which, on the basis of the identifications discussed, could be derived from a multivalued phase function does not in general correspond to the magnetic field obtained from the appropriate quantum-mechanical current $\mathbf{j}(\mathbf{r})$ by applying Maxwell's inhomogeneous equations. [Take, e.g., the state $\psi_{n,l,m}(\mathbf{r})$ of an isolated hydrogen atom whose phase is multivalued around the z axis.] It is the aim of further studies to gain more insight into the nature of this inconsistency.

ACKNOWLEDGMENT

The author would like to thank Professor J. A. R. Coope for many helpful discussions and for reading the manuscript.

PHYSICAL REVIEW D VOLUME 2, NUMBER 4 15 AUGUST 1970

B Meson and Its Current

H. Munczek
Department of Physics, University of Kansas, Lawrence, Kansas 66044
(Received 16 February 1970)

We examine the assumption that the B meson (1220 MeV, $J^{PC}=1^{+-}$) and its chiral partner are gauge fields. This leads to the introduction of an internal $SU(6)$ group which contains $SU(3)\times SU(3)$ as a subgroup. We discuss the appropriate formalism and mass degeneracy breaking by means of the use of nonlinear realizations of the group.

INTRODUCTION

The assumption that hadrons display a chiral $SU(3)\times SU(3)$ symmetry[1] broken by the partially conserved axial-vector current (PCAC) condition has produced many successful results, especially when these results refer to massless pions.[2] However, for physical pions the situation is not that satisfactory. The application of techniques like the use of Ward identities or equivalent phenomenological Lagrangians[3] makes clear that from chiral symmetry and PCAC one can obtain at most correlations between a certain number of low-energy processes and masses relations which are determined only when supplemented by extra assumptions such as, for example, vector-meson dominance[4] and high-energy behavior.[5,6] The possibility of constructing nonlinear representations besides the usual linear ones adds an extra degree of arbitrariness to the task of assigning the experimentally known particles to chiral multiplets. A large number of papers have been devoted to the study of the multiplets characterized by $J^{PC}=(1^{--},1^{++})$, $(0^{-+},0^{++})$, and $(\frac{1}{2}^+)$, and their physics is well understood both in its successes and in its limitations.

In the present article we discuss the inclusion within the chiral scheme of the B meson (1220 MeV, $J^{PC}=1^{+-}$). In doing so one immediately faces the problems of assigning the B to a particular representation, linear or nonlinear, and of finding (or predicting) its partners. Assuming a linear representation, the chiral companion of the B octet (or nonet) should be a 1^- $SU(3)$ multiplet with positive or negative charge conjugation according to whether we have a $(1,8)+(8,1)$ or a $(3,3^*)+(3^*,3)$ representation. The B decay modes and the experimental possibility for a 1^{--} object at a mass around 1650 MeV seem to favor the assignment to a $(3,3^*)+(3^*,3)$. As is shown in this article, this choice offers the possibility of enlarging the symmetry by assuming that all spin-1 particles are gauge-field quanta, the corresponding (internal) symmetry group being $SU(6)$. General considerations about $SU(3)\times SU(3)$ as a subgroup of some larger one have been made earlier,[7,8] and there are many possibilities. The group we choose here appears to be the minimal one that would include the known spin-1 mesons and contain all the results of chiral symmetry.

As a framework for the discussion of the consequences of our assumptions, we shall use the phenomenological Lagrangian approach together with vector-meson dominance in the theoretical form developed by Lee, Weinberg and Zumino.[9] In Sec. I is presented a very compact formalism for the treatment of representations of the chiral symmetry group. Besides its simplicity, it has the value of suggesting that the enlarged group is

[1] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Physics **1**, 63 (1964).

[2] See, e.g., S. L. Adler and R. F. Dashen, *Current Algebras and Their Applications to Particle Physics* (Benjamin, New York, 1968).

[3] The review article by S. Gasiorowicz and D. A. Geffen, Rev. Mod. Phys. **41**, 531 (1969), contains an extensive list of references on the use of these techniques.

[4] J. J. Sakurai, Ann. Phys. (N. Y.) **11**, 1 (1960).

[5] S. Weinberg, Phys. Rev. Letters **18**, 507 (1967).

[6] T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters **18**, 761 (1967).

[7] S. Coleman and S. Glashow, Ann. Phys. (N. Y.) **17**, 41 (1962); S. Coleman, *ibid.* **24**, 37 (1963).

