Spontaneous Breakdown of Weak and Electromagnetic Interaction Symmetry*

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A unified theory of the weak and electromagnetic interactions of leptons and hadrons is constructed. The underlying symmetry group is taken to be the $SU(2)$ generated by the weak lepton currents and the hadronic Cabibbo currents. This symmetry is destroyed by the spontaneous breakdown mechanism. In our theory, the weak coupling constant is the same as the electromagnetic coupling constant, and the mass of the charged intermediate boson is 37.4 GeV.

I. INTRODUCTION

THE similarity of the weak and electromagnetic interactions has attracted much attention since Fermi[1] proposed his β-decay Hamiltonian based on this similarity. Later on, of course, the most straightforward extension of Fermi's model to include parity violation was found[2] to explain the experimental data.

Here we shall be concerned with a model of the type proposed by Glashow[3] and improved by Weinberg.[4] In this model, an $SU(2)$ triplet of vector mesons, as well as a vector-meson singlet, is introduced to explain the interactions of the leptons. The photon corresponds to a mixture of the singlet field with the neutral member of the triplet, and the two charged triplet members are to be identified as charged intermediate vector bosons. There is also a neutral intermediate vector boson which arises from an orthogonal mixture of the singlet and the neutral triplet member.

Since the photon has zero mass and the intermediate bosons must be very massive, it is clear that the symmetry associated with the gauge groups of these four vector mesons must be badly broken. Glashow introduced the symmetry breaking directly, but Weinberg assumed it to come from a spontaneous breakdown mechanism. We shall adopt this latter approach. It involves introducing a complex doublet of auxiliary scalar mesons. The over-all Lagrangian has the gauge symmetries which are then broken for the physical states of the system by requiring one of the scalar mesons to have nonzero vacuum expectation value. Ordinarily, this would imply that the other three scalar mesons be zero-mass (Goldstone) particles, but, as has been pointed out by several authors,[5] in a theory with gauge particles the zero-mass scalar bosons *effectively disappear* by combining with the originally zero-mass vector bosons of the corresponding symmetry gauges to become *massive* vector bosons

The previous work[3,4] only attempted to unify the weak and electromagnetic behavior of the leptons. In this paper, we attempt to give a more complete theory by including the hadrons. Our basic result is that such a unification can be achieved in a theory with spontaneous breakdown of the original symmetry. The difficulties involved in this extension are (i) finding a method of formulation, (ii) arranging for unwanted semileptonic decays to be suppressed, and (iii) arranging for unwanted nonleptonic decays to be suppressed.

Problem (i) is solved by using the quark model to summarize hadron dynamics and noting that the $SU(2)$ gauge group of the leptons is exactly the one that prompted the successful Cabibbo theory[6] of semileptonic decays.

Problem (ii) is solved by introducing another singlet intermediate vector boson field with opposite couplings to hadrons and leptons. For consistency, it is also necessary to take a remarkable limit of the original leptonic theory in which the additional neutral boson initially introduced effectively disappears from the theory by acquiring an infinite mass. As a residue, it leaves a contact interaction. In this limit, the original theory contains a triplet of two charged vector bosons and the photon, the electrical coupling constant is equal to the weak coupling constant, and the mass of the charged intermediate vector boson is 37.4 GeV.

Problem (iii) is circumvented by postulating dynamical suppression of non-octet components of the effective nonleptonic Hamiltonian. This is exactly the same postulate that is normally made in the attempt to use the Cabibbo semileptonic decay theory to also explain nonleptonic decays. We note that this postulate entails the suppression of both $|\Delta I|=\frac{3}{2}$ *and* $|\Delta S|=2$ transitions.

We shall not say much about the problem of *CP* violation in this paper.

Our plan of presentation is first to introduce a compact and symmetrical notation for the "matter" (lepton and spin-$\frac{1}{2}$ fermion) currents (Sec. II) and then to formulate and discuss the part of the Lagrangian which

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[3] Sheldon L. Glashow, Nucl. Phys. **22**, 579 (1961); see also J. Schwinger, Ann. Phys. (N. Y.) **2**, 407 (1957).
[4] S. Weinberg, Phys. Rev. Letters **19**, 1264 (1967).
[5] P. W. Higgs, Phys. Letters **12**, 132 (1964); F. Englert and R. Brout, Phys. Rev. Letters **13**, 321 (1964); G. S. Guralnik, C. R. Hagen, and T. W. B. Kibble, *ibid.* **13**, 585 (1964); P. W. Higgs, Phys. Rev. **145**, 1156 (1966); T. W. B. Kibble, *ibid.* **155**, 1554 (1967). Further discussion and a bibliography are given in the review article by G. S. Guralnik, C. R. Hagen, and T. W. B. Kibble, in *Advances in Particle Physics*, edited by R. L. Cool and R. E. Marshak (Interscience, New York, 1968), Vol. 2.
[6] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963); see also M. Gell-Mann, Physics **1**, 63 (1964).

involves the matter currents coupling to vector bosons (Secs. II–VI). This is the experimentally interesting part and can be discussed without a detailed consideration of the spontaneous-breakdown part of the theory. Finally, the spontaneous-breakdown part of the Lagrangian will be discussed (Sec. VII).

