0$ yield the equation

$$\begin{aligned}0=&\lim_{p_j\to0,\,p_k\to0}\int d^4x_i\, e^{-ip_ix_i}\int d^4x_jd^4x_k\, e^{-ip_jx_j+ip_kx_k}\frac{\langle 0|T(\partial\mathfrak{F}_k{}^5(x_k)\partial\mathfrak{F}_j{}^5(x_j)\partial\mathfrak{F}_i{}^5(x_i))|P_l\rangle}{m_i{}^2F_i}\\ &-\int d^4x_i\, e^{-ip_ix_i}\int d^4x_jd^4x_k\,\delta(t_k-t_i)\delta(t_j-t_i)\frac{\langle 0|[\mathfrak{F}_{0,k}{}^5(x_k),[\mathfrak{F}_{0,j}{}^5(x_j),\partial\mathfrak{F}_i{}^5(x_i)]]|P_l\rangle}{m_i{}^2F_i}\\ &+\lim_{p_j\to0,\,p_k\to0}\int d^4x_i\, e^{-ip_ix_i}\int d^4x_jd^4x_k\, e^{-ip_jx_j+ip_kx_k}\frac{\langle 0|T([\mathfrak{F}_{0,j}{}^5(x_j),\partial\mathfrak{F}_k{}^5(x_k)]_{t_j=t_k}\partial\mathfrak{F}_i{}^5(x_i))|P_l\rangle}{mi^2F_i}. \quad \text{(A2)}\end{aligned}$$

The T matrix is defined in terms of the S matrix as

$$S=1-i\delta^4(p_i+p_j-p_l-p_k)(2\pi)^4T(p_i{}^2\cdots p_l{}^2;\, s,t,u)\,, \quad \text{(A3)}$$

where states are normalized invariantly and $s=-(p_i+p_j)^2$, $t=-(p_i-p_l)^2$, and $u=-(p_i-p_k)^2$. We define the scattering amplitude with three mesons off mass shell in the usual way[27]:

$$\begin{aligned}-i\delta^4(p_i+p_j-p_k-p_l)(2\pi)^4T(s,t,u)=&(i)^3(p_i{}^2+m_i{}^2)(p_j{}^2+m_j{}^2)(p_k{}^2+m_k{}^2)\\ &\times\int d^4x_id^4x_jd^4x_k\, e^{-ip_ix_i-ip_jx_j+ip_kx_k}\frac{\langle 0|T(\partial\mathfrak{F}_i{}^5(x_i)\partial\mathfrak{F}_j{}^5(x_j)\partial\mathfrak{F}_k{}^5(x_k))|P_l\rangle}{m_i{}^2m_j{}^2m_k{}^2F_iF_jF_k}. \quad \text{(A4)}\end{aligned}$$

Equation (A2) can now be rewritten by taking the harmless limit $p_j\to0$ and substituting the definition (A4). The result is

$$\begin{aligned}\lim_{p_k\to0}(2\pi)^4\delta^4(p_i-p_l-p_k)F_iF_jF_km_k{}^2m_i{}^2\frac{T^{ijkl}(p_i{}^2,\,0,\,p_k{}^2,\,-m_l{}^2;\,s,\,t,\,u)}{(p_k{}^2+m_K{}^2)((p_l+p_k)^2+m_i{}^2)}=&-\lim_{p_k\to0}\int d^4x_i\, e^{-ip_ix_i}\int d^4x_k\, e^{ip_kx_k}\\ &\times\langle 0|T([F_i{}^5(t_k),\partial\mathfrak{F}_k{}^5(x_k)]\partial\mathfrak{F}_i{}^5)|P_l\rangle+(2\pi)^4\delta^4(p_i-p_l)\langle 0|[F_k{}^5(0),[F_j{}^5(0),\partial\mathfrak{F}_i{}^5(0)]]|P_l\rangle. \quad \text{(A5)}\end{aligned}$$

In the case $i=l$ and $j=k$, the left-hand side and the first term on the right-hand side each develop a pole in Eq.