[8] M. Gell-Mann and Y. Ne'eman, Ann. Phys. (N. Y.) **30**, 360 (1964).

[9] T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Letters **18**, 1029 (1967).

an internal $SU(6)$. In Sec. II we discuss a Lagrangian displaying a PCAC-broken $SU(6)$ symmetry, while in Sec. III we work out some general consequences of the use of nonlinear representations of $SU(6)$. We refrain in this paper from calculating any specific numerical results, mainly because it is still unclear, both for chiral $SU(3)\times SU(3)$ and for $SU(6)$, what extra assumptions besides symmetry and PCAC have to be imposed to obtain a consistent physical theory. The alternative procedure, consisting in the construction of a general "super-Lagrangian"[3] depending on an experimentally consistent set of parameters (fewer in number, one may hope, than the predictions), is under study.

I. $SU(3)\times SU(3)$ AS $SU(6)$ SUBGROUP

In this section a very convenient formalism for the chiral $SU(3)\times SU(3)$ is presented. Although it is suggested by its relationship to $SU(6)$, we prefer to ignore this fact for the moment in order to make the formalism self-contained. We shall use the following notation: λ^α means any of the nine $SU(3)$ Gell-Mann matrices which satisfy

$$\lambda^\alpha\lambda^\beta=(d^{\alpha\beta\gamma}+if^{\alpha\beta\gamma})\lambda^\gamma. \tag{1}$$

τ^a is a set of Pauli-type matrices with

$$\tau^a\tau^b=i\epsilon^{abc}\tau^c+\delta^{ab}, \tag{2}$$

and we choose a representation in which

$$(\tau^a)^T=-\tau^2\tau^a\tau^2. \tag{3}$$

If we define

$$\Lambda_a{}^\alpha=\tfrac{1}{2}\tau_a\lambda^\alpha, \tag{4}$$

then $\Lambda_0{}^\alpha$ and $\Lambda_2{}^\alpha$ ($\alpha=1,\ldots,8$) have the commutation relations of the vector and axial-vector charges postulated by Gell-Mann[1]:

$$\begin{aligned}[\Lambda_0{}^\alpha,\Lambda_0{}^\beta]&=if^{\alpha\beta\gamma}\Lambda_0{}^\gamma,\\ [\Lambda_0{}^\alpha,\Lambda_2{}^\beta]&=if^{\alpha\beta\gamma}\Lambda_2{}^\gamma,\\ [\Lambda_2{}^\alpha,\Lambda_2{}^\beta]&=if^{\alpha\beta\gamma}\Lambda_0{}^\gamma.\end{aligned} \tag{5}$$

It is trivial to check now that the components of the matrices

$$M_{(8)}=M_0{}^\alpha\Lambda_0{}^\alpha+M_2{}^\alpha\Lambda_2{}^\alpha \tag{6}$$

and

$$M_{(3)}=M_1{}^\alpha\Lambda_1{}^\alpha+M_3{}^\alpha\Lambda_3{}^\alpha \tag{7}$$

transform as representations $(1,8)+(8,1)$ and $(3,3^*)+(3^*,3)$, respectively, if

$$M'=e^{i\theta}Me^{-i\theta}. \tag{8}$$

In Eq. (8),

$$\theta=\theta_0{}^\alpha\Lambda_0{}^\alpha+\theta_2{}^\alpha\Lambda_2{}^\alpha, \tag{9}$$

where $\theta_0{}^\alpha$ are the parameters of a general $SU(3)$ transformation, and $\theta_2{}^\alpha$ are those of a chiral one.

To implement the chirality, we define the parity operation by

$$PM(\mathbf{x})P^{-1}=\tau_3M(-\mathbf{x})\tau_3\times P_M, \tag{10}$$

and $P_M=\pm1$ is the intrinsic parity of the M multiplet. We see from (10) that the components 0 and 2 in Eq. (6), and 1 and 3 in Eq. (7), have opposite parities as they should. The correct behavior under charge conjugation is ensured by the usual prescription

$$CMC^{-1}=M^T\times C_M. \tag{11}$$

C_M is the intrinsic charge conjugation of M, and because of Eq. (3) we see that $M_0\equiv\frac{1}{2}\sqrt{2}M_0{}^\alpha\lambda^\alpha$ has opposite charge conjugation to M_2, while M_1 and M_3 have the same charge conjugation. With the formalism outlined above, it is now very simple to couple representations and to form invariants to be used in a Lagrangian. The coupling of two multiplets is obtained by matrix multiplication in any order; for example,

$$M_{(3)}N_{(3)}=L_{(8)},\quad M_{(3)}N_{(8)}=L_{(3)},\quad M_{(8)}N_{(8)}=L_{(8)}. \tag{12}$$

The trace of a product is an invariant, obviously equal to zero if the product is $L_{(3)}$.