II. LEPTON AND HADRON CURRENTS

We shall take the point of view that weak and electromagnetic processes are generated by the lepton and hadron currents interacting with vector gauge fields. The most familiar lepton current is the electromagnetic (EM) one:

$$l_\mu{}^{\rm EM}=-i\bar{e}\gamma_\mu e-i\bar{\mu}\gamma_\mu\mu\,, \tag{1}$$

where e and μ denote the electron and muon fields, respectively. For discussing weak interactions, the relevant currents are associated with a group we shall denote as the universal left-handed $SU(2)$. Define

$$l_{a\mu}{}^b=i\bar{\psi}_b\gamma_\mu(1+\gamma_5)\psi_a+(e\rightarrow\mu)\,,$$

where $\psi_1=\nu_e$ and $\psi_2=e$. Then we define positive, neutral, and negative left-handed leptonic currents as

$$l_\mu{}^{(+)}=l_{1\mu}{}^2\,,\quad l_\mu{}^{(0)}=\tfrac{1}{2}(l_{1\mu}{}^1-l_{2\mu}{}^2)\,,\quad l_\mu{}^{(-)}=l_{2\mu}{}^1. \tag{2}$$

The integrated fourth components of these currents, namely,

$$K^{(\pm)}=\tfrac{1}{2}i\int d^3x\; l_4{}^{(\pm)},\quad K^{(0)}=\tfrac{1}{2}i\int d^3x\; l_4{}^{(0)}\,,$$

are the generators of an $SU(2)$ group.[7] The commutation relations are

$$[K^{(+)},K^{(-)}]=2K^{(0)}\,,\quad [K^{(\pm)},K^{(0)}]=\mp K^{(\pm)}.$$

Now let us turn to the hadrons. Their structure can be conveniently represented by imagining that all hadrons are made out of three quarks q_1, q_2, and q_3 having electrical charges $\frac{2}{3}$, $-\frac{1}{3}$, and $-\frac{1}{3}$, respectively. The hadron electromagnetic current is

$$h_\mu{}^{\rm EM}=\tfrac{2}{3}i\bar{q}_1\gamma_\mu q_1-\tfrac{1}{3}i\bar{q}_2\gamma_\mu q_2-\tfrac{1}{3}i\bar{q}_3\gamma_\mu q_3\,. \tag{3}$$

For discussing weak interactions, it is better to introduce a notation corresponding to quarks "rotated" through the Cabibbo angle:

$$\begin{aligned} Q_1&=q_1\,,\\ Q_2&=q_2\cos\theta+q_3\sin\theta\,,\\ Q_3&=-q_2\sin\theta+q_3\cos\theta\,, \end{aligned} \tag{4}$$

where $\sin\theta\approx\frac{1}{4}$.

In terms of the combinations,

$$h_{a\mu}{}^b=i\bar{Q}_b\gamma_\mu(1+\gamma_5)Q_a\,,$$

we define positive, neutral, and negative hadronic currents as

$$h_\mu{}^{(+)}=h_{1\mu}{}^2\,,\quad h_\mu{}^{(0)}=\tfrac{1}{2}(h_{1\mu}{}^1-h_{2\mu}{}^2)\,,\quad h_\mu{}^{(-)}=h_{2\mu}{}^1. \tag{5}$$

The assumption is now made that the currents of (5) have the same transformation properties as the currents of (2) with respect to the universal left-handed $SU(2)$ group. This assumption is the one that led to the Cabibbo theory and is the basic one for the discussion that follows.

For each of the four types of currents just introduced, the total current may be written as

$$J_\mu{}^{(i)}=l_\mu{}^{(i)}+h_\mu{}^{(i)}\,, \tag{6}$$

where i stands for $+$, 0, $-$, or EM.

Then the usual electromagnetic interaction is

$$\mathcal{L}^{\rm EM}=|e|J_\mu{}^{\rm EM}a_\mu\,, \tag{7}$$

a_μ being the photon field while $|e|^2/4\pi\simeq1/137$.

Finally, the usual phenomenological weak interaction is

$$\mathcal{L}^W=(G/\sqrt{2})J_\mu{}^{(+)}J_\mu{}^{(-)}\,, \tag{8}$$

where $|G|\simeq1.03\times10^{-5}/M_p{}^2$.

Our goal is to find a unified interaction scheme that gives the same experimental results as (7) and (8).

III. INVARIANT UNIFIED INTERACTION

Let $\mathbf{A}_\mu$ be the gauge field corresponding to the universal left-handed $SU(2)$. Furthermore, let B_μ be a singlet vector field corresponding to a $U(1)$ gauge group. In order to construct invariant Yang-Mills-type interactions,[8] we must specify the transformation properties of the matter fields with respect to these groups. A left-handed $SU(2)$ doublet is

$$L=\tfrac{1}{2}(1+\gamma_5)\begin{pmatrix}\nu_e\\ e\end{pmatrix}, \tag{9}$$

while an $SU(2)$ singlet is

$$R=\tfrac{1}{2}(1-\gamma_5)e\,. \tag{10}$$

If the quantum number associated with the $U(1)$ gauge group is designated weak hypercharge, it turns out to be necessary to assign to R twice as weak a hypercharge as L. Then the invariant lepton Lagrangian density[4] is

$$\begin{aligned} &-\bar{R}\gamma_\mu(\partial_\mu-ig'B_\mu)R\\ &\qquad-\bar{L}\gamma_\mu(\partial_\mu-\tfrac{1}{2}ig\boldsymbol{\tau}\cdot\mathbf{A}_\mu-\tfrac{1}{2}ig'B_\mu)L+(e\rightarrow\mu)\,, \end{aligned} \tag{11}$$

where g and g' are some coupling constants.

