

Spectroscopic Searches for Quarks*

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(Received 2 April 1970)

Cosmological estimates indicate that quarks ought to be found in stellar sources, mainly, as bound to the nuclei of carbon, nitrogen, and oxygen. A method for predicting the electronic transitions for such species is presented and applied. The method of identifying lines in stellar spectra is discussed. Recently reported stellar ultraviolet spectra are inspected for possible coincidences.

INTRODUCTION

A FUNDAMENTAL triplet of constituents, the fractionally charged "quarks" ($Q_{p'} = +\frac{2}{3}e$, $Q_{n'} = Q_{o'} = -\frac{1}{3}e$) was proposed by Gell-Mann¹ and Zweig² as a possible explanation of the approximate $SU(3)$ symmetry of the meson and baryon multiplets. The $SU(6)$ classification³ acquires a particularly simple interpretation in terms of the generalized Pauli principle in this model, although its predictions may be obtained with or without models. The question of whether such triplets actually do exist and are not merely mathematical artifacts seems to acquire further significance as the dimensions of the unitary irreducible representations of the groups grow, while models, though crude, retain a certain amount of specific calculational features.

Cosmological estimates⁴ indicate that $+\frac{2}{3}$ quarks would be less abundant and remain in interstellar space, while $-\frac{1}{3}$ quarks would preferentially attach themselves to carbon, nitrogen, and oxygen and stay in stellar objects. Calculations⁴ have shown the first Bohr orbit of the quark to lie within the nuclear radius, so that the electrons of the atom or ion would essentially "see" a fractional nuclear charge. The "quark" atoms or ions (C' , N' , O'), will thus exhibit distinct electronic spectra.

In a previous note,⁵ based on a suggestion by Sinanoğlu, a search of the far-uv solar spectrum for quarked species was carried out. The present paper discusses the prediction of the electronic transitions of such species, and predicts some additional transitions. We also present the method of identifying transition lines in solar or stellar spectra. The reported line spectra of several stars in the vicinity of Orion have been compared to the predicted quark lines.

* Work supported in part by the Firestone Tire & Rubber Company and by Yale University.

¹ M. Gell-Mann, Phys. Letters, **8**, 214 (1964).

² G. Zweig, CERN report, 1964 (unpublished).

³ F. Gürsey and L. A. Radicati, Phys. Rev. Letters **13**, 173 (1964); B. Sakita, Phys. Rev. **136**, B1756 (1964).

⁴ Ya. B. Zel'dovich, L. B. Okun, and S. B. Pikelner, Usp. Fiz. Nauk **87**, 113 (1965) [Soviet Phys. Usp. **8**, 702 (1966)]; I. D. Novikov and Ya. B. Zel'dovich, Ann. Rev. Astronomy Astrophys. **5**, 627 (1967).

⁵ O. Sinanoğlu, B. Skutnik, and R. Tousey, Phys. Rev. Letters **17**, 785 (1966).

CALCULATIONS

Interpolation techniques similar to Edlén's⁶ are used along isoelectric sequences to predict electronic transitions for C' , N' , O' in various stages of ionization. The basic formula is

$$\Delta E/(\zeta+c) = a\zeta + b, \quad (1)$$

where ΔE is the energy of the transition in cm^{-1} , ζ is the net charge felt by the electron during the transition, and a , b , and c are empirically determined. The net charge ζ is related to the atomic number Z by the relation

$$\zeta = Z - (N - 1). \quad (2)$$

Equation (1) is basically a modified Rydberg expression. Such expressions have been applied extensively in atomic spectroscopy.⁶ Their accuracy is quite excellent, especially for ionic species. The theoretical basis for this behavior has recently been discussed in connection with Z -dependent perturbation theory treatments (for the latter, see Ref. 7 and references cited there). The interpretation becomes evident if we rewrite Eq. (1) as

$$\Delta E = A\zeta^2 + B\zeta + C, \quad (3)$$

where A , B , and C are combinations of the original empirical constants a , b , and c . Note also that ζ can be replaced by Z through Eq. (2). Equation (3) is formally similar to the leading terms of the Z^{-1} expansion of the relativistic energy.⁷

Returning to Eq. (1), we can treat c as a screening parameter, which can be adjusted to fit the observed data to this equation. We choose values of c , therefore, which give the best linear fits for selected data points, generally four or five neighbors in an isoelectronic series. The method of obtaining the best linear plots is as follows: (1) A check is made to see if the transition energies ΔE_i vary linearly with Z or ζ . If they do, Eq. (1) simplifies to

$$\Delta E = a'\zeta + b'. \quad (4)$$

Such a direct linear relationship was found for only two cases, $2p \ ^2P_{1/2} \rightarrow 2s \ ^2S_{1/2}$ and $2p \ ^1S_0 \rightarrow 2s2p \ ^1P_1$. (2) If they do not, the ΔE_i are divided by their respec-

⁶ B. Edlén, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1964), Vol. 27, p. 80.

⁷ Z. Horak, in *Modern Quantum Chemistry—Istanbul Lectures*, edited by O. Sinanoğlu (Academic, New York, 1965), Vol. 2, p. 7; D. Layzer, Z. Horak, M. N. Lewis, and D. P. Thompson, Ann. Phys. (N. Y.) **29**, 101 (1964).

tive net charges ζ_i and the differences between neighbors of the sequence are determined. Since they differ by units of ± 1 charge, these differences in the reduced energies $\Delta E_i^R (= \Delta E_i / \zeta_i)$ are the slopes of lines connecting the reduced energies of neighboring ions of the sequence. (3) From the ΔE^R values obtained in step (2), the sign of c can be determined and several numerical values are assumed for c , e.g., $c=0.200, 0.600, 1.000$. (4) New reduced energies are obtained and the slopes between neighbors are determined again. (5) New values of c are assumed, based on the results of step (4), with smaller increments and the process of obtaining new reduced energies and the corresponding slopes of lines connecting neighbors is repeated. (6) This process is continued until the "best" value of c is obtained within ± 0.001 charge unit. The best value of c is defined as the one which gives the average slope a for all the points whose deviation from the individual slopes connecting ΔE^R of neighboring ions of the sequence is the least.

In Table I we present the relations used to obtain the electronic transitions of the quarked species. The relations are given as differences in order to eliminate any error contributions from the evaluation of b . All the values given in Table I were obtained by the method outlined above.

Using the relations given in Table I, we have predicted a number of transitions of the species C', N', O' in various stages of ionization for $200 < \lambda < 1800 \text{ \AA}$. Several transitions in the visible range of the spectrum have also been calculated. A sample calculation for $O' \text{ VI}$ and $O' \text{ VI}^8$ of the Li I sequence is given in the Appendix. It is compared to the observed values⁹ for the transitions and to the more elaborate method of Edlén.¹⁰ The accuracy of the predicted values is checked by test predictions of known members of the appropriate series. Oscillator strengths are also interpolated.¹¹ Relative intensity ratios of the different ΔJ transitions within the same multiplet are calculated in the usual manner.¹² The species, transition involved, predicted λ 's, and error estimates are shown in Table II. The transitions in the visible spectral region are given at the bottom of the table.