For baryons we have

$$PB(\mathbf{x})P^{-1}=\gamma_0\tau_3B(-\mathbf{x})\tau_3P_B. \tag{13}$$

Therefore, the projection operators $P_\pm=\frac{1}{2}(1\pm\gamma_5\tau_2)$ commute with the parity operation as well as with θ, so the equation

$$P_-B=0 \tag{14}$$

is a covariant irreducibility condition. If B is a $(1,8)+(8,1)$ representation, we have, owing to (14), $B_2=\gamma_5B_0$. If B is a $(3,3^*)+(3^*,3)$, we have instead $B_1=i\gamma_5B_3$.

For completeness we translate into this formalism some of the results of Coleman, Wess, and Zumino[10] on nonlinear representations. If $P\equiv\Lambda_2{}^\alpha P^\alpha$ ($\alpha=1,\ldots,8$) transforms linearly under $SU(3)$, we assign to P the following chiral transformation behavior:

$$e^{iP'}=e^{i\theta_A}e^{iP}e^{-iu}, \tag{15}$$

with $\theta_A=\theta_2{}^\alpha\Lambda_2{}^\alpha$, and $u=u^\alpha\Lambda_0{}^\alpha$. Since, for example, τ_2 anticommutes with τ_1, we get from (15)

$$e^{-iP'}=e^{-i\theta_A}e^{-iP}e^{-iu}. \tag{16}$$

As in Ref. 10, we eliminate e^{-iu} from (15) and (16) to obtain

$$e^{2iP'}=e^{i\theta_A}e^{2iP}e^{i\theta_A}. \tag{17}$$

With the help of P, one can construct representations of $SU(3)\times SU(3)$ out of objects transforming linearly under $SU(3)$. For example, if

$$n=\frac{1}{\sqrt{2}}\sum_{\alpha=0}^{8}n^\alpha\lambda^\alpha,$$

[10] S. Coleman, J. Wess, and B. Zumino, Phys. Rev. **177**, 2239 (1969).

and with

$$n' = e^{iu} n e^{-iu}, \tag{18}$$

we define

$$N_a = e^{iP} \tau_a n e^{-iP}. \tag{19}$$

Therefore, for $a=0$ or 2, N_a transforms like a (1,8) + (8,1). Also, for $a=1$ or 3, N_a transforms like a (3,3*)+(3*,3). In particular, if only $n^0 \neq 0$ and $a=3$ we get the standard form for the representation containing the pseudoscalar octet

$$F e^{2iP} \tau_3 \equiv \tau_1 \pi + \tau_3 (F^2 - \pi^2)^{1/2}. \tag{20}$$

II. B MESON AS GAUGE FIELD

The idea that spin-1 mesons are associated with the generators of local groups of transformations has proven useful in the treatment of the ρ and A_1 mesons and their octet partners. It seems natural then to explore the possibilities and consequences of the assumption that the B meson is also a gauge field. We should ask that by doing so we do not spoil the good results of chiral $SU(3)\times SU(3)$. Therefore, B should belong at least to a (3,3*)+(3*,3) or to a (1,8)+(8,1) representation of this group. Its chiral companion should then be a 1^{--} nonet or a 1^{-+} octet, respectively. We are interested here in the case in which the B and the corresponding R nonets belong to a (3,3*)+(3*,3),[11] and we make the hypothesis that together with the known vector octet V and a nonet containing the A_1 they form a set of 35 spin-1 mesons which make up a regular representation of an (internal) $SU(6)$ group.