The choice of interactions and couplings in Eq. (11) is the unique invariant one that will give rise to the usual electromagnetic interaction when the photon field is identified with the particular mixture of B_μ and the third component of $\mathbf{A}_\mu$ that comes from the spontaneous-breakdown mechanism to be discussed later [see Eqs.

[7] The significance of this group has been stressed by M. Gell-Mann, Ref. 6.

[8] C. N. Yang and F. Mills, Phys. Rev. 96, 191 (1954).

(16) and (39)]. Introduction of the B_μ field and its corresponding gauge group is in the first place required because, without it, we would have vector bosons coupling only to the left-handed (vector plus axial-vector) lepton currents. By adding a boson which couples to the right-handed (vector minus axial-vector) lepton current, we permit the existence of a linear combination which is a pure vector current. This can then be identified with the electromagnetic current.

In the hadron case, we define

$$Q_{aL}=\tfrac{1}{2}(1+\gamma_5)Q_a\,,\qquad Q_{aR}=\tfrac{1}{2}(1-\gamma_5)Q_a\,. \tag{12}$$

A doublet with respect to the universal left-handed $SU(2)$ is

$$\psi_L=\begin{pmatrix}Q_{1L}\\ Q_{2L}\end{pmatrix}, \tag{13}$$

while the following quantities will be taken by analogy with the lepton case to be singlets[9]:

$$Q_{3L},\ Q_{1R},\ Q_{2R},\ \text{and}\ Q_{3R}\,.$$

The possible invariant terms which we can use to construct the Lagrangian are

$$\bar\psi_L\gamma_\mu\boldsymbol{\tau}\cdot\mathbf{A}_\mu\psi_L\,,\quad \bar\psi_L\gamma_\mu\psi_L B_\mu\,,\quad \bar Q_{3L}\gamma_\mu Q_{3L}B_\mu\,,$$
$$\bar Q_{1R}\gamma_\mu Q_{1R}B_\mu\,,\quad \bar Q_{2R}\gamma_\mu Q_{2R}B_\mu\,,\quad \bar Q_{3R}\gamma_\mu Q_{3R}B_\mu\,,$$
$$(\bar Q_{2R}\gamma_\mu Q_{3R}+\bar Q_{3R}\gamma_\mu Q_{2R})B_\mu\,.$$

However, the *unique* invariant hadronic Lagrangian density that reproduces the correct electromagnetic interaction turns out to be[10]

$$-\sum_{a=1}^{3}(\bar Q_{aL}\gamma_\mu\partial_\mu Q_{aL}+\bar Q_{aR}\gamma_\mu\partial_\mu Q_{aR})+ig[\tfrac{1}{2}\bar\psi_L\gamma_\mu\boldsymbol{\tau}\cdot\mathbf{A}_\mu\psi_L$$
$$-\tfrac{1}{6}\tan\phi\,\bar\psi_L\gamma_\mu\psi_L B_\mu-\tfrac{2}{3}\tan\phi\,\bar Q_{1R}\gamma_\mu Q_{1R}B_\mu$$
$$+\tfrac{1}{3}\tan\phi(\bar Q_{2R}\gamma_\mu Q_{2R}+\bar Q_{3R}\gamma_\mu Q_{3R}+\bar Q_{3L}\gamma_\mu Q_{3L})B_\mu]\,, \tag{14}$$

where $\tan\phi$ is a constant to be identified shortly.

Equation (14) is seen to be the most straightforward generalization of (11).

IV. SPONTANEOUS BREAKDOWN

The spontaneous breakdown mechanism will be implemented by introducing a complex doublet of auxiliary scalar mesons which are also coupled through the Yang-Mills mechanism to the gauge fields $\mathbf{A}_\mu$ and B_μ. The details will be discussed later. For the present, it is only necessary to note that the charged fields

$$W_\mu^{(\pm)}=\tfrac{1}{2}\sqrt{2}(A_\mu^1\mp iA_\mu^2) \tag{15}$$

acquire mass M_W and that the photon a_μ and a heavy neutral vector meson Z_μ emerge in the mixture:

$$B_\mu=\cos\phi\ a_\mu+\sin\phi\ Z_\mu\,,\qquad A_\mu^3=-\sin\phi\ a_\mu+\cos\phi\ Z_\mu\,, \tag{16}$$

where

$$\tan\phi=g'/g\,. \tag{17}$$

Furthermore, the mass of Z_μ, M_Z, is related to the mass of $W_\mu^{(\pm)}$ by

$$M_W/M_Z=\cos\phi\,. \tag{18}$$

To see what our interaction looks like after the spontaneous breakdown, we simply substitute (15)–(18) into (11) and (14). The interaction part of the result can be compactly written as

$$\mathcal{L}^{\text{int}}=-g\sin\phi\ J_\mu^{\text{EM}}a_\mu$$
$$+(g/2\sqrt{2})(J_\mu^{(-)}W_\mu^{(+)}+J_\mu^{(+)}W_\mu^{(-)})$$
$$+g(M_Z/M_W)Z_\mu(\tfrac{1}{2}J_\mu^{(0)}-\sin^2\phi\ J_\mu^{\text{EM}})\,, \tag{19}$$

where the total currents J_μ are defined in (6). The first term of (19) is the same as the usual electromagnetic interaction (7) if we identify

$$-g\sin\phi=|e|\,. \tag{20}$$

The second term of (19) gives rise by exchange of $W_\mu^{(\pm)}$ to the usual weak interaction (8) when we identify

$$G/\sqrt{2}=g^2/8M_W^2\,. \tag{21}$$

The M_W^2 in the denominator of (21) comes, of course, from the propagator for a heavy $W_\mu^{(\pm)}$.