[27] Steven Weinberg, Phys. Rev. Letters **17**, 616 (1966).

(A5) when $p_k \to 0$. Requiring cancellation of these singular parts, we obtain

$$m_i^2 F_i F_j^2 T^{ijkl}(0, -m_i^2, 0, -m_i^2; m_i^2, 0, m_i^2)$$
$$= -\lim_{p_k\to 0} 2p^k\cdot p^l (2\pi)^3 \delta^3(\tilde{p}_k) \int_0^\infty dt\, e^{-i(p_0{}^k - i\epsilon)t} \langle 0|[F_j{}^5(0), \partial\mathfrak{F}_j{}^5(0)]|0\rangle\langle 0|\partial\mathfrak{F}_i{}^5(0)|P_i\rangle$$
$$+(2\pi)^3 \int \frac{d^3p_n}{E_n(2\pi)^3} \delta^3(\tilde{p}_k - \tilde{p}_n + \tilde{p}_l) \int_{-\infty}^0 dt\, e^{-i(p_0{}^k - p_0{}^n + p_0{}^l + i\epsilon)t} \langle 0|\partial\mathfrak{F}_i{}^5|P_i\rangle\langle P_i|[F_j{}^5, \partial\mathfrak{F}_j{}^5]|P_i\rangle \quad \text{(A6)}$$

or

$$F_j^2 T^{ijji}(m_i^2, 0, m_i^2) = i\langle P_i|[F_j{}^5(0), \partial\mathfrak{F}_j{}^5(0)]|P_i\rangle - i\langle 0|[F_j{}^5, \partial\mathfrak{F}_j{}^5(0)]|0\rangle\delta^3(0),$$

which is the usual Σ-term theorem with the feature that the vacuum-expectation-value part is explicitly removed on the right-hand side.[22]

We also wish to consider the case of Eq. (A5) for which $m_i \neq m_l$, $m_j \neq m_k$, which is relevant to πK scattering. Because $p_l^2 \neq m_i^2$ and because there are no allowed intermediate states which lead to divergences in the $p_K \to 0$ limit, the theorem (A5) simply reads

$$T^{ijkl}(-m_l^2, 0, 0, -m_l^2; m_l^2, m_l^2, 0) \frac{F_i F_j F_k m_i^2}{(-m_l^2 + m_i^2)}$$
$$= -\int d^4x \langle 0|T([F_j{}^5(t), \partial\mathfrak{F}_k{}^5(x)]\partial\mathfrak{F}_i{}^5(0))|P_l\rangle$$
$$+\langle 0|[F_k{}^5(0), [F_j{}^5(0), \partial\mathfrak{F}_i{}^5(0)]]|P_l\rangle. \quad \text{(A7)}$$

The $\pi^- K^+$ case is particularly interesting, since either π^- or K^+ can be left on shell and the first term on the right-hand side of Eq. (A7) does not contribute if exotic ($I = \frac{3}{2}$, $Y = 1$) content of the Σ terms is ruled out. The theorems for this situation then read

$$T^{\pi^- K^+}(-m_K^2, 0, 0, -m_K^2; m_K^2, m_K^2, 0) \frac{m_\pi^2 F_\pi^2 F_K}{-m_K^2 + m_\pi^2}$$
$$= \langle 0|[F_{\pi^+}{}^5(0), [F_{K^+}{}^5(0), \partial\mathfrak{F}_{\pi^-}{}^5(0)]]|K^+\rangle \quad \text{(A8)}$$

for K on mass shell and

$$T^{\pi^- K^+}(0, -m_\pi^2, -m_\pi^2, 0; m_\pi^2, m_\pi^2, 0) \frac{m_K^2 F_\pi F_K^2}{m_K^2 - m_\pi^2}$$
$$= \langle 0|[F_{K^-}{}^5(0), [F_{\pi^-}{}^5(0), \partial\mathfrak{F}_{K^+}{}^5(0)]]|\pi^-\rangle \quad \text{(A9)}$$