With the notation developed in Sec. I, we can write compactly the commutation relations of the $SU(6)$ generators:

$$[\Lambda_a{}^\alpha, \Lambda_b{}^\beta] = i F_{abc}{}^{\alpha\beta\gamma} \Lambda_c{}^\gamma, \tag{21}$$

where

$$F_{abc}{}^{\alpha\beta\gamma} = S_{abc} f^{\alpha\beta\gamma} + \epsilon_{abc} d^{\alpha\beta\gamma}, \tag{22}$$

with

$$S_{abc} = \delta_{ab}\delta_{c0} + \delta_{ac}\delta_{b0} + \delta_{bc}\delta_{c0} - 2\delta_{a0}\delta_{b0}\delta_{c0}.$$

A representation **35** can be expressed as a matrix:

$$M = \Lambda_a{}^\alpha M_a{}^\alpha = \tfrac{1}{2}\sqrt{2}\, \tau_a M_a. \tag{23}$$

Under a general $SU(6)$ transformation, we have

$$M' = e^{i\theta} M e^{-i\theta}. \tag{24}$$

We define the action of parity and charge conjugation in the same way as in Sec. I. It is obvious that M breaks up into a (1,8)+(8,1) and a (3,3*)+(3*,3) of chiral $SU(3)\times SU(3)$. We write the gauge 35-plet in the form

$$G_\mu = \tfrac{1}{2}\sqrt{2}(\tau_0 V_\mu + \tau_1 B_\mu + \tau_2 A_\mu + \tau_3 R_\mu). \tag{25}$$

The over-all parity and charge conjugation of the multiplet are +1 and −1, respectively. Through the work of Lee, Weinberg, and Zumino,[9] we know how to construct a Lagrangian which will imply the field-current identity leading to the algebra of fields. For example, the Lagrangian for the vector fields alone has the form

$$L_G = -\tfrac{1}{4}\,\mathrm{Tr}(G_{\mu\nu}G^{\mu\nu}) + \tfrac{1}{2} m_0{}^2\, \mathrm{Tr}(G_\mu G^\mu), \tag{26}$$

with

$$G_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu + ig[G_\mu, G_\nu] \tag{27}$$

and

$$G_\mu' = (1/ig) e^{i\theta} \partial_\mu e^{-i\theta} + e^{i\theta} G_\mu e^{-i\theta}. \tag{28}$$

If we want to include a spin-0 35-plet containing the pseudoscalar mesons (π's) and scalar mesons (σ's), we should add

$$L_M = \tfrac{1}{2}\,\mathrm{Tr}(M_\mu M^\mu) - \tfrac{1}{2}\mu_0{}^2\, \mathrm{Tr}(M^2). \tag{29}$$

M_μ is the covariant derivative of M:

$$M_\mu = \partial_\mu M + ig[G_\mu, M]. \tag{30}$$

M breaks into a (3,3*)+(3*,3) and a (1,8)+(8,1) plus a singlet, and if the (3,3*)+(3*,3) contains the π and σ we see that the (1,8)+(8,1) part contains scalar and pseudoscalar octets of positive and negative charge conjugation, respectively, while the singlet is a negative charge conjugation pseudoscalar. Such particles, if they exist, certainly do not have masses in the same range of the π's and σ's. We know, however, that the breaking leading to PCAC induces also the breaking of the masses and we may speculate that in this way we could obtain a realistic removal of the mass degeneracy implied by a purely symmetric Lagrangian. However, if we consider, for example, a simple model in which the vacuum expectation value of $M_3{}^0$ is nonzero, we will find by a repetition of the, by now standard, phenomenological Lagrangian techniques that the pion nonet (M_1) remains degenerate with a 0^{--} (M_2), and that also B and A remain degenerate.

III. NONLINEAR REALIZATIONS OF $SU(6)$

To have the same mass for the B and A_1 is not a bad result since they differ experimentally only by about 10%. We cannot say the same about the pseudoscalars and the yet unobserved mesons associated with M_2. As we shall show, we may solve this difficulty by assigning the spin-0 mesons to nonlinear representations of $SU(6)$. Following the procedure of Coleman, Wess, and Zumino,[10] we choose first an $SU(6)$ subgroup (H) under which all the representations will transform linearly. Although it is not the only possible choice, we shall use the one in which H is the subgroup whose generators are $\Lambda_0{}^\alpha$ and $\Lambda_2{}^\alpha$, i.e., H contains the chiral $SU(3)\times SU(3)$ and the transformations generated by $\Lambda_2{}^0$. Next, we introduce a (3,3*)+(3*,3) linear representation of H:

$$\xi = (\xi_1 \tau_1 + \xi_3 \tau_3)/\sqrt{2}. \tag{31}$$

ξ plays a fundamental role in the process of generating

[11] A different higher symmetry in which the B would belong to a (1,8)+(8,1) has been recently considered by P. J. O'Donnell, Phys. Rev. **184**, 1728 (1969).