The third term in (19) gives rise through exchange of a heavy Z_μ particle to the effective interaction

$$\tfrac{1}{2}(g^2/M_W^2)(\tfrac{1}{2}J_\mu^{(0)}-\sin^2\phi\ J_\mu^{\text{EM}})^2\,. \tag{22}$$

Note that M_W^2 rather than M_Z^2 appears in the denominator. Equation (22) contains some semileptonic and nonleptonic terms that require suppression but, before discussing this, let us consider the limit of the theory as it stands when $M_Z\to\infty$.

In this limit,[11] according to (18), $\cos\phi\to 0$, so that (20) predicts

$$-g=|e|\,, \tag{23}$$

or equality of the weak and electromagnetic coupling constants. From (21), the mass of the charged vector boson is calculated to be

$$M_W\simeq 37.4\ \text{GeV}. \tag{24}$$

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[9] If, for example, Q_{1R} and Q_{2R} are assigned to a doublet, a consistent theory cannot be constructed.

[10] Equation (14) is derived by substituting (16) into the most general linear combination of invariant terms and requiring the resultant photon matter coupling to be the usual one. The equality of lepton and hadron electric charges accounts for the fact that the same g is used in (14) as in (11).

[11] It is important to distinguish our limit from the case where there is no mixing and no spontaneous breakdown of symmetry. In both cases $\sin\phi=1$, so (17) and (18) give $gM_Z=g'M_W$. In our limit, g and M_W remain finite while g' and $M_Z\to\infty$. In the other limit, g and g' remain finite while M_Z and M_W are zero.

Furthermore, (16) shows that the neutral component $A_\mu{}^3$ of the intermediate boson triplet is just $-a_\mu$ in this limit.

Finally, the term (22) becomes

$$\tfrac{1}{2}(|e|^2/M_W{}^2)(\tfrac{1}{2}J_\mu{}^{(0)}-J_\mu{}^{\mathrm{EM}})^2. \quad (25)$$

Thus, if the limit $M_Z\to\infty$ is taken so that the Z_μ particle essentially disappears from the theory, we are left with two charged massive bosons and one massless photon forming a (broken) $SU(2)$ triplet and coupling with the same strength to matter. The only remnant to second order of the Z_μ particle is the appearance of the contact term (25). [In a theory of leptons by themselves, (25) would have no presently objectionable features.] We shall give a reason for taking the limiting case in the next section.

V. SUPPRESSION OF UNWANTED SEMILEPTONIC DECAYS

Equation (22) gives the following contribution to the effective semileptonic interaction:

$$(g^2/M_W{}^2)(\tfrac{1}{2}h_\mu{}^{(0)}-\sin^2\phi\ h_\mu{}^{\mathrm{EM}})(\tfrac{1}{2}l_\mu{}^{(0)}-\sin^2\phi\ l_\mu{}^{\mathrm{EM}}). \quad (26)$$

In terms of the usual quarks, $h_\mu{}^{(0)}$ may be written as

$$h_\mu{}^{(0)}=\tfrac{1}{2}i\bar{q}_1\gamma_\mu(1+\gamma_5)q_1-\tfrac{1}{2}i\cos^2\theta\ \bar{q}_2\gamma_\mu(1+\gamma_5)q_2 -\tfrac{1}{2}i\sin^2\theta\ \bar{q}_3\gamma_\mu(1+\gamma_5)q_3-\tfrac{1}{2}i\sin\theta \times\cos\theta\ [\bar{q}_2\gamma_\mu(1+\gamma_5)q_3+\bar{q}_3\gamma_\mu(1+\gamma_5)q_2].$$

Since the last term of $h_\mu{}^{(0)}$ above gives $|\Delta S|=1$ for hadronic transitions, we see that (26) gives rise to decays like

$$K\to e\bar{e},\quad K\to\pi e\bar{e},\quad K\to\pi\nu\bar{\nu},\quad \Sigma^+\to pe\bar{e},\quad \text{etc.} \quad (27)$$

(Note that the decay $K\to\nu\bar{\nu}$ is prevented by angular momentum conservation.)

There is no experimental evidence for any of the decays of (27), so it is desirable to suppress them in our theory. This can be done by introducing a new $U(1)$ gauge field C_μ that distinguishes between hadrons and leptons by coupling to their currents with opposite sign. Then (26) can be canceled exactly. It is crucial for our theory to make sense that C_μ be a singlet with respect to the universal left-handed $SU(2)$. The most general *invariant* C_μ-lepton coupling is

$$i\beta_1\bar{L}\gamma_\mu LC_\mu+i\beta_2\bar{R}\gamma_\mu RC_\mu+(e\to\mu), \quad (28)$$

where β_1 and β_2 are arbitrary constants. To cancel (26), it is necessary that this be *proportional* to

$$(\tfrac{1}{2}l_\mu{}^{(0)}-\sin^2\phi\ l_\mu{}^{\mathrm{EM}})C_\mu. \quad (29)$$

Equations (28) and (29) can only be proportional if

$$\beta_2=2\beta_1\equiv b,\quad \sin^2\phi=1. \quad (30)$$

This corresponds[12] to the remarkable limiting case previously discussed.