for π on mass shell. Since the difficult term containing the time-ordered product has been eliminated from Eqs. (A8) and (A9) by our assumption that no exotic content is allowed in the Σ commutators, we have two relations which are useful for the exploration of consequences of particular models for the commutators $[F_{\pi,K}{}^5, \partial\mathfrak{F}_{K,\pi}{}^5]$ which satisfy this requirement. For example, it is shown in Sec. IV that Eqs. (A8) and (A9) are badly inconsistent if the $(3^*,3)\oplus(3,3^*)$ breaking is assumed.

This result that standard PCAC and the Veneziano model with constant coefficients are inconsistent with implications of $(3^*,3)\oplus(3,3^*)$ breaking is complementary to the conclusions of Cronin and Kang.[8,26] Our argument relies on a more conventional application of PCAC and charge algebra in that low-energy theorems are used which hold at individual kinematical points which are a distance m_π^2 or m_K^2 off mass shell in one or more variables. In addition, we see that the small value of $(F_K/F_\pi)^2 \approx \pi/4 = 0.8$ which Cronin and Kang obtain is a consequence of extensions off the energy shell of their theorems relating the matrix elements of Σ commutators to the off-mass-shell scattering amplitude. It is *not* to be blamed on the distance of extrapolations involved.[28] More precisely, Eqs. (7) and (8) of Cronin and Kang read, in our notation,

$$\langle 0|[F_{\pi^-}{}^5(0), \partial\mathfrak{F}_{K^+}{}^5(0)]|\pi^-(q)K^+(k)\rangle$$
$$= F_\pi F_K \alpha' \lambda_{\pi K} m_K^2 \frac{\Gamma(\frac{1}{2})\Gamma(1-\alpha^*(s))}{\Gamma(\frac{3}{2}-\alpha^*(s))} \quad \text{(A10)}$$

and

$$\langle 0|[F_{K^+}{}^5(0), \partial\mathfrak{F}_{\pi^-}{}^5(0)]|\pi^-(q)K^+(k)\rangle$$
$$= F_\pi F_K \alpha' \lambda_{\pi K} m_\pi^2 \frac{\Gamma(1-\alpha_\rho(m_K^2))\Gamma(1-\alpha^*(s))}{\Gamma(\frac{3}{2}-\alpha_\rho(s))}, \quad \text{(A11)}$$

respectively. One cannot extend these equations to ones in which three particles are off mass shell by simply ignoring the implied over-all energy-momentum conservation and reducing a pion in (A10) and a kaon in (A11) and replacing S by m_K^2 and m_π^2, respectively. The consequences of these manipulations, when combined with the assumption that the Σ terms carry no exotic quantum numbers, lead to $(F_K/F_\pi)^2 = \cos[\alpha^*(0)(m_K^2 - m_\pi^2)] \approx 0.8$. This contradicts the result of the AW relations $(F_K/F_\pi)^2 \approx 1.6$, which relies on essentially the same assumptions with the exception of the off-energy-shell extension. This latter assumption, which means that Eqs. (A10) and (A11) extend the use of Veneziano to the description of form factors and their off-shell continuations, is too strong. In fact, even if the $\delta^4(P_{\text{in}} - P_{\text{out}})$ factor is retained, the theorems which apply for three off-shell particles cannot be obtained from expressions like (A10) and (A11) but must be derived from the start from forms in which three particles are off-shell and the appropriate limits are taken at the end. Otherwise, one does not pick up all of the relevant equal-time commutators.

[28] That the "long" extrapolation might be the source of difficulty has been suggested by, for example, J. Cronin and K. Kang, Ref. 8, and by F. Hussain and M. S. K. Razmi, University of Islamabad report (unpublished).