$SU(6)$ representations out of representations of H. With the notation

$$\theta_H=\theta_0{}^\alpha\Lambda_0{}^\alpha+\theta_2{}^\alpha\Lambda_2{}^\alpha,\quad \theta_B=\theta_1{}^\alpha\Lambda_1{}^\alpha+\theta_3{}^\alpha\Lambda_3{}^\alpha, \tag{32}$$

we have under a general $SU(6)$ transformation

$$e^{i\xi'}=e^{i\theta}e^{i\xi}e^{-iu}, \tag{33}$$

$$\theta=\theta_H+\theta_B,\quad u=u_0{}^\alpha\Lambda_0{}^\alpha+u_2{}^\alpha\Lambda_2{}^\alpha. \tag{34}$$

Here u is a function of ξ and θ which reduces to θ_H when θ_B is zero. Since θ_B anticommutes with τ_2, we have under a transformation involving θ_B only

$$e^{-i\xi'}=e^{-i\theta_B}e^{-i\xi}e^{-iu}. \tag{35}$$

From (33) and (35) we can eliminate e^{-iu} to obtain

$$e^{2i\xi'}=e^{i\theta_B}e^{2i\xi}e^{i\theta_B}. \tag{36}$$

If n is a linear representation of H, then under B

$$n'=e^{iu}ne^{-iu}, \tag{37}$$

and therefore

$$N\equiv e^{i\xi}ne^{-i\xi} \tag{38}$$

provides a "linearized" $SU(6)$ representation; i.e., in general,

$$N'=e^{i\theta}Ne^{-i\theta}. \tag{39}$$

In particular, if $n=\tau_2$ we can construct a function of ξ only which transforms linearly:

$$\Sigma\equiv e^{2i\xi}\tau_2,\quad \Sigma'=e^{i\theta}\Sigma e^{-i\theta}. \tag{40}$$

We find the use of linearized forms like (38) and (40) very convenient for the construction of phenomenological Lagrangians. This is not the only way of doing it, alternative forms being the ones based on the use of Lagrangians superficially invariant under H.[12] Both procedures are obviously equivalent.

As discussed by Salam and Strathdee[13] on the basis of previous work by Anderson,[14] by Higgs,[15] and especially by Kibble,[16] the particles associated with ξ are Goldstone bosons and they could be made not to appear explicitly in the Lagrangian if we omitted the mass term in L_G, Eq. (26). The B and R would acquire then an induced mass, and A_1 would also, owing to PCAC. The vector-meson octet, however, would remain massless. Besides, we would not have the field-current identity. So we choose to retain the mass term in Eq. (26). The fields ξ will appear explicitly with zero mass in a symmetric Lagrangian. As we shall show, they can be given an arbitrary mass by introducing a breaking which has the favorable property of being invariant under chiral $SU(3)\times SU(3)$. Before proceeding on this line it is useful to discuss the quantum numbers associated with ξ. This can be easily done by considering the covariant derivative of Σ:

$$D_\mu\Sigma=2i[\partial_\mu\xi\tau_2+\tfrac{1}{2}\sqrt{2}ig(B_\mu\tau_3-R_\mu\tau_1)+\cdots]. \tag{41}$$

The center dots represent terms containing at least bilinears in the fields. From Eq. (41) we see that the Σ kinetic-energy contribution to the Lagrangian will contain a mixing between ξ_1 and B_μ and between ξ_3 and R_μ. Therefore, the Goldstone bosons have the same quantum numbers as the corresponding generators of the group. This rules out the possibility of obtaining by this method a nonlinear representation not containing particles of negative charge conjugation. We proceed now to examine a general Lagrangian containing the gauge fields, the ξ's, and a $(3,3^*)+(3^*,3)$ representation of H containing the usual pseudoscalars and their σ partners. More generally, this $(3,3^*)+(3^*,3)$ could be a truly linear or a linearized representation of H.