Only the transitions of $C' \text{ III}$ and $C' \text{ IV}$ could be predicted for the quarked-carbon species. To obtain values for $C' \text{ I}$, we would require knowledge about the related transitions in B^- . Since term level structure has not been observed for negative ions, the use of the equations in Table I cannot be extended beyond C I . The problem in predicting values for $C' \text{ II}$ is similar. The term level

scheme of B I is sufficiently perturbed from that of the higher members of the sequence. The transition energies for B I were not used in deriving the relations given in Table I for the B sequences, and consequently extrapolation beyond C II to $C' \text{ II}$ is not warranted.

We have stressed transitions in the far-uv section of the spectrum, $\lambda < 2000 \text{ \AA}$, for several reasons. While conducting our preliminary search for quarks in the solar spectrum, we learned that below 1900 \AA the spectra yield well-defined emission lines, many of which have been identified with laboratory lines of multiply ionized atoms. They originate from the region between the low chromosphere and the corona, where the temperatures range from 5×10^3 to $2 \times 10^6 \text{ }^\circ\text{K}$ and where carbon, nitrogen, and oxygen are among the most abundant elements after hydrogen and helium. An estimate, based solely on Coulombic forces, indicates that $-\frac{1}{3}$ quarks would remain preferentially attached to the C , N , and O nuclei up to a temperature of $10^7 \text{ }^\circ\text{K}$.

For $\lambda > 2000 \text{ \AA}$ the spectra were found to be photospheric, containing closely packed absorption lines upon a continuum. In a recent search of the visible region around 2700 \AA for the Lyman series of $(q^{+2/3}, e)^{-1/3}$, Leacock, Beavers, and Daub note that the line density is about one line per 0.14 \AA . These authors reported¹³ a 100% probability of finding a sequence resembling a quark Lyman series and consequently they placed more weight on the results of their search of the infrared region for the quark Balmer series. In the latter case, the region around $14\,000 \text{ \AA}$ is relatively clean. Only six lines were found and five of them were previously identified.

The fact is that in the near-uv and visible ranges of the spectrum, there are a large number of lines which have not been identified, and thus a search for quarks in this region alone is not worthwhile. Furthermore, the strongest resonance lines of the various ions of C , N , and O all occur in the far-uv portion of the spectrum. These transitions are often called *raie ultime* and are signified by (1) in Table II.

Let us briefly discuss the method for identifying transition lines in stellar spectra. The standard criteria are as follows: (1) Other ions of the same isoelectronic series have been observed; (2) the complete set of multiplet lines of a particular transition is considered together; (3) all the lines of the multiplet should occur *and* should have the expected separations between them; (4) the relative intensities of the lines should also check with the expected values. For a single set of lines, positive identification requires all four steps to be satisfied. If several sets of lines, all related to the same element, are found, then positive identification can be made if steps 1-3 are satisfied, even though step 4 may not be used because of masking of one or more lines of a particular transition by some other known transition line.

¹³ R. A. Leacock, W. I. Beavers, and C. T. Daub, *Astrophys. J.* **151**, 1179 (1968).

⁸ $O' \text{ VI}$ denotes the quarked-oxygen ion, O^{+4} .

⁹ C. Moore, *Natl. Bur. Std. (U. S.) Circ.* **467**, Vol. I (1949).

¹⁰ B. Edlén, in *Space Age Astronomy*, edited by A. Deutsch and W. Klemperer (Academic, New York, 1962), p. 194.

¹¹ W. L. Weise, M. W. Smith, and B. M. Glennon, *Atomic Transition Probabilities* [Natl. Bur. Std. (U. S.), Washington, D. C., 1966], Vol. I; C. W. Allen, *Astrophysical Quantities* (Athlome, London, 1963), pp. 53-76.

¹² E. U. Condon and G. H. Shortley, *The Theory of Atomic Spectra* (Cambridge U.P., Cambridge, England, 1963), pp. 237-239.

TABLE I. Equations used to obtain predicted "quark" transitions. The transition energies ΔE_i are given in cm^{-1} . $\zeta_i = Z_i - N_i + 1$.