The unique $SU(2)$-invariant C_μ-hadron coupling which will enable us to cancel (26) *completely* is

$$-\tfrac{1}{6}idC_\mu[\bar{\psi}_L\gamma_\mu\psi_L+4\bar{Q}_{1R}\gamma_\mu Q_{1R} -2(\bar{Q}_{2R}\gamma_\mu Q_{2R}+\bar{Q}_{3R}\gamma_\mu Q_{3R}+\bar{Q}_{3L}\gamma_\mu Q_{3L})] =dC_\mu(\tfrac{1}{2}h_\mu{}^{(0)}-h_\mu{}^{\mathrm{EM}}), \quad (31)$$

where d is a constant to be determined.

Actually, (31) is more specific than is required[13] to cancel *just* the decays of (27). However, it is the coupling that is most analogous to the lepton coupling above which *is* required to suppress the unwanted semileptonic modes.

With (28), (30), and (31), the part of the effective Lagrangian responsible for unwanted semileptonic decays is

$$\left(\frac{e^2}{M_W{}^2}+\frac{bd}{M_C{}^2}\right)(\tfrac{1}{2}h_\mu{}^{(0)}-h_\mu{}^{\mathrm{EM}})(\tfrac{1}{2}l_\mu{}^{(0)}-l_\mu{}^{\mathrm{EM}}), \quad (32)$$

where M_C is the mass of the C_μ field. (We assume that C_μ acquires a mass by the same type of spontaneous breakdown mechanism as the other gauge fields.)

The cancellation of (32) evidently gives the condition

$$e^2/M_W{}^2=-bd/M_C{}^2. \quad (33)$$

The most symmetrical choice of coupling constants is the one which assigns opposite "C charge" to hadrons and leptons, namely,

$$b=-d. \quad (34)$$

Although the additional interactions (28) and (31) with the condition (33) make no contribution to semileptonic processes, they do give additional weak corrections to hadron-hadron and lepton-lepton processes. The $e\nu$ scattering reaction is conceivably measurable. Its effective Lagrangian, including the contribution from (19), is

$$\mathfrak{L}_{\mathrm{eff}}(e\nu)=(-e^2/16M_W{}^2)\bar{\nu}_e\gamma_\mu(1+\gamma_5)\nu_e \times\bar{e}\gamma_\mu\{[5+3(bM_W/eM_C)^2] +[1-(bM_W/eM_C)^2]\gamma_5\}e. \quad (35)$$

If the symmetrical choice of coupling constants (34) is made, there are no unknown parameters and we have, noting (33),

$$\mathfrak{L}_{\mathrm{eff}}(e\nu)=-2\sqrt{2}G\bar{\nu}_e\gamma_\mu(1+\gamma_5)\nu_e\bar{e}\gamma_\mu e, \quad (36)$$

where G is the ordinary Fermi constant.

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[12] The same conclusion holds if the field C_μ is allowed to mix with B_μ and $A_\mu{}^3$, corresponding to a generalization of (16).

[13] The most general invariant C_μ-hadron interaction is

$$C_\mu'[d_1\bar{\psi}_L\gamma_\mu\psi_L+d_2\bar{Q}_{1R}\gamma_\mu Q_{1R}+d_3\bar{Q}_{2R}\gamma_\mu Q_{2R}+d_4\bar{Q}_{3R}\gamma_\mu Q_{3R} +d_5\bar{Q}_{3L}\gamma_\mu Q_{3L}+d_6(\bar{Q}_{3R}\gamma_\mu Q_{2R}+\bar{Q}_{2R}\gamma_\mu Q_{3R})],$$

where the $d_1,\ldots,d_6$ are some constants. The suppression of (27) only requires $d_6=0$ and $d_3=d_4$. Thus a certain amount of freedom to modify the theory is available.

Equations (35) and (36) are, of course, different in general from what would be obtained from (19) by itself.

VI. SUPPRESSION OF UNWANTED NONLEPTONIC TRANSITIONS

Our final result for the spontaneously broken $SU(2)$ invariant interaction that contains no unwanted semileptonic pieces is

$$\begin{aligned}\mathcal{L}^{\text{int}}=+|e|J_\mu^{\text{EM}}a_\mu-(|e|/2\sqrt{2})(J_\mu^{(-)}W_\mu^{(+)}+J_\mu^{(+)}W_\mu^{(-)})\\ -(|e|/M_W)(M_ZZ_\mu)(\tfrac{1}{2}J_\mu^{(0)}-J_\mu^{\text{EM}})+(|e|/M_W)\\ \times(M_CC_\mu)[\tfrac{1}{2}(l_\mu^{(0)}-h_\mu^{(0)})-(l_\mu^{\text{EM}}-h_\mu^{\text{EM}})],\end{aligned}\quad(37)$$

where for simplicity we have assumed (34) to hold. Note that Z_μ appears multiplied by M_Z, so that the dependence of any tree-type diagram containing Z_μ as an internal line on M_Z drops out [see (22), for example] in the $M_Z\to\infty$ limit. Furthermore, Z_μ will not appear as an external line since it is infinitely heavy. The only unknown parameter in (37) is M_C, but even this will not appear in processes involving C_μ exchange.

The contribution of (37) to the effective Lagrangian density for nonleptonic transitions is

$$\frac{e^2}{16M_W^2}[h_\mu^{(+)},h_\mu^{(-)}]_+ +\frac{e^2}{M_W^2}(\tfrac{1}{2}h_\mu^{(0)}-h_\mu^{\text{EM}})^2.\quad(38)$$

In (38) the symmetrization of the currents required for CP invariance has been indicated explicitly.