We write L as

$$L=L_G+L_M+L_I+L_{\mathrm{br}}. \tag{42}$$

Here L_G is the $SU(6)$-symmetric Lagrangian for the gauge fields and is given by Eq. (26). L_M is a symmetric form containing the spin-0 fields and their covariant derivatives. L_I is an interaction Lagrangian, also symmetric, and is a function of $G_{\mu\nu}$, the spin-0 fields and their covariant derivatives. Finally, L_{br} contains the symmetry-breaking terms. If $n=\frac{1}{2}\sqrt{2}(\pi\tau_1+\sigma\tau_3)$ is the representation of H mentioned above, we can form with it two linearized representations of $SU(6)$:

$$N=e^{i\xi}ne^{-i\xi} \tag{43}$$

and

$$M=e^{i\xi}i\tau_2ne^{-i\xi}. \tag{44}$$

We can now write[17]

$$L_{\mathrm{br}}=a_1\,\mathrm{Tr}(\Lambda_3{}^0N)-a_2\,\mathrm{Tr}(\Lambda_1{}^0M)+(\sqrt{\tfrac{3}{2}})b\,\mathrm{Tr}(\Lambda_2{}^0\Sigma). \tag{45}$$

The first two terms are responsible for the PCAC condition and induce a vacuum expectation value $\langle\sigma^0\rangle\equiv\sigma_0\neq0$. The last term is chiral invariant and produces a mass contribution for the ξ's. Up to bilinears, L_{br} has the form

$$L_{\mathrm{br}}\simeq(a_1+a_2)\sigma^0-[b+(8/3)a_1\sigma_0]\xi_1{}^\alpha\xi_1{}^\alpha -[b+(8/3)a_2\sigma_0]\xi_3{}^\alpha\xi_3{}^\alpha. \tag{46}$$

We turn now to L_M. Because $\Sigma\Sigma=1$, $i\Sigma N=M$, and $(N)^l=e^{i\xi}(n)^le^{-i\xi}$, L_M consists of a function of n plus a kinetic-energy part. This kinetic energy contains mixings between the spin-0 and the gauge fields, and we show that explicitly by developing the covariant

[12] C. G. Callan, Jr., S. Coleman, J. Wess, and B. Zumino, Phys. Rev. **177**, 2247 (1969).
[13] Abdus Salam and J. Strathdee, Phys. Rev. **184**, 1750 (1969).
[14] P. W. Anderson, Phys. Rev. **130**, 439 (1963).
[15] P. W. Higgs, Phys. Letters **12**, 132 (1964).
[16] T. W. B. Kibble, Phys. Rev. **155**, 1554 (1967).

[17] As discussed by M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968), a more realistic breaking would include also some octet contributions in such a way that L_{br} would be approximately SU(2)×SU(2) invariant. For the purpose of our discussion the form of Eq. (45) is sufficient.

derivatives of N and M:

$$D_\mu N=\partial_\mu n+\tau_2\sigma_0(\sqrt{\tfrac{2}{3}})\partial_\mu\xi_1+g\sigma_0(\sqrt{\tfrac{2}{3}})(B_\mu\tau_2-A_\mu\tau_1)+\cdots, \quad (47)$$

$$D_\mu M=i\tau_2\partial_\mu n+\tau_2\sigma_0(\sqrt{\tfrac{2}{3}})\partial_\mu\xi_3+g\sigma_0(\sqrt{\tfrac{2}{3}})(R_\mu\tau_2-A_\mu\tau_3)+\cdots. \quad (48)$$

We have then, writing the kinetic-energy part L_M' of L_M,

$$L_M'=\alpha_1\,\mathrm{Tr}(D_\mu ND^\mu N)+\alpha_2\,\mathrm{Tr}(D_\mu MD^\mu M)+\alpha_3\,\mathrm{Tr}(D_\mu\Sigma D^\mu\Sigma), \quad (49)$$

that there are in L_M' mixing terms, i.e., bilinears, between $\partial_\mu\xi_1$ and B_μ, $\partial_\mu\xi_3$ and R_μ, and $\partial_\mu\pi$ and A_μ. Taking into account also the mass term from L_G, we can eliminate the mixing by the substitutions