$2p \rightarrow 2s$	$2s2p^3 \rightarrow 2s^22p^2$
${}^2P_{3/2}^\circ \rightarrow {}^2S_{1/2}$: $\Delta E_2 = \Delta E_1 + 16\,171.52(\zeta_2 - \zeta_1)$	${}^3S_1^\circ \rightarrow {}^3P_2$: $\frac{\Delta E_2}{(\zeta_2 + 0.775)} = \frac{\Delta E_1}{(\zeta_1 + 0.775)} - 3727.14(\zeta_2 - \zeta_1)$
${}^2P_{1/2}^\circ \rightarrow {}^2S_{1/2}$: $\frac{\Delta E_2}{(\zeta_2 - 0.039)} = \frac{\Delta E_1}{(\zeta_1 - 0.039)} - 54.74(\zeta_2 - \zeta_1)$	${}^3S_1^\circ \rightarrow {}^3P_1$: $\frac{\Delta E_2}{(\zeta_2 + 0.776)} = \frac{\Delta E_1}{(\zeta_1 + 0.776)} - 3699.29(\zeta_2 - \zeta_1)$
$3s \rightarrow 2p$	${}^3S_1^\circ \rightarrow {}^3P_0$: $\frac{\Delta E_2}{(\zeta_2 + 0.776)} = \frac{\Delta E_1}{(\zeta_1 + 0.776)} - 3688.89(\zeta_2 - \zeta_1)$
${}^2S_{1/2} \rightarrow {}^2P_{3/2}^\circ$: $\frac{\Delta E_2}{(\zeta_2 + 0.470)} = \frac{\Delta E_1}{(\zeta_1 + 0.470)} + 15\,320.3(\zeta_2 - \zeta_1)$	$2s2p^3 \rightarrow 2s^22p^2$
${}^2S_{1/2} \rightarrow {}^2P_{1/2}^\circ$: $\frac{\Delta E_2}{(\zeta_2 + 0.430)} = \frac{\Delta E_1}{(\zeta_1 + 0.430)} + 15\,371.0(\zeta_2 - \zeta_1)$	${}^3D_{3/2}^\circ \rightarrow {}^3P_2$: $\frac{\Delta E_2}{(\zeta_2 + 1.225)} = \frac{\Delta E_1}{(\zeta_1 + 1.225)} - 224.25(\zeta_2 - \zeta_1)$
$2s2p \rightarrow 2s^2$	${}^3D_{2/2}^\circ \rightarrow {}^3P_1$: $\frac{\Delta E_2}{(\zeta_2 + 1.229)} = \frac{\Delta E_1}{(\zeta_1 + 1.229)} - 192.90(\zeta_2 - \zeta_1)$
${}^1P_1^\circ \rightarrow {}^1S_0$: $\frac{\Delta E_2}{(\zeta_2 + 0.475)} = \frac{\Delta E_1}{(\zeta_1 + 0.475)} - 217.34(\zeta_2 - \zeta_1)$	${}^3D_1^\circ \rightarrow {}^3P_0$: $\frac{\Delta E_2}{(\zeta_2 + 1.230)} = \frac{\Delta E_1}{(\zeta_1 + 1.230)} - 179.10(\zeta_2 - \zeta_1)$
$2p^2 \rightarrow 2s2p$	${}^3D_2^\circ \rightarrow {}^3P_2$: $\frac{\Delta E_2}{(\zeta_2 + 1.225)} = \frac{\Delta E_1}{(\zeta_1 + 1.225)} - 222.10(\zeta_2 - \zeta_1)$
${}^1D_2 \rightarrow {}^1P_1^\circ$: $\frac{\Delta E_2}{(\zeta_2 + 0.007)} = \frac{\Delta E_1}{(\zeta_1 + 0.007)} + 44.07(\zeta_2 - \zeta_1)$	${}^3D_1^\circ \rightarrow {}^3P_1$: $\frac{\Delta E_2}{(\zeta_2 + 1.228)} = \frac{\Delta E_1}{(\zeta_1 + 1.228)} - 195.00(\zeta_2 - \zeta_1)$
${}^1S_0 \rightarrow {}^1P_1^\circ$: $\Delta E_2 = \Delta E_1 + 24\,471.0(\zeta_2 - \zeta_1)$	${}^3D_1^\circ \rightarrow {}^3P_2$: $\frac{\Delta E_2}{(\zeta_2 + 1.225)} = \frac{\Delta E_1}{(\zeta_1 + 1.225)} - 221.10(\zeta_2 - \zeta_1)$
$2s2p^2 \rightarrow 2s^22p$	$3p \rightarrow 3s$ (visible range)
${}^2D_{5/2} \rightarrow {}^2P_{3/2}^\circ$: $\frac{\Delta E_2}{(\zeta_2 + 0.885)} = \frac{\Delta E_1}{(\zeta_1 + 0.885)} + 8.40(\zeta_2 - \zeta_1)$	${}^2P_{3/2}^\circ \rightarrow {}^2S_{1/2}$: $\frac{\Delta E_2}{(\zeta_2 - 0.124)} = \frac{\Delta E_1}{(\zeta_1 - 0.124)} + 9.35(\zeta_2 - \zeta_1)$
${}^2D_{3/2} \rightarrow {}^2P_{1/2}^\circ$: $\frac{\Delta E_2}{(\zeta_1 + 0.885)} = \frac{\Delta E_1}{(\zeta_1 + 0.885)} + 9.45(\zeta_2 - \zeta_1)$	${}^2P_{1/2}^\circ \rightarrow {}^2S_{1/2}$: $\frac{\Delta E_2}{(\zeta_2 - 0.151)} = \frac{\Delta E_1}{(\zeta_1 - 0.151)} - 5.53(\zeta_2 - \zeta_1)$
${}^2D_{3/2} \rightarrow {}^2P_{3/2}^\circ$: $\frac{\Delta E_2}{(\zeta_2 + 0.834)} = \frac{\Delta E_1}{(\zeta_1 + 0.834)} - 89.40(\zeta_2 - \zeta_1)$	$2s2p({}^3P^\circ)3d \rightarrow 2s2p({}^3P^\circ)3p$ (visible range)
${}^2S_{1/2} \rightarrow {}^2P_{3/2}^\circ$: $\frac{\Delta E_2}{(\zeta_2 + 0.627)} = \frac{\Delta E_1}{(\zeta_1 + 0.627)} - 633.70(\zeta_2 - \zeta_1)$	${}^4F_{9/2}^\circ \rightarrow {}^4D_{7/2}$: $\frac{\Delta E_2}{(\zeta_2 + 0.026)} = \frac{\Delta E_1}{(\zeta_1 + 0.026)} - 142.82(\zeta_2 - \zeta_1)$
${}^2S_{1/2} \rightarrow {}^2P_{1/2}^\circ$: $\frac{\Delta E_2}{(\zeta_2 + 0.637)} = \frac{\Delta E_1}{(\zeta_1 + 0.637)} - 572.80(\zeta_2 - \zeta_1)$	${}^4F_{7/2}^\circ \rightarrow {}^4D_{5/2}$: $\frac{\Delta E_2}{(\zeta_2 + 0.029)} = \frac{\Delta E_1}{(\zeta_1 + 0.029)} - 135.41(\zeta_2 - \zeta_1)$
$2s^23s \rightarrow 2s^22p$	${}^4F_{5/2}^\circ \rightarrow {}^4D_{3/2}$: $\frac{\Delta E_2}{(\zeta_2 + 0.032)} = \frac{\Delta E_1}{(\zeta_1 + 0.032)} - 131.25(\zeta_2 - \zeta_1)$
${}^2S_{1/2} \rightarrow {}^2P_{3/2}^\circ$: $\frac{\Delta E_2}{(\zeta_2 + 0.017)} = \frac{\Delta E_1}{(\zeta_1 + 0.017)} - 15\,566.6(\zeta_2 - \zeta_1)$	${}^4F_{3/2}^\circ \rightarrow {}^4D_{1/2}$: $\frac{\Delta E_2}{(\zeta_2 + 0.032)} = \frac{\Delta E_1}{(\zeta_1 + 0.032)} - 131.35(\zeta_2 - \zeta_1)$
${}^2S_{1/2} \rightarrow {}^2P_{1/2}^\circ$: $\frac{\Delta E_2}{(\zeta_2 + 0.025)} = \frac{\Delta E_1}{(\zeta_1 + 0.025)} + 15\,626.2(\zeta_2 - \zeta_1)$	${}^4F_{7/2}^\circ \rightarrow {}^4D_{7/2}$: $\frac{\Delta E_2}{(\zeta_2 + 0.018)} = \frac{\Delta E_1}{(\zeta_1 + 0.018)} - 162.29(\zeta_2 - \zeta_1)$
$2s^23d \rightarrow 2s^22p$	${}^4F_{5/2}^\circ \rightarrow {}^4D_{5/2}$: $\frac{\Delta E_2}{(\zeta_2 + 0.024)} = \frac{\Delta E_1}{(\zeta_1 + 0.024)} - 148.96(\zeta_2 - \zeta_1)$
${}^2D_{5/2} \rightarrow {}^2P_{3/2}^\circ$: $\frac{\Delta E_2}{(\zeta_2 - 0.130)} = \frac{\Delta E_1}{(\zeta_1 - 0.130)} + 15\,255.3(\zeta_2 - \zeta_1)$	${}^4F_{3/2}^\circ \rightarrow {}^4D_{3/2}$: $\frac{\Delta E_2}{(\zeta_2 + 0.026)} = \frac{\Delta E_1}{(\zeta_1 + 0.026)} - 142.79(\zeta_2 - \zeta_1)$
${}^2D_{3/2} \rightarrow {}^2P_{1/2}^\circ$: $\frac{\Delta E_2}{(\zeta_2 - 0.127)} = \frac{\Delta E_1}{(\zeta_1 - 0.127)} + 15\,308.7(\zeta_2 - \zeta_1)$	${}^4F_{5/2}^\circ \rightarrow {}^4D_{7/2}$: $\frac{\Delta E_2}{(\zeta_2 + 0.013)} = \frac{\Delta E_1}{(\zeta_1 + 0.013)} - 175.92(\zeta_2 - \zeta_1)$
${}^2D_{3/2} \rightarrow {}^2P_{3/2}^\circ$: $\frac{\Delta E_2}{(\zeta_2 - 0.130)} = \frac{\Delta E_1}{(\zeta_1 - 0.130)} + 15\,252.8(\zeta_2 - \zeta_1)$	${}^4F_{3/2}^\circ \rightarrow {}^4D_{5/2}$: $\frac{\Delta E_2}{(\zeta_2 + 0.018)} = \frac{\Delta E_1}{(\zeta_1 + 0.018)} - 160.58(\zeta_2 - \zeta_1)$
$2p^3 \rightarrow 2s2p^2$	
${}^4S_{3/2}^\circ \rightarrow {}^4P_{5/2}$: $\frac{\Delta E_2}{(\zeta_2 + 1.205)} = \frac{\Delta E_1}{(\zeta_1 + 1.205)} - 91.60(\zeta_2 - \zeta_1)$	
${}^4S_{3/2}^\circ \rightarrow {}^4P_{3/2}$: $\frac{\Delta E_2}{(\zeta_2 + 1.212)} = \frac{\Delta E_1}{(\zeta_1 + 1.212)} - 65.36(\zeta_2 - \zeta_1)$	
${}^4S_{3/2}^\circ \rightarrow {}^4P_{1/2}$: $\frac{\Delta E_2}{(\zeta_1 + 1.216)} = \frac{\Delta E_1}{(\zeta_2 + 1.216)} - 48.73(\zeta_2 - \zeta_1)$	