Now each current appearing in (38) is a member of an octet with respect to the ordinary (strong) $SU(3)$. The symmetrical products in (38), therefore, belong to some mixture of the $\{1\}$, $\{8\}$, $\{8'\}$, $\{10\}$, $\{\overline{10}\}$, and $\{27\}$ representations of $SU(3)$. The statement[14] of "octet dominance" is that when matrix elements of the current-current product are taken between hadron states, the $\{10\}$, $\{\overline{10}\}$, and $\{27\}$ parts give negligible contribution. There is some support of this statement from calculations[15] which try to estimate the current-current matrix elements by the saturation method using experimentally known form factors. There is also some support from dispersion theory calculations.[16]

Since the $\{10\}$, $\{\overline{10}\}$, and $\{27\}$ representations are the only ones of those appearing which contain $\Delta I=\frac{3}{2}$ and $\Delta S=2$ transitions, the postulate of octet dominance will guarantee that our Lagrangian (37) will not give rise to unobserved nonleptonic transitions. We remind the reader that there is no unambiguous evidence for any *intrinsic* $\Delta I=\frac{3}{2}$ nonleptonic decay ($K^+\to\pi^+\pi^0$ may result from electromagnetic breaking of the $\Delta I=\frac{1}{2}$ rule), while the evidence against $\Delta S=2$ transitions [to second order in (37)] comes from the small value of the K_L-K_S mass difference.

Previous treatments[17] of intermediate vector bosons have introduced a number of them in such a way as to eliminate $\Delta S=2$ and $\Delta I=\frac{3}{2}$ transitions without assuming octet dominance. Our procedure is in this respect less aesthetic but, on the other hand, arises from a more unified theory and is in any case no different from the assumption of octet dominance that is necessary when we take the Cabibbo theory seriously for nonleptonic decays.

VII. REMAINING TERMS IN LAGRANGIAN

Here we give the kinematic terms for the $\mathbf{A}_\mu$, B_μ, and C_μ gauge fields, the terms involving the auxiliary scalar fields, and some additional coupling of the scalar fields to the "matter" for the purpose of generating matter field mass terms.

The auxiliary scalar fields consist of a complex doublet[4]

$$\Phi=\begin{pmatrix}\Phi^{(+)}\\ \Phi^{(0)}\end{pmatrix},\quad \bar{\Phi}=(\Phi^{(-)}\bar{\Phi}^{(0)})$$

and a complex singlet X. The remaining part of the invariant Lagrangian density is then

$$\begin{aligned}\mathcal{L}=&-\tfrac{1}{4}(\partial_\mu\mathbf{A}_\nu-\partial_\nu\mathbf{A}_\mu+g\mathbf{A}_\mu\times\mathbf{A}_\nu)^2-\tfrac{1}{4}(\partial_\mu B_\nu-\partial_\nu B_\mu)^2\\ &-\tfrac{1}{4}(\partial_\mu C_\nu-\partial_\nu C_\mu)^2\\ &-\tfrac{1}{2}(\partial_\mu\bar{\Phi}+\tfrac{1}{2}ig\bar{\Phi}\boldsymbol{\tau}\cdot\mathbf{A}_\mu-\tfrac{1}{2}ig'\bar{\Phi}B_\mu)\\ &\times(\partial_\mu\Phi-\tfrac{1}{2}ig\boldsymbol{\tau}\cdot\mathbf{A}_\mu\Phi+\tfrac{1}{2}ig'B_\mu\Phi)\\ &-\tfrac{1}{2}(\partial_\mu X^\dagger-ig''X^\dagger C_\mu)(\partial_\mu X+ig''C_\mu X)-V(\Phi,X)\\ &-[G_e(\bar{L}\Phi R+\bar{R}\bar{\Phi}L)+(e\to\mu)]\\ &+(f_1\bar{\psi}_L\Phi Q_{2R}+f_2\bar{\psi}_L\Phi Q_{3R}\\ &\qquad+f_3\bar{Q}_{3L}Q_{2R}+f_4\bar{Q}_{3L}Q_{3R}+\text{H.c.}).\end{aligned}\quad(39)$$

In (39), $V(\Phi,X)$ is an invariant function of Φ and X. The spontaneous breakdown of symmetry comes about because $V(\Phi,X)$ is chosen so that its minimum does not occur at $\Phi=X=0$. We choose the minimum at

$$\Phi=\begin{pmatrix}0\\ \lambda\end{pmatrix},\quad X=\lambda',\quad(40)$$

where λ and λ' are two real C numbers. The second derivatives of $V(\Phi,X)$ with respect to Φ and X, evaluated at the minimum, determine the masses of the auxiliary mesons which remain in the theory. We shall assume that these masses are so high that the auxiliary particles should not yet have been observed.

A shorthand prescription for finding the Lagrangian after spontaneous breakdown (if we are not interested

[14] See, e.g., R. Dashen, S. Frautschi, M. Gell-Mann, and Y. Hara, in *The Eightfold Way*, edited by M. Gell-Mann and Y. Ne'eman (Benjamin, New York, 1964).

[15] Y. T. Chiu, J. Schechter, and Y. Ueda, Phys. Rev. **150**, 1201 (1966); S. Biswas, A. Kumar, and R. Saxena, Phys. Rev. Letters **17**, 268 (1966); Y. Hara, Progr. Theoret. Phys. (Kyoto) **37**, 710 (1967); W. Simmons, Phys. Rev. **164**, 1956 (1967); S. Nussinov and G. Preparata, *ibid.* **175**, 2180 (1968).