$$B_\mu=b_\mu+\lambda_1\partial_\mu\xi_1, \quad (50a)$$

$$A_\mu=a_\mu+\lambda_2\partial_\mu\pi, \quad (50b)$$

$$R_\mu=r_\mu+\lambda_3\partial_\mu\xi_3, \quad (50c)$$

with

$$\begin{aligned}\lambda_1&=-g(2\alpha_3+\tfrac{2}{3}\alpha_1\sigma_0^2)[\tfrac{1}{4}m_0^2+g^2(2\alpha_3+\tfrac{2}{3}\alpha_1\sigma_0^2)]^{-1},\\ \lambda_2&=\tfrac{1}{3}\sqrt{3}g\sigma_0(\alpha_1+\alpha_2)[\tfrac{1}{4}m_0^2+(\alpha_1+\alpha_2)g^2\tfrac{2}{3}\sigma_0^2]^{-1},\\ \lambda_3&=-g(2\alpha_3+\tfrac{2}{3}\alpha_2\sigma_0^2)[\tfrac{1}{4}m_0^2+g^2(2\alpha_3+\tfrac{2}{3}\alpha_2\sigma_0^2)]^{-1}.\end{aligned} \quad (51)$$

When the mixing is eliminated, the vector mesons emerge, with masses

$$\begin{aligned}m_V^2&=m_0^2,\\ m_A^2&=m_0^2+(8/3)g^2\sigma_0^2(\alpha_1+\alpha_2),\\ m_B^2&=m_0^2+4g^2(\tfrac{2}{3}\alpha_1\sigma_0^2+2\alpha_3),\\ m_R^2&=m_0^2+4g^2(\tfrac{2}{3}\alpha_2\sigma_0^2+2\alpha_3).\end{aligned} \quad (52)$$

Relations of the type displayed in (52) are not the most general ones compatible with symmetry and $\langle\sigma^0\rangle\neq 0$. By forming L_I with terms like

$$L_I=a_1\,\mathrm{Tr}(G_{\mu\nu}\Sigma G^{\mu\nu}\Sigma)+a_2\,\mathrm{Tr}(G_{\mu\nu}NG^{\mu\nu}N)+a_3\,\mathrm{Tr}(G_{\mu\nu}MG^{\mu\nu}M), \quad (53)$$

we can produce even more complicated mass relations among the spin-1 mesons. Therefore, the actual values of the masses can not be derived by the above type of arguments. Recalling the familiar case of the A_1 meson,[5] the relation $m_{A_1}=\sqrt{2}m_\rho$ is based on the KSRF relation, on PCAC, and on high-energy behavior assumptions, besides that of $SU(2)\times SU(2)$ symmetry. For the sake of completeness, let us apply asymptotic symmetry arguments to the currents' spectral functions. We restrict ourselves to the sector of internal $SU(6)$ which contains chiral $SU(2)\times SU(2)$. We assume the currents are dominated by the isospin-1 mesons of spin 1 and 0. A repetition of the superconvergence considerations of Das, Mathur, and Okubo[6] leads to two sets of first and second Weinberg's sum rules. Extending Weinberg's[5] notation in an obvious way, we obtain

$$g_\rho^2=g_A^2=g_B^2=g_R^2 \quad (54)$$

and

$$\begin{aligned}g_\rho^2m_\rho^{-2}&=g_A^2m_A^{-2}+F_\pi^2\\ &=g_B^2m_B^{-2}+F_{\xi_1}^2\\ &=g_R^2m_R^{-2}+F_{\xi_3}^2.\end{aligned} \quad (55)$$

We see from the preceding formulas that the knowledge about g_ρ and F_π permits one to determine the A_1 mass. Because of the closeness of the masses of B and A_1, we see also that $F_{\xi_1}^2\simeq F_\pi^2$. We cannot, however, say very much about $F_{\xi_3}^2$ because of the still uncertain mass of R and the possibility of a ρ-R mixing.

An experimental consequence of the Lagrangian (42) should be the determination of the B and R decay rates. An examination of the resulting three-particle terms show that, for example, the B meson couples to R and π, but not to V and π, as it should. The observed decay $B\to\omega+\pi$ can be obtained by adding *ad hoc* $SU(6)$-symmetric terms like

$$\mathrm{Tr}(\{G_{\mu\nu},D^\mu\Sigma\}D^\nu M) \quad (56)$$

and

$$\mathrm{Tr}(G_{\mu\nu}G^{\mu\nu}N). \quad (57)$$

A term like (57) produces, besides, a mixing between R and V, and, through it, the decay $R\to 2\pi$. Finally, an R-V mixing would be relevant to the discussion of the electromagnetic structure of hadrons.