TABLE II. Predicted quark lines.

Species ^a	Transition	λ (Å)	$ f_{ki} $	Relative intensity	Species ^a	Transition	λ (Å)	$ f_{ki} $	Relative intensity
Ultraviolet region ^b					Ultraviolet region ^b				
C' IV (1)	${}^2P_{3/2}^{\circ} \rightarrow {}^2S_{1/2}$	1689.1 ± 0.2	0.102	2	(5)	${}^2D_{3/2}^{\circ} \rightarrow {}^2P_{3/2}^{\circ}$	273.876 ± 0.008	0.28	9
	${}^2P_{1/2}^{\circ} \rightarrow {}^2S_{1/2}$	1691.2 ± 0.2	0.102	1		${}^2D_{3/2}^{\circ} \rightarrow {}^2P_{1/2}^{\circ}$	273.661 ± 0.008	0.23	5
(6)	${}^2S_{1/2} \rightarrow {}^2P_{3/2}^{\circ}$	501.50 ± 0.03	0.081	2		${}^2D_{3/2}^{\circ} \rightarrow {}^2P_{3/2}^{\circ}$	273.886 ± 0.008	0.046	1
	${}^2S_{1/2} \rightarrow {}^2P_{1/2}^{\circ}$	501.32 ± 0.03	0.041	1	(6)	${}^4S_{3/2}^{\circ} \rightarrow {}^4P_{5/2}$	667.965 ± 0.004	0.24	3
C' III (1)	${}^1P_1^{\circ} \rightarrow {}^1S_0$	1077.9 ± 0.1	0.29	...		${}^4S_{3/2}^{\circ} \rightarrow {}^4P_{3/2}$	667.323 ± 0.004	0.16	2
(8)	${}^1D_2 \rightarrow {}^1P_1^{\circ}$	2584.4 ± 0.2	0.32	...		${}^4S_{3/2}^{\circ} \rightarrow {}^4P_{1/2}$	666.861 ± 0.004	0.075	1
(9)	${}^1S_0 \rightarrow {}^1P_1^{\circ}$	1412.5 ± 0.5	0.30	...	O' III (1)	${}^3D_3^{\circ} \rightarrow {}^3P_2$	904.34 ± 0.05	0.090	84
N' V (1)	${}^2P_{3/2}^{\circ} \rightarrow {}^2S_{1/2}$	1327.4 ± 0.3	0.084	2		${}^3D_2^{\circ} \rightarrow {}^3P_1$	902.91 ± 0.05	0.072	45
	${}^2P_{1/2}^{\circ} \rightarrow {}^2S_{1/2}$	1330.7 ± 0.3	0.084	1		${}^3D_1^{\circ} \rightarrow {}^3P_0$	902.16 ± 0.05	0.053	20
(4)	${}^2S_{1/2} \rightarrow {}^2P_{3/2}^{\circ}$	306.42 ± 0.02	0.068	2		${}^3D_2^{\circ} \rightarrow {}^3P_2$	904.15 ± 0.05	0.023	15
	${}^2S_{1/2} \rightarrow {}^2P_{1/2}^{\circ}$	306.24 ± 0.02	0.034	1		${}^3D_1^{\circ} \rightarrow {}^3P_1$	902.90 ± 0.05	0.039	15
N' IV (1)	${}^1P_1^{\circ} \rightarrow {}^1S_0$	824.6 ± 0.2	0.23	...		${}^3D_1^{\circ} \rightarrow {}^3P_2$	904.14 ± 0.05	0.0026	1
(7)	${}^1D_2 \rightarrow {}^1P_1^{\circ}$	1876.2 ± 0.2	0.25	...	(3)	${}^3S_1^{\circ} \rightarrow {}^3P_2$	544.38 ± 0.01	0.34	5
(8)	${}^1S_0 \rightarrow {}^1P_1^{\circ}$	1036.0 ± 0.3	0.24	...		${}^3S_1^{\circ} \rightarrow {}^3P_1$	543.94 ± 0.01	0.20	3
N' III (1)	${}^2D_{5/2} \rightarrow {}^2P_{3/2}^{\circ}$	1084.65 ± 0.05	0.12	9		${}^3S_1^{\circ} \rightarrow {}^3P_0$	534.67 ± 0.01	0.068	1
	${}^2D_{3/2} \rightarrow {}^2P_{1/2}^{\circ}$	1082.76 ± 0.05	0.10	5	Visible region				
	${}^2D_{3/2} \rightarrow {}^2P_{3/2}^{\circ}$	1084.58 ± 0.05	0.021	1	C' IV, $3p \rightarrow 3s$				
(2)	${}^2S_{1/2} \rightarrow {}^2P_{3/2}^{\circ}$	836.57 ± 0.10	0.22	2		${}^2P_{3/2}^{\circ} \rightarrow {}^2S_{1/2}$	6353.0 ± 0.3	0.175	2
	${}^2S_{1/2} \rightarrow {}^2P_{1/2}^{\circ}$	835.85 ± 0.10	0.11	1		${}^2P_{1/2}^{\circ} \rightarrow {}^2S_{1/2}$	6361.78 ± 0.05	0.175	1
N' III (4)	${}^2S_{1/2} \rightarrow {}^2P_{3/2}^{\circ}$	547.1 ± 0.4	0.091	2	N' V, $3p \rightarrow 3s$				
	${}^2S_{1/2} \rightarrow {}^2P_{1/2}^{\circ}$	546.6 ± 0.4	0.047	1		${}^2P_{3/2}^{\circ} \rightarrow {}^2S_{1/2}$	4945.6 ± 0.2	0.141	2
(5)	${}^2D_{5/2} \rightarrow {}^2P_{3/2}^{\circ}$	448.04 ± 0.01	0.20	9		${}^2P_{1/2}^{\circ} \rightarrow {}^2S_{1/2}$	4959.06 ± 0.07	0.141	1
	${}^2D_{3/2} \rightarrow {}^2P_{1/2}^{\circ}$	447.80 ± 0.01	0.17	5	N' III, $2s2p({}^3P^{\circ})3d \rightarrow 2s2p({}^3P^{\circ})3p$				