[16] See, e.g., R. Dashen and S. Frautschi, Phys. Rev. **140**, B698 (1965).

[17] T. D. Lee and C. N. Yang, Phys. Rev. **119**, 1410 (1960); B. D'Espagnat, Phys. Letters **7**, 209 (1963); S. Okubo, *ibid.* **8**, 362 (1964).

in the Φ and X couplings) is simply to replace Φ and X in (39) by (40). Doing this lets us make the identifications that result in (16)–(18), as well as

$$M_W=\tfrac{1}{2}g\lambda\,,\quad M_C=g''\lambda'\,,\qquad m_e=G_e\lambda\,,\quad m_\mu=G_\mu\lambda\,, \tag{41}$$

where m_e is the electron mass and m_μ is the muon mass. Note that G_e and G_μ are fixed since

$$\lambda=2M_W/|e|\,.$$

The mixing given in (16) was necessary so that the vector-meson mass terms resulting from the fourth term of (39) be diagonal.

In (39) we have also written some invariant weak and electromagnetic contributions to quark-mass-type terms. There are four unknown constants $f_1\cdots f_4$, so we cannot really say too much. Nevertheless, expansion of the quark terms in (39) shows that no term like $\bar{q}_1q_1$ appears, so that the mass of q_1 cannot come from the above mechanism.

The shorthand prescription mentioned above can be formally justified and a more complete discussion given by using the approach[5] of Higgs and of Kibble. A brief treatment of this kind follows. Introduce the "polar decompositions" of the scalar fields:

$$\Phi=\exp(i\mathbf{\Theta}\cdot\boldsymbol{\tau})\begin{pmatrix}0\\ \rho\end{pmatrix},\quad X=e^{i\xi}r\,, \tag{42}$$

where the fields $\mathbf{\Theta}$ and ξ will disappear from the theory while the neutral fields

$$\tilde{\rho}=\rho-\lambda\,,\quad \tilde{r}=r-\lambda' \tag{43}$$

will remain. From (42) we identify

$$\begin{aligned}
&\rho^2=\bar{\Phi}\Phi\,,\\
&\Theta_1=\frac{1}{2i}(\Phi^{(+)}-\Phi^{(-)})\frac{|\mathbf{\Theta}|}{\rho\sin|\mathbf{\Theta}|}\,,\\
&\Theta_2=\tfrac{1}{2}(\Phi^{(+)}+\Phi^{(-)})\frac{|\mathbf{\Theta}|}{\rho\sin|\mathbf{\Theta}|}\,,\\
&\Theta_3=\frac{1}{2i}(\bar{\Phi}^{(0)}-\Phi^{(0)})\frac{|\mathbf{\Theta}|}{\rho\sin|\mathbf{\Theta}|}\,,
\end{aligned} \tag{44}$$

and

$$\rho\sin|\mathbf{\Theta}|=\frac{1}{\sqrt{2}}\left[\left(\frac{\Phi^{(+)}+\Phi^{(-)}}{\sqrt{2}}\right)^2+\left(\frac{\Phi^{(+)}-\Phi^{(-)}}{\sqrt{2}i}\right)^2+\left(\frac{\bar{\Phi}^{(0)}-\Phi^{(0)}}{\sqrt{2}i}\right)^2\right]^{1/2}.$$

The polar decomposition which would make sense in a C-number theory must be interpreted in terms of power-series expansions for the quantized case. Note that division by ρ, for example, is meaningless unless $\langle\rho\rangle_0\neq0$. The physical (primed) vector-meson fields are defined as

$$\begin{aligned}
M_\mu'&=U^{-1}M_\mu U-(2/ig)U^{-1}\partial_\mu U\,,\\
C_\mu'&=C_\mu+(1/g'')\partial_\mu\xi\,,
\end{aligned} \tag{45}$$

where we have set $U=\exp(i\mathbf{\Theta}\cdot\boldsymbol{\tau})$ and introduced the matrices

$$M_\mu=\boldsymbol{\tau}\cdot\mathbf{A}_\mu\,,\quad M_\mu'=\boldsymbol{\tau}\cdot\mathbf{A}_\mu'\,.$$

We note that the transformations of (42) and (45) have the same form as gauge transformations under which $\mathcal{L}$ is, by construction, invariant. Thus we expect that the fields $\mathbf{\Theta}$ and ξ which appear formally as gauge parameters will drop out. Explicitly, (39) becomes

$$\begin{aligned}
\mathcal{L}=&-\tfrac{1}{2}W_{\mu\nu}{}^{(+)\prime}W_{\mu\nu}{}^{(-)\prime}-\tfrac{1}{4}(a_{\mu\nu}')^2-\tfrac{1}{4}(Z_{\mu\nu}')^2-\tfrac{1}{4}(C_{\mu\nu}')^2\\
&-M_W{}^2W_\mu{}^{(+)\prime}W_\mu{}^{(-)\prime}-\tfrac{1}{2}M_Z{}^2(Z_\mu')^2-\tfrac{1}{2}M_C{}^2(C_\mu')^2\\
&-\tfrac{1}{2}(\partial_\mu\tilde{\rho})^2-\tfrac{1}{2}(\partial_\mu\tilde{r})^2-m_e\bar{e}'e'-m_\mu\bar{\mu}'\mu'-V(\rho,r)\\
&+g^2[a_\mu'W_\mu{}^{(+)\prime}a_\nu'W_\nu{}^{(-)\prime}-a_\mu'a_\mu'W_\nu{}^{(+)\prime}W_\nu{}^{(-)\prime}]\\
&-2ig[a_{\mu\nu}'W_\mu{}^{(+)\prime}W_\nu{}^{(-)\prime}+W_{\mu\nu}{}^{(+)\prime}a_\nu'W_\mu{}^{(-)\prime}\\
&+W_{\mu\nu}{}^{(-)\prime}a_\mu'W_\nu{}^{(+)\prime}]\\
&-[M_W{}^2W_\mu{}^{(+)\prime}W_\mu{}^{(-)\prime}+\tfrac{1}{2}M_Z{}^2Z_\mu'Z_\mu']\lambda^{-1}(2\tilde{\rho}+\lambda^{-1}\tilde{\rho}^2)\\
&-\tfrac{1}{2}M_C{}^2C_\mu'C_\mu'(1/\lambda')[2\tilde{r}+(1/\lambda')\tilde{r}^2]-(m_e/\lambda)\bar{e}'e'\tilde{\rho}\\
&-(m_\mu/\lambda)\bar{\mu}'\mu'\tilde{\rho}+(\text{quark mass terms})\,,
\end{aligned} \tag{46}$$

where

$$a_{\mu\nu}'=\partial_\mu a_\nu'-\partial_\nu a_\mu'\,,\ \text{etc.}$$

and

$$L'=\tfrac{1}{2}(1+\gamma_5)\begin{pmatrix}\nu_e'\\ e'\end{pmatrix}=U^{-1}L,\ \text{etc.}$$

From (46) it is seen that the W_μ, C_μ, and Z_μ fields have become massive and that some interaction terms involving $\tilde{\rho}$ and $\tilde{r}$ have appeared. The $\mathbf{\Theta}$ and ξ fields have dropped out. Note that (46) also contains the electromagnetic interaction of the W meson.

Finally, in order to demonstrate the invariance of the interactions (11), (14), (28), and (31) under the transformations (42) and (45), we must redefine all the physical matter fields to be the ones that have been suitably gauge transformed with gauge parameters $\mathbf{\Theta}$ and ξ. The previous results hold but the fields appearing in them should be taken to be the transformed ones.

VIII. CONCLUDING REMARKS

(1) We have demonstrated that a unified weak electromagnetic theory for leptonic and hadronic processes case be constructed using the left-handed $SU(2)$ connected with the weak currents as well as two more $U(1)$ gauge groups. This is the main conclusion since it was not clear at the beginning that such a scheme is possible.

(2) The limiting case where $M_Z\to\infty$ can also be applied in a theory of leptons by themselves. In this case, it is not required but does give the theory a greater degree of elegance. The behavior of non-tree-type diagrams in this limit seems to be worth investigating.

(3) We regard this theory as a tentative step in the right direction rather than a final result. In particular, it would be nice to introduce *CP* violation. It would also be nice not to have to require dynamical suppression of the non-octet parts of the nonleptonic interaction. Perhaps this could be achieved if strong interactions were taken into account at the outset.

(4) Since our interaction contains some more terms than the usual one, their presence may be tested with the help of other theoretical models or in several hard to observe reactions. We shall postpone detailed discussion of these points.

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Veneziano Amplitudes for $\pi\pi$, πK, and KK Scattering and Chiral Symmetry Breaking*

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$\pi\pi$, πK, and KK single- and multiple-term Veneziano amplitudes are studied as a coupled system. Adler and Adler-Weisberger conditions are imposed, and it is found that the single-term system cannot satisfy all of the PCAC (partial conservation of axial-vector current) and charge-algebra constraints. The multiple-term system, constructed to satisfy these constraints, results in much improved width predictions. These improved amplitudes are used to study chiral symmetry breaking by investigating the Σ terms. It is found that a single $(3,3^*)\oplus(3^*,3)$ representation is not sufficient to explain the symmetry breaking, whereas a mixture of $(3,3^*)\oplus(3^*,3)$ and $(1,8)\oplus(8,1)$ is sufficient (but not necessary). The admixture of $(1,8)\oplus(8,1)$ is considerable.

I. INTRODUCTION

CONSIDERABLE interest has been focused on the elegant amplitude construction of Veneziano.[1] Work has proceeded in many directions, including two in which we shall be most interested, namely, the comparison of Veneziano forms with (1) experimental data and (2) current-algebra off-mass-shell predictions.[2] For the latter, the Lovelace conjecture[3] has often been taken as a working hypothesis, that is, that the Veneziano amplitude with constant coefficients is the correct off-mass-shell extrapolator.

Much of this effort, however, has had somewhat of a patchwork quality with emphasis on a single amplitude at a time[4] (say, $\pi\pi$ elastic scattering), ignoring other systems (such as KK and $K\pi$ elastic scattering) which share common trajectories and are jointly constrained by factorization and current-algebra requirements. In this study we shall consider the Veneziano amplitudes for $\pi\pi$, πK, and KK[5] elastic scattering as a coupled system and attempt simultaneously and consistently to satisfy these constraints. (We have not included $\eta\eta$, $\eta\pi$, and ηK in our system because of the mixing problem.[6])

Initially, we investigate the single-term Veneziano forms (STV) constructed according to the duality diagram rules of Harari and Rosner.[7] These amplitudes have been constructed by Kawarabayashi, Kitakado, and Yabuki.[5] The $\pi\pi$ and πK system have been studied from the point of view of low-energy theorems and chiral symmetry breaking by several authors.[8] We find that we cannot consistently satisfy the Adler[9] and Alder-Weisberger[10] theorems with this single-term set of

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