	${}^2D_{3/2} \rightarrow {}^2P_{3/2}^{\circ}$	448.05 ± 0.01	0.034	1		${}^4F_{9/2}^{\circ} \rightarrow {}^4D_{7/2}$	5432.5 ± 0.10	0.242	350
(8)	${}^4S_{3/2}^{\circ} \rightarrow {}^4P_{5/2}$	837.982 ± 0.005	0.26	3		${}^4F_{7/2}^{\circ} \rightarrow {}^4D_{5/2}$	5427.26 ± 0.08	0.206	240
	${}^4S_{3/2}^{\circ} \rightarrow {}^4P_{3/2}$	837.574 ± 0.005	0.17	2		${}^4F_{5/2}^{\circ} \rightarrow {}^4D_{3/2}$	5424.93 ± 0.04	0.180	157
	${}^4S_{3/2}^{\circ} \rightarrow {}^4P_{1/2}$	837.269 ± 0.005	0.085	1		${}^4F_{3/2}^{\circ} \rightarrow {}^4D_{1/2}$	5424.79 ± 0.02	0.169	98
N' II (1)	${}^3D_3^{\circ} \rightarrow {}^3P_2$	1207.6 ± 0.2	0.087	84		${}^4F_{7/2}^{\circ} \rightarrow {}^4D_{7/2}$	5448.0 ± 0.10	0.0345	40
	${}^3D_2^{\circ} \rightarrow {}^3P_1$	1206.5 ± 0.2	0.072	45		${}^4F_{5/2}^{\circ} \rightarrow {}^4D_{5/2}$	5438.45 ± 0.08	0.058	51
	${}^3D_1^{\circ} \rightarrow {}^3P_0$	1206.0 ± 0.2	0.047	20		${}^4F_{3/2}^{\circ} \rightarrow {}^4D_{3/2}$	5432.48 ± 0.04	0.067	39
	${}^3D_2^{\circ} \rightarrow {}^3P_2$	1207.4 ± 0.2	0.021	15		${}^4F_{5/2}^{\circ} \rightarrow {}^4D_{5/2}$	5446.03 ± 0.04	0.00228	2
	${}^3D_1^{\circ} \rightarrow {}^3P_1$	1206.5 ± 0.2	0.036	15		${}^4F_{3/2}^{\circ} \rightarrow {}^4D_{7/2}$	5445.76 ± 0.04	0.00480	3
	${}^3D_1^{\circ} \rightarrow {}^3P_2$	1207.4 ± 0.2	0.0023	1	O' VI, $3p \rightarrow 3s$				
(4)	${}^3S_1^{\circ} \rightarrow {}^3P_2$	717.21 ± 0.01	0.40	5		${}^2P_{3/2}^{\circ} \rightarrow {}^2S_{1/2}$	4044.3 ± 0.2	0.118	2
	${}^3S_1^{\circ} \rightarrow {}^3P_1$	716.91 ± 0.01	0.24	3		${}^2P_{1/2}^{\circ} \rightarrow {}^2S_{1/2}$	4065.21 ± 0.08	0.118	1
	${}^3S_1^{\circ} \rightarrow {}^3P_0$	716.73 ± 0.01	0.081	1	O' IV, $2s2p({}^3P^{\circ})3d \rightarrow 2s2p({}^3P^{\circ})3p$				
O' VI (1)	${}^2P_{3/2}^{\circ} \rightarrow {}^2S_{1/2}$	1092.7 ± 0.10	0.069	2		${}^4F_{9/2}^{\circ} \rightarrow {}^4D_{7/2}$	4045.93 ± 0.08	0.186	350
	${}^2P_{1/2}^{\circ} \rightarrow {}^2S_{1/2}$	1097.77 ± 0.08	0.065	1		${}^4F_{7/2}^{\circ} \rightarrow {}^4D_{5/2}$	4038.79 ± 0.06	0.160	240
(3)	${}^2S_{1/2} \rightarrow {}^2P_{3/2}^{\circ}$	206.667 ± 0.008	0.060	2		${}^4F_{5/2}^{\circ} \rightarrow {}^4D_{3/2}$	4035.89 ± 0.02	0.140	157
	${}^2S_{1/2} \rightarrow {}^2P_{1/2}^{\circ}$	206.483 ± 0.008	0.030	1		${}^4F_{3/2}^{\circ} \rightarrow {}^4D_{1/2}$	4035.86 ± 0.02	0.131	98
O' V (1)	${}^1P_1^{\circ} \rightarrow {}^1S_0$	668.84 ± 0.08	0.19	...		${}^4F_{7/2}^{\circ} \rightarrow {}^4D_{7/2}$	4066.04 ± 0.08	0.0264	40
(7)	${}^1P_2 \rightarrow {}^1P_1^{\circ}$	1470.47 ± 0.08	0.20	...		${}^4F_{5/2}^{\circ} \rightarrow {}^4D_{5/2}$	4053.44 ± 0.06	0.0451	51
(8)	${}^1S_0 \rightarrow {}^1P_1^{\circ}$	826.7 ± 0.2	0.20	...		${}^4F_{3/2}^{\circ} \rightarrow {}^4D_{3/2}$	4045.95 ± 0.04	0.0515	39
O' IV (1)	${}^2D_{5/2} \rightarrow {}^2P_{3/2}^{\circ}$	846.24 ± 0.04	0.093	9		${}^4F_{5/2}^{\circ} \rightarrow {}^4D_{7/2}$	4063.58 ± 0.02	0.00178	2
	${}^2D_{3/2} \rightarrow {}^2P_{1/2}^{\circ}$	845.06 ± 0.04	0.080	5		${}^4F_{3/2}^{\circ} \rightarrow {}^4D_{5/2}$	4063.37 ± 0.02	0.00370	3
	${}^2D_{3/2} \rightarrow {}^2P_{3/2}^{\circ}$	846.17 ± 0.04	0.015	1					
(2)	${}^2S_{1/2} \rightarrow {}^2P_{3/2}^{\circ}$	653.27 ± 0.08	0.20	2					
	${}^2S_{1/2} \rightarrow {}^2P_{1/2}^{\circ}$	651.99 ± 0.08	0.10	1					
(4)	${}^2S_{1/2} \rightarrow {}^2P_{3/2}^{\circ}$	324.1 ± 0.2	0.097	2					
	${}^2S_{1/2} \rightarrow {}^2P_{1/2}^{\circ}$	323.8 ± 0.2	0.049	1					

^a In A', atom A has one $-\frac{1}{2}$ quark associated with its nucleus.^b The number in the parentheses after the species designation in this section is the multiplet number as in C. E. Moore, Natl. Bur. Std. (U. S.) Circ. 488, Vol. I (1950), Sec. 1. It specifies the electronic configurations involved.

Masking is one of the major problems associated with assigning transitions to observed spectral lines. In the case of a doublet, the occurrence of a known line at the

position expected for one of the lines, obviously, allows one to make at best only a tentative identification of the remaining line. Singlet transitions are very difficult to

assign because they give rise to only single lines. In such cases, only step 1 applies and positive identification rests rather solely on the observation of other lines from the same element, and thus requires very careful checking whether other species might produce spectral lines in the same region of the spectrum.

The problems of positively identifying transition lines without the aid of steps 2–4 were brought out rather well by Bennett.¹⁴ He took all the available, critically evaluated energy-level data for C, N, and O and used a computer sorting program to print out tables of possible transitions, based on a prescribed selection rule over a given wavelength range. Bennett was able to show that 24 of the predicted quark lines,⁵ including all three tentatively assigned quark lines in the observed data, coincided with transition lines of the normal species, as printed out by his program. He did not search the spectra for the other multiplet lines of the predicted normal transitions, and thus these identifications are really no less tentative than the quark ones. The selection rules which he used,¹⁴ i.e., requiring only that parity changes and that $\Delta J = \pm 1, 0$, appear to be sufficient, since the predicted quark lines are expected to be at least 10^{-4} times less intense than the corresponding normal transitions, which are strongly allowed ones.

In the original search⁵ we found that a total of 11 lines actually coincided with known transition lines of Si and Fe as well as of C, N, and O. It is quite probable that if the energy-level data of these two elements, whose solar abundance relative to H is about the same magnitude as that of N, were included in the computer sorting program, many more of the predicted quark lines would be found to coincide with transition lines of the normal species. For this reason we *stress* the importance of satisfying the criteria of at least steps 2 and 3, necessarily, before positively identifying any transition line in stellar spectra and especially before judging a predicted quark line to be present.

In Table II we have included several transitions in the visible range of the spectrum, in order to allow an independent check in this region to tentative quark line assignments from the uv searches.

RECENT STELLAR SPECTRA

In the past three years far-uv spectra from stellar sources other than our sun have been reported. Morton and Spitzer¹⁵ have reported the line spectra of δ and π Scorpii in this region. The resolution of the spectra was about 1 Å. Of the 29 lines found and reported, 10 were unidentified. None of the seven predicted quark lines, which fall within the spectral limits of the experiment (1260–2180 Å), were found to coincide with the reported lines. The lines differed by at least 2–3 Å. For both stars, the Doppler shift was too small to significantly affect the reported wavelengths.¹⁵

¹⁴ W. R. Bennett, Phys. Rev. Letters **17**, 1196 (1966).

¹⁵ D. C. Morton and L. Spitzer, Astrophys. J. **144**, 1 (1966).

Carruthers¹⁶ has reported the far-uv spectra, range 1050–1362 Å, for several stars in the vicinity of Orion, namely γ Vel, ζ Pup, β CMa, π Ori, ι Ori, ζ Ori, and λ Ori. The 2–3 Å resolution was too poor to warrant a search of his data.

Morton¹⁷ and Jenkins and Morton¹⁸ have also reported spectra from this section of the sky. The resolution of their data is about 1–1.5 Å. The reported spectra covered essentially the same spectral range as the Scorpii spectra. None of the predicted quark lines for this spectral region ($1215 \text{ \AA} < \lambda < 2600 \text{ \AA}$) were found to coincide with the reported lines. The Doppler shifts were much larger for the Orion stars than for those of Scorpio.¹⁵ The spectral shifts were to shorter wavelengths by 5.6–19.6 Å. Allowing for these shifts, only the resonance lines for C' IV (1) at 1689.1 and 1691.2 Å might overlap with the reported lines. However, since the separations of the coincident lines is 5 Å, it is unlikely that the observed lines could be ascribed to C' IV.¹⁹

Recently, Huang and Edwards²⁰ identified all the observed spectral lines, including those previously unidentified, of quasar 3C191 by assuming a model quasar containing both ordinary and quarked atoms. They define quarked atoms as those atoms whose nuclear charge has been reduced by $\frac{1}{3}e$ or $\frac{2}{3}e$ through a loss of quarks. This differs from our definition (see Ref. 5 and our introduction). Their results are somewhat ambiguous. "Accuracies" of the quark lines are reported to lie between 10 and 200 Å, and the criteria for positive identification, which we mentioned above, appears to have been applied only to the selection of the Lyman- α line for quarked hydrogen. On the other hand, the reduced red shift ($z=0.31$), which was found, lowers the required luminosity of 3C191 to less than that of the brightest galaxies, thus solving a major dilemma concerning the energy output of quasars. Finally, estimates of the abundance ratio of quarked species, A'/A , are not given, nor is any reference to earlier⁴ cosmological estimates made.²¹

SUMMARY

In this paper we presented an empirical method for predicting electronic transitions of quarked species. The method is based on a Rydberg-type equation. Values for various transitions of C', N', and O' were calculated. Precisions (accuracies) ranged from ± 0.004

¹⁶ G. R. Carruthers, Astrophys. J. **151**, 269 (1968).

¹⁷ D. C. Morton, Astrophys. J. **147**, 1017 (1967).

¹⁸ E. B. Jenkins and D. C. Morton, Nature **215**, 1257 (1967).

¹⁹ A detailed search of the original photographs was not carried out after Dr. Donald C. Morton (private communication) informed us that the lowest detectable intensity increment in the stellar uv spectra is considerably larger than for the solar uv spectra. This means that the upper limits on the ratio A'/A , set earlier (Ref. 5), have to be exceeded in order to observe a quarked species in the stellar spectra.

²⁰ J. C. Huang and T. W. Edwards, Phys. Rev. **171**, 1331 (1968).

²¹ Further comments on this paper were published separately; B. Skutnik, Phys. Rev. **181**, 2145 (1969).

to 0.3 Å in the regions covered. The standard criteria for positive identification of transition lines in stellar spectra have been given and discussed. We have extended our search for quarks to the spectra of several stars of Scorpio and Orion. The results are inconclusive because the actual spectra are not as good as the solar spectra. The trend remains negative, however. The recent results for quasars²⁰ seem to indicate that these sources may be fruitful places to continue the spectroscopic search for quarks.

ACKNOWLEDGMENTS

The author would like to thank Dr. O. Sinanoğlu for bringing this problem to his attention and for many helpful discussions. Thanks are also due to Dr. D. Morton for sharing his observations with the author.

APPENDIX

Sample Calculation on Li I Sequence

From Table I, we have the following equations for $2p \rightarrow 2s$:

$${}^2P_{3/2}^{\circ} \rightarrow {}^2S_{1/2}:$$

$$\Delta E_2 = \Delta E_1 + 16\,171.5(\zeta_2 - \zeta_1),$$

$${}^2P_{1/2}^{\circ} \rightarrow {}^2S_{1/2}:$$

$$\Delta E_2 / (\zeta_2 - 0.039) = \Delta E_1 / (\zeta_1 - 0.039) - 54.74(\zeta_2 - \zeta_1),$$

where $\zeta_i = Z_i - 2$, and ΔE is in cm^{-1} .

Calculation for O VI

In both cases, $\zeta_1 = \zeta_N = 5.0$ and $\Delta E_1 = \Delta E(N_V)$ = observed energy.⁹

$${}^2P_{3/2}^{\circ} \rightarrow {}^2S_{1/2}:$$

$$\Delta E_1 = 80\,723.3 \text{ cm}^{-1}, \quad \zeta_1 = 5.0, \quad \zeta_2 = 6.0,$$

$$\Delta E_2 = 80\,723.3 + 16\,171.5(1) = 96\,894.8 \text{ cm}^{-1} \Rightarrow \lambda = 1032.05 \text{ Å}.$$

$${}^2P_{1/2}^{\circ} \rightarrow {}^2S_{1/2}:$$

$$\Delta E_1 = 80\,464.9 \text{ cm}^{-1}, \quad \zeta_1 - 0.039 = 4.961,$$

$$\zeta_2 - 0.039 = 5.961,$$

$$\Delta E_1 / (\zeta_1 - 0.039) = 16\,219.49 \text{ cm}^{-1} = \Delta E_1^R,$$

$$\Rightarrow \Delta E_2^R = 16\,219.49 - 54.74(1) = 16\,164.75 \text{ cm}^{-1}$$

$$\Rightarrow \Delta E_2 = 96\,358.1 \text{ cm}^{-1} \Rightarrow \lambda = 1037.80 \text{ Å}.$$

Calculation for O' VI

In both cases $\zeta_1 = \zeta_O = 6$ and $\Delta E_1 = \Delta E(O \text{ VI})$ observed energy.⁹

$${}^2P_{3/2}^{\circ} \rightarrow {}^2S_{1/2}:$$

$$\Delta E_1 = 96\,907.5 \text{ cm}^{-1}, \quad \zeta_1 = 6.000, \quad \zeta_2 = 5.667,$$

$$\Delta E_2 = 96\,907.5 + 16\,171.5(-\frac{1}{3}) = 96\,907.5 - 5390.5,$$

$$= 91\,517.0 \text{ cm}^{-1} \Rightarrow \lambda = 1092.7 \text{ Å}.$$

$${}^2P_{1/2}^{\circ} \rightarrow {}^2S_{1/2}:$$

$$\Delta E_1 = 96\,375.0 \text{ cm}^{-1}, \quad \zeta_1 - 0.039 = 5.961,$$

$$\zeta_2 - 0.039 = 5.628,$$

$$\Delta E_1 / (\zeta_1 - 0.039) = 16\,167.59 \text{ cm}^{-1} = \Delta E_1^R,$$

$$\Rightarrow \Delta E_2^R = 16\,167.59 - 54.74(-\frac{1}{3}) = 16\,167.59 + 18.25 = 16\,185.84 \text{ cm}^{-1}$$

$$\Rightarrow \Delta E_2 = 91\,093.9 \text{ cm}^{-1} \Rightarrow \lambda = 1097.77 \text{ Å}.$$

Formulas Reported by Edlén¹⁰ for Use in Li I Sequence

Sample calculation for O VI and O' VI

$$2p \rightarrow 2s$$

$${}^2P_{3/2}^{\circ} \rightarrow {}^2S_{1/2}:$$

$$\Delta E = 15\,393 \zeta + 6053 - 19\,752 / (\zeta + 2) + 0.4565(Z - 1)^4 - \frac{1}{4} [{}^2P_{3/2}^{\circ} - {}^2P_{1/2}^{\circ}].$$

$$\zeta_i = (Z_i - 2); \text{ units are } \text{cm}^{-1}.$$

$$[{}^2P_{3/2}^{\circ} - {}^2P_{1/2}^{\circ}] = \frac{1}{16} R_i \alpha^2 (Z - s')^4,$$

$$s' = s - \frac{1}{32} s \alpha^2 (Z - s')^3, \quad s = 1.720 + 0.782 / (Z - 0.4),$$

$$\alpha^2 = 5.32492 \times 10^{-5},$$

$$R_i = \text{Rydberg constant of element } i.$$

$$\text{For O VI, } Z = 8, \quad \zeta = 6, \quad R_O = 109\,733.54 \text{ cm}^{-1},$$

$$s = 1.720 + 0.782 / 7.6 = 1.823,$$

$$s_{(1)}' = 1.823 - 8.32019 \times 10^{-6} (6.177)^3 = 1.823 - 0.00196 = 1.821,$$

$$s_{(2)}' = 1.823 - 8.32019 \times 10^{-6} (6.179)^3 = 1.823 - 0.00196 = 1.821,$$

$$\Rightarrow s' = 1.821,$$

$$[{}^2P_{3/2}^{\circ} - {}^2P_{1/2}^{\circ}] = \frac{1}{16} 109\,733.54 \times 5.32492 \times 10^{-5} (6.179)^4 = 532.4 \text{ cm}^{-1},$$

$${}^2P_{3/2}^{\circ} \rightarrow {}^2S_{1/2} = 15\,393(6) + 6053 - \frac{1}{8} (19\,752) + 0.4565(7)^4 - \frac{1}{4} (532.4) = 96\,905.0 \text{ cm}^{-1} \rightarrow 1031.94 \text{ Å} = \lambda,$$

$${}^2P_{1/2}^{\circ} \rightarrow {}^2S_{1/2} = 96\,372.6 \text{ cm}^{-1} \rightarrow 1037.64 \text{ Å} = \lambda.$$

$$\text{O' VI with } m_q = 10m_p, \quad R_{O'} = 109\,735.02 \text{ cm}^{-1},$$

$$Z = 7.667, \quad \zeta = 5.667,$$

$$s = 1.720 + 0.782 / 7.267 = 1.828,$$

$$s_{(1)}' = 1.828 - 8.32019 \times 10^{-6} (5.839)^3 = 1.828 - 0.00166 = 1.826,$$

$$s_{(2)}' = 1.828 - 8.32019 \times 10^{-6} (5.841)^3 = 1.828 - 0.00166 = 1.826,$$

$$\Rightarrow s' = 1.826.$$

$$[{}^2P_{3/2}^{\circ} - {}^2P_{1/2}^{\circ}] = \frac{1}{16} 109\,735.02 \times 5.32492 \times 10^{-5} (5.841)^4 = 425.1 \text{ cm}^{-1},$$

$${}^2P_{3/2}^{\circ} \rightarrow {}^2S_{1/2} = 15\,393(5.667) + 16\,053 - 19\,752/7.667 \\ + 0.4565(6.667)^4 - \frac{1}{4}(425.1) \\ = 91\,504.5 \text{ cm}^{-1} \rightarrow 1092.84 \text{ \AA} = \lambda,$$

$${}^2P_{1/2}^{\circ} \rightarrow {}^2S_{1/2} = 91\,079.4 \text{ cm}^{-1} \rightarrow 1097.94 \text{ \AA} = \lambda.$$

The following table gives a comparison of the O VI

values with experiment.⁹

	Calculated	Observed (Ref. 9)	Edlén (Ref. 10)
${}^2P_{3/2}^{\circ} \rightarrow {}^2S_{1/2}$	$\bar{\nu}$ 96 874.8 cm^{-1}	96 907.5 cm^{-1}	96 905.0 cm^{-1}
	λ 1032.05 \AA	1031.91 \AA	1031.94 \AA
${}^2P_{1/2}^{\circ} \rightarrow {}^2S_{1/2}$	$\bar{\nu}$ 96 358.1 cm^{-1}	96 375.0 cm^{-1}	96 3726 cm^{-1}
	λ 1037.80 \AA	1037.61 \AA	1037.64 \AA

Source of the Kerr Metric*

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(Received 16 March 1970)

Assuming that the Kerr-Newman metric is the field of a layer of mass and charge distributed over the equatorial disk spanning the ring singularity, the source distribution on the disk is computed explicitly. In the uncharged case, this interpretation automatically excises the noncausal parts of the manifold, so that one obtains the unique source of the causally maximal extension of the vacuum metric. A Newtonian field which gives the same source distribution is exhibited, and shown to be closely analogous to the relativistic case. In particular, the Newtonian particle orbits show the same avoidance of the ring singularity that is such a remarkable feature of geodesics in the Kerr geometry. In the charged case, we examine how the gyromagnetic moment (which is equal to that of the Dirac electron) is reflected in the character of the source distribution.

I. INTRODUCTION

THE gravitational collapse of a star is a highly complex phenomenon whose details depend sensitively on the nature of the asymmetries involved. No matter how diverse the initial conditions, however, it is now widely believed¹⁻³ that, in the terminal black-hole stage of an irreversible collapse, the external field depends on only two parameters, and in fact is identical with the Kerr vacuum field having the appropriate mass and angular momentum.

In the case of zero angular momentum, this conjecture is already on fairly firm ground. It is known⁴ that the Reissner-Nordström spherically symmetric space-times are the only electromagnetic vacuum (EMV) fields which are static, asymptotically flat and possess nonsingular event horizons with the topology of a 2-sphere. Therefore, external asymmetries due to internal sources (such as mass quadrupoles and magnetic dipoles) cannot be statically supported by a black hole. This suggests⁴ that in the gravitational collapse of a nonrotating electrically neutral star all such asymmetries should

rapidly leak away—partly by radiation to infinity, and partly by falling in through the event horizon after the star itself has collapsed—leaving Schwarzschild's vacuum field as the sole external manifestation of the collapsed object. Direct support for these ideas has come from recent dynamical studies⁵ of idealized collapse models with small departures from spherical symmetry.

Justification for extending these arguments to the case of nonvanishing angular momentum hangs at present on two rather slender lines of evidence. First, the charged Kerr-Newman fields⁶ are (in several senses) the natural stationary generalizations of the Reissner-Nordström fields; they have the same simple algebraic structure (Petrov Type D) and are the only stationary, asymptotically flat EMV fields having this structure. Secondly, a study of small, axisymmetric stationary EMV perturbations of the Reissner-Nordström fields^{2,7-9} shows that the only perturbations which preserve asymptotic flatness and a nonsingular event horizon are members of the Kerr-Newman family.

These considerations, inconclusive though they still are, lend a special interest to recent efforts to

* Work partially supported by the National Research Council of Canada.

¹ B. Carter, *Phys. Rev.* **174**, 1559 (1968).

² K. S. Thorne, in *Proceedings of the International School of Physics "Enrico Fermi"* (Academic, New York, to be published), Course 47.

³ J. M. Bardeen, *Nature* **226**, 64 (1970).

⁴ V. L. Ginzburg and L. M. Ozernoi, *Zh. Eksperim. i Teor. Fiz.* **47**, 1030 (1964) [*Soviet Phys. JETP* **20**, 689 (1965)]; A. G. Doroshkevich, Ya. B. Zel'dovich, and I. D. Novikov, *Zh. Eksperim. i Teor. Fiz.* **49**, 170 (1965) [*Soviet Phys. JETP* **22**, 122 (1966)]; W. Israel, *Commun. Math. Phys.* **8**, 245 (1968).

⁵ V. de la Cruz, J. E. Chase, and W. Israel, *Phys. Rev. Letters* **24**, 423 (1970).

⁶ E. T. Newman, E. Couch, K. Chinnapared, A. Exton, A. Prakash, and R. Torrence, *J. Math. Phys.* **6**, 918 (1965).

⁷ V. de la Cruz and W. Israel, *Phys. Rev.* **170**, 1187 (1968), Ref. 15.

⁸ K. S. Thorne, 1968 (unpublished).

⁹ W. Israel, in *Proceedings of the Berne Seminar on the Bearings of Topology upon General Relativity, Gravitation et Relativité Generale*, Vol. 1, No. 3 (to be published).